NUMERICAL STUDY OF A PTT VISCOELASTIC FLUID FLOW THROUGH A CONCENTRIC ANNULAR

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Abstract. *In the oil industry is usual to circulate non-Newtonian fluids through annular geometries during the well drilling operation. The prediction of the stress and velocity fields inside the gap between the drill string and the rock formation is essential for the success of the operation. Among the rheological models for non-Newtonian fluids, the viscoelastic models allow a more realistic prediction of the physical phenomena related to the fluid flow. This work intends to describe the flow pattern of the drilling mud through an annular gap. The numerical study of a laminar and steady state non-Newtonian flow through a concentric annular is presented. The drilling mud is modeled as a viscoelastic fluid, based on the non-linear rheological model developed by Phan-Thien-Tanner (PTT). The mathematical formulation and the methodology for the numerical implementation is presented. The discretization of the PTT differential equations was developed by the Finite Volume Method and the numerical simulation was performed in the commercial software PHOENICS-CFD. The velocity field, the viscoelastic properties and the normal and shear stress fields are obtained and discussed in this work. Thereafter is evaluated the influence of the rheological parameters and the annular size in the flow behavior. The results were analyzed and validated with the analytical solution for the fully developed flow.*

Keywords*: axial annular flow, viscoelastic fluid, numerical simulation, Phan-Thien-Tanner constitutive equation.*

1. INTRODUCTION

The oil extraction in deep waters has become necessary because the inevitable scarcity of the petroleum, stimulating studies to solve new well drilling problems and creating new challenges for engineers. During the drilling operation, fluid is pumped through the column until the bottom well and returns to the surface through the annular gap between the drilling column and the wall of the well. Among the functions of the drilling fluid are the pressure control, the carrying of the gravel and the refrigeration and lubrication of the drill and the column. Machado (2002) discusses the importance of velocity and pressure control inside the annular gap. High velocities imply in possibilities of erosion, while low velocities can be insufficient to carry the gravel. Extreme pressures can break the rock formation. On the other hand, insufficient pressures make the pumping difficult and can allow fluid flow from the formation to the well.

The determination of the stress and velocity fields in the annular during the drilling operation involves many variables, such as geometric factors of the well and the fluid's rheological properties. However, the problem can be simplified approaching the well elliptical geometry by a perfect circle and neglecting the annular eccentricity and the column movements. As a result, the problem becomes the study of axial flow through a concentric annular.

Currently the drilling fluids are available in different formulations. In recent years, the study of fluids with viscoelastic properties has been of particular interest, because the possibility of formulate them to help the kick control.

Literature is still scarce on annular flow of viscoelastic fluids. Escudier et al. (2002) state that there are few works that determine experimentally the velocity distribution for non-Newtonian laminar flows through annuli. Besides, many of these studies include the influence of the inner cylinder rotation.

The first studies found in literature on the annular axial flow are analytical, experimental and numerical solutions for the simplest non-Newtonian fluids models, namely, Power-Law and Bingham Plastic. The most recent works explore viscoelastic models. These studies seek a more detailed modeling of the physical phenomena involved in this kind of flow. Analytical solution of viscoelastic fluid flow in simple geometries using the Giesekus' model (Giesekus, 1982) and Phan-Thien-Tanner's classical model (Phan-Thien and Tanner,1977) can be derived.

Mostafaiyan et al. (2004) used the Giesekus' model to determine an analytical solution for the velocity profile in the fully developed axial flow through a concentric annular. They observed the influences of the fluid proprieties and the annular aspect ratio in the solution. The solution obtained allows evaluating the normal stress and the radial pressure distribution.

Pinho and Oliveira (2000) performed a similar analyze to that one done by Mostafaiyan et al. (2004) but they used the Phan-Thien-Tanner's viscoelastic model (PTT model). The results are presented exploring the influence of geometric, dynamic and constitutive non-dimensional parameters. The authors justified the use of the PTT model due to its capacity of simulating adequately the material functions of some casting polymers.

Brondani et. al. (2006) presented a numerical study of the fully developed flow of a PTT fluid through a concentric annular. The simulations were performed in a commercial program and the PTT model was implemented using the fluid apparent viscosity. The results were validated using the solution of Pinho and Oliveira (2000).

Quinzani et al. (1995) presented an experimental study to evaluate the capacity of several differential constitutive models can be found. They conclude that the Phan-Thien-Tanner's model is the one that better characterizes the extensional viscosity. The PTT model is considered able to represent the shear and the elongational behavior for diverse polymeric solutions, as discussed by Peter et al. (1999).

Tanner (2000) also studied the performance of some viscoelastic models for several classes of flows. Among the studied models, the PTT model is considered the best one for simulating shear flows. Therefore, it is recommended for the study of the axial annular flow.

There are many advantages for using CDF commercial softwares, among them: a robust and optimized solver, facilities to assemble complex geometries, different kinds of meshes, many convective schemes and the possibility to choose between staggered and non-staggered grids. Few CFD softwares are able to solve viscoelastic problems, probably because the additional complexity to resolve the new set of equations. Therefore, in this paper a general structure to support diverse differential viscoelastic constitutive equations in the CFD commercial software PHOENICS-CFD (Spalding, 1994) is implemented. The problem investigated is the viscoelastic flow of a PTT fluid through a concentric annular. The numerical results obtained are compared with the analytical solution given by Pinho and Oliveira (2000).

2. MATHEMATICAL FORMULATION

2.1. Geometry

Figure 1 depicts the geometry of the concentric annular pipe and the cylindrical coordinate system utilized. L is the total length of the annular pipe, r_i is the inner radius and r_o the outer radius.

The non-dimensional annular radius is defined by:

$$\bar{r} = \frac{r - r_i}{r_o - r_i} \tag{1}$$

The parameter $k = r_i / r_o$ represents the ratio between the inner and the outer radii.

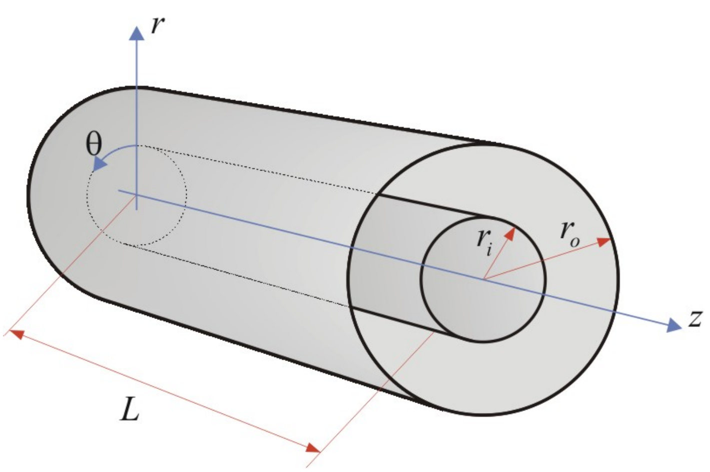

Figure 1. Axial annular geometry and the cylindrical coordinate system.

2.2. Governing equations

For the solution of the present problem, the following hypotheses are assumed: steady state, laminar and bidimensional flow with a constant fluid density. Besides, the consideration of symmetrical stress tensor is also adopted. Thus, the momentum equation in the θ direction became null. The continuity equation and the r and z components of the momentum equation result in (Bird, 1987):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rV_r\right)+\frac{\partial}{\partial z}V_z=0 \tag{2}$$

$$\rho\left(V_r\frac{\partial V_r}{\partial r}+V_z\frac{\partial V_r}{\partial z}\right)=\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)+\frac{\partial}{\partial z}\tau_{zr}-\frac{\tau_{\theta\theta}}{r}\right]-\frac{\partial p}{\partial r} \tag{3}$$

$$\rho\left(V_r\frac{\partial V_z}{\partial r}+V_z\frac{\partial V_z}{\partial z}\right)=\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\frac{\partial}{\partial z}\tau_{zz}\right]-\frac{\partial p}{\partial z} \tag{4}$$

where ρ is the fluid density, V_j is the velocity component in the j direction and p is the thermodynamic pressure. The stress field is represented by τ_{ij}. Thus, there are a system with three equations (Eq.(2), Eq.(3) and Eq.(4)) and seven variables ($p,V_r,V_z,\tau_{rz},\tau_{rr},\tau_{zz},\tau_{\theta\theta}$). For solving this problem, the stress and velocity fields must be linked to the four constitutive equations from the PTT fluid model.

2.3. Constitutive equation – PTT model

The PTT viscoelastic model is given by the following equation in the tensor form (Tanner, 2000):

$$\overset{\triangledown}{\boldsymbol{\tau}}+\xi\left(\mathbf{D}.\boldsymbol{\tau}-\boldsymbol{\tau}.\mathbf{D}^T\right)+\frac{Y}{\lambda}.\boldsymbol{\tau}=2G\mathbf{D} \tag{5}$$

where the Oldroyd's upper convective derivative $\overset{\triangledown}{\boldsymbol{\tau}}$, is defined by:

$$\overset{\triangledown}{\boldsymbol{\tau}}=\frac{D\mathbf{V}}{Dt}-\boldsymbol{\tau}.\nabla\mathbf{V}-\nabla\mathbf{V}^T.\boldsymbol{\tau} \tag{6}$$

where $\mathbf{V}$ represents the velocity matrix, $\mathbf{V}^T$ is the transpose of the velocity matrix and $D(\mathbf{V})/Dt$ is the material derivative of the velocity matrix.

Analyzing Eq.(5), the first term represents the stress tensor transport and the transient part of the flow. In the second term, the slipping between the fluid polymeric chains is computed. The third term includes the elastic effects. Finally, the term on the right side of the equality represents the diffusive effects. $\boldsymbol{\tau}$ is the stress tensor, $\mathbf{D}$ the deformation-rate tensor, λ the fluid relaxation time and G the relaxation module. The parameter ξ controls the amount of movements between the fluid polymeric chains. For $\xi=0$ the model is named PTT Affine and the slipping between the polymeric chains are neglected. The function Y depends on the rate of creation and destruction of the links between the chains. This function can assume a linear or an exponential form. In the present work, the linear form will be used, given by:

$$Y=\left(1+\frac{\varepsilon}{G}\mathrm{tr}(\boldsymbol{\tau})\right) \tag{7}$$

where $\mathrm{tr}(\boldsymbol{\tau})$ is the trace of the stress tensor and ε the parameter that governs the extensional fluid response.

To simplify the mathematical analysis and the numerical implementation, the slipping effect between the polymeric chains will be neglected ($\xi=0$). Thus, the PTT Affine model is considered:

$$\overset{\nabla}{\boldsymbol{\tau}} + \frac{Y}{\lambda}.\boldsymbol{\tau} = 2G\mathbf{D} \tag{8}$$

The apparent viscosity for the null shear rate (η_0) is defined as:

$$\eta_0 = \lambda G \tag{9}$$

The shear viscosity is a shear rate function and it is calculated by:

$$\eta(\dot{\gamma}) = \frac{\tau_{rz}}{\dot{\gamma}} \tag{10}$$

where $\dot{\gamma}$ is the fluid shear rate.

From the three normal stresses, it can be defined two normal stresses differences. For the axial annular flow, the first (N_1) and the second (N_2) normal stresses differences are:

$$N_1 = \tau_{zz} - \tau_{rr} \tag{11}$$

$$N_2 = \tau_{rr} - \tau_{\theta\theta} \tag{12}$$

These functions depend on the shear rate and can define two normal stresses coefficients. The first (ψ_1) and the second (ψ_2) normal stresses coefficients are defined by:

$$\psi_1(\dot{\gamma}) = \frac{N_1}{\dot{\gamma}^2} \tag{13}$$

$$\psi_2(\dot{\gamma}) = \frac{N_2}{\dot{\gamma}^2} \tag{14}$$

The normal stresses coefficients (ψ_1 and ψ_2) and the shear viscosity (η) are material functions that represent the fluid viscometric properties.

The Reynolds number represents the relation between the inertia and viscous strengths. For the annular axial flow, Re is given by:

$$\mathrm{Re} = \frac{2\rho U \delta}{\eta_0} \tag{15}$$

where $\delta = r_o - r_i$ is the annular gap and U is the flow average velocity.

The Deborah number is the ratio between the elastic and viscous strengths. It is defined by:

$$De = \frac{\lambda U}{\delta} \tag{16}$$

where δ/U represents the flow characteristic time and λ the fluid relaxation time.

2.4. The conservation equations in the general form

The governing equations are written in the following generic conservative form:

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla.(\rho u \phi) = \nabla.(\Gamma_\phi \nabla \phi) + S_\phi + P_\phi \tag{17}$$

where ϕ is a scalar or vectorial variable, Γ_ϕ is the diffusive coefficient and S_ϕ and P_ϕ are the source terms. Γ_ϕ, S_ϕ and P_ϕ assumes different forms depending on the variable ϕ. The first term of the general differential equation, Eq. (17), represents the transient component of the phenomenon, after that appears the convective part, the diffusive part and the source terms. The term P_ϕ represents the source term dependent on the pressure and S_ϕ includes the others source terms. Table 1 shows the coefficients of the govern equations in the general form.

Table 1. Coefficients of the governing equations in the general form for the bidimensional laminar flow of a PTT viscoelastic fluid in cylindrical coordinates.

Equation	ϕ	Γ_ϕ	S_ϕ	P_ϕ
Mass conservation [Eq.(2)]	1	0	0	0
Momentum conservation in r direction [Eq.(3)]	V_r	0	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)+\frac{\partial}{\partial z}\tau_{rz}-\frac{\tau_{\theta\theta}}{r}$	$-\frac{\partial p}{\partial r}$
Momentum conservation in z direction [Eq.(4)]	V_z	0	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\frac{\partial}{\partial z}\tau_{zz}$	$-\frac{\partial p}{\partial z}$
PTT equation, rr	τ_{rr}/ρ	0	$2G\frac{\partial V_r}{\partial r}-\frac{Y}{\lambda}\tau_{rr}+2\left(\tau_{rr}\frac{\partial V_r}{\partial r}+\tau_{rz}\frac{\partial V_r}{\partial z}\right)$	0
PTT equation, $\theta\theta$	$\tau_{\theta\theta}/\rho$	0	$2G\frac{V_r}{r}-\frac{Y}{\lambda}\tau_{\theta\theta}+2\tau_{\theta\theta}\frac{V_r}{r}$	0
PTT equation, zz	τ_{zz}/ρ	0	$2G\frac{\partial V_z}{\partial z}-\frac{Y}{\lambda}\tau_{zz}+2\left(\tau_{zr}\frac{\partial V_z}{\partial r}+\tau_{zz}\frac{\partial V_z}{\partial z}\right)$	0
PTT equation, rz	τ_{rz}/ρ	0	$G\left(\frac{\partial V_z}{\partial r}+\frac{\partial V_r}{\partial z}\right)-\frac{Y}{\lambda}\tau_{rz}+\left(\tau_{zz}\frac{\partial V_r}{\partial z}+\tau_{rr}\frac{\partial V_z}{\partial r}-\frac{V_r}{r}\tau_{rz}\right)$	0

In Table 1 can be observed that not all the diffusive flows are governed by the gradient of the variable. Thus, whenever a term cannot be adjusted to the general differential equation in the form of divergent and gradient, it must be explicit as part of the source term. Patankar (1980) states that sometimes is convenient to make $\Gamma_\phi = 0$, and include all the diffusive part in the source term.

3. NUMERICAL METHOD

The Finite Volume Method developed by Patankar (1980) was used for the discretization of the differential equations that govern the problem, shown in Table 1. The elliptical formulation with staggered grid is used. The computational mesh is structured and orthogonal. The Hybrid scheme is used for the convective terms interpolation. The resultant system of algebraic equations is solved with the commercial software PHOENICS-CFD, with the SIMPLEST solution algorithm (Spalding, 1994).

The momentum equations in the PHOENICS-CFD are written for Newtonian fluids. Therefore, the equations are not written in the form of tensors, but in form of velocity derivatives of second order. For viscoelastic fluids, this form is not suitable and the momentum equations must be rewritten as function of the stress field. Moreover, is necessary to implement the constitutive equations linking the velocity and stress fields.

The computational mesh used was chosen after a mesh test performed to verify the amount of elements necessary to make the solution independent of the mesh.

The numerical boundary conditions are: no-slip at the pipe walls, a constant velocity profile at the inlet and a reference pressure at the outlet.

4. RESULTS

In this section we present the numerical results obtained. To validate de methodology, the results are compared with the analytical solution given by Pinho and Oliveira (2000). The results show the performance of the algorithm for some sizes of annular and several types of fluid.

The values of velocity are made non-dimensional by the flow average velocity ($\overline{V_z} = V_z/U$) and the stresses in the form $\overline{\tau_{ij}} = \tau_{ij}/\left(\eta_0 U/\delta\right)$.

4.1. The effect of annular aspect ratio (k)

Analyzing the profiles presented in Figs. (2) and (3), it is clear that the numerical results report to the analytical solution for any size of the annular space. The small differences between numerical and analytical solutions could be eliminated refining the mesh in the radial direction.

The effect of the annular aspect ratio in the velocity profile is shown in Fig. (2). The reduction of k, increasing the annular curvature, causes the shift of the axial velocity profile toward the inner radius direction and increases the maximum velocity.

Figure (3) illustrates the influence of k in the stresses profiles, for constants values of De and ε. At the inner wall ($\bar{r}=0$), the shear ($\overline{\tau_{rz}}$) and the normal ($\overline{\tau_{zz}}$) stresses increase when k decreases. However at the outer wall ($\bar{r}=1$) the stresses rise in magnitude when k increases. At the walls it is also observed that the normal stress is greater than the shear stress for any aspect ratio of the annular.

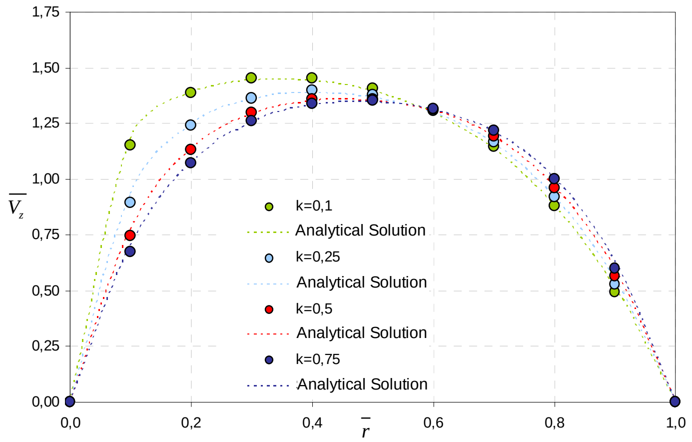

Figure 2. Transversal velocity profile ($\overline{V_z}$) for different annular aspect ratios ($k=r_i/r_o$), $De=1$ and $\varepsilon=0,25$.

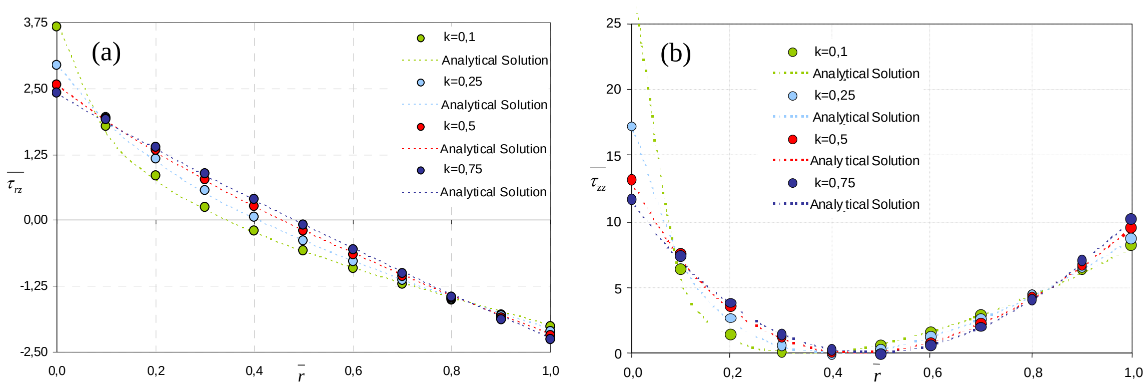

Figure 3. Stress profiles for different annular aspect ratios ($k=r_i/r_o$), $De=1$ and $\varepsilon=0,25$. (a) Shear stress ($\overline{\tau_{rz}}$) and (b) normal stress ($\overline{\tau_{zz}}$).

The behavior of the fluid rheologycal properties (ψ_1 and η) as a function of the shear rate can be seen in Fig. (4). Together with the curves of ψ_1 and η are displayed the stress profiles. In this case, the stresses are made non-dimensional by its value at the inner wall. We can see that the stresses decrease with the shear rate reduction, while ψ_1 and η increase. This behavior is typical of a shear-thinning fluid, for which viscosity diminishes with the increase of the shear rate. When $\dot{\gamma}\to 0$, η tends to the value of the zero-shear viscosity (η_0).

Figure (5) shows that ψ_1 and η depends only on the shear rate for any annular aspect ratio. One can see that for high shear rate ψ_1 and η tend to assume a constant value.

In Figs. (4) and (5), we can observe a discrepancy in ψ_1 results at low shear rates. Considering that ψ_1 is given by $N_1/\dot{\gamma}^2$, it is necessary a very refined mesh to allow accurate determination of $\dot{\gamma}$, in order to avoid numerical errors in the calculation of ψ_1. When $\dot{\gamma}$ comes closer to zero, its dependence on the mesh is still more critical. In this case, small errors in the approach of $\dot{\gamma}$ cause great deviations in the value of ψ_1.

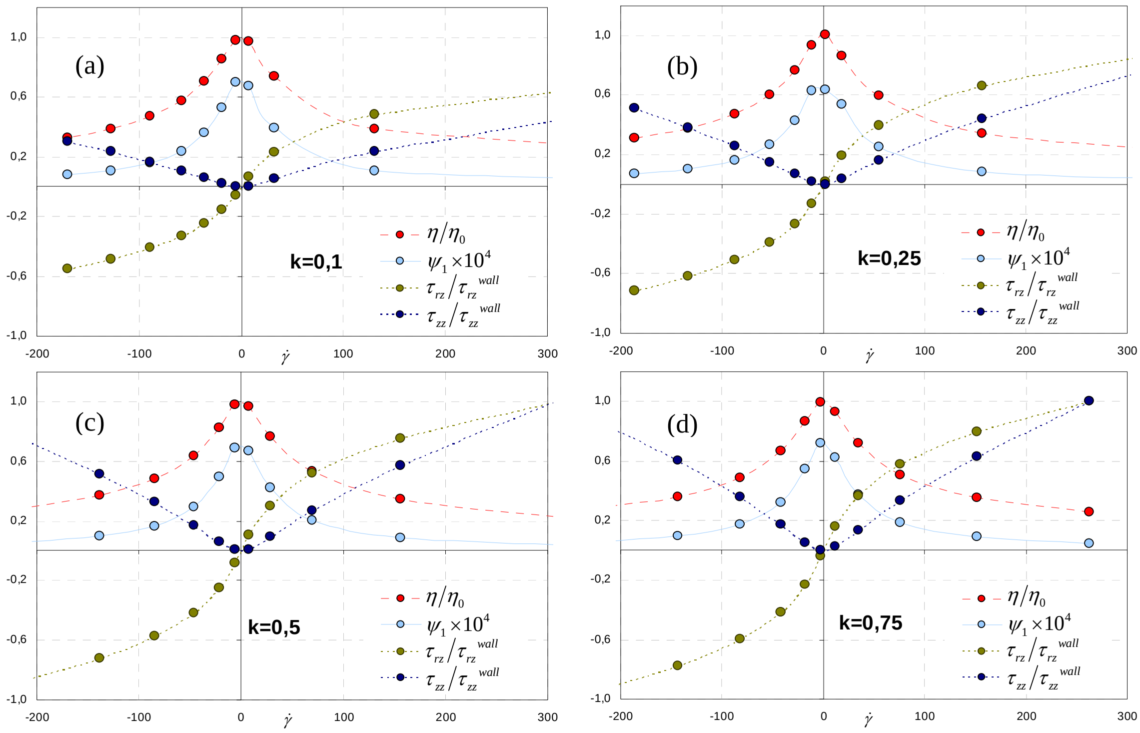

Figure 4. The behavior of the rheologycal properties (ψ_1 and η) and of the stresses (τ_{rz} and τ_{zz}) as a function of the shear rate for different annular aspect ratios: a) $k = 0,1$, b) $k = 0,25$, c) $k = 0,5$ e d) $k = 0,75$.

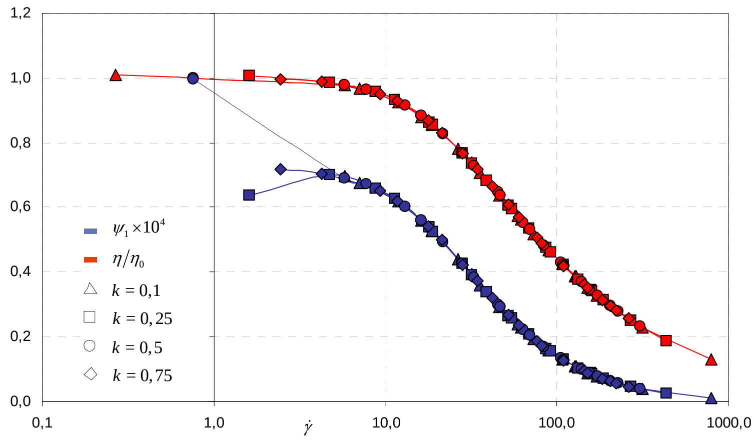

Figure 5. The ψ_1 and η variation with the shear rate ($\dot{\gamma}$), for $De = 1$ and $\varepsilon = 0,25$.

4.2. The effect of Deborah number (De) and of extensional parameter (ε)

The influence of the fluid properties on the transversal velocity profile is given in Fig (6). Increasing values of De and ε the velocity profiles become flatter and the gradients near the walls enlarge. This behavior indicates the shear-thinning effect and the elastic characteristics of the fluid.

In the stress profiles, it is evident that either shear stresses (Fig. 7) or normal stresses (Fig. 8) are greater at the inner wall than at the outer wall. This behavior is a result of the higher gradients in the inner region of the annular, as observed when the annular aspect ratio is varied (Fig. 3). The shear stress (Fig.7a) decreases when De or ε increases. The fluid behavior moves away from the purely viscous behavior of a Newtonian fluid toward a shear-thinning behavior, where the elastic characteristics become dominant.

When only the extensional parameter (ε) varies the normal stress decreases with the increase of ε (Fig. 8b). However, for variation of De (Fig. 8a) this relation is not monotonic. $\overline{\tau_{zz}}$ grows up for $0 \le De \le 1$ and then decreases. Pinho and Oliveira (2000) justify this behavior due to the shear-thinning effect in the wall. A monotonic relation between the normal stress and the Deborah number can be obtained whether the stresses are scaled with the inner wall shear stress value.

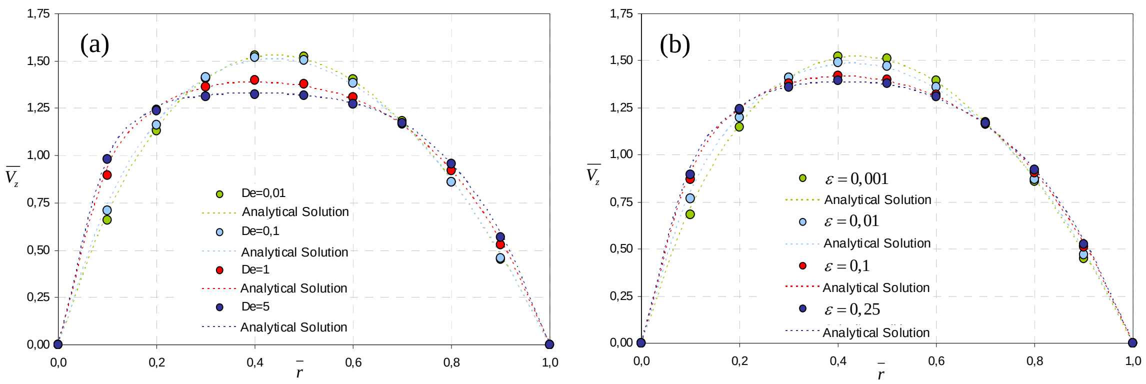

Figure 6. Transversal velocity profile ($\overline{V_z}$) for: (a) different De, $\varepsilon = 0,25$ and $k = 0,25$, (b) various ε, $De = 1$ and $k = 0,25$.

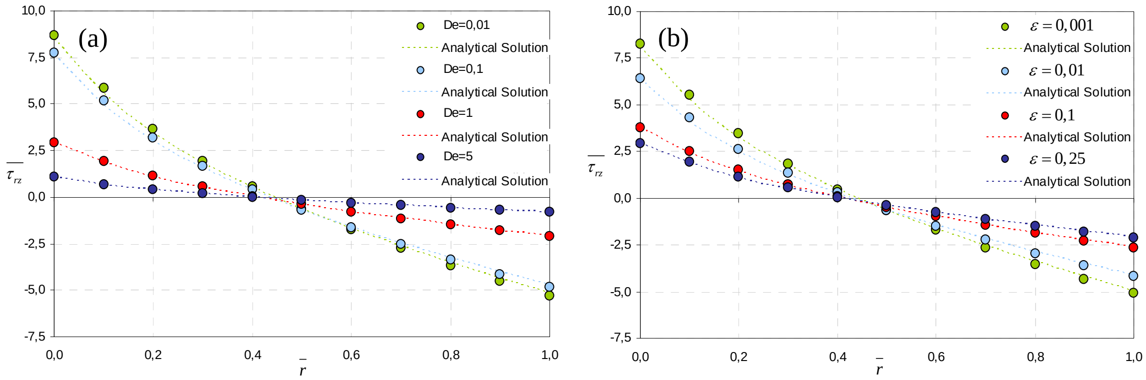

Figure 7. Shear stress profiles ($\overline{\tau_{rz}}$) for (a) different De, $\varepsilon = 0,25$ and $\varepsilon = 0,25$, (b) various ε, $De = 1$ and $k = 0,25$.

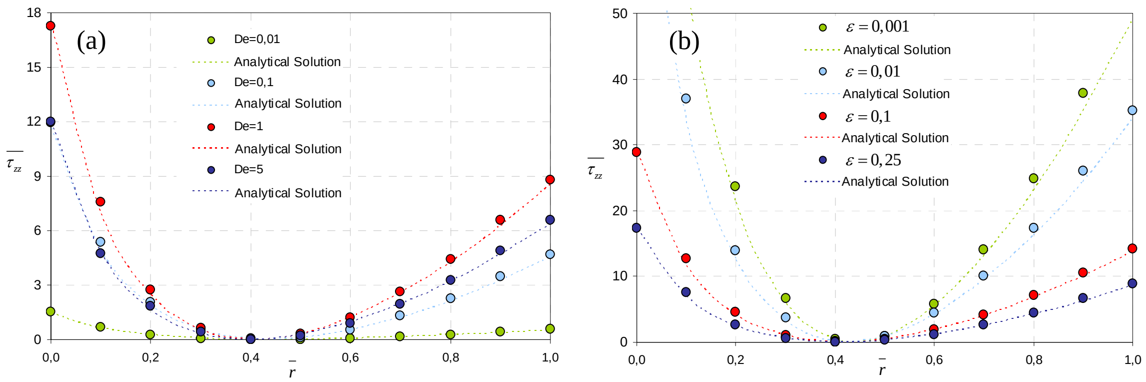

Figure 8. Normal stress profiles ($\overline{\tau_{zz}}$) for (a) different De, $\varepsilon = 0,25$ and $\varepsilon = 0,25$, (b) various ε, $De = 1$ and $k = 0,25$.

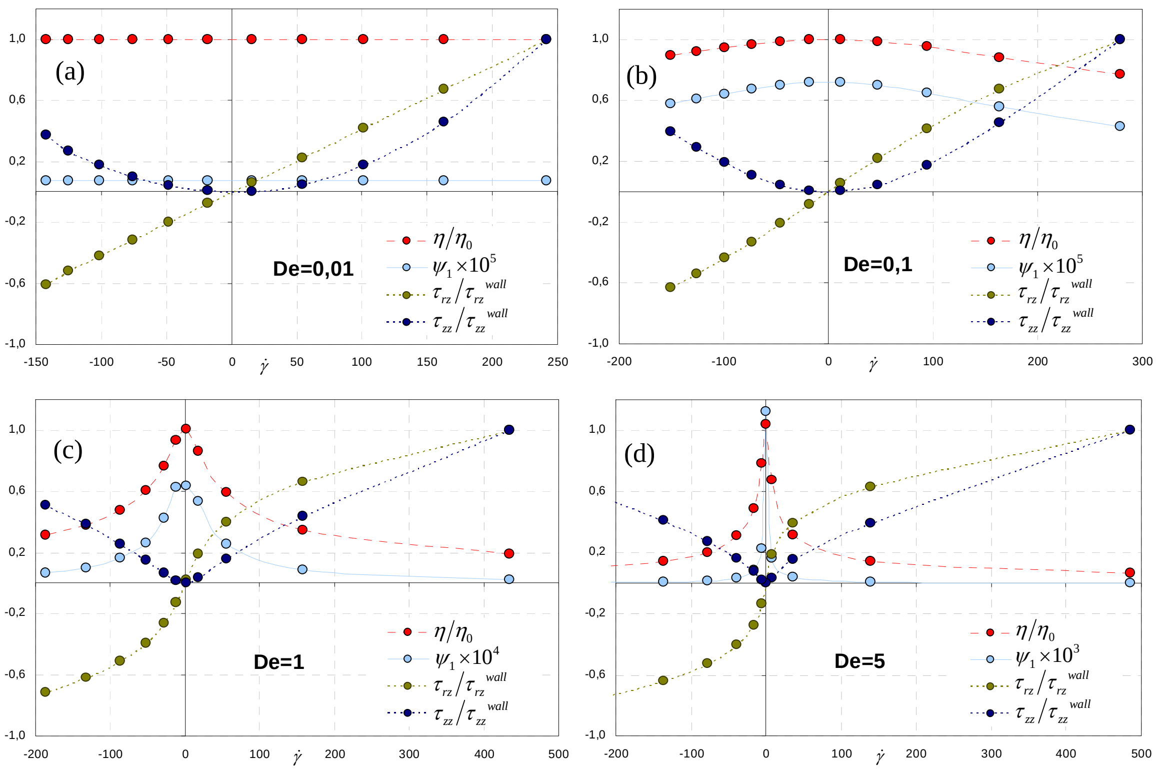

Figure 9. The dependence of the rheologycal properties (ψ_1 and η) and of the stresses (τ_{rz} and τ_{zz}) on the shear rate for different Deborah number: a) $De = 0{,}01$, b) $De = 0{,}1$, c) $De = 1$ e d) $De = 5$.

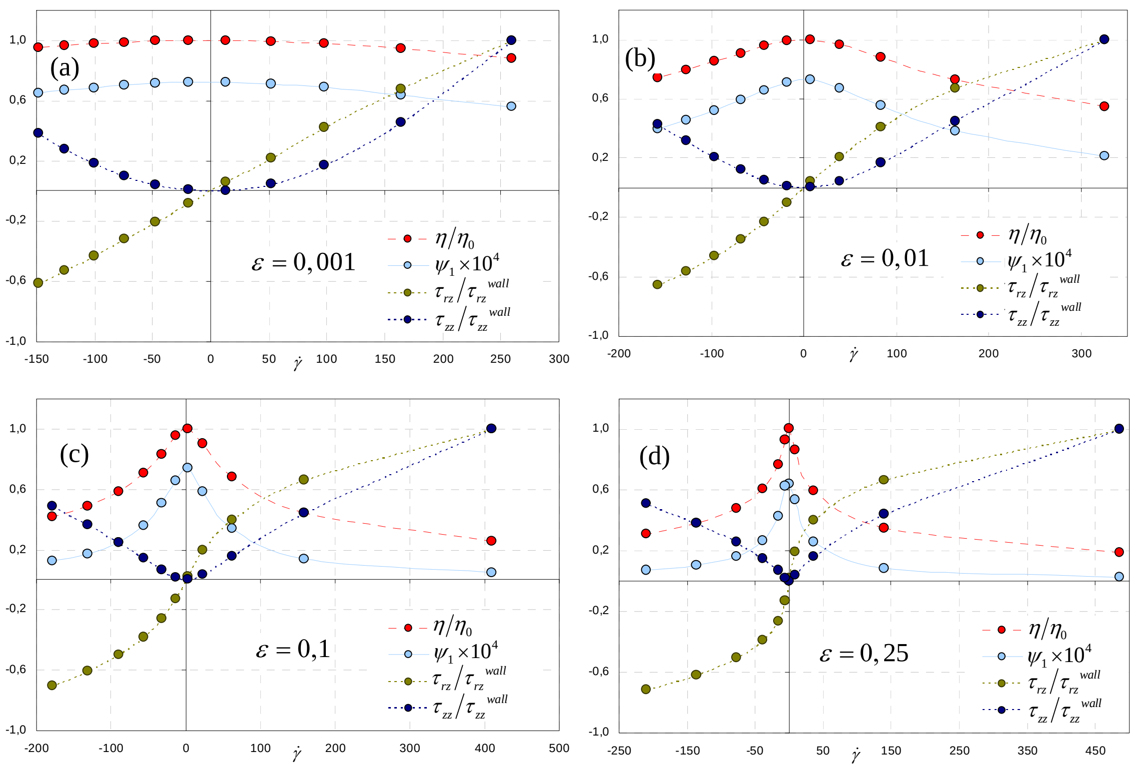

Figure 10. The dependence of the rheologycal properties (ψ_1 and η) and of the stresses (τ_{rz} and τ_{zz}) on the shear rate for different extensional parameters: a) $\varepsilon = 0{,}001$, b) $\varepsilon = 0{,}01$, c) $\varepsilon = 0{,}1$ e d) $\varepsilon = 0{,}25$.

The dependence of the shear viscosity and the first normal-stress difference coefficient on the shear rate is given in Figs. (9) and (10). Together with material functions curves are plotted the stress profiles. Becoming the fluid elasticity greater, by raising the values of De and ε, the higher the value of $\dot{\gamma}$ the smaller the fluid flow resistance. This behavior is confirmed by the quick decrease of ψ_1 and η with the increase of $\dot{\gamma}$. It is also observed that the curves ψ_1 vs. $\dot{\gamma}$ and η vs. $\dot{\gamma}$ have similar forms for De and ε constants.

The coherence of the results obtained shows that the developed structure is valid independently of the fluid properties. The small differences observed when the numerical results are compared with the analytical solution are eliminated with a more refined mesh.

Numerical converge control is always critical during numerical simulations of viscoelastic flows. Since the differential equations system that governs the problem is nonlinear and coupled, the use of small under-relaxation factors is essential to assure numerical stability.

5. CONCLUSIONS

The creation of a support structure for the differential constitutive equations in the commercial software PHOENICS-CFD was discussed in this work. The Phan-Thien-Tanner viscoelastic differential model was implemented and solved for axial flow through a concentric annular. The assessment and the numerical validation was carried out through the solution and discussion of the ε, De and k values upon the laminar fluid flow.

The performance of the numerical results obtained was excellent when compared with the analytical solution available. The convergence control was the main difficult found during the numerical solution, mainly because the strong non-linear coupling between the governing equations. Detailed attention should be given to this point.

The structure developed allows other differential constitutive equations to be easily implemented in the PHOENICS-CFD, making possible to utilize all the advantages of a CFD commercial software.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE