STRUCTURAL-ACOUSTIC SENSITIVITY ANALYSIS OF CLOSED SYSTEM UNDER LOCAL STRUCTURAL DEFECTS

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Abstract. *The behavior of coupled fluid-structural systems can be modified due to presence of located structural defects. The study of these modifications allows the engineers, using numerical methods of simulation, to foresee the new state of the system from the evaluation of structural and acoustics responses. The sensitivity analysis of the acoustic and structural response with respect to structural geometry modification allows to verify the viability of the application of present techniques in fault detention, noise control and optimization procedures. This work presents a formulation for structural-acoustic sensitivity analysis of an aircraft fuselage model under local structural defects. An efficient sensitivity analysis technique of acoustic-structural response (Sensitivity of the direct frequency response) with respect to structural modification is implemented and its precision is tested comparing with results obtained by finite difference technique. This formulation describes the dependence of acoustic variables (sound pressure, fluid speed), with respect to structure geometry modification (thickness, length, height). The finite element method is used to acoustic-structural simulation, with nonsymmetrical formulation adopting structural displacements and fluid pressure as variables. Iterative strategies are tested to solve the coupled problem. A model of aircraft fuselage will be proposed and its direct frequency response and sensitivities under structural modifications will be present.*

Keywords: *Fluid-structure interaction, Sensitivity analysis, Structural damage, Finite element method.*

1. INTRODUCTION

Acoustic-structural coupling has been an important research topic for many years. In particular, for aircraft structures, the interior sound radiation problems have been an active research area [3]. The vibro-acoustic behavior of cabin-fuselage systems can be modified due to presence of located structural defects. The study of these modifications allows the engineers, using numerical methods of simulation [1][4], to find the new state of the system. In general, at the low-frequency range (below approximately 400 Hz) high interior acoustic noise can be generated and transmitted by structural-borne paths. At these conditions, the structural modifications caused by damages in the structure can modify the dynamic behavior of acoustic-structural coupled system. In this work, the sensitivity analysis is used to estimate the influence of the structural damages in the behavior of the whole system. A partitioned solution technique is implemented to solve the problem in the frequency domain [4] [7]. The present techniques can be applied in damage detention, noise control and optimization procedures. In this article, the numerical results in a simplified aircraft fuselage are presented.

2. STRUCTURAL-ACOUSTIC SIMULATION

2.1 Structural domain

We consider a structure domain Ω_s, fixed on Γ_1, subject to surface harmonics forces $F_i(\mathbf{x},\omega)$ on Γ_2, showed in Figure 1. Assuming harmonics vibrations, u_i satisfies the boundary value problem:

$$\begin{aligned} \sigma_{ij,j}(u_i) - \rho\omega^2 u_i = 0 \quad & in \quad \Omega_s \\ \sigma_{ij,j}(u_i)n_j = F_i \quad on \quad \Gamma_2 \quad ; \quad & u_i = g_{si} \quad on \quad \Gamma_1 \end{aligned} \tag{1}$$

werein g_{si} is a know function, σ_{ij} is Cauchy stress tensor, ρ is structural domain density, n_j indicates exterior normal direction and u_i point out the structural desplacement.

It is assumed that all field variables behave stationary. Hence, it's applied a steady-state approach separating time-dependent variables $F_i(\mathbf{x},t)$ and $u_i(\mathbf{x},t)$ at field point $\mathbf{x}$ into a frequency-dependt spatial part $\bar{F}_i(\mathbf{x},\omega)$ and a function $e^{i\omega t}$ with ω being the circular frequency:

$$F_i(\mathbf{x},t) = \bar{F}_i(\mathbf{x},\omega)e^{i\omega t} \tag{2}$$

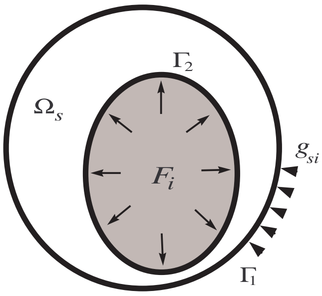

Figure 1. Elastic structure domain

$$u_i(\mathbf{x},t) \quad = \quad \bar{u}_i(\mathbf{x},\omega)e^{i\omega t} \tag{3}$$

By using finite-element discretization for n elements; all components $\bar{u}_i$ of spatial structural displacement vector $\bar{\mathbf{u}}_i$ yields $\bar{\mathbf{u}}(\mathbf{x},\omega) = [\mathbf{N}_s(\mathbf{x})]\{\mathbf{u_s}(\omega)\}$. wherein $[\mathbf{N_s}]$ represents the matrix of nodal interpolation functions and $\{\mathbf{u_s}(\omega)\}$ contains the nodal displacements. Using equaton 1 aproxiamtion, the matrix equations are:

$$[\mathbf{K}_s]\{\mathbf{u}\} - \omega^2[\mathbf{M}_s]\{\mathbf{u}\} \quad = \quad \{\mathbf{f}_s\} \tag{4}$$

where, $\{\mathbf{f}_s\}$ is the nodal equivalent forces vector.

The discretized expressions involved in the previous formulation yield nxn matrices of mass $[\mathbf{M_s}]$ (symmetric, definite positive), of stiffnes $[\mathbf{K_s}]$ (symmetric, semi-definite positive). For details of the finite elements, used in this work, the construction of stiffness and mass matrices of structures modelled by beams, plates or shell, see [8],[2].

2.2 Acoustic domain

Keeping the steady-state approach, the basic diferential equations for the harmonic response of an inviscid imcompressivel fluid (air) occupying a bounded domaind Ω_f, and subject to an prescribed normal displacement u_n on Γ, is given by the Helmholtz equation:

$$\begin{aligned} \nabla^2 p(\mathbf{x},\omega) + k^2 p(\mathbf{x},\omega) &= 0 \qquad \Omega_f \\ \frac{\partial p}{\partial n} &= \omega^2 \rho_f u_n \qquad \Gamma \end{aligned} \tag{5}$$

wherein $k = \omega/c$ is the wave number, c is the speed of sound into Ω_f ρ_f is the fluid density , ω is the circular frequency and $p(\mathbf{x},\omega)$ is the pressure fluid.

By using finite-element discretization for n elements, the pressure is discretized at the fluid domain as: $\bar{p}(\mathbf{x},\omega) = [\mathbf{N}_f(\mathbf{x})]\{\mathbf{p}(\omega)\}$, introducing the row matrix $[\mathbf{N_f}]$ of the interpolation functions and the vector of nodal sound pressures $\{\mathbf{p}(\omega)\}$. The matrix equation gotten from equation 5 is:

$$[\mathbf{K}_f]\{\mathbf{p}(\omega)\} - \omega^2[\mathbf{M}_f]\{\mathbf{p}(\omega)\} = \{\mathbf{f}_f\} \tag{6}$$

wherein $[\mathbf{M_f}]$ and $[\mathbf{K_f}]$ are the fluid volumetric and inercial matrix respectively.

2.3 Coupled structural-acoustic system

We consider a coupled structural-acoustic system. Ω_s and Ω_f are separate by Γ. The resulting from the action of pressure forces exerted by the fluid on the structure is:

$$\sigma_{ij,j}(u)n_j^s = -pn_i^s = -pn_i \tag{7}$$

these forces produce adicional forces in the $\{\mathbf{f}_s\} = [\mathbf{K}_{fs}]\{\mathbf{p}\} + \{\mathbf{f}\}$ and $\{\mathbf{f}_f\} = -\omega^2[\mathbf{M}_{fs}]\{\ddot{\mathbf{u}}\} + \{\mathbf{p}\}$ vector forces in Equations 1 and 5.

$$\begin{aligned} \{\mathbf{f}_s\} &= [\mathbf{K}_{fs}]\{\mathbf{p}\} + \{\mathbf{f}\} \\ \{\mathbf{f}_f\} &= -\omega^2[\mathbf{M}_{fs}]\{\ddot{\mathbf{u}}\} + \{\mathbf{p}\} \end{aligned} \tag{8}$$

The finite element equations for treating a coupled acoustic-structural system are:

$$\begin{bmatrix} [\mathbf{K}_s] - \omega^2[\mathbf{M}_s] & [\mathbf{K}_{fs}] \\ -\omega^2[\mathbf{M}_{sf}] & [\mathbf{K}_f] - \omega^2[\mathbf{M}_f] \end{bmatrix} \begin{Bmatrix} \{\mathbf{u}_s\} \\ \{\mathbf{p}\} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{f}_s\} \\ \{\mathbf{f}_f\} \end{Bmatrix} \tag{9}$$

and the compact form of equation 9 is:

$$[\mathbf{Z}]\{\mathbf{u}\} = \{\mathbf{f}\} \tag{10}$$

Harmonic problem frequency response can solved using direct method [4] by equation 11 or particioned method [4] to reduce computacional cost to matrix equations calculation.

$$\{\mathbf{u}\} = [\mathbf{Z}]^{-\mathbf{1}}\{\mathbf{F}\} \tag{11}$$

2.4 Frequency Response Sensitivity

The objective of the sensitivity analysis is to find the frequency response derivatives (sensitivity) with respect to the structural variable (e_k), like a objective function first step toward, defined in equation 12. Although there are various uses for sensitivity information, the main motivation is to estimate the influence of the structural damages in the behavior of structural-acoustic coupled system.

$$\frac{dI}{de_k} = \frac{\partial I}{\partial e_k} + \frac{\partial I}{\partial\{\mathbf{u}\}_i}\frac{d\{\mathbf{u}\}_i}{de_k} \tag{12}$$

There are a several different methods for sensitivity analysis like finites differences methods and analytical methods. Factors that affect the choice of method include: the ratio of the number of outputs to the number of inputs, the importance of computational efficiency and the amount of human effort that is required in the implementation.

Finite-difference formulate are commonly used to estimate sensitivities. Although these approximations are neither particularly accurate nor computationally efficient, the greatest advantage of this method resides in the ease of implementation. A common estimate for the first derivative is the forward difference which is given by:

$$\frac{d\{\mathbf{u}\}_i}{de_k} \approx \frac{\{\mathbf{u}\}_i(e_k+\delta) - \{\mathbf{u}\}_i(e_k)}{\delta} \tag{13}$$

wherein $\frac{d\{\mathbf{u}\}_i}{de_k}$ is the frequency response sensitivity, e_k is the kth structural variable or defect. In the limiting case $\delta \to 0$ the frequency response will reach the analytical derived sensitivity. This aproximation requires $k+1$ calculatios of $\{\mathbf{u}\}_i$ to gain all sensitivities.

Analytic approaches such as adjoint and direct methods [4],[1] are the most accurate and efficient for sensitivity analysis. They are, however, more involved than the other approaches presented so far because they require knowledge of the governing equations and the algorithm that is used to solve them, in order to derive and implement a program that solves the corresponding sensitivity equations. Adjoint methods are particularly attractive since the cost of computing the gradient of a given function is independent of the number of design variables.

Performing a partial derivative operation on equation 9 with respect to the number to independent variables (e_k), results in:

$$[\mathbf{Z}']_k\{\mathbf{u}\}_i + [\mathbf{Z}]_k\frac{d\{\mathbf{u}\}_i}{de_k}=0 \tag{14}$$

where $[\mathbf{Z}']$ is the first derivative of impedance matrix, with respect to e_k variables, given by:

$$[\mathbf{Z}']_k = \begin{bmatrix} [\mathbf{K}'_s] - \omega^2[\mathbf{M}'_s] & [\mathbf{K}'_{fs}] \\ -\omega^2[\mathbf{M}'_{fs}] & [\mathbf{K}'_f] - \omega^2[\mathbf{M}'_f] \end{bmatrix} \tag{15}$$

wherein, [**K'**] and [**M'**] are the derivatives of stiffness and mass matrix, with respect to e_k.

In order to calculate objective function sensitivity of the equation 12, following expression is obtained from equation 14:

$$\frac{d\{\mathbf{u}\}_i}{de_k} = -[\mathbf{Z}]_k^{-1}[\mathbf{Z}']_k\{\mathbf{u}\}_i \tag{16}$$

and replacing this result into the total derivative equation 12 to obtain:

$$\frac{dI}{de_k} = \frac{\partial I}{\partial e_k} + \frac{\partial I}{\partial \mathbf{u}_i} - [\mathbf{Z}]_k^{-1}[\mathbf{Z}']_k\{\mathbf{u}\}_i \tag{17}$$

The approach using Equation 17 is called direct methods. Note that solving for $\frac{d\{\mathbf{u}\}_i}{de_k}$ requires the solution of the matrix equation 16 for each variable e_k.

Returning to the total sensitivity equation 12, there is an alternative option when computing total sensitivity $\frac{dI}{de_k}$, given by:

$$\frac{dI}{de_k} = \frac{\partial I}{\partial e_k} + \frac{\partial I}{\partial u_i}\Psi_k[\mathbf{Z}']_k \tag{18}$$

The auxiliary vector Ψ_k can be obtained by solving the adjoint equations, given by:

$$[\mathbf{Z}]_k\Psi_k = -\frac{\partial I}{\partial\{\mathbf{u}\}_i} \tag{19}$$

The vector Ψ_k is usually called the adjoint vector and is substituted into equation 18 to find the total sensitivity. In contrast with the direct method, the adjoint vector does not depend on the design variables, e_k, but instead depends on the function of interest, I.

Therefore, if the number of design variables is greater than the number of functions for which we seek sensitivity information, the adjoint method is computationally more efficient. Otherwise, if the number of functions to be differentiated is greater than the number of design variables, the direct method would be a better choice.

3. APLICATION EXAMPLE

The structural response of the fuselage section and associated interior noise were calculated using BEAM elements for solid domain and QUAD4 elements for fluid domain. It is considered approximated dimensions and materials properties of a full scale aircraft fuselage section. The pre and post-processing of the numerical model were made using the GID program. The fuselage Frequency Response Function (FRF) and operational modes shapes of the coupled system are obtained using a finite element system called MEFLAB. In Figure 2. a $2m$ of diameter fuselage model is represented, and its dimensions and properties are given.

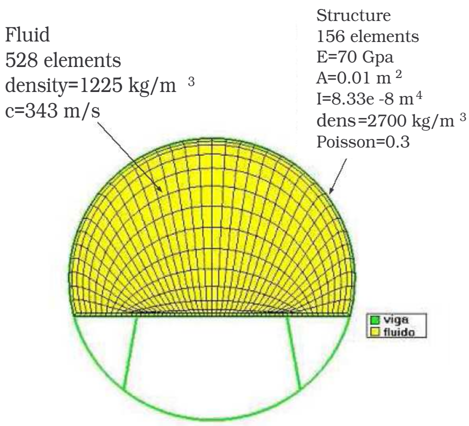

Figure 2. Bi-dimensional model of aircraft fuselage

In Figure 3. and figure 4. the system frequency response to a point excitation force and three first vibration modes and the pressure field of coupled system are represented. The three firsth mode take place in $8.3Hz$, 16.5 and $20Hz$ respectively, and they are structural predominant.

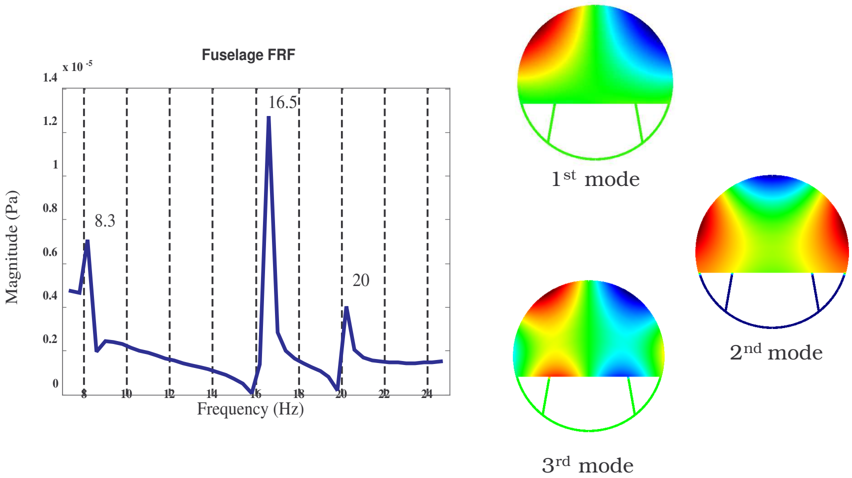

Figure 3. Acoustic frequency response and pressure field.

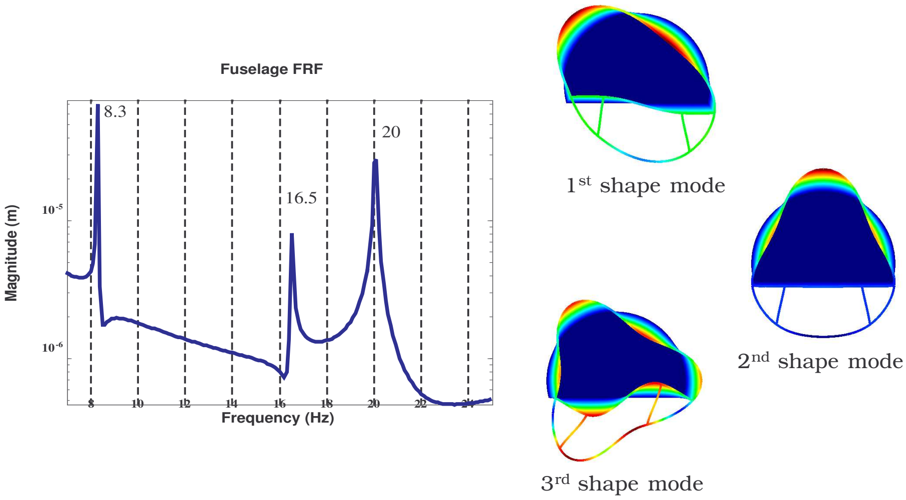

Figure 4. Structural frequency response and shape mode deformation.

Structural modifications, like beam thickness reduction for example, cause changes in acoustic harmonic response. Figure 5. shows frequency response and pressure field changes with a beam element, located in the circle, has 50 % thickness reduction.

In the same manner, preceding structural modifications cause changes in frequency structural response and deformation of shape, showed in Figure 6.

The Figure 7. shows the comparison between analytical and finite differences method to calculate the fluid pressure sensitivity of fuselage model, is represented. The Figure 7. shows the frequency response sensitivity of fuselage model for a local defect in one beam element.

In Figure 8. the rate of change in the sound presure is presented for different local defects. Each defect consists in to change the cross-sectional height of each element of beam. For damping coupled system, it is possible to see the defect that produces greater sensitivity in a given acoustic point. Figure 8. shows acoustic Sensitivity for different defect locations in the fuselage structure for each operational frequency and sensitivity field of fluid pressure with respect to local defect in the circulated beam element.

4. CONCLUSIONS

Using sensitivity analysis procedure is possible to know the response variations produced by structural defects of an acoustic-structural system. For the particular bi-dimensional fuselage simplified model and excitation frequency con-

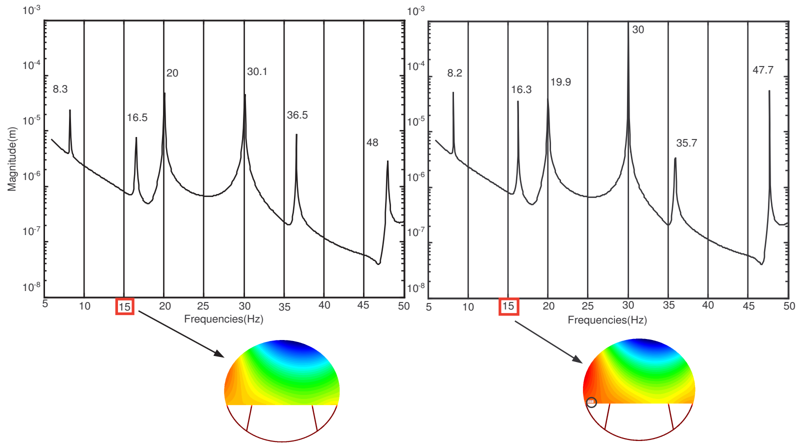

Figure 5. FRF and interior pressure field to a point force excitation without and whit local defect

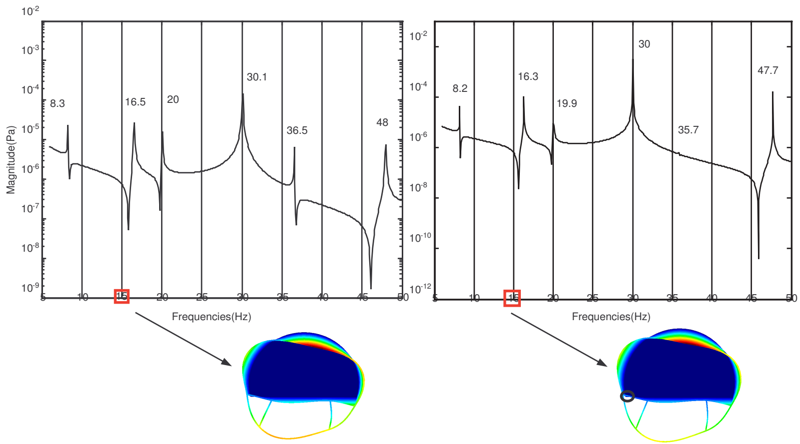

Figure 6. FRF and deformation shape to a point force excitation without and with local defect

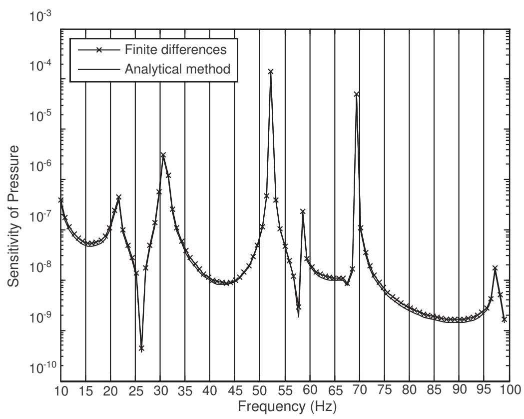

Figure 7. Validity of analytical sensitivity analysis of fluid pressure

sidered, sensitivity of the acoustic response with respect to defect locations and beam section sizes, was found. This procedure can be applied in structural damage detention. This work is in progress to extend it to use of faster and computational cheaper methods to acoustic-structural resolution procedure and sensitivity analysis. Likewise the study of three-dimensional fuselage model under structural defect, like cracks.

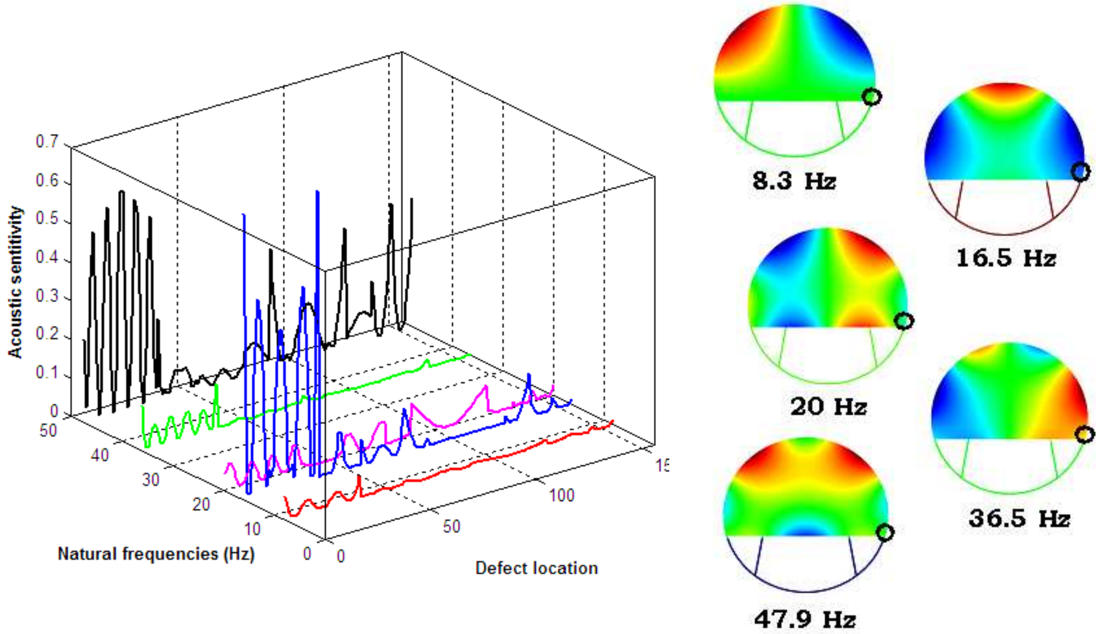

Figure 8. Acoustic Sensitivity for diferent local defects for each operational frequency

5. ACKNOWLEDGMENTS

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