NUMERICAL STUDY OF THE MIXING PROCESS IN LAMINAR CONFINED OPPOSING JETS

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Abstract. *The mixing process in laminar confined opposing jets is studied by a finite volume method. In the present work, two different chemical species enter in a parallel flat plate channel through different slots positioned on each channel wall. In the entrance region of the channel, a counterflow is formed. The properties are assumed constants. The model is based on mass, momentum and mixture fraction conservation equations. The numerical solution uses a WUDS interpolation in a structured regular non-staggered grid. The SIMPLEC algorithm is employed to solve the pressure field. An implicit scheme is used. The proposed model considers a laminar incompressible flow formulation. A parametric study is conducted for Reynolds number, Peclet number, Lewis number, nozzle thickness ratio and jet velocities. The influences of the parameters on mixture fraction behavior and on the mixing process are analyzed.*

Keywords: *mixture fraction, finite volume, flat plate channel, opposing jets, mixing process.*

1. INTRODUCTION

The configuration of opposing jets is present in many studies of mixing process. “Opposing jet impacting head-on provides a simple in-line mixer configuration with potential industrial applications for rapid mixing of viscous fluids” (Wang *et al*., 2005). “Compared with mechanically agitated mixers, mixers which use the opposing jet technique to create high turbulence, high shear rates and vortex motion caused by collision of the opposing jets offer several advantages in achieving rapid mixing over a short distance in the mixing chamber without use of moving parts of internal baffles” (Wang and Mujumdar, 2007). Furthermore, “due to their favorable mixing features opposing jet configurations have found applications in several different processes such as reaction injection molding (R.I.M.) and combustion” (Hosseinalipour and Mujumdar, 1997).

Hosseinalipour and Mujumdar (1997a) had developed a numerical model for confined and equal opposing jets inside a parallel flat plate channel with isothermal or adiabatic walls, considering laminar flow in steady-state. This work was extended to unequal opposing jets condition (Hosseinalipour and Mujumdar, 1997b). The unequal opposing jets velocities modify the flow behavior creating intense eddy zones, improving the mixing process. Wang *et al*. (2005) introduced in the Hosseinalipour and Mujumdar (1997a and 1997b) works the effect of baffles in the channel walls. They observed that the baffles improve the mixing process. Roseira Jr (2005) had studied the confined opposing jets configuration, observing the effects of different Reynolds number and the nozzle thickness on the mixing process. Wang *et al*. (2006) had analyzed the effect of the inlet opposing jets temperatures in different geometries and operational conditions.

There is an important difference in the mixing problem studied in the present work and the mixing process studied in other works (Hosseinalipour and Mujumdar, 1997a, 1997b, Wang *et al*., 2005, 2006). The mixing process in the present work is related with the chemical species while the others are related with the temperature field. Mathematically, both problems are similar since the systems of equations and the boundary conditions are the same.

The present work is devoted to the numerical investigation of the mixing process in laminar confined opposing jets inside a parallel flat plate channel. Two different chemical species enter through different slots (nozzles) positioned on each channel wall. The proposed model considers a laminar incompressible flow formulation. A parametric study is conducted for Reynolds number, Peclet number, Lewis number, nozzle thickness ratio and jet velocities. The influences of the parameters on mixture fraction field behavior and on the mixing process are analyzed.

2. PHYSICAL AND MATHEMATICAL MODELS

The physical and the mathematical model involve the mixing process of two initially separate streams of different non-reacting chemical species in two-dimensional confined laminar opposing jets. The jets enter in the channel through different slots positioned on each channel wall. In the entrance region of the channel, the opposing jets create a counterflow. The Lewis number is considered constant and equal for both chemical species. The thermo-physical properties are assumed constant. The model is based on the mass, momentum and mixture fraction conservation equations.

The figure 1 shows the simplified geometry of the proposed problem, representing the entrance and exit regions of the channel, the parallel flat plate channel walls, the symmetry plane and the mathematical domain.

The geometry adopted to represent the mathematical domain is described in Fig.2. The left boundary represents the symmetry plane and the nozzle separation width (L^*), the right boundary is the outflow region, as well as the truncation position in the horizontal direction. The top and bottom boundaries (P^*) include the nozzles thickness, respectively, A_i^* and D^*, and the adiabatic solid walls.

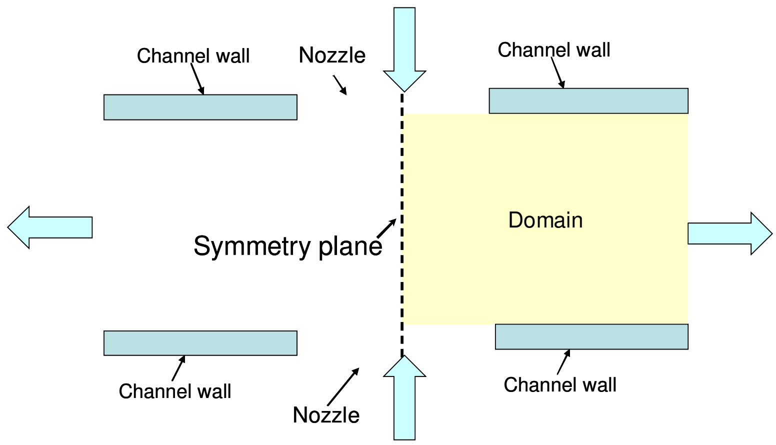

Figure 1. Simplified geometry of the problem.

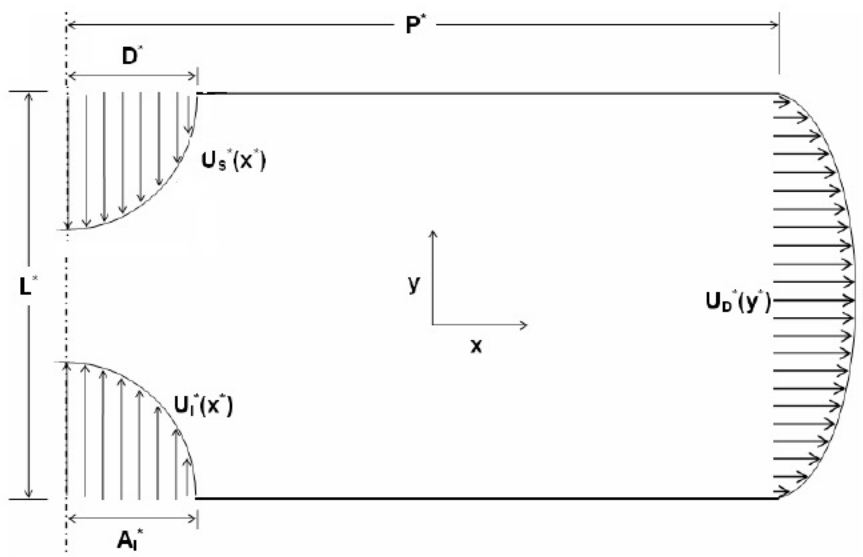

Figure 2. Geometry of the mathematical domain.

The transient, laminar and incompressible flow formulation is presented below. However, in the present work only the steady-state results will be investigated.

Mass conservation equation:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

Momentum conservation equation:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + Re^{-1}\, \Delta \boldsymbol{u} \tag{2}$$

Energy conservation equation:

$$\frac{\partial Z}{\partial t} + \boldsymbol{u} \cdot \nabla Z = (\, LePe \,)^{-1}\, \Delta Z \tag{3}$$

In the conservation equations ∇ and Δ denote the nondimensional gradient and the Laplace operator, respectively. In addition Z is the mixture fraction.

The definitions of the dimensionless variables and parameters employed in the mathematical model are given by Eqs. (4).

$$x=\frac{x^*}{L^*},\quad y=\frac{y^*}{L^*},\quad P=\frac{P^*}{L^*},\quad p=\frac{p^*}{\rho^2\left(U^*_{DMAX}\right)^2},$$
$$\boldsymbol{u}=\frac{\boldsymbol{u}^*}{U^*_{DMAX}},\quad R_b=\frac{A_i^*}{D^*},\quad R_v=\frac{R_b\,U^*_{I\,MAX}}{U^*_{S\,MAX}},\quad R_{DL}=\frac{D^*}{L^*},\tag{4}$$
$$Re=\frac{U^*_{DMAX}L^*}{\upsilon^*},\quad Pe=\frac{U^*_{DMAX}L^*}{\alpha^*},\quad Pr=\frac{Pe}{Re},\quad Le=\frac{\alpha^*}{D^*_{ij}}.$$

In equation (4) U^*_{DMAX} is the maximum outlet channel flow velocity, $\boldsymbol{u}^*$ is the flow velocity vector and L^* is the height of the channel. The thermo-physical properties of the fluid are the thermal diffusivity (α^*), the kinematic viscosity (υ^*) and the species diffusivity (D_{ij}^*). The dimensionless numbers are the Reynolds number (*Re*), the Prandlt number (*Pr*), the Peclet number (*Pe*), and the Lewis number (*Le*). Other parameters are defined: the nozzle thickness ratio (R_b), the confinement ratio (R_{DL}) and the inlet jet velocities ratio (R_v).

The boundary conditions are written in Eqs. (5-10). These equations represent the symmetry plane in the left boundary of the domain, Eq. (5), the developed flow and mixing condition in the right boundary of the domain, Eq. (6), the inlet jet condition in the upper and lower nozzles, respectivaly, Eq. (7) and Eq. (9), and the no-slip and impenetrability boundary condition on the adiabatic solid walls, Eq. (8) and Eq. (10). The inlet jet conditions consider a parabolic function with maximum velocity in the symmetry plane and null at the solid wall.

$$\frac{\partial u}{\partial x}=0\quad,\frac{\partial v}{\partial x}=0\quad,\frac{\partial Z}{\partial x}=0;\quad x=0,\quad 0<y<L\tag{5}$$
$$\frac{\partial u}{\partial x}=0\quad,\frac{\partial v}{\partial x}=0\quad,\frac{\partial Z}{\partial x}=0;\quad x=P,\quad 0<y<L\tag{6}$$
$$u=0\quad,v=-\frac{(R_{DL}{}^2-x^2)}{R_{DL}{}^3(1+R_v)}\quad,Z=1;\quad 0<x<R_{DL},\quad y=L\tag{7}$$
$$u=0\quad,v=0\quad,\frac{\partial Z}{\partial y}=0;\quad R_{DL}<x<P,\quad y=L\tag{8}$$
$$u=0\quad,v=\frac{R_v\left[(R_bR_{DL})^2-x^2\right]}{(R_bR_{DL})^3(1+R_v)}\quad,Z=0;\quad 0<x<R_bR_{DL},\quad y=0\tag{9}$$
$$u=0\quad,v=0\quad,\frac{\partial Z}{\partial y}=0;\quad R_bR_{DL}<x<P,\quad y=0\tag{10}$$

3. NUMERICAL PROCEDURE

In the present work, the finite volume method was used for the numerical solution of the governing equations previously described. The semi-infinite physical domain was truncated at $x = L$ where the outlet boundary conditions were applied. Numerical tests were performed for different values of L in order to guarantee the solution independence of the truncated domain length. It was observed that a good criterion to truncate the domain is

$$P=0.2\,Re.\tag{11}$$

The equation (11) is based on the fluid dynamic development problem inside a parallel flat plate channel. It is a modified expression for the development length (Kays and Crawford, 1980). The finite volume method is employed in a non-staggered grid (Maliska, 1995) and the pressure-velocity coupling is solved by the SIMPLEC algorithm (Van Dormaal and Raithby, 1984). The WUDS (Raithby and Torrance, 1974) is used as the interpolation function in an implicit scheme. The linear systems of equations are solved by the GMRES algorithm (Press *et al.* 1992). A uniform structured mesh with 120 x 50 volumes is adopted. Numerical tests were performed for different number of volumes to guarantee that maximum relative discrepancy is 1% in the velocities and mixture fraction solutions. The pseudo-transient approach is considered until the steady-state condition is reached.

4. RESULTS

The numerical solution of Roseira Jr (2005) for the heat transfer problem of mixing of one hot and one cold streams in a confined opposing jet configuration inside a parallel flat plate channel is used to validate the proposed numerical procedure. It is important to note that the heat transfer problem described by Roseira Jr (2005) is equivalent to the mixture fraction problem studied in the present work when the Lewis number is equal to 1. Nevertheless, the dimensionless procedures in the present work and in the Roseira Jr (2005) are different. So, the results of Roseira Jr (2005) are transformed to the same dimensionless form adopted here.

Figure 3 shows the mixture fraction solutions obtained in the present work and the ones obtained by Roseira Jr (2005) for different values of the Reynolds number. These cases consider the following parameters values: $R_{DL} = 0.5$, $R_v = 1$, $R_b = 1$, $Le = 1$, $Pr = 0.7$. The mixture fraction results in Fig. 3 are evaluated at $x = 0.4$. From Fig. 3 is possible to observe that there is not relevant discrepancy between the present results and the results of Roseira Jr (2005). In addition, for low values of the Reynolds number the mixture fraction profile is smoother. This fact is explained because the diffusion is more intense for low values of the Reynolds number.

A parametric study of the mixing problem is conducted in order to identify the influences of the Reynolds number (*Re*), the Lewis number (*Le*), the Peclet number (*Pe*), the nozzle thickness ratio (R_b) and the inlet jet velocities ratio (R_v) on the obtained solutions. It is noteworthy that, as can be seen in Eq. (3), the product *Le Pe* is the parameter presents in the mixture fraction conservation equation. So, the analysis of the *Le* and *Pe* numbers can be combined in the analyses of different values of the product *Le Pe*.

One reference case is chosen to perform the parameter analysis. Then, in the following results each parameter above described will be modified in relation to the reference case, permitting the evaluation of each parameter influence on the solution. The values of the reference case parameters are presented in Tab. 1. The values of the physical and geometric parameters shown in Tab. 1 are defined with basis on the Roseira Jr (2005) work.

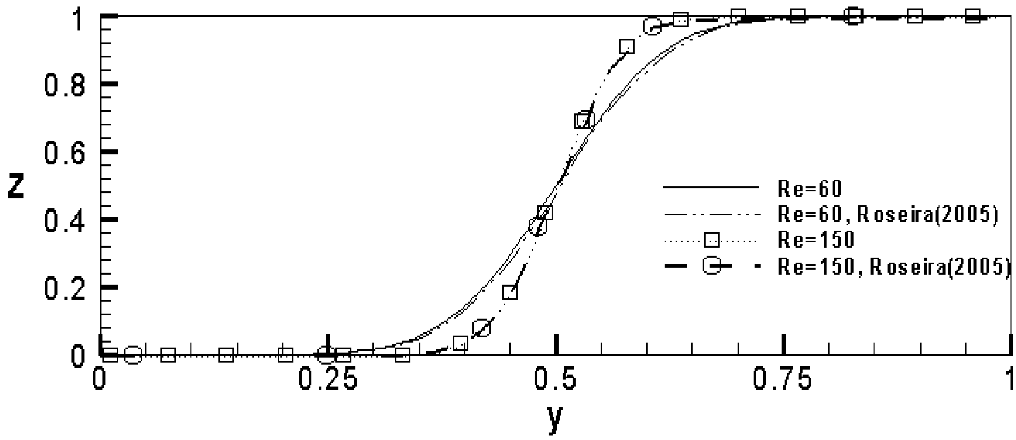

Figure 3. Mixture fraction profile at $x = 0.4$.

Table 1. Values of the reference case parameters.

Re	*Le*	*Pe*	R_b	R_v	R_{DL}
60	1	42	1	1	0.5

The streamlines for the reference case are shown in Fig. 4. From this figure the counterflow configuration is identified in the channel region near of the nozzles where the opposing jets are deflected. Downstream, a fluid dynamic developed condition is reached.

The mixture fraction field for the reference case is presented in Fig. 5. In this figure it is observed the simmetry of the figure in relation to a longitudinal line positioned at $y = 0.5$. This behavior is promoted by symmetric boundary conditions. At the and of the channel the mixture fraction reaches a equilibrium values approximatedely equal to 0.5. In the same way, this result is consequence of the equal inlet mass conditon for each chemical specie. The equilibrium condition reveals the ideal mixture fraction. In the present work, the better mixing process is obtained when the equilibrium mixture fraction condition is achitivied.

The effect of the Reynolds number parameter on the mixture fraction field can be analysed comparing Fig. 5 and 6. In figure 6 the results for *Re* = 150 case are reported. The increase in the Reynolds number implies in the increase in the mixture fraction development region. For high values of the Reynolds number the flow drags the species downstream, disturbing the mixing process. So, large channel lengths are required to reach the equilibrium mixture fraction condition. However, the simmetry of the figure is preserved.

In figure 7 the effect of the product *Le Pe* parameter is depicted. Comparing, Fig. 5 with Fig. 7 is possible to note that decreasing the Lewis number (or the Peclet number) the development region of the mixture fraction field is reduced. The mixture fraction reaches the equilibrium condition in short lengths of the channel. In other words, the species diffusivity became more important in the case presented in Fig. 7, improving the mixing process. The symmetry of the Fig. 7 is kept as observed in Fig. 5 and Fig. 6.

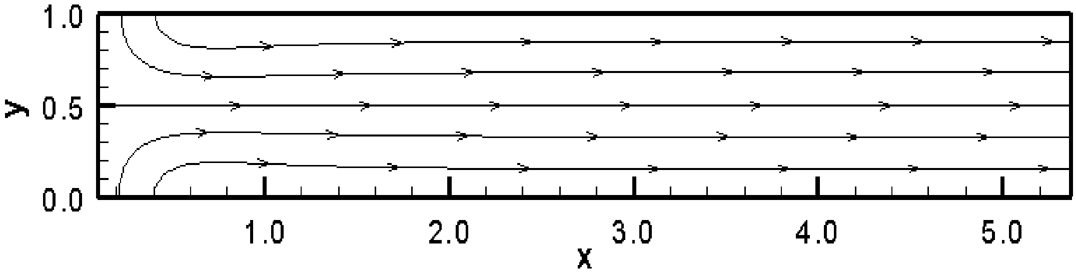

Figure 4. Streamlines of the reference case (Re = 60, $Le.Pe$ = 42, R_b =1, R_v =1 and R_{DL} = 0.5).

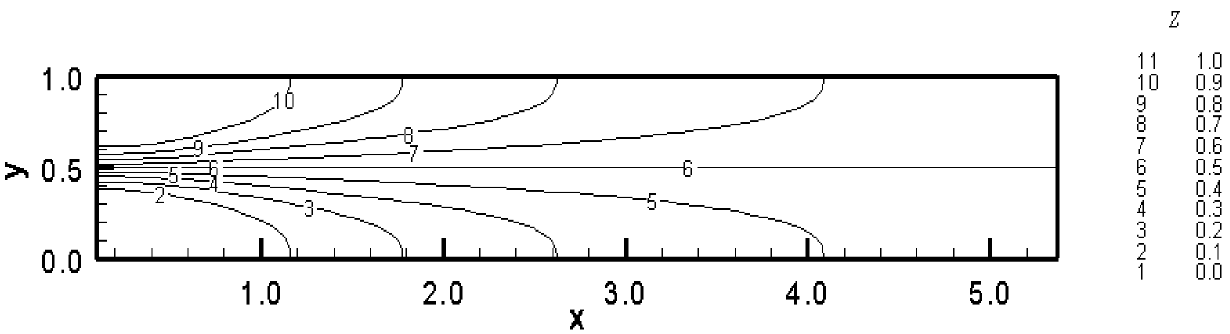

Figure 5. Mixture fraction field of the reference case (Re = 60, $Le.Pe$ = 42, R_b = 1, R_v = 1 and R_{DL} = 0.5).

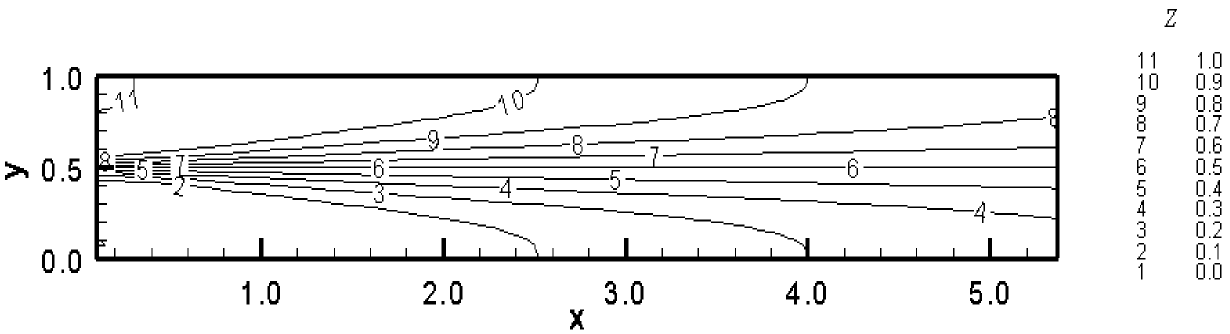

Figure 6. Mixture fraction field of the case with Re = 150 ($Le.Pe$ = 42, R_b = 1, R_v = 1 and R_{DL} = 0.5).

The inlet jet velocities ratio (R_v) is investigated in Fig. 8. When the inlet jet velocities ratio is not equal to one, the amount of mass of the chemical species that enter in the channel is unequal. In consequence, downstream of the channel the mixture fraction equilibrium condition is modified and the value of the mixture fraction is not equal to 0.5. The ideal mixing condition represented by the mixture fraction isoline Z = 0.5 starts from the symmetry plane, deflecting and touching the upper channel wall in vicinity of the x = 2.5 position. So, increasing the inlet jet velocities ratio parameter the ideal mixing condition is obtained in short length of the channel. The mixing process is improved. On the other hand, a higher concentration of the chemical specie that enters through the inferior nozzle is found downstream. The symmetry discussed above is not preserved for $R_v \neq 1$.

The figure 9 shows the streamlines for the R_v = 2 case. Comparing this figure with streamline figure of the reference case (Fig. 4) is verified that the increase in the inlet velocities ratio disturbs the flow and the streamlines. Eddy zones appear in the vicinity of the inferior nozzle. The eddy zone affects the mixture fraction field.

The effects of nozzle thickness ratio parameter (R_b) on the mixing process are presented in Fig. 9. The symmetry of the mixture fraction field observed in Fig. 4-7 disappears when the value of R_b is reduced. Diminishing the value of the R_b parameter the velocity of the inferior jet stream increases, but the amount of mass that enters through the superior and inferior nozzles are equal. In figure 10 a strong deformation of the isoline Z = 0.1 is verified. It is related with the intense eddy zone formed in the vicinity of the inferior nozzle (Fig. 11).

Comparing the isoline Z = 0.5 behavior in Fig. 8 and 10 it is possible to note that the ideal mixing condition line starts from the symmetry plane deflects and touches the upper channel wall in the vicinity of the x = 4 position for R_b = 2 case. Then the ideal mixing condition is improved for both cases depicted in Fig. 8 and 10, but it is better when the R_v parameter is reduced.

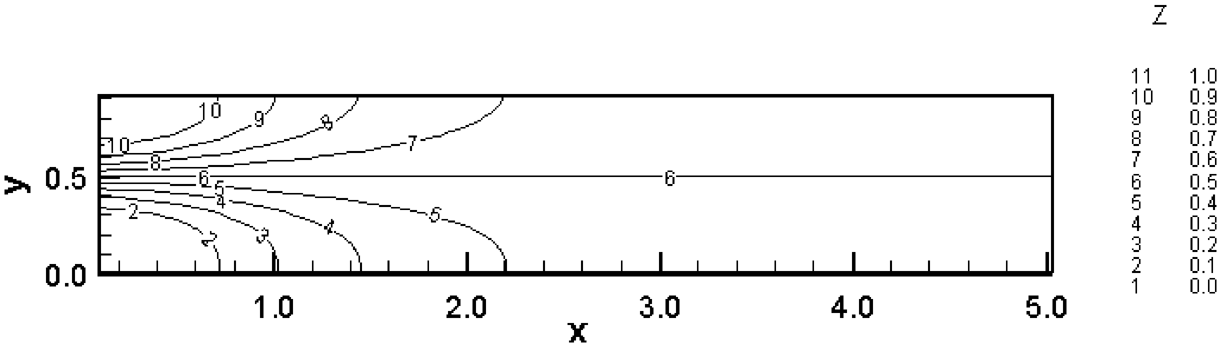

Figure 7. Mixture fraction field of the case with $Le.Pe$ = 21 (Re = 60, R_b = 1, R_v = 1 and R_{DL} = 0.5).

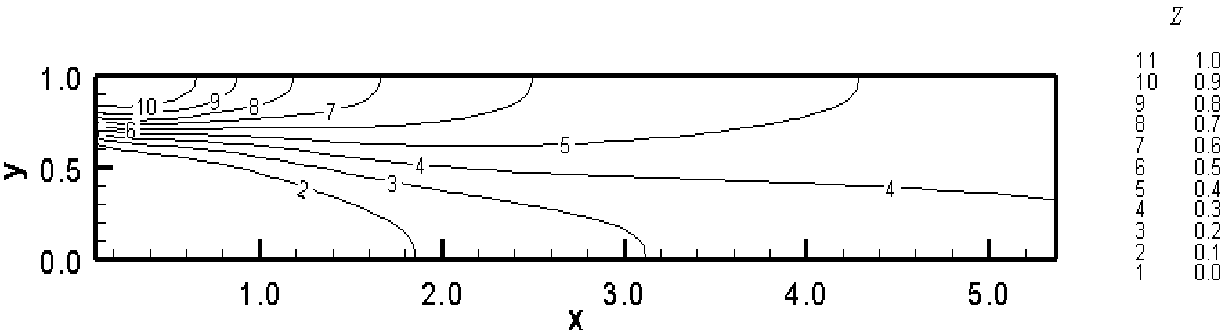

Figure 8. Mixture fraction field of the case with R_v = 2 (Re = 60, Le = 1, Pe = 42, R_b = 1 and R_{DL} = 0.5).

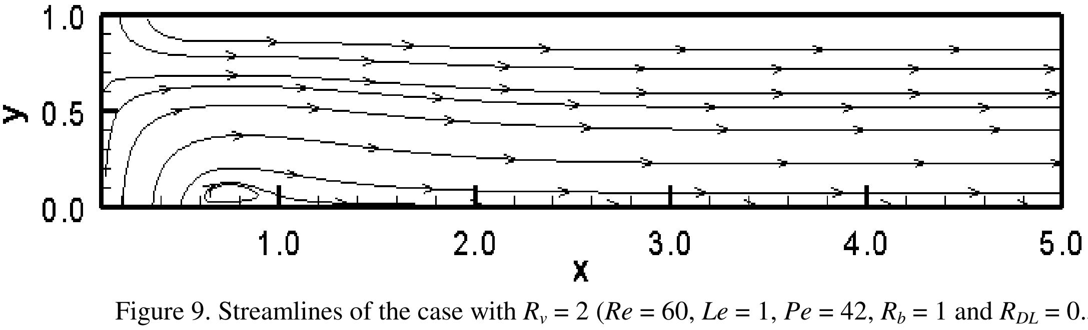

Figure 9. Streamlines of the case with R_v = 2 (Re = 60, Le = 1, Pe = 42, R_b = 1 and R_{DL} = 0.5).

The Fig. 11 shows the streamlines for the R_b = 0.5 case. Comparing this figure with the reference case streamline (Fig. 4), it is verified that the reduction in the nozzle thickness ratio disturbs the flow and the streamlines. A large eddy zone is created in the vicinity of the inferior nozzle. The eddy zone affects the mixture fraction isoline Z = 0.1 as discussed previously.

There is a strong relation between the mixture fraction field and the flow field. In addition, the eddy zones are important phenomena for the mixing process.

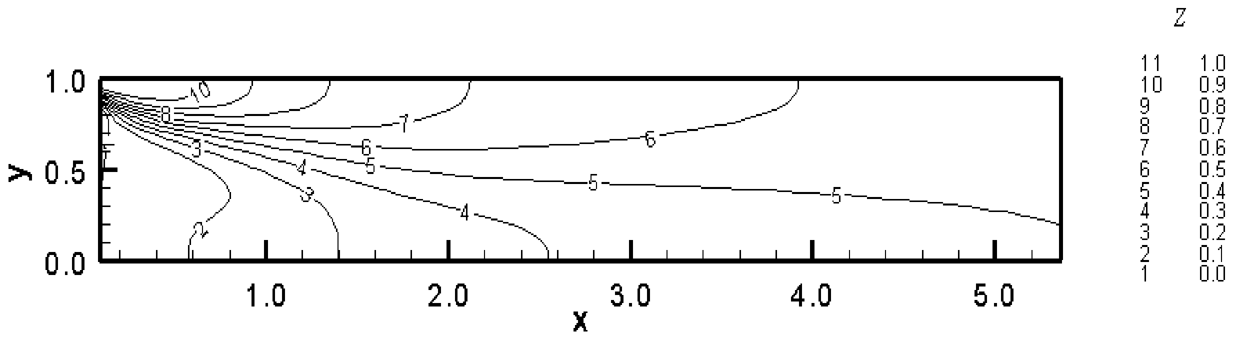

Figure 10. Mixture fraction field of the case with R_b = 0.5 (Re = 60, Le = 1, Pe = 42, R_v = 1 and R_{DL} = 0.5).

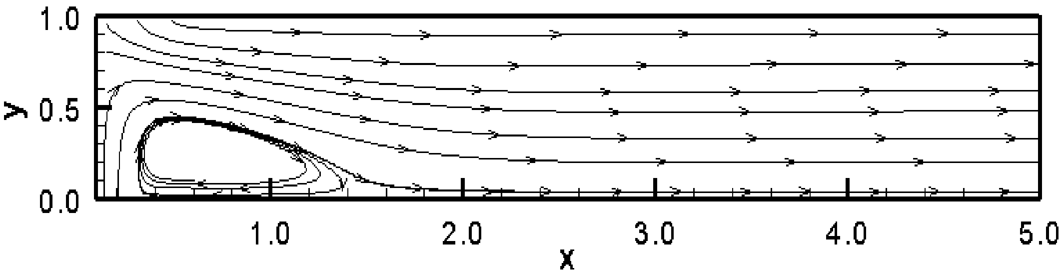

Figure 11. Streamlines of the case with R_b = 0.5 (Re = 60, Le = 1, Pe = 42, R_v = 1 and R_{DL} = 0.5).

5. CONCLUSIONS

The present study shows the influence of physical and geometric parameters on the mixing process in laminar confined opposing jets inside a parallel flat plate channel. The studied physical parameters are Reynolds number, Lewis number, Peclet number and the inlet jet velocities ratio. The geometric parameter is the nozzle thickness ratio. The results show that there is an important relation between the flow field and the mixture fraction field. The presence of intense eddy zones disturbs the mixture fraction field. Furthermore, increasing the Reynolds number the ideal mixing condition is pushed downstream. The ideal mixing condition is obtained in short lengths of the channel when the inlet jet velocities ratio is increased or when the nozzle thickness ratio is reduced. In both cases, intense eddy zones are shown. In these cases, the ideal mixing line touches the channel wall, but for the other studied cases the ideal mixing line build a symmetry plane in the mixture fraction field. It was observed that reducing the Lewis or Peclet number a large region with mixture fraction values near the equilibrium condition is formed in short lengths of the channel, improving the mixing process. The finite volume solution is compared with a reference solution obtained by Roseira Jr (2005) that had employed the vorticity-stream function formulation. No significant discrepancies are observed between present and results of Roseira Jr (2005).

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