THE EFFECTS OF SHORT WAVELENGTH MODES ON THE ACTIVE CONTROL OF VIBRATION IN LATTICE STRUCTURES

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Abstract. *Lattice structures consists of many similar one-dimensional coupled elements. These members can have four different types of motion; longitudinal, torsion and bending in two orthogonal planes. Generally, bending natural frequencies tend to be low compared to natural frequencies associated with longitudinal and torsional motion. Because of this, lattice structures have a particular dynamic characteristic; the coexistence of independent long and short wavelength modes. Long wavelength modes are associated with member axial elongation of the elements and occur when the length of the structure is a multiple of half wavelength of waves propagating axially in the structural members, while the short wavelength behaviour is related to the bending of the structural members occurring when the length of a element is a multiple of half flexural wavelength. In this paper the effects of short wavelength modes in the performance of some acive control strategies is analysed. Feedforward and feedback control strategies are considered in the analysis and a lattice structure consisting of 93 members connected through 33 joints is used as an example.*

Keywords: *Lattice structures, Vibration Control, feedback, feedforward*

1. INTRODUCTION

In the literature, the terms Truss and Frame are often used to describe lattice structures. Both terms come from the static analysis field. They refer to two- or three-dimensional structures composed of one-dimensional elements. A one dimensional element is long and thin, so that all its properties can be reasonably defined by a single axial coordinate orthogonal to the member cross section. Lattice structures have attracted the attention of engineers for many years. The first applications of these structures can be found in the civil engineering field. (The American Society of Civil Engineers, 1972, 1976) has published extensive survey about the theme prior to 1975. In the aerospace engineering field, lattice structures are employed in applications such as solar energy panels, solar sails, large astronomical telescopes, communication antennae and space station structures. These structures have to keep their weight to a minimum and lattice structures have an easy way of packing, deployment and construction in space. Important developments in large space structures were done in the nineteen eighties and some examples of initial wishes in (The journal of Astronautics and Aeronautics 1978). During the mid 1980's, National Aeronautics and Space Administration (NASA) developed the program Assembly Concept for Construction of Erectable Space Structures (ACCESS) to test construction in space (Rogers and Tutterow, 1986). The lattice structure considered in this work is based on the structure assembled by NASA in 1984 during the program ACCESS. The structure is a satellite boom consisting of 93 members connected by 33 joints as illustrated in figure 1. The structural members have Young's modulus of 6.8967×10^{10} N/m, density of 2.6840×10^3 Kg/m^3 and diameter of 0.00635 m. The structure has 10 equal units called bays with length of 0.45 m. A structure with the same characteristics have been used in previous works, where (Moshrefi-Torbati et al., 2003), (Moshrefi-Torbati et al., 2006) discuss feedforward control strategies for vibration control.

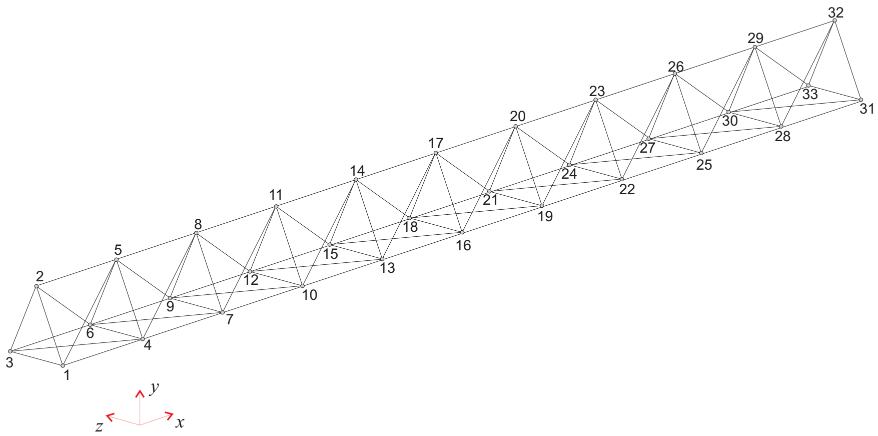


Figure 1. The satellite boom with 93 structural members and 33 joints with respective numbering scheme and global reference system showing the directions x, y and z.

1.1 The dynamic behaviour of lattice structures

Lattice structures consists of many similar coupled one-dimensional elements. A lattice member in space can have four different types of motion; longitudinal, torsion and bending in two orthogonal planes. In general, bending natural frequencies tend to be low compared to natural frequencies associated with longitudinal motion or torsion of beams. This leads to a particular dynamic characteristic in lattice structures: the coexistence of very different short and long wavelength modes of vibration. These dynamic features have already been studied by (Flotow, 1986) where he discussed the design of lattice structures taking into account the short/long wavelength behaviour in order to enhance the performance of a active control strategy. The LWM (Long Wavelength Mode) comprises of axial elongation of the structural members and they occur when a dimension of the structure, such as length, is a multiple of half wavelength of waves propagating axially in the structural members. In most cases this mode can be compared to the mode of a continuous system such as a beam or a plate. An example of this mode is shown in figure 2 for the structure considered in this work where this mode shape can be compared to the first bending mode shape of a continuous beam with free-free (f-f) boundary conditions. The other

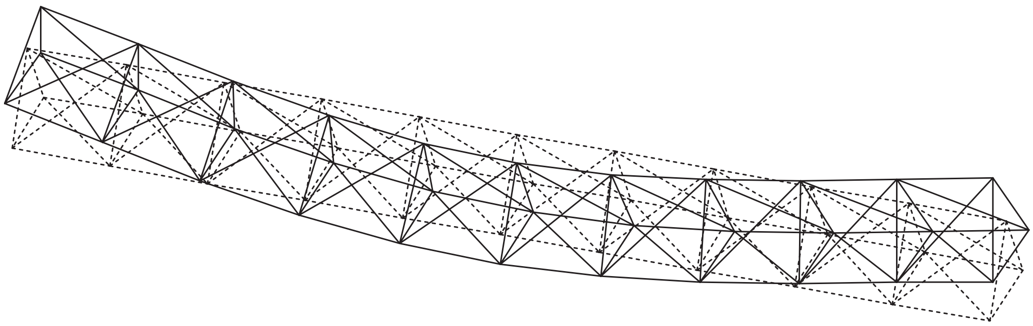


Figure 2. An example of a long wavelength mode showing the bending of a satellite boom.

behaviour of lattice structures is the SWM (Short Wavelength Mode). The SWM are generally dominated by bending of the structural members. They occur when the length of a structural member is a multiple of half flexural wavelengths. The natural frequencies related to bending motion tend to be low when compared to natural frequencies related to longitudinal motion. Because lattice structures are assembled using many similar coupled subsystems, many SWM may occur in narrow frequency bands. Moreover manufacturing imperfections and other small non-linearities (joint backlash, member buckling, joint friction), nominal properties can diverge from one member to the others, increasing even more the number of modes and causing a phenomenon called mode localization, where the motion is confined in a few regions of the system (Anderson, 1958) (Lust et al., 1995). Short wavelength modes can be beneficial in terms of vibration, if they work as absorbers as discussed by (Flotow, 1986) or they may be undesirable if they interfere with an active control strategy, for example. In typical mechanical systems, short wavelength modes usually occur only in mid and high frequencies, however, lattice structures may exhibit this behaviour in frequencies even lower than the long wavelength modes shapes. An example of a SWM is illustrated in figure 3 for the satellite boom structure considered in this work. This mode involves bending of the structural members only. The properties that control the dynamics of SWM are associated with the characteristics of the members such as material and geometric properties and also the boundary condition imposed by the structure joints. Some of these properties can be changed without affecting the LWM. Increasing the ratio of second moment of area to the cross section area of the members, for example, would increase only the natural frequencies of short wavelength mode shapes, while the frequencies of long wavelength mode shapes would remain fixed. Depending on the behaviour of the lattice structure (short or long wavelength), the placement of actuators and the performance an active control strategy can be affected. In this paper, two strategies for vibration control are considered; Feedforward and Integral Force Feedback (*IFF*).

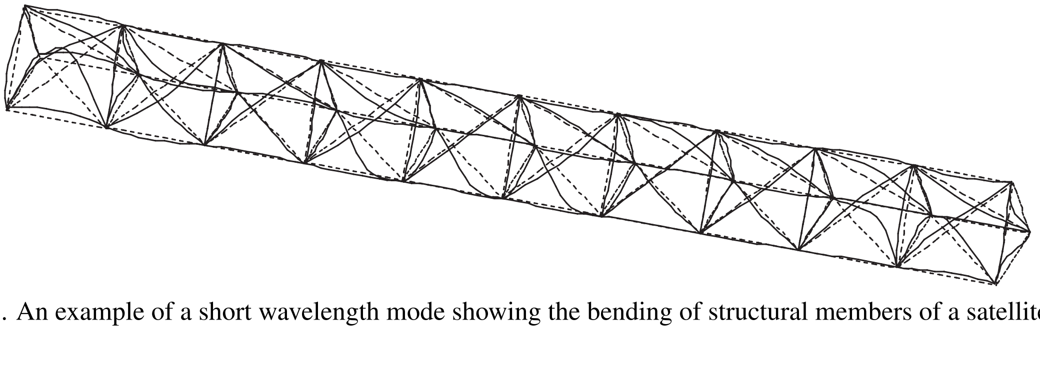


Figure 3. An example of a short wavelength mode showing the bending of structural members of a satellite boom.

2. THE CONTROL OF VIBRATION IN LATTICE STRUCTURES

Lattice structures are used in space to keep the spacecraft weight to a minimum during launch. Some of the desirable characteristics for a space structure are good wave dispersion, rapid decay of transient disturbances, desired frequency spectrum/mode shapes and adaptability to active and passive control. It is known, however, that materials used for space applications are in some cases in contrast to these characteristics. Most materials used in space are light and flexible with inherent low values of structural damping. Active vibration control is used to reduce vibration levels with minimum addition of mass to the structure. Most applications of active control in lattice structures make use of piezoelectric actuators where members (or part of them) are replaced by actuators which work by applying two axial forces to the member. In terms of control architectures, two different strategies are generally used in vibration control: feedforward and feedback control. One of the differences between them is the reference signal driving the controller. In feedforward control, a reference signal correlated to the disturbances forces is used to generate a control force. In the feedback control strategy, a sensor measures a signal that is feed back to a controller in order to modify this signal. In terms of disturbance characteristics, feedforward is usually used to control tonal disturbances and their harmonics, while feedback can be applied to a variety of disturbances types. In figure 4, the block diagrams illustrate the differences between these two forms of active control.

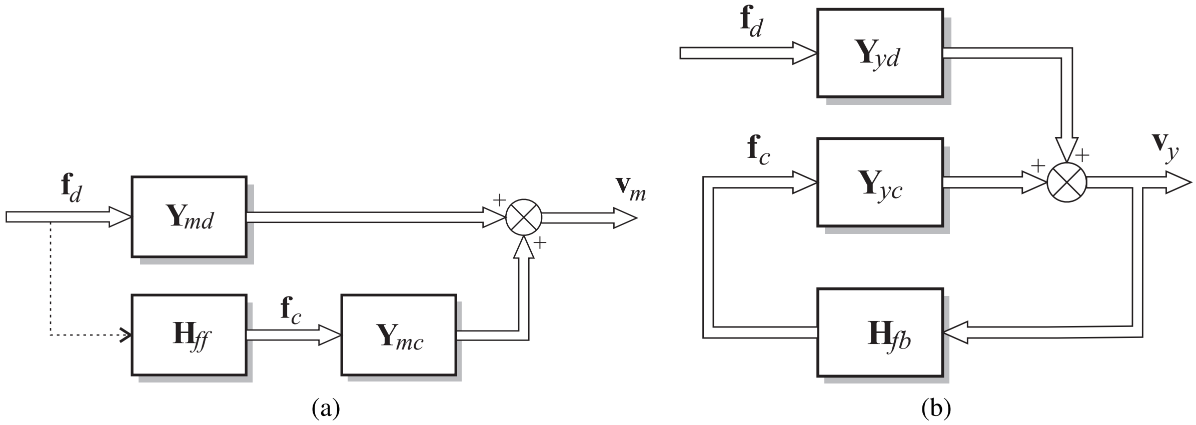


Figure 4. Block diagrams describing the different strategies of active control, (a) feedforward, (b) feedback

In figures 4(a) and 4(b), $\mathbf{f}_d$ and $\mathbf{f}_c$ are the vectors of disturbance forces and control forces, respectively. $\mathbf{Y}$ is the mobility relating either the disturbance or control force to the velocities at positions y and m. $\mathbf{H}_{fb}$ and $\mathbf{H}_{ff}$ are the matrices of feedback and feedforward control gains, respectively. A difference between the two controllers is that the feedforward control is applied at position c is designed to minimize variables at positions m while, in the case of collocated feedback control, the variable that is minimized is that at position y where the controller c is applied. This gives no guarantee that the variables at position m are also minimized. In the work of (Preumont et al., 1992) an active control system using integral force feedback (*IFF*) is described where actuators are placed in a collocated manner with force sensors. The aim of the controller is to reduce vibration levels of the two first long wavelength mode shapes. Because actuators and sensors are collocated and dual, the active control system is always stable, and the *IFF* control acts similar to an active damper reducing the resonance peaks in the system frequency response function. Similar applications for lattice structures are also described by (Gawronski, 1998) and (Meirovitch, 1990) where modal based active control is used. The success of a feedback active control system for a lattice structure based on the mode shapes assumes that short wavelength mode shapes are not present while in an implementation of feedforward control, this does not affect the performance of a active controller, as discussed in the following sections.

3. MODELLING THE LATTICE STRUCTURE

In order to predict the short and long wavelength modes, a model of the lattice structure was developed using the dynamic stiffness method, where the dynamic stiffness matrices of the individual members were calculated by means of the exact solutions of the beam wave equations. The procedure for calculating the dynamic stiffness matrix of the whole system is similar to the finite element method procedure and it is described in reference (Richards and Leung, 1977). In this method the joints displacements are related to the external loads applied to the joints in the frequency domain as

$$\mathbf{q}(\omega) = \mathbf{D}^{-1}(\omega)\mathbf{f}(\omega) \tag{1}$$

where, $\mathbf{q}$ and $\mathbf{f}$ are the vectors containing the joints displacements and external forces, respectively. Considering a three-dimensional structure, these vectors have dimension $6n_j \times 1$, where n_j is the number of joints in the structure. The dynamic stiffness matrix $\mathbf{D}$ has dimension $6n_j \times 6n_j$. This means that six variables are used to describe the motion of one joint, where three variables refer to linear displacements and the other three to angular displacements. Similarly, for the vector of external loads, three variables are forces and three variables are moments applied in the joints.

3.1 Objective function for vibration control

Often in applications of active vibration control it is common to use cost functions based on the square of a quantity such as acceleration or velocity. The formulation of sensible cost functions for similar application has been discussed by (Anthony et al., 2000) and (Moshrefi-Torbati et al., 2006). In this work, the objective function is defined as the sum of linear square velocities of the joints 31, 32 and 33 of the lattice structure showed in Figure 1. This objective function, or Cost Function (J) is proportional to the kinetic energy at these joints and it is calculated from

$$J(\omega) = \mathbf{v}_m^H(\omega)\mathbf{v}_m(\omega) \tag{2}$$

where, ω is the frequency and the simbol $()^H$ denotes the Hermitian forms of a vector. The vector $\mathbf{v}_m$ is given by

$$\mathbf{v}_m = \begin{bmatrix} v_x^{31} & v_y^{31} & v_z^{31} & v_x^{32} & v_y^{32} & v_z^{32} & v_x^{33} & v_y^{33} & v_z^{33} \end{bmatrix}^T \tag{3}$$

where $\mathbf{v}_x^{31}$ is the velocity at joint 31 on direction x, for example. The frequency dependency has been drooped for simplicity. The results of a model obtained by numerical simulation of the cost function J has been compared to the results of a laboratory experiment

4. EXPERIMENTAL TEST

Experimental tests were conducted in a lattice structure in order to compare the results of a mathematical model obtained by the dynamic stiffness method. The experimental rig consists of a lattice structure of 93 aluminium members and 33 aluminium spherical joints of mass 0.022 kg. A picture of the experimental rig is shown in figure 5. To simulate the free-free boundary condition, the structure was suspended using elastic wires. The suspension apparatus with the structure has natural frequencies bellow 4 Hz. The system was excited with an electromagnetic shaker mounted on foam rubber. The system was excited with white noise in the frequency band 20 - 1kHz. Acceleration at joints 4, 31, 32 and 33 were measured using PCB-Model 352A24 ICP accelerometers (with nominal sensitivity of 10 mV/ms^2). The mass of the accelerometers is about 0.8 grams and they have not been take into account in the modelling of the system. The input force was measured using a PCB-Model 208C01 ICP force sensor (with nominal sensitivity about 100 mV/N). Using a frequency analyser model HP35650, the transducer signals were recorded and acceleration frequency response functions (FRFs) were calculated and recorded. The FRFs were calculated with a resolution of 0.25 Hz. Hanning windows was used in the calculation of the FRFs. The FRFs were smoothed by averaging the data 10 times using no time overlapping.

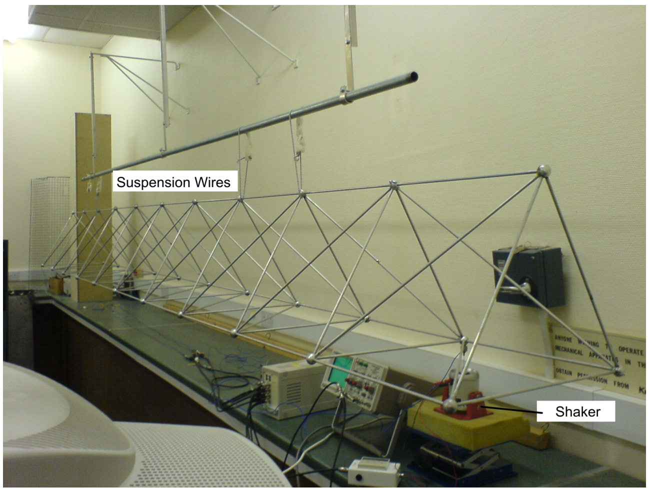


Figure 5. The lattice structure experimental rig. Structure is suspended by elastic wires with electromagnetic shaker attached to joint 4.

4.1 Comparison of numerical and experimental results

The results of the model obtained by numerical simulation of the cost function J has been compared to the results of a laboratory experiment. In these results, a harmonic force was applied at joint 4 in the y direction in the frequency range 20-1kHz using steps of 1 Hz. The comparison is given in Figure 5 which shows a good agreement between results

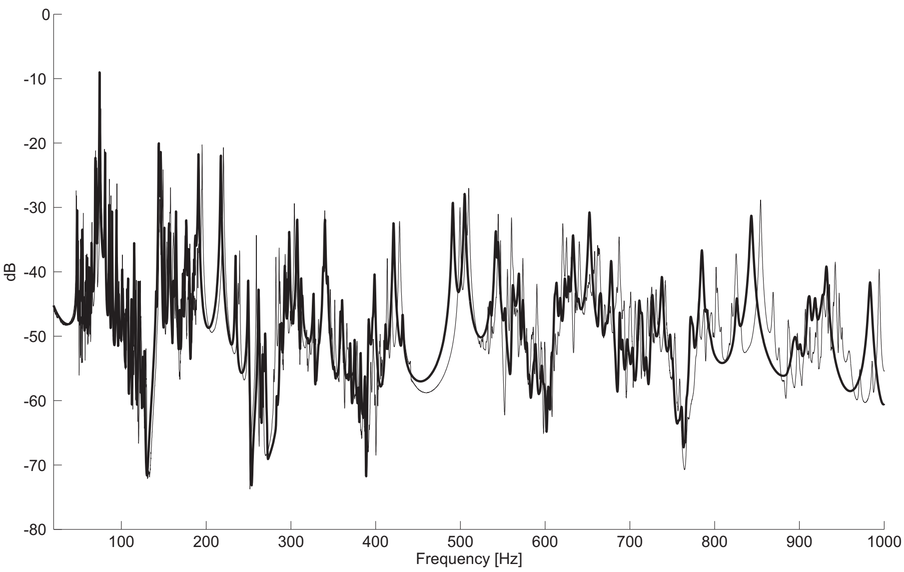


Figure 6. Cost function. The thick line is the theoretical. The thin line is the experimental. dB ref. 1 m^2/Ns^2.

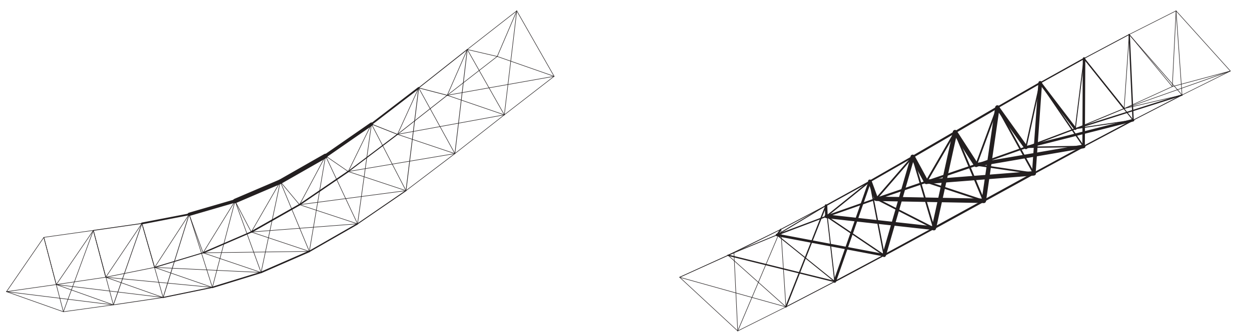
obtained numerically and experimentally. This result shows that the model is capable of predicting the various resonant peaks associated to the bending of the structural members.

5. FEEDBACK CONTROL

In this article, integral force feedback (IFF) control is considered as one of the control strategies. The advantage of using this form of feedback control or the direct velocity feedback is if the sensor-actuator pair is placed in a collocated manner, stability is guaranteed. This form of active control is explained in the work of (Preumont et al., 1992), in which a force sensor is used to measure the force acting axially in a structural member. The sensor signal passes through a signal integrator and an amplifier which drives voltage/current to a piezoelectric actuator producing an unconstrained deformation δ in the actuator. The unconstrained deformation in the actuator is related to the measured force f_y as

$$\delta = HF_y = \frac{g}{j\omega} f_y \tag{4}$$

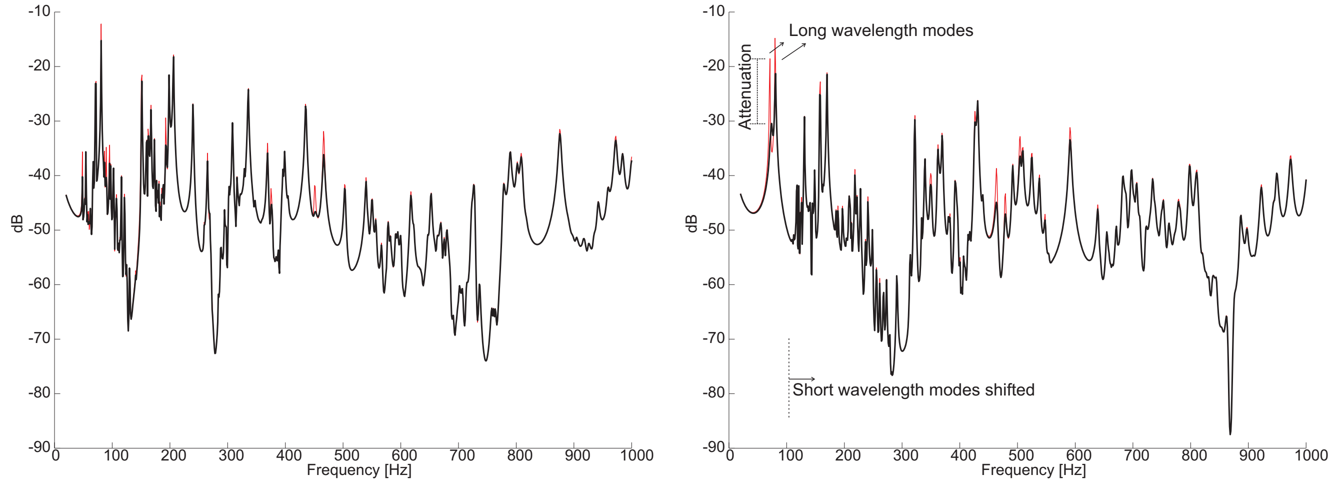
For small deformations, δ can be written as $\delta = n_d d_{33} V$, where n_d is the number of piezoelectric stacks in the actuator, d_{33} is a piezoelectric constant and V is the voltage applied. The placement of actuators in a lattice structure for *IFF* can be done based in a index based on the *fraction of modal strain* as defined in the work of (Preumont et al., 1992) this methodology assumes that the behaviour of the structure can be explained by modal parameters, or in other words, that the dynamics of the structure is controlled by long wavelength mode shapes. Two examples of long wavelength mode shapes showing the members with a larger fraction of modal strain are showed in figure 6.



(a) (b)

Figure 7. Placement of actuators for IFF control. Lines plotted with thicker lines indicate members with larger index of fraction of modal strain for that mode: (a) bending, (b) torsion.

For structures, where the low frequency range is dominated by short wavelength modes, it may be difficult to use the fraction of modal strain to defined good positions for placing actuators. As a numerical example, two situations are considered. The first uses the nominal properties of the structure discussed previously in this article and the second situation is for a structure with the same properties, except that the ratio between the second moment of area and the cross section area (I/S) has been increased by 5 times the nominal value. By doing this, the long wavelength natural frequencies remain unchanged, while short wavelength natural frequencies are shifted to higher frequencies. An actuator is placed in the member defined by joints [17-20] where, according to figure 6, it is an optimum place to control the first bending mode shape, which is the first long wavelength mode of the lattice structure. Figures 7(a) and 7(b) show the result of the simulation for the two situations. The value of gain for the feedback controller in both situation was set to $g = 0.0004$ m/N.



(a) (b)

Figure 8. The cost functions with (solid line) and without IFF control (faint line) for the structure with the nominal properties (a) and the structure where short wavelength mode shapes have been shifted (b). dB ref. 1 m^2/Ns^2.

The results of figure 7(a) shows that the cost function remains almost unchanged with the implementation of IFF in the presence of short wavelength mode shapes, while in the figure 7(b), where the short wavelength mode shapes have been shifted to higher frequencies, the IFF is capable of controlling the peak associated to the first long wavelength bending mode shape with a attenuation of approximate 12 dB for that mode.

6. FEEDFORWARD CONTROL

Feedforward control can be applied in situations where there exists advanced information about the disturbance forces in a structure. In feedforward control, each frequency is considered separately (this is suitable for tonal vibration and their harmonics) and therefore the problem of non-causality is avoided (Nelson and Elliott, 1992). In this work, the translational velocity components at three joints in the structure are used. In this way, the vibration on the "end-face" plane on which the joins are located is controlled (equations 2 and 3). The vector of velocities vm can be written in terms of transfer mobilities, disturbances and control forces as

$$\mathbf{v}_m = \mathbf{Y}_{md}\mathbf{f}_d + \mathbf{Y}_{mc}\mathbf{f}_c \tag{5}$$

where, $\mathbf{Y}_{md}$ and $\mathbf{Y}_{mc}$ are the transfer mobility from the disturbance forces $\mathbf{f}_d$ and the transfer mobility from the control forces $\mathbf{f}_c$ to the velocity vector $\mathbf{v}_m$, respectively. The idea of the feedforward control is to reduce the velocities at position m by destructive interference of waves. The control force in this case is a linear weighted combination of the disturbance force given as

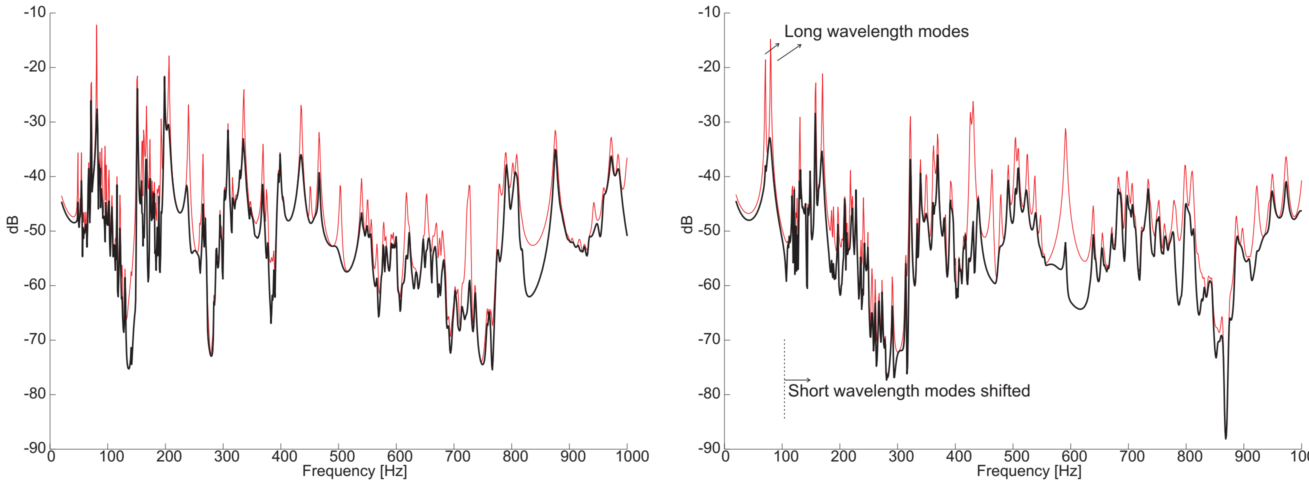
$$\mathbf{f}_c = \mathbf{H}_{ff}\mathbf{f}_d \tag{6}$$

where, $\mathbf{H}_{ff}$ is the feedforward controller. Using the quadratic representation of the cost function found in equation 2 it is possible to define a optimum controller that minimizes the sum of squared velocities at position m. The optimum controller is given by (Fuller et al., 1996) as

$$\mathbf{H}_{ff}^{\text{optimum}} = -\left(\mathbf{Y}_{mc}^{H}\mathbf{Y}_{mc}\right)^{-1}\mathbf{Y}_{mc}^{H}\mathbf{Y}_{md} \tag{7}$$

Results of numerical simulation of an implementation of feedforward control are shown in figure 8 for the system with nominal properties and for the system where the SWM have been shifted as in the previous example. The position of the actuator is the same of that used in IFF control (member defined by joints [17-20]).

The results in figure 8 show that the performance of the feedforward control is not influenced greatly by the short wavelength modes shapes.



(a) (b)

Figure 9. The cost functions with (solid line) and without feedforward control (fainted line) for the structure with the nominal properties (a) and the structure where short wavelength modes have been shifted (b). dB ref. 1 m^2/Ns^2.

7. CONCLUDING REMARKS

The dynamics of lattice structures related to the different short and long wavelength regimes have been discussed. A numerical model obtained by dynamic stiffness method using the solutions of wave equations was shown to be capable of predicting the dynamics of short wavelength modes. This model has been used to predict the results of two forms of active control; feedforward and feedback. It has been shown that the short wavelength modes play an important role in the performance and placement of an actuator for feedback control. The results of feedforward control, however, do not depend greatly on the existence of the short wavelength modes shapes.

8. ACKNOWLEDGEMENTS

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