# THE EXTENDED RESIDUAL ERROR METHOD FOR JET ENGINES IN-FLIGHT THRUST DETERMINATION

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***Abstract.*** *A new formulation to the Residual Error Method for in-flight thrust measurement is proposed. The new formulation minimizes a cost functional derived from the Maximum Likelihood function by choosing more parameters than the unique Fan Pressure Correlation defined in the Standard Residual Error Method. To do that the Output-Error Method is used and a Newton-Raphson multi-directional search algorithm replaces the one-directional search formulation of the standard method. The new method is called the Extended Residual Error and has been motivated by the difficulties the Standard Residual Error Method presents when applied to long cowl jet engines with mixer. It is demonstrated the feasibility of the new technique and has been identified that the difficulties of the Residual Error Method application to long cowl engines may be associated to errors in the total temperature estimation at the mixer station. This error propagates to the total pressure at the same station and to the nozzle total pressure and may be corrected by identifying a bias to the calculated total temperature, among other possible solutions.*

***Keywords****: In-Flight Thrust Measurement, Output-Error, Residual Error Method.*

## NOMENCLATURE

| | |
|---|---|
| A | Nozzle area, $m^2$ |
| ATF | Altitude Test Facility |
| C | Unknowns of the identification process |
| Cd | Nozzle mass flow coefficient, dimensionless |
| Cv | Nozzle thrust coefficient, dimensionless |
| CPC | Core Pressure Correlation, dimensionless |
| $D_1$ | Weighting matrix, dimensionless |
| Deck | Engine cycle model |
| EREM | Extended Residual Error Method |
| FAR | Federal Air Regulation |
| FPC | Fan Pressure Correlation, dimensionless |
| FTB | Flying Test Bed |
| $FG$ | Calculated gross thrust, $N$ |
| GLTF | Ground Level Test Bed |
| IFTD | In-flight thrust, determination |
| N | Samples number |
| $n_{comb}$ | Combustor efficiency, dimensionless |
| $\eta_{mix}$ | Mixer efficiency, dimensionless |
| P | Pressure, $Pa$ |
| $Pt_{16}$ | Total pressure at station 16, $Pa$ |
| $Pt_6$ | Total pressure at station 6, $Pa$ |
| REM | Residual Error Method |
| $Tt_{16}$ | Total temperature at station 16, $°F$ |
| $Tt_6$ | Total temperature at station 6, $°F$ |
| $\hat{x}$ | Model output |
| Z | Measurements used in the cost functional |
| W | Air mass flow, $kg/s$ |
| $W_c$ | Calculated air mass flow, $kg/s$ |
| $W_m$ | Measured air mass flow, $kg/s$ |

**Subscripts**

| | |
|---|---|
| c | Calculated |
| m | Measured in load cell |

## 1. INTRODUCTION

For the development of an aircraft it is required a process allowing the determination of the engine installed thrust for the aircraft operating envelope. The FAR 25 (1999), through AC 25-7A (1999), requires the measurement of the installed thrust for a range of altitude, airspeed, fan speed, and power extraction to be used in the calibration of the engine cycle model (Deck). The FAR 23 (2005) is less severe and may accept a calibrated Deck based on inlet and nozzle characterization, GLTF and FTB tests. At the end it is still required to demonstrate that the model matches the actual engine lapse rate.

In principle the engine installed thrust cannot be directly measured. In fact it is indirectly determined by a process that requires the nozzle characterization from nozzle model tests and the measurement of actual engine pressures and temperatures at the same stations measured in the model during the nozzle model characterization. It is required a refined plan, an accurate and expensive instrumentation, nozzle model calibration, engine calibration on GLTF and ATF, tests on FTB, several hours of expensive flights and many hours of engineering analysis. The literature presents several methods for measuring engines installed thrust. Today, the SAE AIR 1703A (2006) is the main guide and reflects the state of the art and industry standard on the subject. Among the methods there described the more accurate and used in the industry are the denominated $W\sqrt{T}$ (Weight temperature), $Ap$ (Area pressure) and the Residual Error Method (REM), which are of the '*gas flow path*' group.

The process of measuring thrust in-flight may be seen like a modeling process which may accept different levels of complexity and accuracy and, consequently associated costs. The mathematical model is formulated and latter calibrated by engine nozzle scale model test in a calibration stand. The calibration process is done by determining the nozzle coefficients Cv, Cd (or Cg, Cf, Cx) which are dimensionless groups that relates actual nozzle thrust to ideal nozzle thrust and actual mass flow to ideal mass flow (ideal means what may be produced by an ideal nozzle as presented in gas turbine books). In-flight the ideal thrust and mass flow are calculated and then, via the coefficients, the actual thrust is determined. Before this step it is required to calculate the nozzle total pressure and temperature what is done for example via energy, mass, and momentum conservation and/or component maps.

## 2. THE RESIDUAL ERROR METHOD

The Residual Error Method, presented in SAE AIR 1703A (2006), has been developed to overcome the problem of testing very large turbofan engines in ATF which is of difficult logistics and very expensive. When an engine is tested on a GLTF it is common not reproduce the thrust and flow coefficients obtained in the model calibration, i.e., the model calibrated in the test facility does not matches accurately the real engine. One way found to overcome this problem was recalibrating the model in a way that it reproduces the GLTF results and this has been done for large turbofans by searching for a multiplier that applied to the fan pressure makes the model match more closely the GLTF results while minimizing the cost functional below, Eq. (1):

$$REM = \sqrt{\left(\frac{FG_c}{FG_m} - 1\right)^2 - \left(\frac{W_c}{W_m} - 1\right)^2} \tag{1}$$

This multiplier has been denominated Fan Pressure Correlation (FPC). Using GLTF data a set of FPC is determined for the range of engine pressure ratio (in general engine overboost is required to reach the full required range). Figure 1 shows the minimization of the Residual Error.

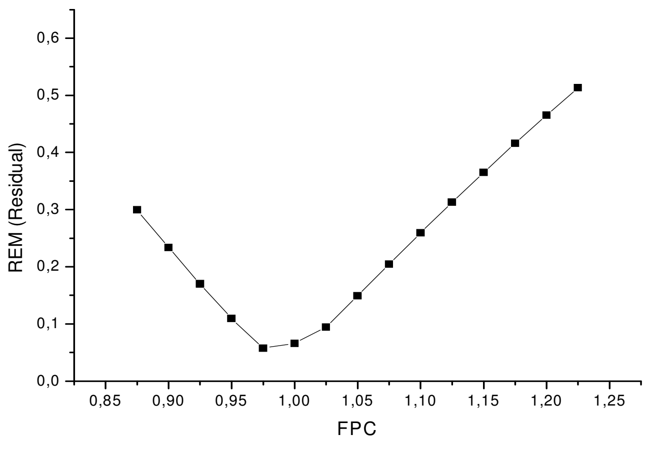


Figure 1. Minimizing the Residual Error via FPC

## 3. DEVELOPMENT

Attempts of the author to use the REM resulted in gross thrust errors ranging from 2 % to 5 % when using the REM procedure over the $W\sqrt{T}$ and $Ap$ methods applied to the same data set extracted from a cycle model of a long cowl turbofan. In other words, the other two methods reproduced satisfactorily the cycle model data while the REM didn't. The small and medium size turbofans and other long cowl engines in general do not have the high by pass ratio of the large turbofan engines which are in general of separated flow. For the large turbofan group, mainly those of separated flow, the fan is the main responsible by the produced thrust. For the other groups the core also produces a significant parcel of thrust and there is today the mixer that difficult a little bit the temperature and pressure calculation at its station. So, if the FPC is not enough to model the probable errors in the model why not to introduce other corrections to the model, Core Pressure Correlation (CPC) for instance, or still more such us mixer efficiency ($\eta_{mix}$), combustor efficiency ($n_{comb}$), etc., that are internal to the formulated model used in this research. Under this view the problem may be formulated as a system identification problem and a multi-direction search algorithm is needed. As the engine thrust and mass flow doesn't have specific equations or state space models the Output-Error seems to be the ideal method to be used to identify the unknowns. The problem is stated as an optimization problem where given the GLTF measured thrust and air mass flow and other measured quantities as fan and core pressures and temperatures, fuel flow, etc., the algorithm inserts values for FPC, CPC, etc., by minimization of a cost functional based on the calculated and measured thrust and air mass flow. The cost functional (Eq. 2) comes from the negative logarithm of the *Likelihood Function,* which is derived from the probability density function of a Gaussian process whose minimization is analogous to maximize the conditional probability $p(c/Z)$ .

$$J = \frac{1}{N-1}\sum_{K=1}^{N}(z-\hat{x})^T D_1 (z-\hat{x}) \tag{2}$$

Equation (2) is here minimized by a Newton-Raphson type algorithm and the unknowns are calculated iteratively from a given initial value:

$$c_L = c_{L-1} - \left[\frac{\partial^2 J}{\partial c^2}\right]^{-1}\frac{\partial J}{\partial c} \tag{3}$$

The derivatives of $J$ to the unknowns are calculated by:

$$\begin{aligned}\frac{\partial J}{\partial c} &= \frac{2}{N-1}\sum_{k=1}^{N}(z-\hat{x})^T D_1 \frac{\partial (z-\hat{x})}{\partial c}\\ \frac{\partial^2 J}{\partial c^2} &= \frac{2}{N-1}\sum_{k=1}^{N}\frac{\partial (z-\hat{x})^T}{\partial c} D_1 \frac{\partial (z-\hat{x})}{\partial c}\end{aligned} \tag{4}$$

In Eq. (4) above $D_1$ is a diagonal weighting matrix that may be unitary. However, the best option is to use in its main diagonal the covariance of the measurements of thrust and air mass flow from the GLTF. If several samples of noisy data are available, $D_1$ may be made equal to the inverse of the residual R calculated by Eq. (5) below.

$$R = \frac{1}{N-1}\sum_{K=1}^{N}(z-\hat{x})(z-\hat{x})^T \tag{5}$$

The estimates accuracy may be evaluated by the Cramér-Rao lower bound (variance theoretical minimum values), which can be calculated from terms of Eq. (3), or its improved version for *colored* noise as presented in Morelli, Klein (1994).

**4. RESULTS**

Several experiments were carried out to investigate if improvements that would be introduced to the Residual Error Method by analyzing the influence of other parameters in addition to the Fan Pressure Correlation. Data obtained from cycle model were used in the calculations.

➢ First experiment: calculation of FPC, CPC, $\eta_{mix}$ and $n_{comb}$ using the Extended REM. The data was processed with the Standard REM and the EREM procedure. Table 1 below presents the errors of the two procedures relative to the cycle model data and Fig. 2 to 4 presents the estimated parameters.

Table 1. Data comparison relative to the Deck

| | Standard REM | Extended REM |
|---|---|---|
| Gross Thrust Error | -5.73% | +0.13% |
| Air Mass Flow Error | +1.40% | +2.38% |

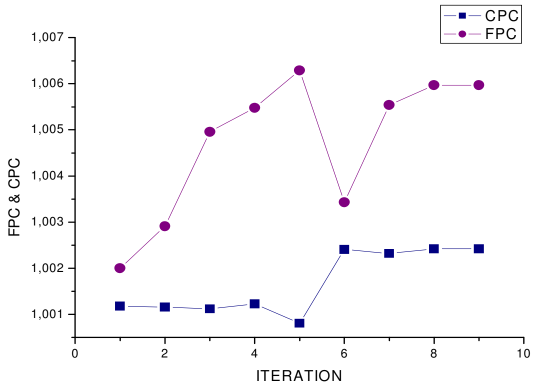


Figure 2. Extended REM Fan and Core Pressure Correlation

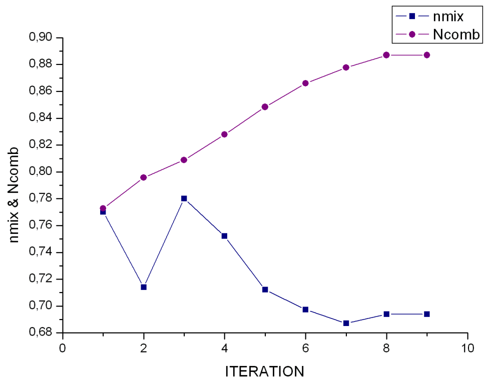


Figure 3. Extended REM Mixer and Combustor Efficiencies

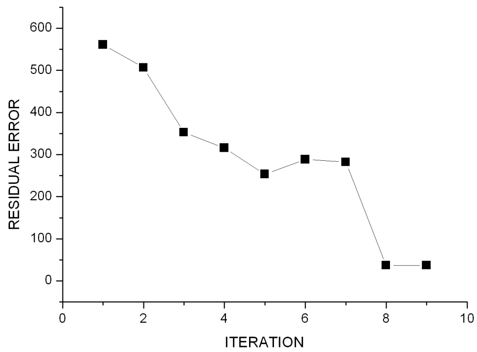


Figure 4 – Extended REM Iterations Versus Cost Functional Values

The estimated value of $n_{comb}$ and $\eta_{mix}$ are below their expected values, however there is no indication that these estimated parameters are biased. Regarding identifiability the unknown number is within the identification algorithm limits, however moderate to high correlation levels have been observed between the identified parameters indicating difficulties in the identification process. A second experiment was performed to observe if the identifiability improved by reducing the number of unknowns.

- Second experiment: the parameter $n_{comb}$ was discarded leaving only three unknowns to be identified (FPC, CPC, $\eta_{mix}$). The experiment resulted in a $\eta_{mix}$ well above 100% instead of the expected value of 75% approximately. The estimated thrust and mass flow matched well the cycle model, as in the first case, and the correlation decreased to acceptable levels.
- Third Experiment: estimating only FPC and $\eta_{mix}$. The parameter $\eta_{mix}$ also converged to a value close to the value obtained in the second experiment. A dipper analysis of the estimates indicated that there is no indication of biased estimation and the data reflected what has been modeled. If $\eta_{mix}$ may assume a value that has no physical meaning in the two experiments then it is possible to say that in what concerns the temperature and pressure calculation at the mixer station the adopted model is not good (or the model differs significantly from the cycle model).
- Fourth experiment: leaving in the model only the FPC and a BIAS to the total temperature at the mixer station. This temperature is a significant parameter to the mixer model and from which the total pressure at the mixer station is calculated and whose value is then used to calculate the nozzle total pressure. Again the new values were calculated, the algorithm converged well and the data reproduced very well the cycle model. It has been observed that the calculated BIAS plus the calculated total temperature at the mixer station resulted in the same total temperature calculated in the second and third experiments. This result validated the calculation performed in the previous experiments and reinforced the concept of mixer improper modeling.
- Last experiment: the conclusion above lead to a last experiment that was the Standard Residual Error minimized for a multiplier of the total temperature at the mixer station. The experiment resulted in calculated thrust and mass flow situated between the results using the Standard REM, i.e., using only FPC, and the original cycle model data. For the analysis performed it was better to search for a temperature correction than for a fan pressure correction term.

Among all the experiments carried out the best result came from the fourth, the one the calculated simultaneously the FPC and the BIAS. The FPC takes into account for errors in the fan pressure measurements (or in the pressure averaging process) while the BIAS corrects for mixer modeling errors. Figure 5 below present the FPC and BIAS to the mixer total temperature calculated for three test point of different engine pressure ratios.

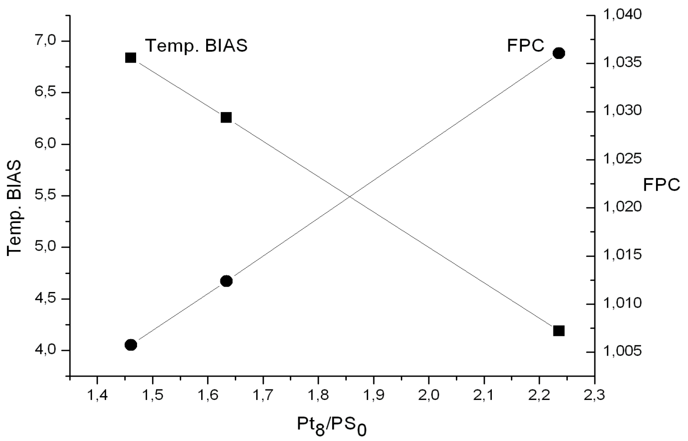


Figure 5. FPC and Bias to the Mixer Total Temperature

## 5. FINAL COMMENTS

A new approach to the Residual Error Method of determining thrust in-flight has been formulated. The technique applied to the cycle model presented good results and point to further and dipper investigations using real data. The sequence of experiments suggested that the REM may be formulated also for the total temperature at the mixer station instead of the fan pressure for the type of engine analyzes. However, better results have been obtained when using the Output-Error method and identifying simultaneously the fan pressure correlation and a total temperature bias at the mixer station. The use of the CPC does not contributed positively in any of the experiments and its elimination improved the estimates of the remaining unknown parameters. It is not known the differences between the thrust calculation process used in the experiments and the Deck thrust calculation process what can justify the large values calculated for $\eta_{mix}$ and even for the BIAS, however they should be equal. In the analysis of real GLTF data it is suggested to analyze several configurations of unknowns privileging parameters that allows for corrections of pressure and/or temperature at mixer station where in fact modeling uncertainty prevails.
Notice that the algorithm was used in a sub-optimal condition since only one sample was used in the calculation while it has stochastic characteristics and is formulated to handle more data samples, for example, few seconds of engine ground test calibration data. Algorithm identifiability improves as the number of unknowns decrease to two where the Cramer-Rao Bound is representative of the actual results. Otherwise, the minimization of Eq. 2 is equivalent to minimize REM (Eq. 1).

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## 7. RESPONSIBILITY

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