

## APPLICATION OF THE GENERALIZED EXTREMAL OPTIMIZATION (GEO) ALGORITHM IN AN ILLUMINATION INVERSE DESIGN

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**Abstract.** *This work applies a stochastic algorithm, named generalized extremal optimization (GEO) method, for the illumination design of a three-dimensional rectangular environment. As will be shown, the illumination design is inherently an inverse problem, in which the design surface is subjected to two conditions – the luminous flux and null luminous power – while the light sources are left unconstrained. The design requires the determination of the locations and of the luminous power inputs of the light sources to satisfy the prescribed uniform luminous flux on the design surface. The GEO method is specially advantageous to be applied in complex problems where traditional gradient-based methods may become inefficient, such as when applied to a nonconvex or disjoint design space, or when there are different kinds of design variables in it. The present study presents the solution for the luminous power required in the lamps to satisfy the condition of uniform illumination in the design surface. The results are compared with those obtained from the Truncated Singular Value Decomposition (TSVD) regularization of the system of equations resulting from the radiative exchange analysis. Although the analysis considers a fixed position of the lamps, the methodology can be extended to determine the optimal locations of the lamps.*

**Keywords:** *Inverse analysis, illumination design, radiation exchanges, design optimization, stochastic algorithm*

### 1. INTRODUCTION

The first methods for the analysis and design of artificial lighting of environments were established in the beginning of the 20th century. Since in this early age, it was already known that the luminous flux on a given working area was not only dependent on the power of the light sources, but also on the absorbing and reflecting effect of the remaining surfaces of the enclosure. Later advances provided methods for calculation of light radiation exchanges as well as for the characterization of the light sources behavior.

In addition, many studies have been carried out to provide recommending lighting for the many possible applications, from human and industrial activities to plants growth (Boast, 1953; Mark, 2000). In general, not only the intensity of light (luminous flux intensity) is specified, but also it is required uniformity of the lighting. The major goal of the illumination designer is to determine the position and power of the light sources so that a uniform luminous flux, at a specified value, is achieved on the working area.

The first works to deal with the illumination design were presented by Harrison and Anderson (1916 and 1920), who proposed an experimental procedure, the now called lumen method, in which the luminous flux on a working plane was determined from a combination of a series of proposed assembling of punctual and continuous light sources. In the forties, Moon (1941) and Moon and Spencer (1946a, 1946b) proposed the interreflection method for the design of three-dimensional rectangular enclosures having any aspect ratio and being formed by diffuse surfaces. The method presented the advantage of allowing the calculation of the brightness of a surface, accounting for the reflection of light. Due to the complexities of the required calculations, the method required the use of tables. The lumen method (Phillips, 1981, IESNA, 2000, and OSRAM, 2005) is probably the most widely employed for the design of illumination, for its algebraic relations provide a rapid, simple procedure to determine the power of the lamps, although the method lacks on precision. A more elaborate solution can be achieved by the WinElux code (EEE, 2002), which contains a database of different types of lamps. In spite of their widespread use, both the lumen method and the WinElux code are in general not capable of providing solutions that can satisfy uniformity of luminous flux on the design surface (Seewald, 2006). A new approach has been proposed in the works of Smith Schneider and França (2004) and Seewald et al. (2006), in which the illumination design is treated as an inverse problem. Starting from the radiation exchange relations within an enclosure, these two works proposed a methodology based on the regularization of the ill-conditioned system of equations that resulted from the numerical treatment of the luminous exchange relations. Both works employed the truncated singular value decomposition (TSVD), obtaining a luminous flux on the design that was satisfactory both in terms of intensity and uniformity.

This paper considers an inverse illumination design of a three-dimensional rectangular enclosure. The objective is to find the luminous fluxes on the light sources located on the top surfaces that satisfy the specified uniform luminous flux on the design surface, located on the bottom of the enclosure. All the surfaces that form the enclosure are assumed diffuse and having spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum. The work considers first a thermal energy balance in the visible light region, which is then converted into luminous quantities. A zonal type formulation is applied for the discretization of the radiation exchanges. The results are obtained from the coupling between the forward solution radiation exchanges heat transfer and the Generalized Extremal Optimization (GEO) algorithm (Sousa et al., 2003). The proposed methodology is capable of providing satisfactory solutions for the required luminous power in the lamps. As in the works of Smith Schneider and França (2004) and Seewald et al. (2006), the locations of the lamps are fixed, although the present optimization technique could be used to determine the optimum locations of the lamps in the analysis.

## 2. PHYSICAL AND MATHEMATICAL MODELING

### 2.1. Luminous flux and thermal radiation

Incandescent lamps are a common source of light. Their main component is a resistance device that reaches very high temperatures (typically around 2900 K) under the passage of electric current. At such temperatures, a considerable amount of thermal radiation is dissipated in the visible region of the wavelength spectrum,  $0.4 \mu\text{m} \leq \lambda \leq 0.7 \mu\text{m}$ . The luminous flux, in units of lumens/m<sup>2</sup> or lux, can be related to the thermal radiation flux, in units of W/m<sup>2</sup>, by means of the following relation:

$$q^{(l)} = CV_{\lambda}q^{(w)} \quad (1)$$

where  $q^{(l)}$  and  $q^{(w)}$  correspond respectively to the luminous flux (lumens/m<sup>2</sup>) and to the thermal radiation flux (W/m<sup>2</sup>) for a specific wavelength  $\lambda$ ,  $C$  is a conversion factor constant, equal to 683 lumens/W, and  $V_{\lambda}$  is the so called photopic spectral luminous efficacy of the human eye, which takes into account the human eye sensitivity to the thermal radiation comprehended in the visible region of the spectrum. As shown in Fig. 1, the spectral luminous efficacy peaks with a value of 1.0 for a thermal radiation in the wavelength of 0.555  $\mu\text{m}$ , and then decay monotonically to zero as the lower and upper limits of the visible region, 0.4  $\mu\text{m}$  and 0.7  $\mu\text{m}$ , are approached. In general, a source of light is composed by radiation covering the entire range of the visible region. In such a case, Eq. (1) must be applied to each infinitesimal amount of the spectral energy and then be integrated in the visible spectrum. Such procedure will be demonstrated in the next section.

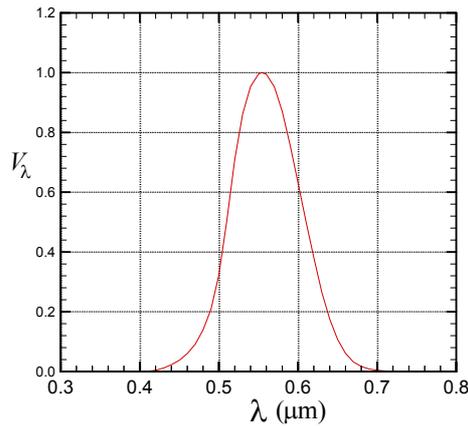


Figure 1. Photopic luminous efficacy of the human eye.

### 2.2. Radiation exchanges in an enclosure

A procedure for the evaluation of radiation exchanges in an enclosure is well established in the modern literature (Siegel and Howell, 2002). In this work, this will be accomplished by first subdividing the enclosure into sufficiently small elements, so that all thermal quantities can be assumed uniform. Next, the energy balance will be applied to each element  $j$  of the enclosure. Depending whether the temperature or the net radiative heat flux are known, the energy balance can take the following forms, respectively:

$$q_{\lambda,o,j}^{(w)} = \varepsilon_{\lambda,j} e_{\lambda,b,j}^{(w)} + (1 - \varepsilon_{\lambda,j}) \sum_k F_{j-k} q_{\lambda,o,k}^{(w)} \quad (2)$$

or

$$q_{\lambda,o,j}^{(w)} = q_{\lambda,r,j}^{(w)} + \sum_k F_{j-k} q_{\lambda,o,k}^{(w)} \quad (3)$$

where  $q_{\lambda,o}^{(w)}$  is the spectral outgoing radiative heat flux, in  $W/(m^2\mu m)$ , which takes into account both emission and reflection;  $q_{\lambda,r}^{(w)}$  is the spectral net radiative heat flux, in  $W/(m^2\mu m)$ , which takes into account emission minus absorption;  $e_{\lambda,b}^{(w)}$  is the spectral blackbody emissive power, in  $W/(m^2\mu m)$ , which is solely dependent on the temperature of the surface element;  $\varepsilon_{\lambda}$  is the spectral emissivity; and  $F_{j-k}$  is the view factor between surface elements  $j$  and  $k$ . In the above equations the superscript  $(w)$  was maintained in all energy terms to indicate thermal energy quantities. In a next step, such terms will be converted into luminous quantities. To accomplish this, it should be first noted that the thermal energy flux ( $W/m^2$ ) in the wavelength  $\lambda$  centered in the interval  $d\lambda$  can be related to the spectral energy flux by:

$$dq^{(w)} = q_{\lambda}^{(w)} d\lambda \quad (4)$$

The corresponding luminous flux (lumens) in the wavelength  $\lambda$  centered in the interval  $d\lambda$  follows from Eq. (1):

$$dq^{(l)} = CV_{\lambda} q_{\lambda}^{(w)} d\lambda \quad (5)$$

Finally the total luminous flux is obtained through the integration of the above equation in the entire visible region:

$$q^{(l)} = \int_{\lambda=0.4\mu m}^{0.7\mu m} CV_{\lambda} q_{\lambda}^{(w)} d\lambda \quad (6)$$

Applying steps (4) to (6) to all the spectral thermal energy fluxes of Eqs. (2) and (3), one obtains the energy balance in terms of the luminous fluxes:

$$q_{o,j}^{(l)} = \varepsilon_j e_{b,j}^{(l)} + (1 - \varepsilon_j) \sum_k F_{j-k} q_{o,k}^{(l)} \quad (7)$$

or

$$q_{o,j}^{(l)} = q_{r,j}^{(l)} + \sum_k F_{j-k} q_{o,k}^{(l)} \quad (8)$$

where  $q_o^{(l)}$  is the outgoing luminous flux, in lumens/m<sup>2</sup> or lux, which takes into account both emission and reflection;  $q_r^{(l)}$  is the net luminous flux, in lumens/m<sup>2</sup>, which takes into account emission minus absorption;  $e_b^{(l)}$  is the blackbody luminous power, in lumens/m<sup>2</sup>, which is solely dependent on the temperature. To obtain Eqs. (7) and (8), it was assumed that the spectral emissivity  $\varepsilon_j$  was independent of the wavelength in the visible region of the spectrum.

### 3. THE GENERALIZED EXTREMAL OPTIMIZATION ALGORITHM

The generalized extremal optimization (GEO) algorithm (Sousa et al., 2003) is a new evolutionary algorithm devised to improve the Extremal Optimization method (Boettcher and Percus, 2001) so that it could be easily applicable to virtually any kind of optimization problem. Both algorithms were inspired by the evolutionary model of Bak and Sneppen (1993). Following the Bak and Sneppen (1993) model, in GEO  $L$  species are aligned and for each species it is assigned a fitness value that will determine the species that are more prone to mutate. One can think of these species as bits that can assume the values of 0 or 1. Hence, the entire population would consist of a single binary string. The design variables of the optimization problem are encoded in this string that is similar to a chromosome in a genetic algorithm (GA) with binary representation (see Fig. 2).



$$x_j = x_j^l + (x_j^u - x_j^l) \frac{I_j}{(2^m - 1)} \quad (10)$$

where  $I_j$  is the integer number obtained in the transformation of the variable  $j$  from its binary form to a decimal representation.

### 3.1. Taking into account discrete and integer variables

As we have seen above, continuous variables are represented in the GEO in binary form, with precision  $p$ . Integer variables have precision  $p = 1$  and may be treated such as presented in Lin and Hajela (1992) for a binary coded GA. If the relation  $(x_j^u - x_j^l) = 2^N - 1$  is satisfied, there is a string of bits that will encode all variables biunivocally. If there is not a direct correspondence between one sequence of bits and the variables, the smallest number  $m$  that satisfies  $2^m > (x_j^u - x_j^l) + 1$  is calculated and for each of the  $N$  variables it is associated one sequence of bits. To the remaining  $2^m - N$  strings, integers out of the range of the variables are attributed, which are treated as unfeasible solutions. (How GEO deals with constraints is described in the next sub-section). Integers within the feasible interval may also be used. In this case, one or more variables will be associated with more than one sequence of bits. Although this last option avoids the need of imposing additional constraints to the problem, it implies, in the case of the GEO, a non-uniform probability for the selection process of the bit to be mutated in step 4.

Discrete variables may be treated in the same way as the integer variables. The process is carried out in two steps: first, to each discrete variable an integer number is associated and, second, one of the approaches described before is used to code them into binary form.

### 3.2. How GEO deals with constraints

Constraints in design optimization can be handled by many different ways. A simple, and probably the most common, way to deal with constraints in algorithms such as the GA and the simulated annealing (SA) is to incorporate them into the objective function via penalties. In evolutionary algorithms the penalty function approach have been extensively used in different types of implementations. Methods that deal directly with the constraints have also been proposed in order to avoid the process of setting the penalty parameters, since their values are highly problem dependent and if not properly set can lead to sub-optimal designs. Alternatively, adaptive penalty schemes have been proposed in such a way that the parameters are set automatically, without the need of fine tuning them for a particular application.

For GEO, side constraints (the bounds on the design variables) are directly incorporated when the design variables are encoded in binary form. Equality and inequality constraints are easily incorporated into the algorithms by simply setting a high (for a minimization problem) or low (for a maximization problem) fitness value to the bit that, when flipped, leads the configuration to an unfeasible region of the design space. For example, in a minimization problem, when the fitness values are attributed to the bits in step 2, the ones that when flipped result in a non-feasible configuration receive a high value for  $\Delta V_i$  (the same value is attributed to all bits in which this occurs). This means that those bits will be considered well adapted and will have a low probability to be flipped in step 4. However, they are not forbidden to be flipped, what makes the algorithm able to walk through infeasible regions of the design space. This gives a great flexibility to the algorithm that can, for example, be applied to design spaces that present disconnected feasible regions. In fact, the GEO can even start from an infeasible solution. In this case a dummy value is attributed to  $V_{best}$  in the initialization of the algorithm, which is replaced by the first feasible value of  $V$  found during the search.

It must be pointed out here, that other ways to take into account constraints in GEO may also be easily implemented, including the penalty function approach. However, the approach described above is very simple to apply and does not introduce any new adjustable parameter in the algorithm.

## 4. PROBLEM DEFINITION

Figure 3 presents a schematic view of a three-dimensional enclosure, which is formed by surfaces that are diffuse and have spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum. The design surface, where a luminous flux is to be specified, is located at the bottom of the enclosure; the incandescent lamps, the light sources, are located on the top surface. The remaining of the enclosure is formed by walls that do not emit but reflect incident light. The length, width and height of the enclosure are designated by  $L$ ,  $W$  and  $H$ , respectively.

As depicted in Fig. 4, the enclosure is divided into finite-sized square elements,  $\Delta x = \Delta y = \Delta z$ , in which the luminous energy balance will be applied. For designation of elements in the design surface, lamps and wall, indices  $jd$ ,  $jl$  and  $jw$  will be used throughout this analysis.

In this analysis, it is considered that a uniform luminous flux (in lumens/m<sup>2</sup> or lux), designated by  $q_{specified}^{(l)}$ , is specified on the design surface. The problem consists of finding the luminous power on each light source element located on the top surface so that this requirement is achieved. The luminous exchanges in the enclosure are governed by Eqs. (7) and (8).

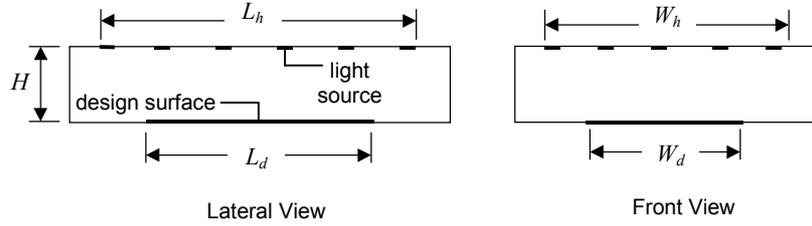


Figure 3. Three-dimensional rectangular enclosure.

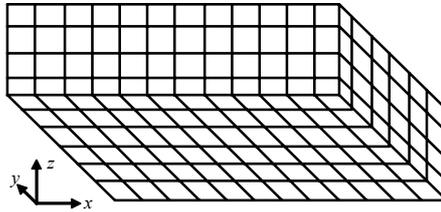


Figure 4. Division of the bottom and two side surfaces of the enclosure into finite size elements.

For the elements on the side walls, one single condition is known,  $e_b^{(l)} = 0$ , since they do not emit light. On the other hand, no condition is known for the light source elements. In fact, the conditions on the light sources are to be found from the two required conditions ( $q^{(l)} = q_{specified}^{(l)}$  and  $e_b^{(l)} = 0$ ) on the design surface.

The use of an optimization technique such as GEO allows the adoption of the conventional forward solution technique. In this approach, the net luminous fluxes  $q_{r,j}^{(l)}$  (alternatively, it could be the blackbody luminous power,  $e_{b,j}^{(l)}$ ) are imposed on the light sources, and the condition of null luminous powers,  $e_b^{(l)} = 0$ , are imposed on the remaining elements on the design surface and walls. This leads to a system of equations on the unknown outgoing luminous flux of each surface  $j$ ,  $q_{o,j}^{(l)}$ , formed by Eqs. (7) and (8). This system is in general well-conditioned and can be solved by any standard matrix inversion technique, such as Gaussian elimination, or by iterative techniques, such as the Gauss-Seidel method. Once the system is solved for the outgoing luminous fluxes, Eqs. (7) and (8) can be again applied to find the unknown conditions (the net luminous flux or the blackbody luminous power) of each surface element. At this point in the solution, the net luminous fluxes on the design surface elements are compared with the imposed value,  $q_{specified}^{(l)}$ . Different solutions can then be chosen with the aid of GEO, and the designer can select the solutions that better satisfy the illumination specifications on the design surface. It is important to stress that in inverse design, contrarily to inverse determination of parameters, the availability of more than one solution for the problem is a positive aspect rather than a difficulty.

## 5. SOLUTION PROCEDURE

The problem consists in minimizing the error function  $F$ , which is a measure of the difference between the specified luminous flux on the design surface,  $q_{specified}^{(l)}$ , and the luminous fluxes on the design surface that are obtained from a given choice of net luminous flux on the light sources,  $q_{r,jd}^{(l)}$ , that is:

$$F = \sum_{jd} \sqrt{|q_{specified}^{(l)} - q_{r,jd}^{(l)}|^2} \quad (11)$$

To minimize the above relation, the following procedure is followed:

1. Define the positions of the light sources;
2. Define the required precision,  $p$ , to find minimum number of bits,  $m$ , using Eq. (9);
3. Start with a given set of luminous powers on the light sources;
4. Solve the system of equations described in Section 3 to find the net luminous fluxes on the design surface elements,  $q_{r,jd}^{(l)}$ ;
5. Choose a new set of luminous powers on the light sources according to the GEO algorithm;
6. Repeat from step 4 until satisfactory solutions for the luminous powers on the light sources are found.

## 6. RESULTS AND DISCUSSION

The case considered in this work consists of a three-dimensional enclosure as shown in the schematic representation in Fig. 3. The aspect ratio of the enclosure base is  $W/L = 0.8$ ; the dimensionless height is  $H/L = 0.2$ . The selection of the other dimensions of the enclosure will require a few considerations. First, the design surface ought not to cover the entire extension of the base, since the portions close to the corners would be mainly affected by the reflections from the side walls, not from the luminous radiation from the light source elements on the top surface. Therefore, the design surface dimensions are taken as  $L_d/L = 0.8$  and  $W_d/L = 0.6$ . The amount and positions of the light sources will be proposed later on.

The boundary conditions are: for the design surface elements, no boundary condition was prescribed but the expected dimensionless net luminous flux (defined as  $Q_{r,jd} = q_{r,jd}^{(l)} / q_{specified}^{(l)}$ ) is  $Q_{r,jd} = -1.0$ ; for the wall elements, the luminous emissive power is zero; the light source elements are left unconstrained to be found by application of the GEO algorithm. The hemispherical emissivities in the visible light region of the design surface, of the light sources and of the walls are  $\epsilon_d = 0.9$ ,  $\epsilon_l = 0.9$  and  $\epsilon_w = 0.5$ , respectively. The problem is at this point completely defined except for the location of the light source elements, which will be discussed next.

To illustrate the proposed methodology, it is considered that the light source elements are distributed on the top surface according to the schematic of Fig. 5. The light sources are indicated by circular dots. The shaded area represents the design surface at the bottom surface. This configuration, proposed in the works of Smith Schneider and França (2004) and Seewald et al. (2006), has proved an interesting choice in the sense that it allowed the determination of luminous powers of the light sources that satisfied the prescribed luminous flux on the design surface with acceptable accuracy. For this reason, this same configuration is adopted in the present work, although the methodology can be applied to any other light sources locations. Due to the problem symmetry, indicated by the dashed lines, only a quarter of the domain needs to be solved:  $0 \leq x/L \leq 0.5$ ,  $0 \leq y/L \leq 0.4$ .

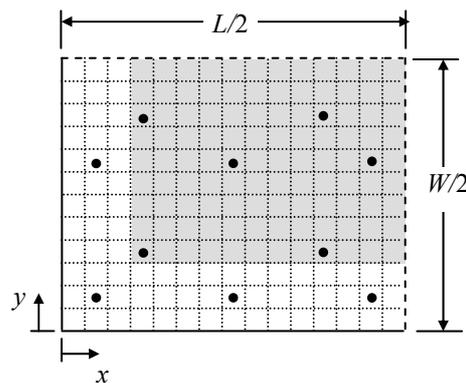


Figure 5. Locations of the design surface (shaded area) and light source elements (circular dots) in one quarter of the bottom and top surfaces for case 1: light sources covering the entire top surface. Dashed lines indicate symmetry.

The procedure presented in Section 4 was then applied to find the required net luminous flux on the light source elements. The interval specified in the GEO algorithm was  $[0, 50]$  for each light source shown in Fig. 5, with a precision of 0.2. These results were compared with a previous work (Smith Schneider and França, 2004), in which the same case was solved with the TSVD regularization method. Table 1 shows the required dimensionless net luminous flux on the light sources for the best four cases found by GEO and the result found in Smith Schneider and França (2004). With exception of cases 1 and 4, the solutions are considerably different, showing a typical characteristic of this type of problem, that is, different solutions can satisfy the specified conditions on the design surface. Another typical aspect was the oscillatory behavior of the results for the luminous power. In the case of the solutions via the

regularization of the matrices that describe the inverse relations (as in Smith Schneider and França, 2004, and Seewald et al., 2006), it is often found negative values for the luminous power of the light sources, which is not a physically acceptable solution. On the other hand, treating the inverse design as an optimization problem using the GEO algorithm, since the interval for the values of each luminous power of the light sources are defined by the designer, reaching undesirable, non-physically meaningful results is naturally avoided. This is an important advantage of the optimization approach.

Figure 6 presents the resulting net luminous flux distribution on the design surface for the four solutions obtained from the application of the GEO algorithm. Figure 7 presents the same result with the use of the TSVD solution (Smith Schneider and França, 2004). All the solutions are capable of satisfying the net luminous on the design surface (specified as  $Q_{r,jd} = -1.0$ ) with an error of 3.0% or less, which would be very difficult to obtain using a trial-and-error approach. This indicates the usefulness of the inverse analysis as a design tool for illumination systems.

Table 1: Required dimensionless net luminous flux on the light source elements

$j\backslash i$	$i$	$j$	$Q_{r,jl}$ (case 1)	$Q_{r,jl}$ (case 2)	$Q_{r,jl}$ (case 3)	$Q_{r,jl}$ (case 4)	$Q_{r,jl}$ (TSVD)
1	2	2	50.0000	26.6667	0.9804	50.0000	17.4009
2	2	8	10.0000	12.5490	43.9217	9.8039	11.7953
3	4	4	12.3529	25.4902	31.5686	12.3529	31.1068
4	4	10	37.4510	37.6471	24.1176	37.2549	36.6861
5	8	2	31.3725	33.3333	47.6471	31.3725	35.6220
6	8	8	21.7647	19.0196	12.1569	21.5686	20.8525
7	12	4	18.8235	23.9216	4.1176	20.7843	15.7124
8	12	10	31.1765	34.5098	25.0980	34.3137	31.2191
9	14	2	25.0980	18.4314	21.3725	25.0980	24.5422
10	14	8	7.4510	4.7059	25.0980	2.9412	9.1672

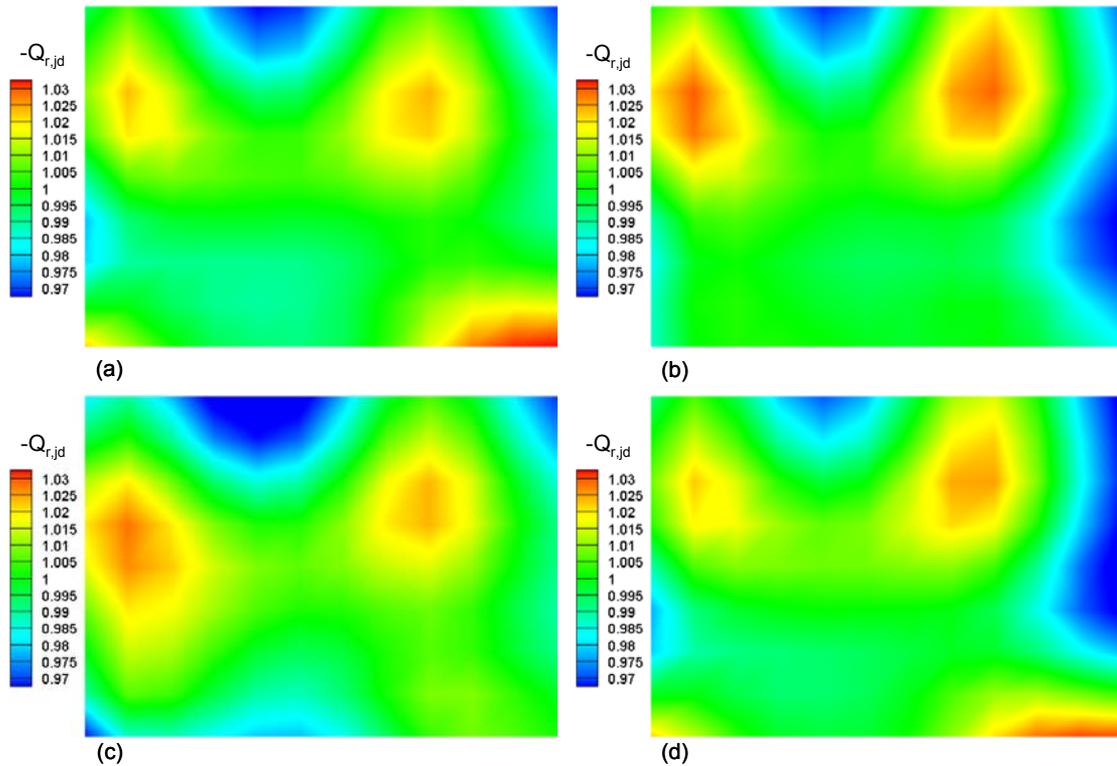


Figure 6: Dimensionless net luminous flux on the design surface (results obtained with GEO)

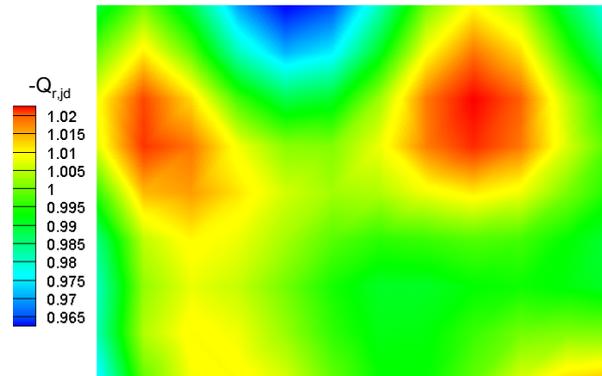


Figure 7: Dimensionless net luminous flux on design surface (result obtained with TSVD (Smith Schneider et. al. 2004))

Table 2: Minimized error function (in dimensionless form) with GEO

$$F = \sum_{jd} \sqrt{|Q_{required} - Q_{r,jd}|^2}$$

<b>Case 1</b>	0.1408
<b>Case 2</b>	0.1553
<b>Case 3</b>	0.1600
<b>Case 4</b>	0.1653
<b>TSVD</b>	0.1325

Table 2 presents the value of the error function defined by Eq. (11), but computed with the dimensionless net luminous fluxes, for all the net luminous flux distributions on the design surface that are shown in Figs. 6 and 7. Note that the result obtained with the TSVD regularization method was the one with the smallest error. For a given level of regularization of the system of equations, or alternatively for a given level of perturbation of the original problem, the TSVD method selects the smallest-norm solution that leads to the smallest value for the error function. However, depending of the level of regularization, the solution via an optimization method can present a smaller error. Table 2 indicates that the solutions obtained from the GEO algorithm also presented small errors, and can be considered satisfactory, as already illustrated in Figs. 6(a) to 6(d). In terms of computational time, the TSVD solution, based on the direct regularization of an inverted system of equations, required only a few minutes while, for the same computer machine, the GEO algorithm, based on the solution of a forward system of equations a large number of times, required a few days. However, this computation time can be reduced with an increased experience in the selection of optimum parameters in the GEO algorithm. Another attractive aspect of an optimization technique such as the GEO algorithm is that it allows the choice of a variety of solutions (as shown in Table 1), and can be extended to find the optimum position of the light sources, contrarily to the regularization techniques, for which it would be very difficult to incorporate the geometry parameters as unknowns of the problem.

## 7. CONCLUSIONS

In this paper, the optimization algorithm named Generalized Extremal Optimization was applied to an illumination design. Its application to a real design problem highlighted its characteristic of being easy to implement and effective to find satisfactory solutions for complex design problems. This method was applied to an inverse design problem in which the net luminous fluxes on the light source elements were determined to satisfy a specified uniform luminous flux on the design surface. The GEO algorithm presented results that compared well with the result obtained previously using the TSVD regularization of the system of equations formed from an inverse analysis of the problem. Despite the GEO algorithm requiring a larger computational effort, as typical of stochastic methods, it allowed finding a larger amount of satisfactory solutions, and in general can be extended more easily to non-linear problems than it would be possible with direct regularization methods. As possible next steps, the proposed inverse design analysis can be applied to consider the effect of external illumination, to include surfaces that present both specular and diffuse reflection characteristics, to take into account the directional and/or wavelength dependency of the surface emissivities, and to consider the problem of finding the optimum location of the light sources.

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