DESIGN OF A FLIGHT CONTROL SYSTEM BASED ON THE TOTAL HEADING CONTROL SYSTEM PHILOSOPHY

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Abstract. *This work presents an application of the Total Heading Control System strategy to the project of a lateral-directional autopilot for a 50-seat airplane, designed as part of the Engineering Specialization Program held by Empresa Brasileira de Aeronáutica (EMBRAER) in partnership with the Instituto Tecnológico de Aeronáutica (ITA). The control loops were built with the aid of MATLAB and SIMULINK computational tools. Optimization algorithms were used extensively for equilibrium and feedback gains calculation. THCS results were then compared with those got from the more classical approach, widely diffused in the literature. A comparative analysis was carried out in the time domain only.*

Keywords*: THCS, Flight, Control, System*

1. INTRODUCTION

The development of automatic flight control systems is permeated by old practices that have been present since the very beginning of such métier. As an example, one could cite the control of aircraft speed by modulation of thrust and the control of path angle by elevator actuation. It is easy to see that if more thrust is applied, the aircraft will not only accelerate but also climb if everything else is left unchanged.

Efforts were then carried out in order to try to develop new techniques that would rely more on the mechanics behind the system than the ones listed above. From those efforts emerged the concepts known as Total Energy Control System (TECS) and Total Heading Control System (THCS), which base their control loop topologies on physical principles rather than on old practices (Lambregts, 1996).

It is important to mention that new, revolutionary control strategies are introduced very slowly in civil aviation, due to the certification impacts. That explains, in some sense, why the old practices have lasted until these days.

2. THE RJ2

All control loops listed in this work were designed using a 50-seat airplane (called RJ2) as the plant. Such airplane was the outcome of the effort made by 30 engineers during 4 months. Its fundamental aeronautical characteristics (like dimensions, weight and control surfaces) were defined. However, its systems remained in a very incipient level of detail. Figure 1 shows some views of the aircraft (Carvalho Júnior, 2006).

Figure 1. RJ2 views

2.1. Mathematical formulation

In this work the earth was considered an inertial frame of reference and the airplane a rigid body. Under these assumptions, the equations describing the six degree-of-freedom of the aircraft are well know and easily found in the literature (Stevens and Lewis, 2003) and therefore will not be reproduced here.

Optimization routines were used extensively in this work. Equilibrium state determination, model linearization and feedback gains calculation are common-place and they all employ such routines.

The linear model for the airplane lateral-directional movement was extracted from the complete linear model determined from the complete set of equations. For a rectilinear, leveled flight (which are the cases considered in this work), such assumption is pretty much true.

2.2. Traditional approach review

Nowadays, when someone talks about lateral-directional control of an aircraft, terms like yaw damper, roll damper, roll angle hold, turn coordination and heading hold always appear. Such control strategies are, for sure, effective but are more the result of a historical evolution than of a physical assessment of the problem.

Historically, the yaw damper was conceived earlier than the roll damper. They are based on the feedback of the yaw and roll rate to the rudder and ailerons, respectively. The result is, fundamentally, an increase in the damping of roll and dutch-roll modes.

The roll angle hold functionality was first created to guarantee a leveled flight, what is very important to prevent the aircraft from entering a spiral movement towards the ground, not always noticed by the pilots. Afterwards it was applied to the execution of coordinated turns, together with sideslip and pitch control. The control idea is just to feedback the roll angle so that its error regarding the reference will generate a command to the ailerons after being multiplied by a gain.

Coordinated turns provide comfort to the passengers, maximize aerodynamic efficiency (no sideslip) and minimize the aerodynamic load on the structure. For such characteristics, it is crucial for an airplane to be able to perform coordinated turns.

There are several control strategies that allow the execution of coordinated turns. The most intuitive approach (and the one adopted in this work) is to feedback the sideslip angle, β, so that a sideslip generates a rudder deflection that will tend to cancel it.

2.3. THCS – Total Heading Control System

As mentioned before, the THCS was developed on the mechanics behind the controlled flight. In what concerns the lateral directional movement, a negative aileron deflection (left wing with higher lift) causes both a positive sideslip (if the rudder is not used) and a steady increase in the heading angle, ψ. This way, the total heading, $\psi + \beta$, augments.

Now, starting from a total heading of zero, if the rudder is deflected positively (towards the left side of the airplane, aft looking forward) the aircraft yaws to the left, creating a positive sideslip and a negative heading angle, with approximately the same magnitudes. It can be stated then that the total heading of zero degrees was “distributed” between ψ and β, the first receiving $-X$ and the second X degrees.

The architecture found in Fig. 2 was conceived based on the basic principles just explained.

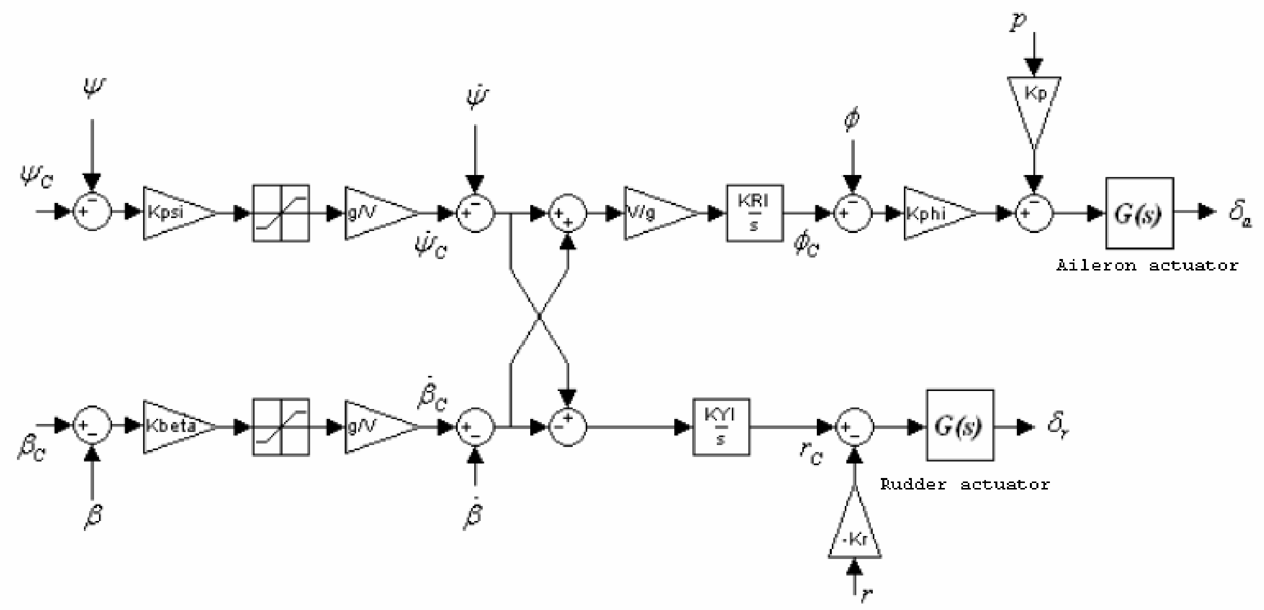

Figure 2. THCS basic architecture (Lambregts, 1996)

Where ψ_C = commanded heading angle, β_C = commanded sideslip angle, Kpsi = feedback gain for the error in the heading angle, Kbeta = feedback gain for the error in the sideslip angle, g = acceleration of gravity, V = true aircraft speed, KRI = KYI = general gains, r_C = commanded yaw rate, Kphi = feedback gain for the error on the roll angle, $G(s)$ = actuator transfer function, ϕ = roll angle, ϕ_C = commanded roll angle, δ_a = aileron deflection and δ_r = rudder deflection.

The actuators were modeled as first order transfer functions, with a time constant equal to 0.05 seconds. Rate and amplitude limiters were also added to the actuator models, so that these would be more representative of real actuators. The values were defined as 20 degrees of maximum amplitude and 40 degrees/second of maximum rate of deflection.

As it can be seen, the commanded roll angle depends on the total heading error, while the commanded yaw rate depends on the heading distribution. The equations bellow will clarify this point.

$$\phi_c = KRI\frac{V_{true}}{g}\int\left(\dot{\psi}_{erro} + \dot{\beta}_{erro}\right)dt = KRI\frac{V_{true}}{g}\left(\psi_{erro} + \beta_{erro}\right) \tag{1}$$

$$r_c = KYI\int\left(\dot{\psi}_{erro} - \dot{\beta}_{erro}\right)dt = KYI\left(\psi_{erro} - \beta_{erro}\right) \tag{2}$$

Where

$$\dot{\psi}_{erro} = \dot{\psi}_c - \dot{\psi} \tag{3}$$

$$\beta_{erro} = \beta_c - \beta \tag{4}$$

As the roll angle is easily controlled by the ailerons and the yaw rate by the rudder, it is possible to conclude that the error in the total heading controls the ailerons while the error in the distribution of this heading controls the rudder. By controlling the sum and the difference between two variables (ψ and β), their individual values can also be controlled.

Although Figure. 2 allows a good understanding of the general algorithm, it is not very realistic since signals like $\dot{\beta}$ and $\dot{\psi}$ are used. In what follows it will be shown that this algorithm can be implemented without such differentiations.

The equation below is well know from the courses in flight dynamics:

$$\dot{v} = -ru + pw + g\sin\phi\cos\theta + \frac{Y}{m} \tag{5}$$

Where u, v and w are the components of the velocity vector on the aircraft body axes x, y and z, respectively. The yaw rate is r, p is the roll rate and Y is the sum of forces in the y axis.

By making some simplifying assumptions it is possible to gain some insight in the idea behind the THCS. Consider, for example, that both w and θ are zero (very small angle of attack), ϕ is also small (leveled flight) and Y is only composed of aerodynamic terms (thrust aligned with the x axis). Under those assumptions, (5) can be re-written as:

$$\dot{v} = -ru + g\phi + \frac{Y_A}{m} \tag{6}$$

Considering that $v = V\beta$, where V is the aircraft total velocity (assumed to be constant), and that $u \approx V$, (6) becomes:

$$\phi = \frac{u(\dot{\beta} + r)}{g} - \frac{Y_A}{mg} \tag{7}$$

For a coordinated turn, $\dot{\beta} = 0$ and $Y_A = 0$. If one takes $\dot{\psi} \approx r$, then:

$$\phi = \frac{V\dot{\psi}}{g}$$

Conclusion: a positive roll angle leads to an increase in the heading angle. This intuitive result was therefore demonstrated.

In the case where the yaw rate is zero, (5) becomes:

$$\phi + \frac{Y_A}{mg} = \frac{V\dot{\beta}}{g} \tag{8}$$

As Y_A takes some time to become significant, one can assume temporarily that:

$$\phi \approx \frac{V\dot{\beta}}{g} \tag{9}$$

And the following conclusion can be drawn: a positive roll angle forces the sideslip angle to increase in a rate that is similar to the one verified in the heading angle.

In order to finish the analysis, the case where a yaw rate is commanded shall be studied. Considering that the roll angle is kept at zero and that the lateral forces do not appear instantaneously, one can write $\dot{v} \approx ru$. As $v = V\beta$ and $u \approx V$, then $\dot{\beta} \approx -r$. As it was assumed that $\dot{\psi} \approx r$, then it can be said that a positive yaw rate causes a decrease in the sideslip angle and an increase in the heading angle of approximately the same magnitude, which is a fairly intuitive result.

2.4. Aircraft modeling, model linearization and feedback gains calculation

RJ2 stability derivatives were calculated by the aerodynamics team for several flight conditions. In order to simplify the work without penalizing the results, such derivatives were expressed as a polynomial function of those parameters that revealed themselves to be relevant for each derivative. For example, for a clean configuration (no flaps), forward CG and altitude between 0 and 41000 ft, it was possible to consider:

$$C_{L\alpha} = 0.0000493Ve^2 - 0.0076194Ve + 5.4513 \tag{10}$$

Where *Ve* refers to the airplane equilibrium velocity, expressed in m/s. During this whole work, only clean configurations were considered.

In order to complete the aircraft model, a very simple propulsive model was developed and the inertia properties were calculated.

The next step was related to the model linearization, and for that it was first necessary to determine an equilibrium state, around which a linear model is valid if small disturbances are considered. Such equilibrium state was calculated with the aid of the *fmincon* optimization routine available in MATLAB, which permits to solve optimization problems with constraints. Once the equilibrium state was known, the model linearization was accomplished numerically.

Once the linear model was obtained, the feedback gains were calculated also using the *fmincon* optimization routine.

Although it is theoretically possible to calculate the feedback gains using the non-linear model, such approach is not practical because it would demand too much time, since thousands of simulations can be run in order to attain the minimum value of the optimization index.

Another central point in this work was the use of the *sim* routine, which allows running Simulink models from a MATLAB script. The idea is to take advantage of the visual language of Simulink instead of closing the several loops on the script itself, what is a procedure very prone to mistakes. Also, the choice of optimization index becomes a very straight-forward procedure, as illustrated below.

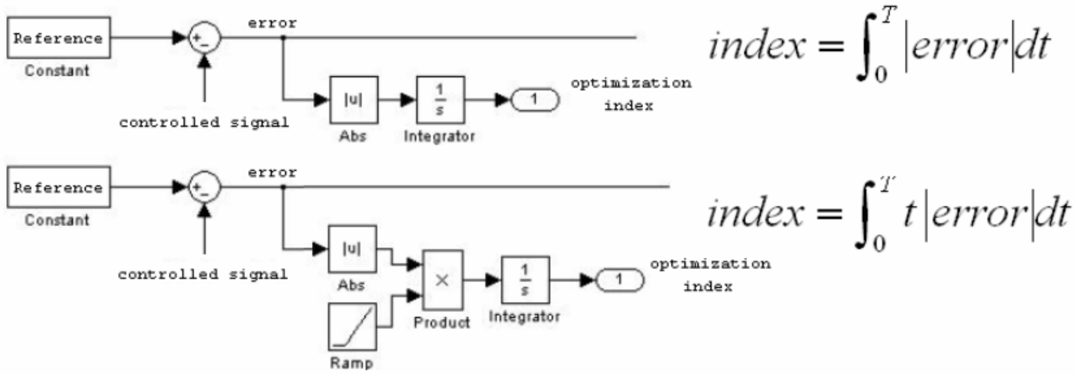

Figure 3. Example of how Simulink can be helpful

The index is calculated in Simulink and the exported to the script file, which then calculates a new set of feedback gains as directed by the optimization routine. Those new gains are then considered for the next simulation, generating a new index and reinitiating the cycle.

2.5. Summary of the work

Initially, the inner loops, which are common to the classical and THCS strategies, were designed. These are the yaw and roll dampers and the roll angle hold. The optimum feedback gains were calculated for a large number of different flight conditions so that simple patterns could be extracted for each one of them. This approach proved to be very helpful in the outer loops design phase.

When designing the outer loops, care was taken so that the results comparison would be fair. This means that the same number of integrators was used in both approaches, classical and THCS. As before, the feedback gains were calculated using *fmincon*. However in order to make a more practical comparison between the two approaches, only four flight conditions were considered. This comparison was made only in the time domain, using criteria like overshoot, stabilization time and response time.

3. RESULTS AND CONCLUSIONS

3.1. Inner loop design

Using the architecture showed in Fig. 6 (but excluding the sideslip and roll angle feedback paths), the optimum gains (*Kr* and *Kp*) were calculated for the following flight conditions:

Altitude (m): 0, 1500, 3000, …, 12000

Velocity (m/s): 50, 75, 100, …, 250

The optimization index considered was the time integral of the sideslip angle absolute value (initially zero). The initial disturbance in the sideslip angle was taken as five degrees. The plots in Fig. 4 were then generated.

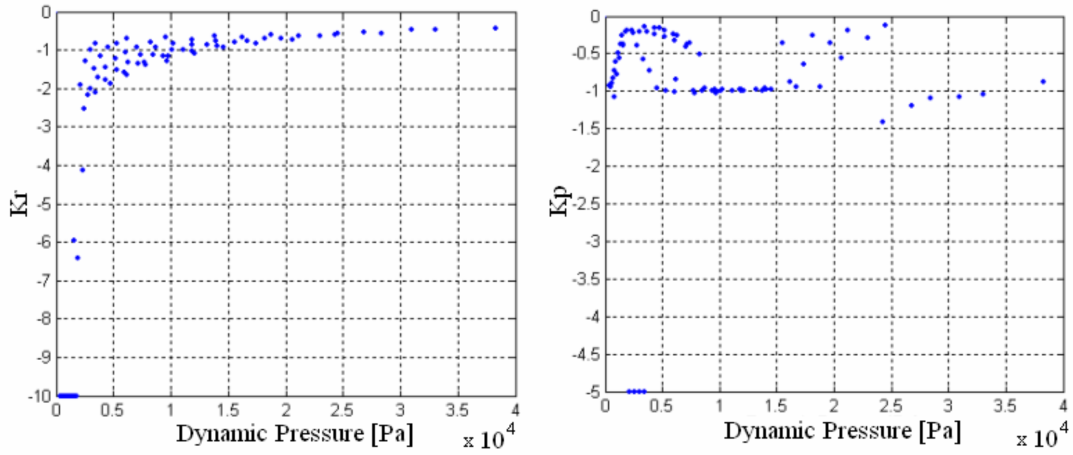

Figure 4. *Kr* and *Kp* versus dynamic pressure

Disregarding the very low dynamic pressure conditions, *Kr* and *Kp* were taken as -1 and -0.75 respectively. The results will show that such drastic simplification led to good results. Furthermore, this work's main focus is on the outer loops.

As for the roll angle hold, the control strategy considered was the one showed in Fig. 6, but without the sideslip feedback path. The controller was taken as a simple gain, K_ϕ, which was calculated for the same flight conditions used in *Kr* and *Kp* calculation (and already considering the values previously calculated for these two gains).

The optimization index considered for the roll angle hold was the time integral of the absolute value of the difference between the roll angle reference and the actual roll angle. From an actual angle of zero degrees, the reference was set to twenty degrees. Figure 5 shows the results of the many calculations, which led to the choice of $K_\phi = -3$.

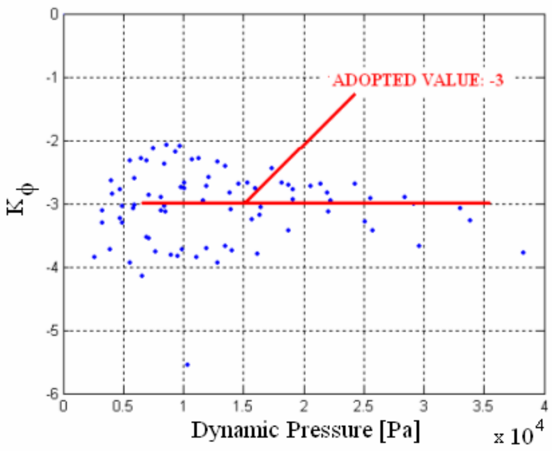

Figure 5. K_ϕ versus dynamic pressure

3.2. Outer loop design (classical approach)

In this item the feedback loops for sideslip and heading control will be designed. In order to ease the comparison between the results presented here and the ones obtained with the THCS approach, four flight conditions were defined, as presented in Tab. 1. Those four flight conditions cover a wide range of dynamic pressure values.

Table 1. Flight conditions considered

Flight Condition	Altitude (m)	Velocity (m/s)	Mach	Dynamic Pressure (kPa)
I	10000	150	0.5	4.6
II	5000	140	0.437	7.2
III	10000	250	0.83	13
IV	0	180	0.53	19.8

3.2.1. Sideslip control

The architecture considered in this topic is shown in Fig. 6. It is important to notice that only one additional loop was created in order to feedback the sideslip angle. The inner loops are already well defined. Another important point is that a proportional-integral type of controller was used. This is in accordance with the idea of fairness (same number of integrators used in the THCS), previously discussed.

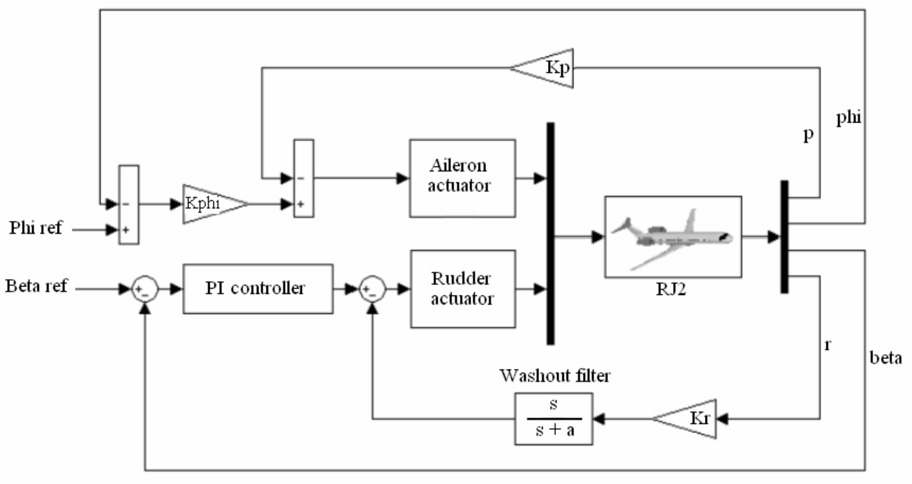

Figure 6. Sideslip control loop

The controller has a transfer function of the kind $\frac{K_{prop}s+K_{Int}}{s}$.

The washout filter is necessary because during an execution of a coordinated turn the yaw rate is steady but not zero. The washout filter time constant was chosen to be 1 second.

In the figure above, K_p and K_r are the feedback gains for p (roll rate) and r (yaw rate), respectively.

As always done in this work, K_{prop} and K_{Int} were calculated as to minimize an optimization index, which in this case equals the time integral of the sideslip angle absolute value, since initially $\beta = 0$. A command of ten degrees was given in the roll angle while keeping the sideslip reference equal to zero (the goal is to perform a coordinated turn). The results are in Tab. 2.

Table 2. Proportional-integral controller gains for sideslip control

Flight Condition	K_{prop}	K_{Int}
I	5	0.61
II	5	0.74
III	5	0.91
IV	5	1.72

It is interesting how K_{Int} increases with the dynamic pressure. Just to show how the system is working fine, Fig. 7 brings the results for the flight condition I.

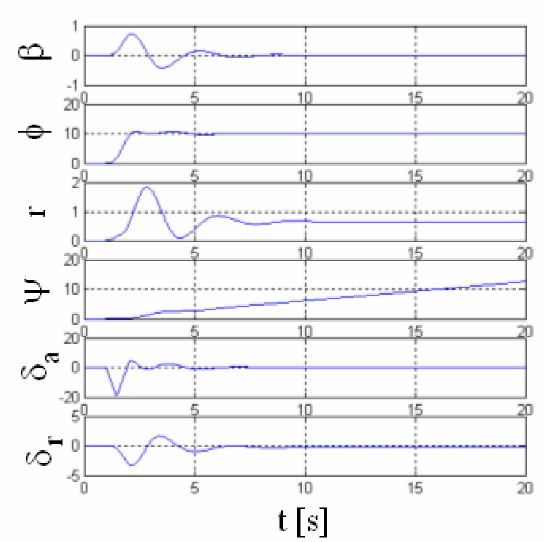

Figure 7. Coordinated turn. Classical approach, flight condition I.

In the figure above and on the many others that will follow, all variables are measured in degrees, with the exception of the yaw rate, r, which is measured in degrees/second.

The roll angle followed the command very well and the sideslip was brought to zero shortly after the roll command.

3.2.2. Heading angle control

Since a roll angle different than zero will cause the heading angle to change, a good idea to control ψ is to force its error to generate a reference in the roll angle that acts in the direction of canceling the error. The only thing one needs to change is to include a PI controller (with gains K_{prop2} and K_{Int2}) that has as input the error between the heading reference and the actual aircraft heading and as output the commanded roll angle reference.

The optimization index was made equal to $\int_0^T \left(\left|\beta_{ref} - \beta\right| + \left|\psi_{ref} - \psi\right| + \delta_{aileron}\right)dt$ and the optimum gains calculated for a command of five degrees in the heading angle while keeping the sideslip reference at zero.

Table 3 lists the results.

Table 3. Proportional-integral controller gains for heading control

Flight Condition	K_{prop2}	K_{Int2}
I	1.41	1.17
II	2.92	1.18

III	5	2.25
IV	4	0.66

3.3. Outer loop design (THCS approach)

As in the previous items, the THCS feedback gains were calculated by using optimization routines. The optimization index considered was $\int_0^T \left(\left|\beta_{ref} - \beta\right| + \left|\psi_{ref} - \psi\right|\right)dt$ and the command given was five degrees in the sideslip angle and minus five degrees in the heading angle (from an initial condition of both being zero), a maneuver named “crab”.

According to Figure, four gains should be calculated for the THCS. However, it can be demonstrated that, in order to guarantee a good command coordination between ailerons and rudder, *Kpsi* can be made equal to *Kbeta* and *KRI* equal to *KYI*. This reduces the workload in 50%, what is a tremendous gain.

Table 4 shows the optimum feedback gains for the four flight conditions considered.

Table 4. Feedback gains for THCS

Flight Condition	K_{psi} / K_{beta}	*KRI/KYI*
I	4.6	1.5
II	6	1.72
III	5.65	1.3
IV	5	1.28

It is notorious how the feedback gains varied only a little for the four flight conditions. This represents a considerable advantage since the gain scheduling algorithms can be simplified. Fig. 8 below illustrates the crab maneuver for flight condition II.

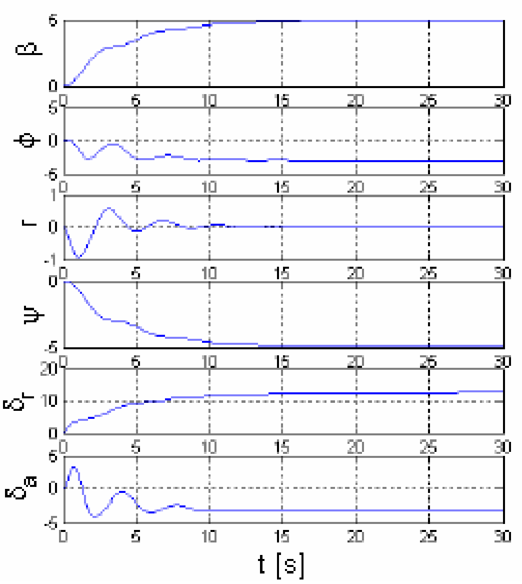

Figure 8. Crab maneuver for flight condition II

3.4. Comparison between the THCS and classical approaches

Two scenarios were considered for this comparison: a disturbance of five degrees in the sideslip angle (which will allow comparing the yaw damper functionality) and the realization of a coordinated turn where the heading reference was being incremented at a rate of sixty degrees/s (a complete turn in six minutes).

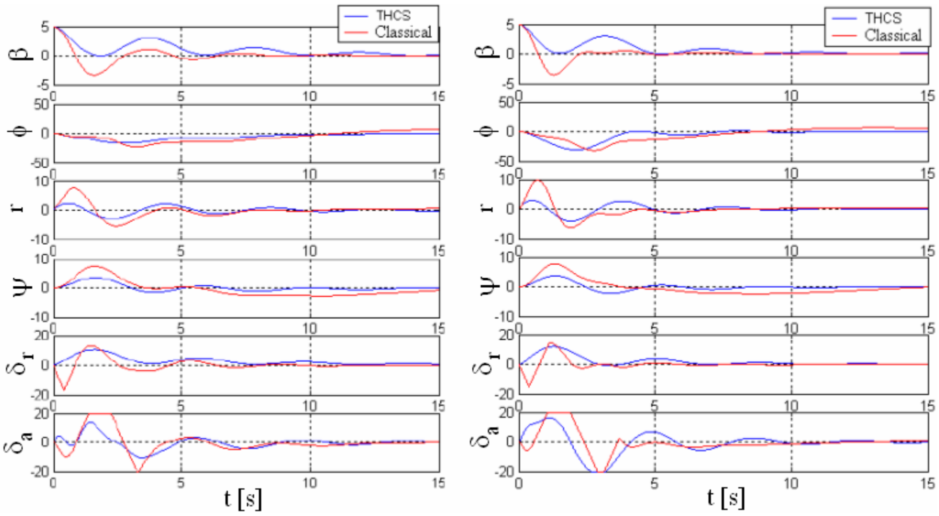

Figure 9. Yaw damper functionality. Flight conditions I (left) and II (right).

No doubt the classical approach yielded a shorter response time in what concerns sideslip elimination. However, such speed was not for free. The actuators were much more demanded (showing signals of saturation) in the classical approach. For the THCS, they remained far from the saturation point.

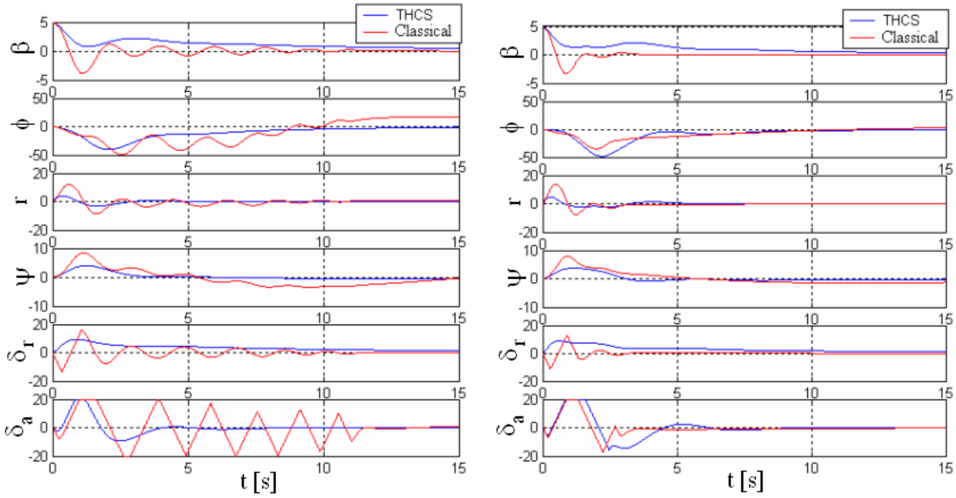

Figure 10. Yaw damper functionality. Flight conditions III (left) and IV (right).

For flight condition III, the THCS was clearly superior, despite of its much bigger stabilization time. The classical approach almost led to a system instability. Another point: the roll angle was driven to zero by the THCS.

For flight condition IV, the two systems behaved in a very similar way.

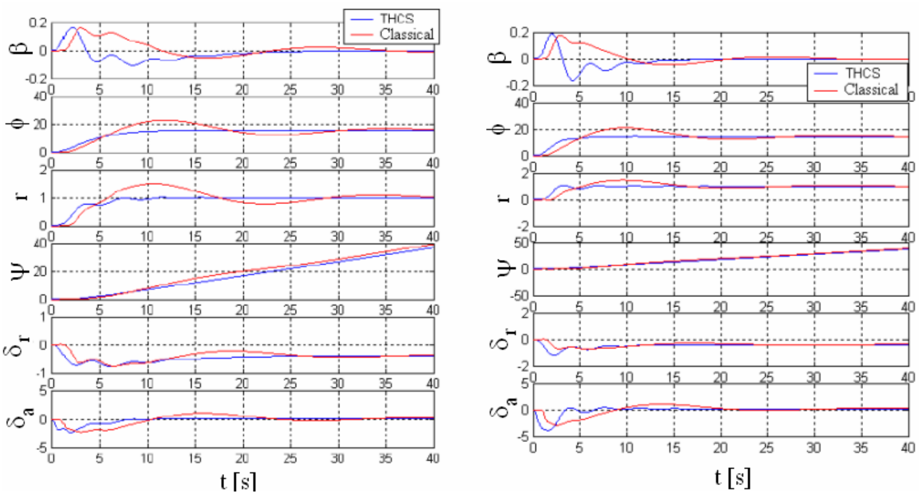

Figure 11. Turn coordination. Flight conditions I (left) and II (right).

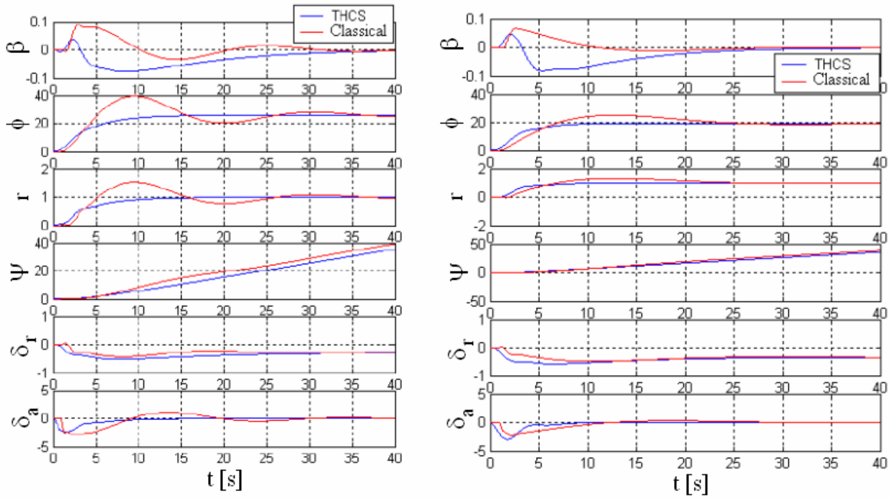

Figure 12. Turn coordination. Flight conditions III (left) and IV (right).

In what concerns the execution of coordinated turns, the THCS showed better results for the roll angle and yaw rate stabilization time.

3.5. Conclusions

Based on the obtained results it can be said that, in what concerns the dynamic response of the controlled system, the two approaches are very effective. There were some occasions where the THCS was superior and others where the classical methods led to better results.

Although this work gives good insight in the THCS capabilities, a more exhaustive investigation must be carried out considering several different optimization indexes, maneuvers and optimization algorithms (especially those who can guarantee that a global minimum will be attained).

It should be pointed out, however, that the THCS seems to be a more simple approach since there are fewer feedback gains to calculate and there is less variation in those gains as one takes different points in the aircraft flight envelope. Such considerable advantages should be taken into account when developing new flight control systems.

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