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ACTIVE VIBRATION CONTROL AS A FUNCTION OF ROTOR FLEXIBILITY

Luiz de Paula do Nascimento, depaula@dem.feis.unesp.br
Romildo Gonçalves Júnior, midojunior@yahoo.com.br
Departament of Mechanical Engineering, São Paulo State Universty – UNESP
Av. Brasil, 56 – Ilha Solteira – SP - Brazil

Abstract. *This work presents a theoretical analysis of the performance of a system of active control of rotor vibrations using magnetic actuators. The proposed scheme of active control uses a feedforward active control strategy superimposed on the feedback control system of the magnetic actuators. The performance of this control system has been analyzed as a function of the rotor flexibility regarding the impact and optimization of the actuators and error sensors placement on the reduction of vibration levels of the rotors, in terms of global vibration as well as in terms of local vibration. The control system was applied to a theoretical rotor model developed by the matrix impedance method.*

Keywords*: Magnetic actuator, Active control, Rotor dynamics, Vibration*

1. INTRODUCTION

Active magnetic bearings are feedback mechanisms that support a spinning shaft by levitating it in a magnetic field. Besides several other applications, they also have the added capability for active vibration control allowing for the reduction of rotor vibrations, Kasarda (2000). A considerable amount of literature exists on control algorithms used for reduction of rotor vibration using magnetic actuator. Shi *et al.* (2004) used a new adaptive vibration control algorithms developed for minimizing selected vibration performance measures by adjusting the amplitude and phase of a synchronous signal injected at the summing junction of the magnetic bearing feedback control loop. Two methods have been investigated, i.e., the filtered-x adaptive filtering techniques and the method that minimize the magnitude of the magnetic bearing system error signal, both for synchronous disturbance attenuation.

An interesting work was published by Piper *et al.* (2005). They present a novel approach to reducing blade-rate noise of axial-flow fans. By using magnetic bearings as a noise control actuator, it was possible to create the anti-noise source for the disturbance noise source. This approach allows for global noise reduction throughout the sound field. Also, it can be mentioned other works that use magnetic bearings as actuator (Jang *et al.* 2005 and Kasarda *et al.* 2004).

There are several interesting publications where the authors use active control systems with magnetic actuator together with systems of fault control or fault diagnosis in turbomachinery or rotor system, for instance (Zhu *et al.,* 2003 and Aenis *et al.* 2002).

The performance of a vibration control system can be different depending of the rotor shaft flexibility. Thus, the contribution of this work is analyzing the performance of an active control system using magnetic actuator to reduce unbalance vibration of a rotor considering several shaft diameters. It was used a feedforward control strategy superimposed on the feedback control strategy of the actuators. The influence of the actuators and error sensors position also has been analyzed. The analysis is accomplished using a theoretical rotor model.

2. THEORETICAL FUNDAMENTALS

2.1. Rotor modeling

In this work the rotor model is developed employing two methods together. First, the rotor shaft is modeled as a free-free beam without any discrete mass, stiffness and damping elements connected to it. Only the continuous mass and stiffness of the beam, as well as the structural damping are regarded at this time. The free-free beam model is carried out using the theory that establishes the relation between the external forces applied on the beam and its vibratory motion through the computation of the effect of a finite number of natural frequencies and mode shapes, Johnson *et al*. (2003). In the following step the effects of stiffness and damping of the magnetic actuators, as well as the discrete mass of the disks are added onto the free-free beam model employing the impedance matrix method, Bonello and Brennan (2001). The detailed procedure of that rotor modeling methodology can be seen in the work published by Nascimento and Hipólito, 2006.

2.2. Feedback control system of magnetic actuators

The magnetic actuators operate using an electronic feedback circuit to provide the necessary electric current to the actuator to control the rotor vibrations at point where it is located. Thus, a global transfer function expressing the relationship between the output control current and the input shaft position must be established. This relationship can be given by the equation,

$$i_c(j\omega) = G(j\omega)x(j\omega) \tag{1}$$

where $i_c(j\omega)$ is the electric current of control, $G(j\omega)$ is the global transfer function obtained from the characteristic of all components of electronic circuit and $x(j\omega)$ is shaft displacement. The transfer function can be expressed as,

$$G(j\omega) = a_G(\omega) + jb_G(\omega) \tag{2}$$

where $a_G(\omega)$ and $b_G(\omega)$ are the real and imaginary part of the transfer function, respectively. From a mathematical model, the dynamic characteristics of a magnetic actuator can be given by,

$$K_{eq}(\omega) = K_x + K_i a_G(\omega) \tag{3}$$

$$C_{eq} = \frac{K_i b_G(\omega)}{\omega} \tag{4}$$

where K_{eq} and C_{eq} are the equivalent stiffness and damping of the magnetic actuator. K_x and K_i are defined as position stiffness and current stiffness, respectively, which are obtained from the geometric and constructive characteristics of the actuator. For more details of active magnetic actuator theory the reader can see the publication of Nascimento (2002).

2.3. Feedforward active control

The goal of a feedforward active control system is to attenuate vibrations at point where the error sensors are placed. In this work a feedforward control system using the LMS algorithm was applied superimposed on the feedback control system of the actuator. The algorithm will attempt to minimize a cost function given by the sum of the squared outputs from the error sensors. That procedure can be seen in Nascimento (2006). With the process of minimizing the cost function, the optimal control forces are determined. Using these optimal forces, the velocity at any set of observation points $\mathbf{x}_i$ along the rotor can then be calculated before $\mathbf{u}_{ib}$ and after $\mathbf{u}_{ia}$ optimal control as,

$$\mathbf{u}_{ib} = \widehat{\mathbf{T}}_{ip}\mathbf{f}_p \tag{5}$$

$$\mathbf{u}_{ia} = \widehat{\mathbf{T}}_{ip}\mathbf{f}_p + \widehat{\mathbf{T}}_{ic}\mathbf{f}_{c\_opt} \tag{6}$$

The two matrices $\widehat{\mathbf{T}}_{ip}$ and $\widehat{\mathbf{T}}_{ic}$ contain augmented rotor mobilities describing the velocities at the observation locations $\mathbf{x_i}$ due to forces acting at the locations $\mathbf{x_p}$ of the unbalance forces (primary disturbance) and the locations $\mathbf{x_c}$ of the control forces. That procedure can be seen with more details in Nascimento (2006).

3. THEORETICAL ROTOR FOR ACTIVE CONTROL ANALYSIS

In this section a theoretical rotor supported by magnetic bearings is presented. The magnetic bearings will also be used as actuators of the active control of rotor vibrations induced by unbalance forces. There are two disks with different masses (m_3 and m_4) mounted on the rotor, in witch the unbalance forces of unitary magnitude are generated. Also, it is regarded the discrete masses of the ferromagnetic material (m_1 and m_2) which are part of the actuators. A scheme of that rotor is shown in Fig. 1. To vary the rotor flexibility it was taken different shaft diameters, i.e., 3.0, 5.0, 7.5, 10.0, 12.5 and 15.0 mm, corresponding to the cases A to F, respectively. The equivalent stiffness and damping of the magnetic bearings (denoted by k_1, k_2, c_1 and c_2 in Fig. 1) were obtained using the theory of the section 2.2, Eqs. (3) and (4), for a specific bearing and a set of parameters for the components of the electronic circuit of control, Guiráo and Nascimento (2004). The physical characteristics for this theoretical model and data for simulations are given in Tab. 1.

The rotor vibration in the horizontal direction will not be considered in this model and will be assumed to be independent of the vertical vibration. Any vibration in the horizontal direction can be controlled in the same way as the vertical vibration using another uncoupled active control system producing similar results. The rotor model was discretized in 20 nodal points that can be used to investigate the vibration reduction after the active control system to be applied and does not include gyroscopic effects. The simulations will be presented for exciting frequencies between 0 and 200 Hz such that the vibration level at each frequency represents the vibration as if the rotor was run at that frequency and the excitation was caused by unit unbalance forces. Also, that frequency range is supposed to contain at least the first three vibration modes of the rotor, which have higher probability to be excited in the practice.

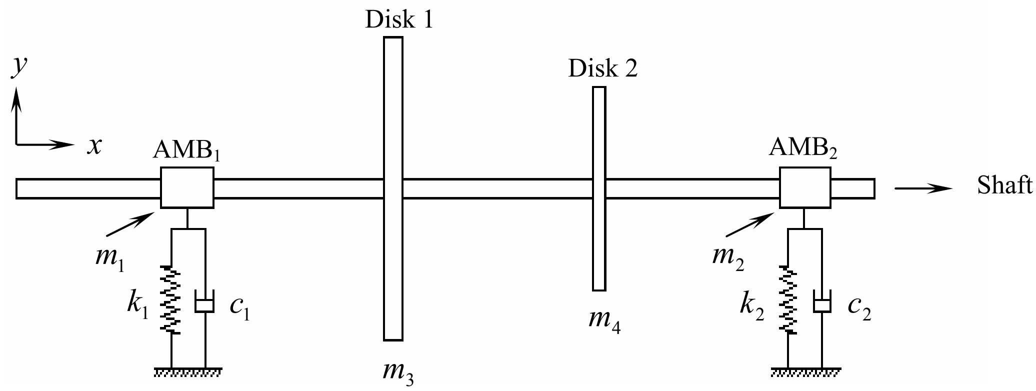

Figure 1. Schematic of rotor system used in theoretical model

Table 1. Physical properties of rotor and bearings used for the simulation.

Rotor		**Bearing 1**		**Bearing 2**		**Disk 1**		**Disk 2**	
Length (l)	500 mm	Mass	0.25 kg	Mass	0.25 kg	Mass	1.3 kg	Mass	0.5 kg
Diameter (d)	Variable	Stiffness k_1	20 KNm^{-1}	Stiffness k_2	20 KNm^{-1}	- -	-	- -	-
Modulus of elasticity (E)	207GNm^{-2}	Damper c_1	10 $Nm^{-1}s$	Damper c_2	10 $Nm^{-1}s$	- -	-	- -	-
Density (ρ)	7850kg^{-3}								
Structural Damper (ξ)	0.001	Position x_1	100 mm	Position x_2	460 mm	Position x_3	220 mm	Position x_4	340 mm

4. THE EFFECT OF SHAFT DIAMENTER

A factor which controls critical speeds of the rotors is the shaft stiffness (or shaft diameter). However, we must look at this effect as a function of the support stiffness. The augmented mobility of the rotor given in Eq. (5) can be considered to contain augmented natural frequencies and mode shapes. Table 2 shows the three first critical speeds of the rotor for all the analyzed cases (different shaft diameters). As it can be seen the first critical speed increases strongly until the case C. After that, only very small variations in this critical speed are observed. Similar tendency also is observed in the second critical speed. On the other hand, the critical speeds of superior order (as the third) increase continually in all the analyzed cases. In the cases A, B and C (smaller diameters), the support stiffness is higher than the shaft stiffness and the first critical speed is completely dependent of the geometry of the shaft. So, in those cases we can say that the rotor is in the "shaft dependent" region. In the cases D, E and F the first critical speed approaches of an asymptotic value and it will only vary as a function of the support stiffness. Then, we can say the rotor is in the "support dependent" region.

Table 2. Critical speeds (Hz) of the rotor for the simulated cases.

Critical speed	**Case A**	**Case B**	**Case C**	**Case D**	**Case E**	**Case F**
1st	4.0	9.6	16.0	18.4	19.2	19.2
2nd	16.8	33.6	34.4	32.8	30.4	28.0
3rd	44.0	48.0	60.8	87.2	121.6	160.8

5. THEORETICAL VIBRATION CONTROL ANALYSIS

The performance of the control system implemented on the rotor will be limited by the actuator and error sensors placement, and the analysis of that performance will be accomplished using the theory presented in section 2. The term "global vibration level" will be used in this section and refers to the mean displacement squared level averaged over the entire length of the rotor (at the nodal points). Although it is often more important to reduce the vibration levels at particular positions along the rotor, the global vibration level will provide some indication as to the overall performance of the system and highlight potential problems.

5.1. Perfomance as a function of the actuator position

Normally the rotors are not symmetric and the mass distribution on the length is not uniform. Then the magnetic bearings support different dynamic loads and when they also are used as actuators of a control system the performance of each actuator can be distinct. To analyze this it will be assumed that the control system is designed to minimize the global vibration level. For the purposes of investigating the effect of the actuator location alone it is assumed that a "perfect" error sensor, that can directly measure the global vibration level, is used (not realizable in practice). The global vibration level, with and without control, using the actuator 1 and actuator 2 separated, and both actuator simultaneously, for different flexibility of the rotor (case A to F) are presented in Fig. 2.

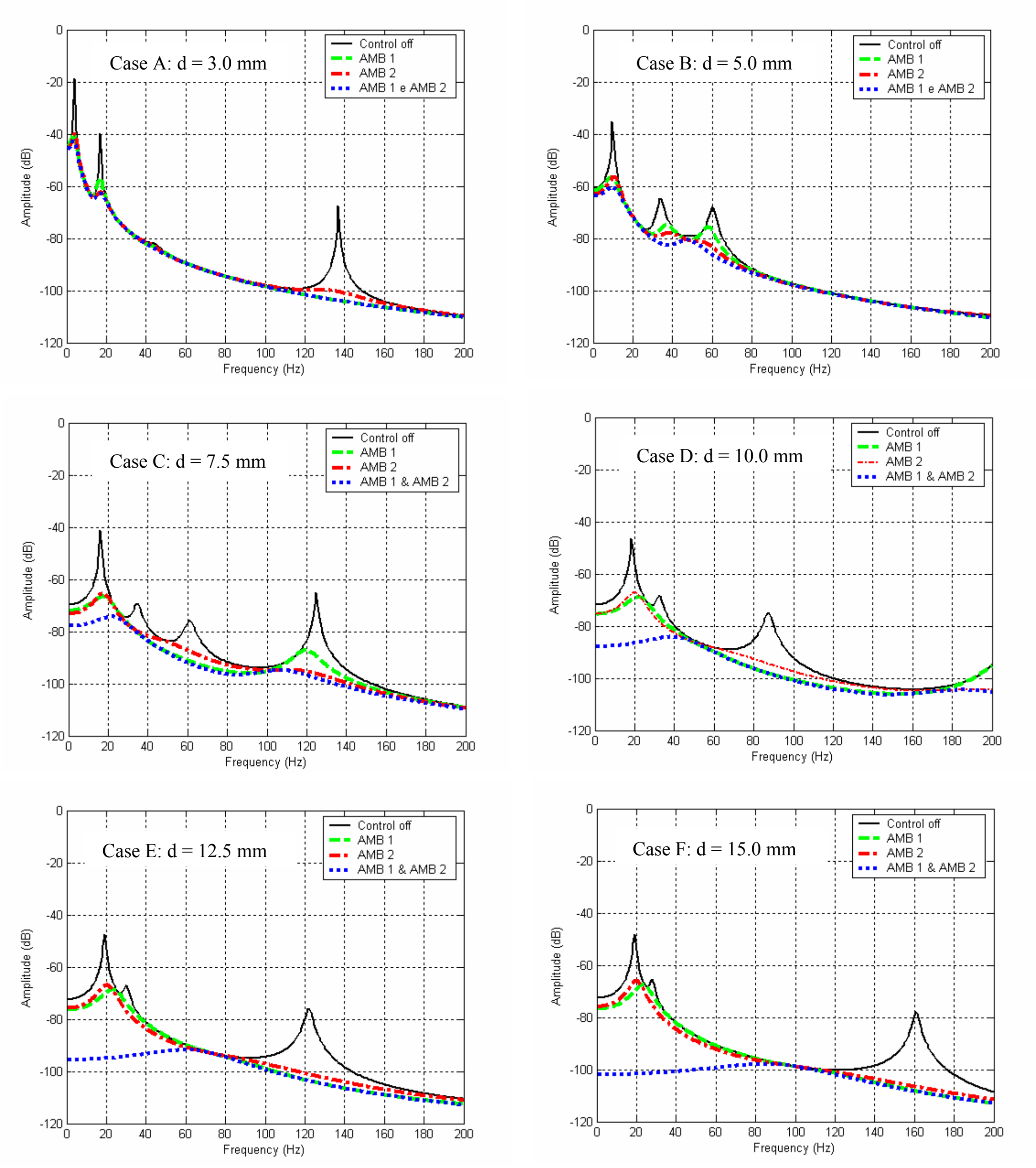

Figure 2. Global vibration reduction using the actuator 1, actuator 2 and both actuators, cases A to F.

The first thing to be noted from these results is the similar performance of the control system for the actuator 1 and actuator 2 for all analyzed cases. In terms of global vibration it was possible attenuating all critical speeds amplitudes.

At frequencies away from resonance or critical frequencies the performance can be poor. It can also be seen that the performance when the architecture of control uses both actuators simultaneously is higher and increases significantly when the rotor leaves the “shaft dependent” region and goes to “support dependent” region (cases D, E and F of the Fig. 2). At the “support dependent” region the control system is more effective to attenuate the lower critical speed.

5.2. Perfomance using single actuator and single error sensor

The simplest feedforward system of active control is the strategy that uses single actuator and single error sensor for its operation. This strategy is more economically suitable in terms of hardware costs and computer capacity. In terms of local vibration, this strategy will be conditioned to attenuate vibration at the point where the error sensor is placed on the rotor. However, employing an accuracy analysis, points for placing error sensor along of rotor can be determined, in which the global vibration of the rotor decreases at an acceptable level. Also, when the aim of the active control is to attenuate vibrations when the rotor operates in critical speed conditions, this analysis will indicate the best position for the error sensor getting the maximum performance of the control.

By using an accuracy analysis it has been found that the vibration control system, in general, provides non satisfactory performance with the error sensor placed at several nodal points. However, it has been verified that there are some positions for the error sensor where high performances were reached. In terms of global vibration, Fig. 3 shows the results of the vibration reductions for these positions for the error sensor. It can be observed that the best positions for error sensor are the points next to the magnetic bearing opposite to the magnetic bearing used as actuator. These results indicate that this strategy of control does not have enough efficiency to attenuate vibrations when the shaft is very high flexibility, inside the “shaft dependent” region (case A and B in Fig. 3). In those cases very high amplifications in rotor vibration are verified at higher frequencies and at frequencies out of critical speeds. These amplifications are bigger when the actuator 1 is employed.

When the rotor goes to the “support dependent” region, this strategy become more efficient and it is possible to reach large attenuation at critical speeds without amplifying significantly other frequencies except at very low frequencies where high amplification is observed (case D, E and F in Fig. 3).

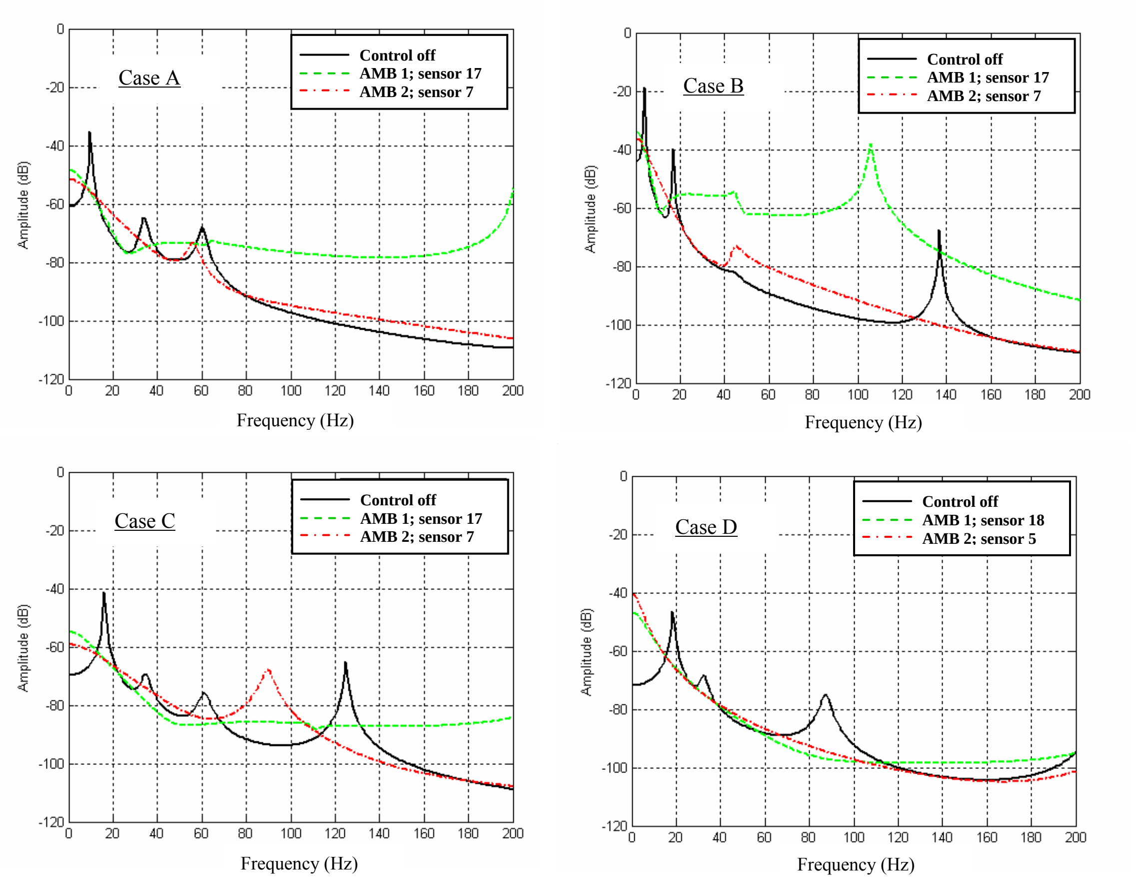

Figure 3. Global vibration reduction using control system with single actuator and single error sensor

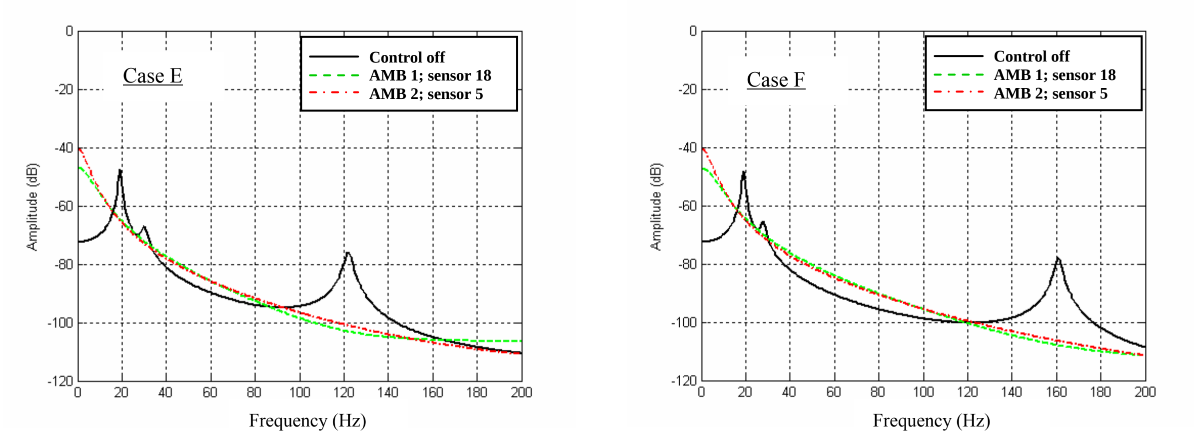

Figure 3. Global vibration reduction using control system with single actuator and single error sensor (Cont.)

5.3. Perfomance using single actuator and two error sensors

When an active control system with single actuator and single error sensor is used to control rotor vibration, the level at critical frequencies can be attenuated, but at other range of frequencies the vibration level increases of an undesirable way. Thus, to try minimizing this problem an active control system using single actuator and two error sensors normally is employed. Figure 4 presents the results when an active control system this type is used. Here, the best performance is obtained when the error sensors also are placed close to the extremities of the rotor shaft, nearby of the supports.

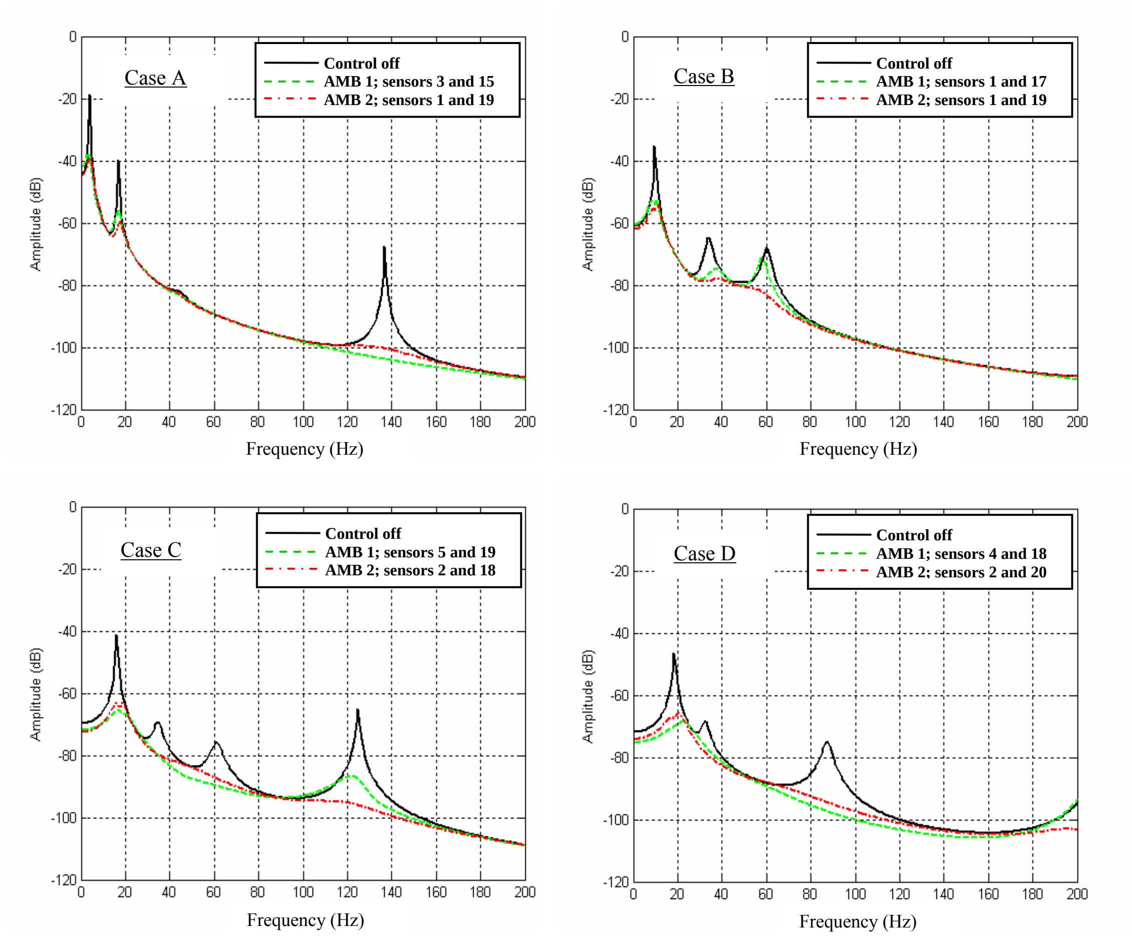

Figure 4. Global vibration reduction using control system with single actuator and two error sensors

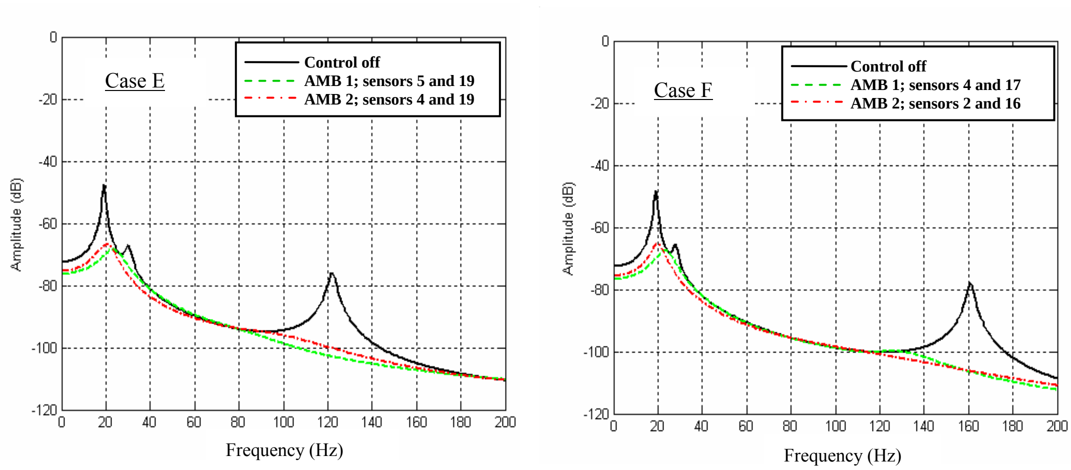

Figure 4. Global vibration reduction using control system with single actuator and two error sensors (Cont.)

The results of the Fig. 4 show that this strategy of control can attenuate a lot the resonance amplitudes and no important amplification is verified at frequencies away from the critical resonances. Also, a better performance in reducing the modal amplitudes is observed on the rotor "support dependent".

Although the control strategies have reduced the global vibration at critical speeds, that does not guarantee there will be reduction of vibration at all points along the rotor. In terms of local vibration, Fig. 5 and Fig. 6 present the modal displacement along of rotor before and after vibration control, for the cases B ("shaft dependent") and E ("support dependent"), respectively. As it can be seen, the displacements at critical frequencies for both cases were significantly attenuated. However, at critical frequency of 48 Hz of the case B the displacement increases a little at right extremity.

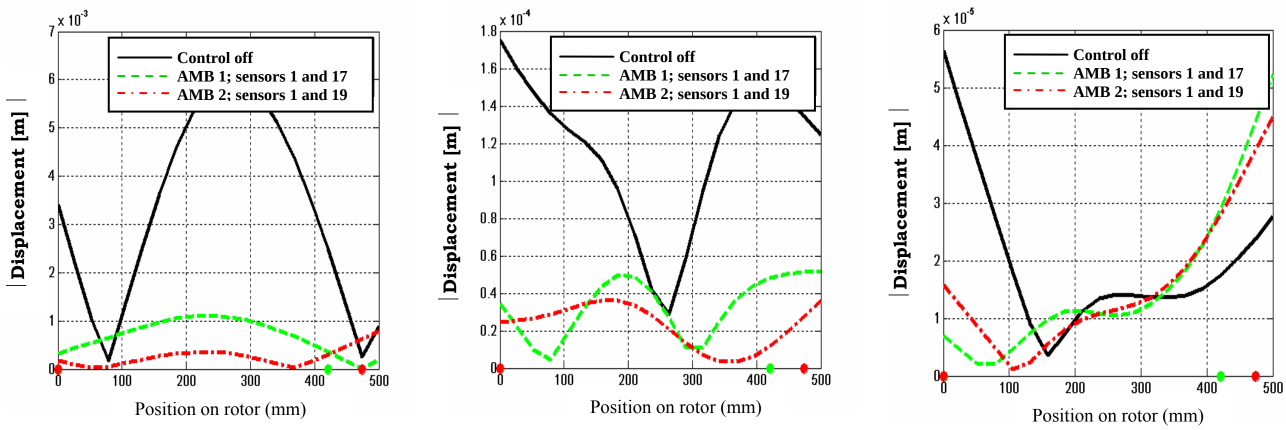

Figure 5. Local vibration along of rotor at critical frequencies 9.6, 33.6 and 48 Hz, case B

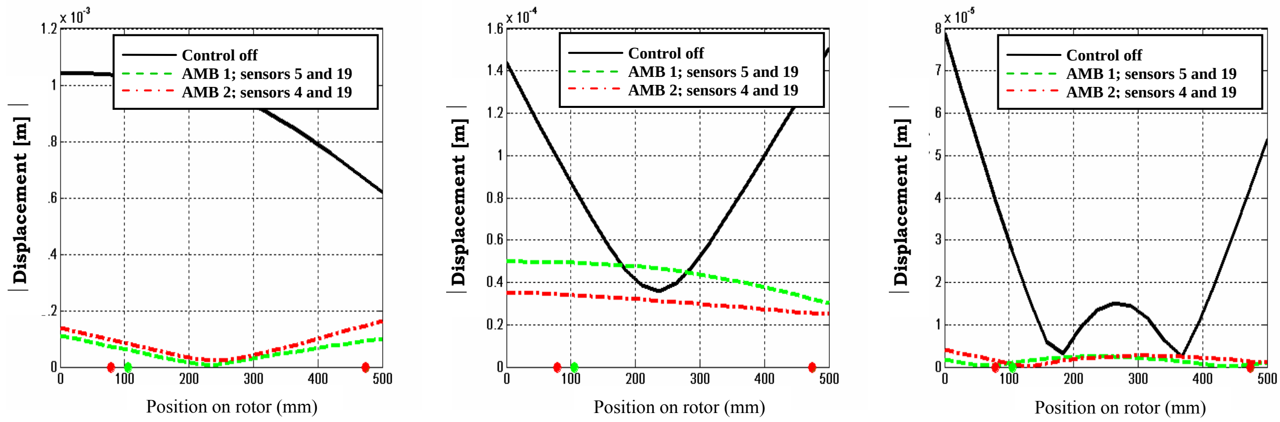

Figure 6. Local vibration along of rotor at critical frequencies of 19.2, 30.4 and 121.6 Hz, case E

As it was seen in the global vibration analysis, the performance of the active control system increases when the rotor is “support dependent”, as it can be verified comparing the results of Fig 5 and 6. Besides, in this strategy with two error sensors the performance is practically the same when the magnetic bearing 1 or the magnetic bearing 2 is used as actuator.

Figure 7 and 8 show the vibrations along of the rotor at frequencies away from the critical frequencies for the case B and D, respectively. At these frequencies the global vibration analysis, Fig. 4, indicates that the vibration level is practically the same before and after the active control is applied, but the local vibration analysis shows that there are regions along of the rotor shaft where the amplitude increase when the control is applied.

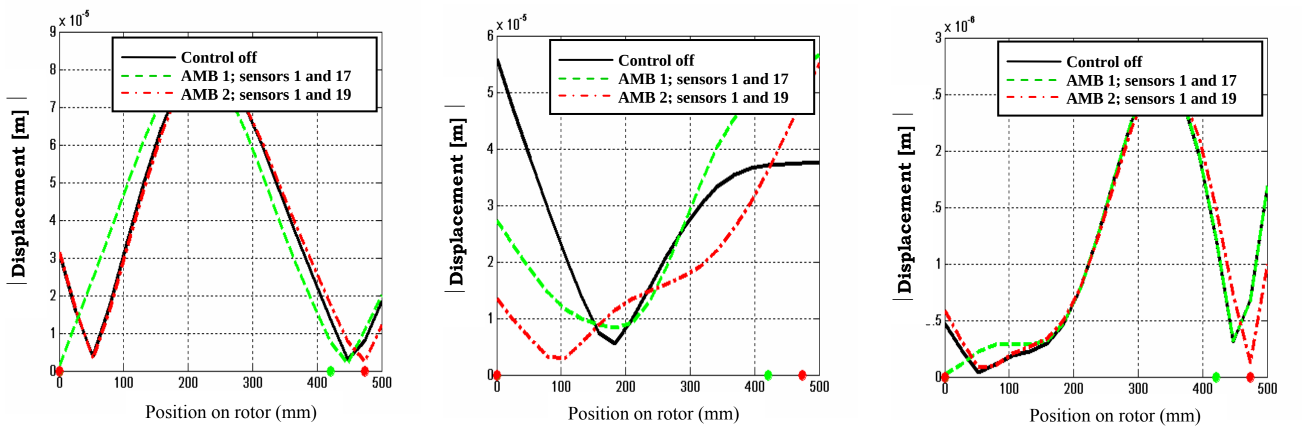

Figure 7. Local vibration along of rotor at frequencies 21.6, 40.8 and 140 Hz, case B

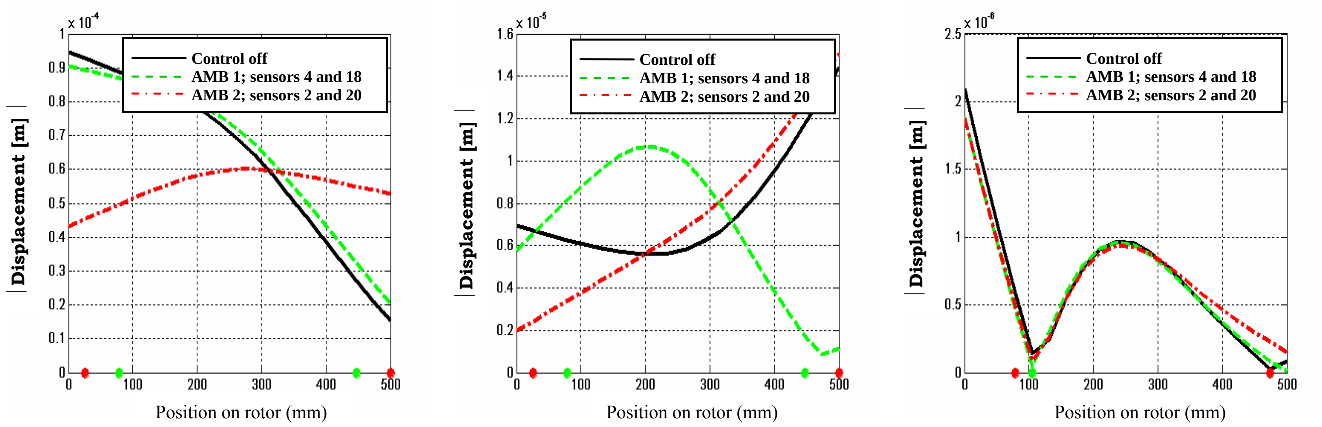

Figure 8. Local vibration along of rotor at frequencies 25.6, 60 and 160 Hz, case D

6. CONCLUSIONS

A theoretical investigation into the effect of the location of error sensors and actuators on both global and local vibration reduction along a rotor using a feedforward control scheme with magnetic actuator was accomplished. The strategy with single actuator and single error sensor is not efficient to control vibration of the rotor “shaft dependent”. Very good reduction of global and local vibration at modal or critical frequencies can be achieved with an active control working with a single actuator and two error sensor, but away from these frequencies the performance can be poor and amplifying vibration level at some points of the rotor shaft can occur. A more sophisticated control system using a larger number of error sensors and actuators can be necessary to attenuate satisfactorily vibrations at frequencies away form resonances. In general, the performance of the control system increases as the rotor goes from “shaft dependent” region to “support dependent” region.

7. ACKNOWLEDGEMENTS

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