

## THE INFLUENCE OF SOIL AND ROTOR DAMPING MECHANISMS ON THE STATIONARY RESPONSE OF A ROTOR-FOUNDATION-SOIL SYSTEM

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**Abstract.** *In this article the stationary response of a 3D rotor-block-foundation-soil system is analyzed. Particular emphasis is placed on the role of distinct damping mechanisms present in the system. The system is a Laval (Jeffcott) rotor with 2 displacement degrees of freedom (DOF) with external damping mechanisms. The foundation is a rigid block interacting with a three-dimensional half-space. Surface and embedded foundations are considered. The soil is a homogeneous and viscoelastic half-space, presenting internal and geometric damping mechanisms. The soil dynamics is characterized by frequency dependent impedance or compliance matrices relating the external excitations to the foundation degrees of freedom. The rigid foundation dynamic compliance matrices are synthesized by a direct version of the Boundary Element Method (DBEM) using the frequency domain, stationary, full-space fundamental solution. In a first analysis the soil behavior is considered to be a spring element with the coefficient of the soil-static solution. This study allows to discuss the effect of soil internal damping as well as rotor internal and external damping mechanisms on the rotor and foundation unbalance response. In a second step the soil geometric damping mechanism is added to the system. This has a major influence on the resonance frequencies and amplitudes. The changes in the geometric damping due to increase in the foundation embedment ratio are addressed. The relative effect of each damping mechanism is discussed in detail.*

**Keywords:** *Dynamic soil-structure interaction, Rotor dynamics, Foundation vibration, Boundary Element Method, Damping mechanisms.*

### 1. INTRODUCTION

Dynamic analysis of rotating machines has recently experienced a great development in methods and research topics [Lalanne and Ferraris, 1990, Childs, 1993], but there is a topic which has not received the attention it may deserve. The rotor supporting structures are bonded to a foundation system and the foundation interacts with the surrounding and supporting soil. This issue, describing the influence of foundation system and surrounding soil on the rotor response has not been intensively studied. Former attempts to describe rotor-foundation-soil systems were limited to very simple soil models, i.e., half-space and surface foundations with no layers [Gasch et al., 1984, Mesquita, 1990]. To describe the dynamics of realistic soil models and soil-foundation systems it is necessary to develop numerical tools which can take into account the so called Sommerfeld radiation condition [Gazetas, 1983]. Physically, the Sommerfeld radiation condition implies that the energy and wave sources are not placed at the infinity, that no energy or wave travels from the outer boundaries into the analyzed system. The Boundary Element Method (BEM) has become an efficient and accurate technique to describe the dynamics of unbounded domains [Manolis and Beskos, 1988, Dominguez 1992]. The BEM can naturally account for the radiation condition and has become the standard numerical technique applied to model the dynamic behaviour of unbounded domains and soils [Beskos, 1987, Beskos 1997].

The present article addresses the dynamic response of a rotor over a block foundation interacting with the supporting soil. Harmonic time behavior is considered leading to a stationary response. The soil is a linear, isotropic and homogeneous media. Material or internal damping is also introduced in the soil by means of the elastic-viscoelastic correspondence principle [Christensen, 1982]. The internal soil damping is considered hysteretic with a frequency independent coefficient [Findley, 1989]. In the present analysis both soil damping mechanisms, material and geometric, are present. Distinct soil profiles are considered: the homogeneous half-space, a horizontal layer over rigid bedrock and a non-horizontal layer also over a rigid bottom. Surface and embedded foundations are analyzed.

The soil dynamic response is given in terms of a dynamic compliance matrix. This dynamic compliance matrix describes the response of a rigid and massless foundation interacting with the prescribed soil profile [Gazetas 1983, Hall&Olivetto 2003]. The compliance matrices for the present article were obtained by a direct version of the Boundary Element Method (DBEM) based on the work of Carrion (2002). Surface and embedded foundations are considered.

Although the complete system (rotor-foundation-soil) is three-dimensional (3D), the analysis presented in this article, that is, the excitations and the determined degrees of freedom, are restricted to a plane parallel to the rotor and transversal to the soil horizontal free surface. The equations of motion of the soil-foundation system, presents a structure with several non-diagonal elements. These off-diagonal elements represent a coupling of the system degree of freedom.

## 2. EQUATIONS OF MOTION OF THE ROTOR-FOUNDATION-SOIL SYSTEM

The complete mechanical rotor-foundation system can be divided in two parts. The first subsystem is a Laval rotor modelled as a viscoelastic beam with round cross section, stiffness  $k^R$ , external damping coefficient  $d_E$ , internal damping coefficient  $d_i$  and a concentrated mass  $m_R$ , centered between the bearings. The rotor is attached to the foundation by rigid (roller) bearings and there is a mass eccentricity  $\varepsilon$ , so that the system is excited by unbalanced forces, (see Figure 1). The rotor presents the circular frequency  $\Omega$ . This rotor model has already been considered by Tondl (1965), Gasch (1975), Gasch et. alli. (1984), Krämer (1993) and Lee (1993).

The second component is a rigid foundation block. The foundation has width  $2a$ , length  $2b$  and height  $h_F$ . The foundation mass centre lies at the distance  $h_G$  from the soil surface, where the origin of the coordinate system is fixed. The foundation presents density  $\rho_F$ , mass  $m_F$  and moment of inertia  $I_{yy}^F$  about the  $y$  axis passing through the mass center. The distance between the foundation mass center and bearings center is given by  $h_B$ , as shown in Figure 1. The foundation is considered to be square with  $a = b$ . The soil is considered to be isotropic and viscoelastic. It is characterized by the shear modulus  $G$ , the density  $\rho$ , the Poisson ratio  $\nu$  and the hysteretic internal damping coefficient  $\eta$  [Barros et. alli, 1999].

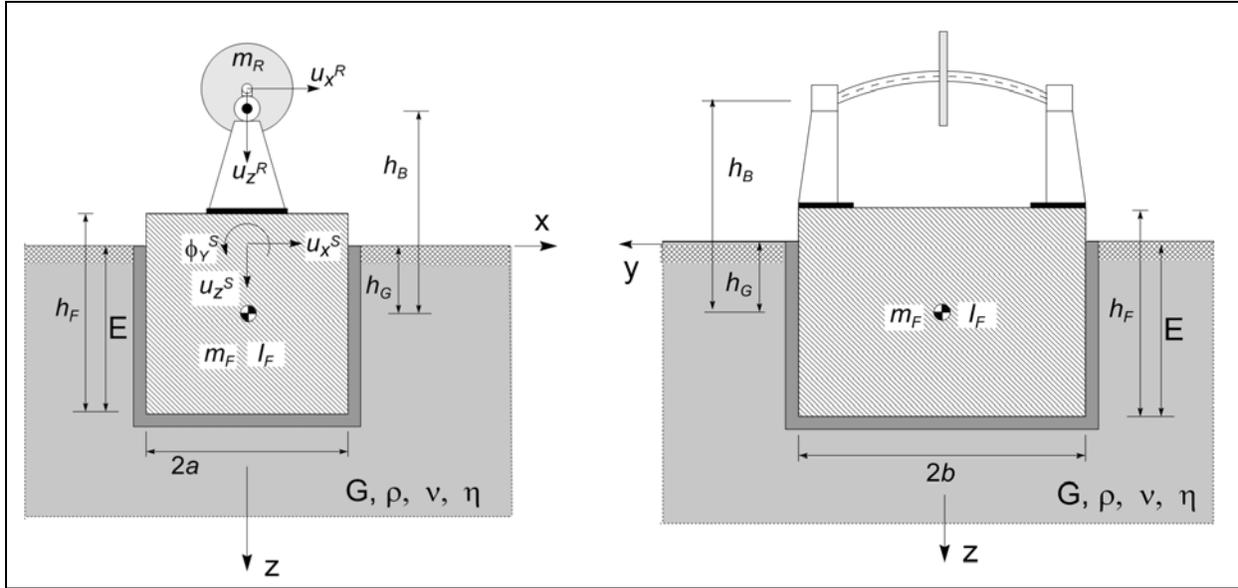


Figure 1. Rotor–foundation–soil system.

In the  $x$ - $z$  plane the degrees of freedom of the rotor are: the horizontal and vertical rotor displacements  $u_X^R$  and  $u_Z^R$  (see figure 1). The degrees of freedom of the block foundation measured with respect to the point S, at the level of the soil surface, are the horizontal and vertical displacement of the foundation and the rocking degree of freedom about the  $y$ -axis, respectively,  $u_X^S$ ,  $u_Z^S$ ,  $\phi_Y^S$ . For this system the equations of motion are [Gasch et alli, 1984, Sousa, 2007]:

$$[-\Omega^2[M] + i\Omega[C] + [K]] \begin{Bmatrix} u_z^R \\ u_X^R \\ u_z^S \\ u_X^S \\ \phi_Y^S \end{Bmatrix} = \begin{Bmatrix} m_R \varepsilon \Omega^2 \\ i m_R \varepsilon \Omega^2 \\ F_z^S \\ F_x^S \\ M_y^S \end{Bmatrix} \quad (1)$$

In the equation (1), the matrix  $[M]$  represents the system inertia,  $[C]$  the damping matrix and  $[K]$  the rigidity. The imaginary unit is  $i^2 = -1$ . The vector containing the resulting components of the generalized soil forces is  $\{F^S\} = \{F_Z^S \ F_x^S \ M_y^S\}^T$ . A vector containing the foundation DOFs may also be defined:  $\{U^S\} = \{u_Z^S \ u_x^S \ \phi_y^S\}^T$ . The detailed structures of the matrices are [Gasch et alli, 1984, Sousa, 2007]:

$$[M] = \begin{bmatrix} m_R & 0 & 0 & 0 & 0 \\ 0 & m_R & 0 & 0 & 0 \\ 0 & 0 & m_F & 0 & 0 \\ 0 & 0 & 0 & m_F & m_F h_G \\ 0 & 0 & 0 & m_F h_G & I_{yy}^F + m_F h_G^2 \end{bmatrix} \quad (2)$$

$$[C] = \begin{bmatrix} (d_I + d_E) & 0 & -(d_I + d_E) & 0 & 0 \\ 0 & (d_I + d_E) & 0 & -(d_I + d_E) & -(d_I + d_E)(h_B + h_G) \\ -(d_I + d_E) & 0 & (d_I + d_E) & 0 & 0 \\ 0 & -(d_I + d_E) & 0 & (d_I + d_E) & (d_I + d_E)(h_B + h_G) \\ 0 & -(d_I + d_E)(h_B + h_G) & 0 & (d_I + d_E)(h_B + h_G) & (d_I + d_E)(h_B + h_G)^2 \end{bmatrix} \quad (3)$$

$$[K] = \begin{bmatrix} k^R & -d_I \Omega & -k^R & d_I \Omega & d_I \Omega (h_B + h_G) \\ d_I \Omega & k^R & -d_I \Omega & -k^R & -k^R (h_B + h_G) \\ -k^R & d_I \Omega & k^R & -d_I \Omega & -d_I \Omega (h_B + h_G) \\ -d_I \Omega & -k^R & d_I \Omega & k^R & k^R (h_B + h_G) \\ -d_I \Omega (h_B + h_G) & -k^R (h_B + h_G) & d_I \Omega (h_B + h_G) & k^R (h_B + h_G) & k^R (h_B + h_G)^2 \end{bmatrix} \quad (4)$$

### 3. BOUNDARY ELEMENT SYNTHESIS OF THE RIGID FOUNDATION COMPLIANCE MATRIX

This section reports the synthesis of the dynamic compliance  $[N]$  and impedance  $[S]$  matrices for rigid and massless foundations. Inertia properties of the block foundations are incorporated through equations (1) to (4). The direct version of the Boundary Element Method (DBEM) is used to model and solve the stationary dynamic soil-structure interaction problem [Carrion 2002, Dominguez 1995]. The soil, discretized by the BEM, leads to the system of linear algebraic equations, that in matrix form may be expressed as:

$$[H]\{u\} = [G]\{T\} \quad (5)$$

In Eq. (5) the vectors  $\{u\}$  and  $\{t\}$  represent, respectively, the displacements and the tractions on the BE nodes. These vectors can be divided into soil-foundation interface nodes  $\{u_f\}$  and  $\{t_f\}$  and the remaining soil nodes  $\{u_s\}$  and  $\{t_s\}$ , with  $\{u\} = \{u_f\} \cup \{u_s\}$  and  $\{t\} = \{t_f\} \cup \{t_s\}$ . The influence matrices  $[H]$  and  $[G]$  are result from the numerical integration, over the area of each Boundary Element, of the fundamental solutions  $t_{ij}^*$  e  $u_{ij}^*$  multiplied by the interpolation functions and the proper Jacobian [Carrion 2002, Dominguez 1995]. After the matrices  $[H]$  and  $[G]$  have been synthesized, rigid body kinematics compatibility restrictions  $[CC]$  may be applied between the nodes of the soil foundation interface  $\{u_f\}$  and the vector of the rigid foundation degrees of freedom  $\{U^S\}$ . Analogously, equilibrium equations  $[EQ]$  may be applied between the tractions at the nodes of the soil foundation interface  $\{t_f\}$  and the vector of the external excitation  $\{F^S\}$  leading to [Carrion, 2002]:

$$\{u_f\} = [CC]\{U^S\} \quad \text{and} \quad \{F^S\} = [EQ]\{t_f\} \quad (6)$$

To synthesize the rigid and massless foundation dynamic compliance matrix, additional boundary conditions must be prescribed. For deep soil profiles, which can be modeled as a half-space, usually it is assumed that the tractions at the soil free surface vanish,  $\{t_s\} = \{0\}$ . Under these assumptions equation (5). and (6) may be combined to yield:

$$\begin{bmatrix} [H_{ff}] & [CC] & [H_{fs}] & [-G_{ff}] \\ [H_{sf}] & [CC] & [H_{ss}] & [-G_{sf}] \\ [0] & [0] & [EQ] & \end{bmatrix} = \begin{Bmatrix} \{U^S\} \\ \{u_s\} \\ \{t_f\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{F^S\} \end{Bmatrix} \quad (7)$$

Equation (7) delivers the rigid foundation degrees of freedom (DOF)  $\{U^S\}$ , the displacements at the half-space surface  $\{u_s\}$  an the tractions at the soil-foundation interface  $\{t_f\}$ . It may be used to synthesize a stationary frequency dependent compliance matrix  $[S(A_0)]$  for the rigid foundation, relating  $\{U^S\}$  to the foundation to the vector containing external forces applied at the foundation  $\{F^S\}$ :

$$\begin{Bmatrix} F_Z^S \\ F_x^S \\ M_y^S \end{Bmatrix} = Ga \begin{bmatrix} S_{UzFz} & S_{UzFx} & S_{UzMy} \\ S_{UxFz} & S_{UxFx} & S_{UxMy} \\ S_{\Phi yFz} & S_{\Phi yFx} & S_{\Phi yMy} \end{bmatrix} \begin{Bmatrix} u_z^S \\ u_x^S \\ \phi_y^S \end{Bmatrix} \quad (8)$$

The vector of the generalized forces acting on the rigid foundation  $\{F^S\}$  shown in equation (1) may now be substituted by the impedance relation  $Ga[S(A_0)]\{U^S\}$  given in equation (8), leading to a complete system describing the dynamic response of the rotor-foundation-soil due to a rotor unbalance response with eccentricity  $\varepsilon$ . The dimensionless frequency parameter  $A_0$  is defined by:  $A_0 = \Omega a / c_s$ , with  $c_s$  being the shear wave velocity of the elastic soil. For other soil profiles, like a layer over rigid base, the resulting equations may be found in Thomazo et. alli (2007).

#### 4. STRUCTURE AND FREQUENCY CONTENT OF THE SOIL IMPEDANCE MATRICES

In this section both the structure of the impedance matrix and the frequency content of its elements will be addressed. Three distinct soil profiles are described, namely, the half-space, a layer over horizontal rigid bottom and a layer over inclined rigid bedrock. Constant rectangular BE elements were used throughout the present study.

Figures 2a to 2c show the soil profiles that have been used in the present investigation, respectively, a) the half-space, b) the horizontal layer over a rigid base and c) a non-horizontal layer over a rigid base. The same figure also shows the resulting structure of the compliance matrix. Notice that for the half-space and for the horizontal layer, the vertical degree of freedom (DOF) is uncoupled from the horizontal and rocking DOFs. On the other hand, the non horizontal layer yields a fully populated matrix, indicating that all DOFs in the plane of analysis are coupled. Moreover, it can be shown that for surface foundations over the half-space and over a horizontal layer the compliance matrices are symmetric.

The Boundary Element meshes employed to synthesize the compliance matrices for each soil profile are shown in figures 3a to 3b. These figures also indicate the number of elements used to discretize de soil-foundation interface,  $n_f$  and the number of elements used to discretize the soil free surface and the rigid base (for the case of layers)  $n_s$ . For all simulations considered, the soil free surface was discretized within the range  $x \leq |5a|$  and  $y \leq |5a|$ . The constitutive parameters used in the numerical synthesis are foundation half width  $a = 1\text{m}$ , soil shear modulus  $G = 1\text{N/m}^2$ , Poisson ratio of the soil  $\nu = 0.25$  and soil density  $\rho = 1\text{kg/m}^3$ .

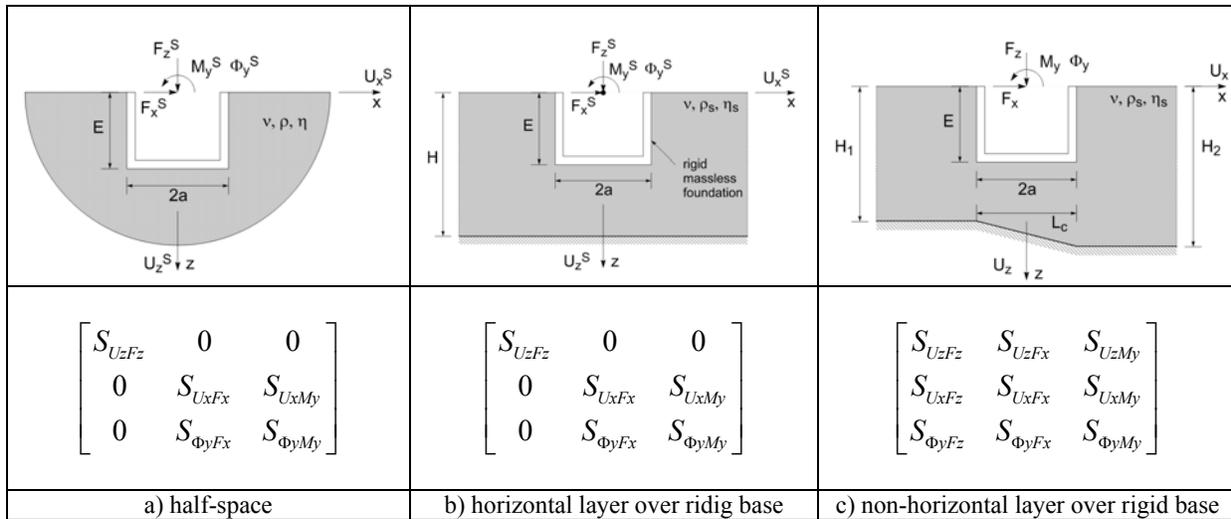


Figure 2: Analyzed soil profiles and resulting structure of rigid foundation impedance matrices

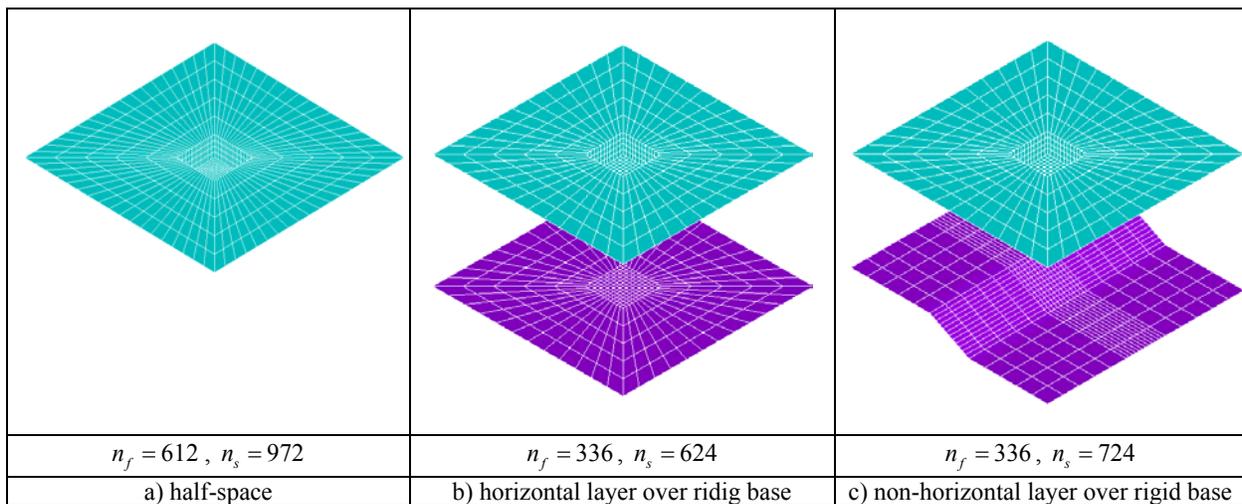


Figure 3: BE meshes used to synthesize the compliance matrices for embedded rigid foundations

Figures 4a and 4b show, respectively, real and imaginary parts of the vertical  $N_{UzFz}(A_0)$  and horizontal  $N_{UxFx}(A_0)$  compliance functions for rigid foundations with an embedment ratio  $E=a$ , for distinct values of the soil hysteretic damping coefficient  $\eta = 0.0$  and  $\eta = 0.05$ . The complete set of results can be found in [Sousa, 2007].

Figures 5a and 5b show the real and imaginary parts of the vertical compliance function  $N_{UzFz}(A_0)$  for foundations embedded in a horizontal and a non-horizontal layer, respectively. For the horizontal layer the constant thickness, the depth is  $H = 5a$ , see figure 2b. For the non-horizontal layer the geometric parameters are:  $H_1 = 5a$ ,  $H_2 = 6a$  and  $L_c = 2a$ , see figure 2c. The internal hysteretic damping coefficient  $\eta = 0.05$  was employed. The remaining constitutive parameters are the same used to determine the previous results of figure 4.

The key issue when addressing the compliance functions of the layers is that they present a limited dimension,  $H$ . This fact has profound implications in the dynamic behavior of the domain. Layers of finite depth present resonances which are clearly visible in figures 5a and 5b. A comparison between half-space solutions (figure 4a) and solutions for layers (figs. 5a and 5b) show the dramatic change in the soil response due to distinct profiles. These comparisons anticipate the influence of soil profile on the dynamic behavior of the rotor-foundation soil system.

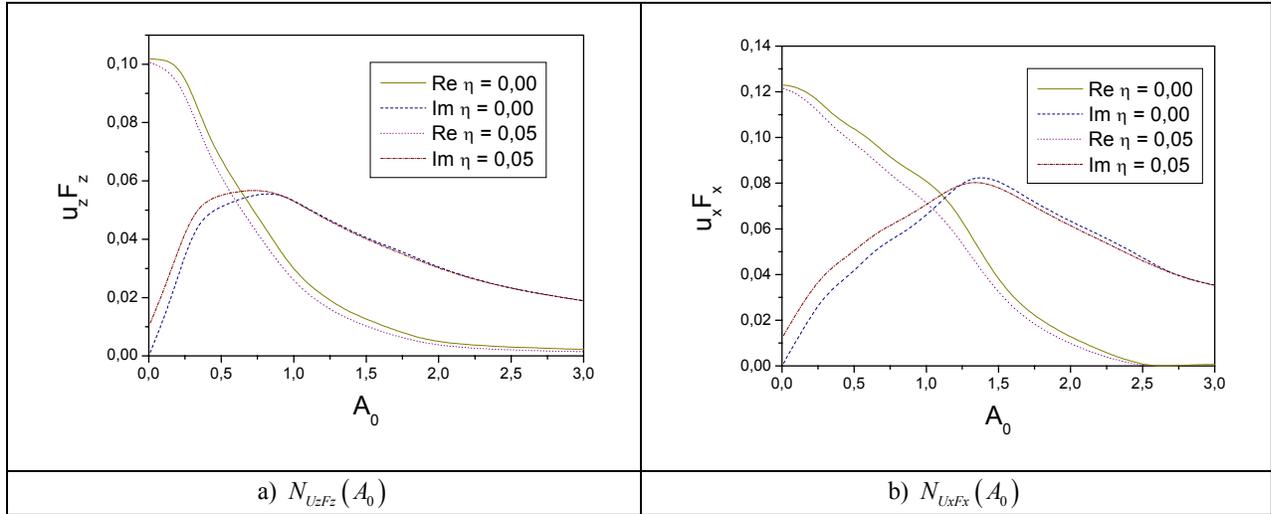


Figure 4: Vertical  $N_{U_z F_z}(A_0)$  and horizontal  $N_{U_x F_x}(A_0)$  compliance functions rigid foundation embedded in a half-space for distinct soil internal damping coefficients

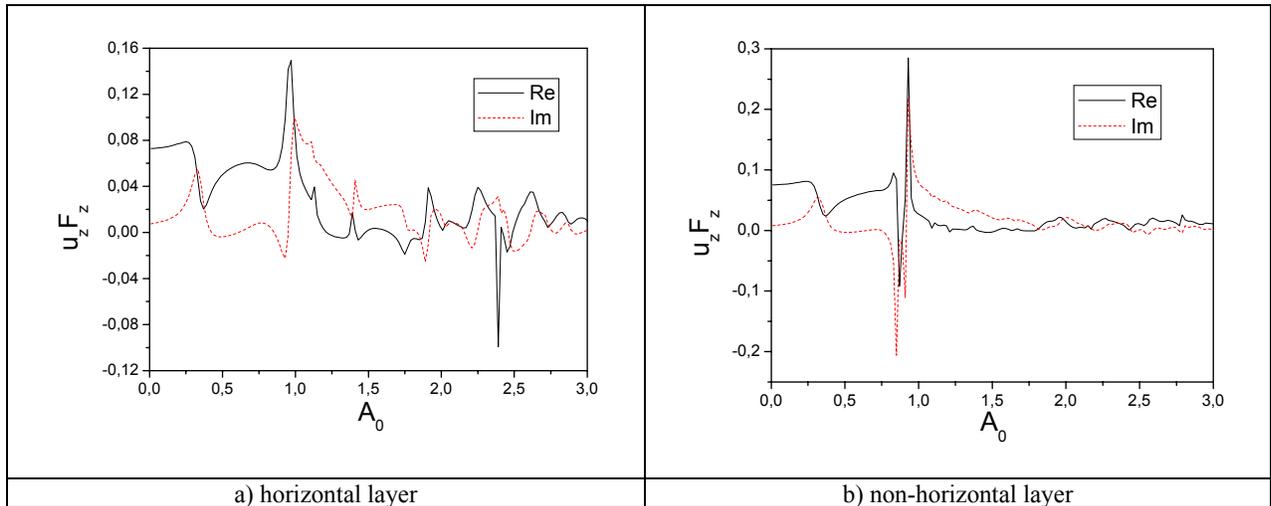


Figure 5: Vertical compliance functions  $N_{U_z F_z}(A_0)$  for rigid embedded foundation interacting with horizontal and non-horizontal layer over rigid base.

## 5. STATIONARY DYNAMIC RESPONSE OF ROTORS AND FOUNDATION INTERACTING WITH SOIL

### 5.1. Influence of the geometric and material damping.

As already described there are 4 damping mechanisms present in the analyzed system. The first mechanism is the soil geometric damping, associated to the Sommerfeld radiation condition. The second mechanism is the soil internal damping, characterized by the hysteretic damping coefficient of the soil  $\eta$ . The remaining two mechanisms are the internal and external damping of the rotor, characterized by the parameters  $d_E$  and  $d_I$  in equations (3) and (4).

Initially the role of the geometric damping on the rotor and foundation response is studied. For this purpose the soil model is substituted by coefficient of the static solution, that is, by the real part of the compliance functions for a vanishing frequency,  $N_{ij}(A_0 \rightarrow 0)$ . The rotor damping coefficients are also set equal to zero. Figures 6a and 6b show, respectively, the vertical response of the rotor  $u_z^R$  and foundation  $u_z^S$  embedded in a half-space  $E = a$ . Two situations

are examined. In the first case the soil response is a spring with the static coefficient of the corresponding compliance function. In the second case the frequency dependent soil solution (impedances) are included. The parameters used in the calculations are:  $\rho_F / \rho = 2.5$ ,  $m_R / m_F = 0.1$ ,  $h_F = 2a$ ,  $h_G = 0$ ,  $h_B = a$ ,  $d_E = d_I = 0$  and  $\eta = 0.05$ . An analysis of the results show clearly the two resonances related to the vertical degrees of freedom of rotor and foundation. The geometric damping has a far larger effect on the lower resonance, controlled by the foundation mass, than on the higher resonance, controlled by the rotor inertial properties.

**5.2. Influence of the external rotor damping..**

In the sequence the effect of the external ( $d_E$ ) rotor damping is investigated. Figures 7a and 7b show the rotor  $u_Z^R$  and foundation  $u_Z^S$  vertical response for the case of a foundation embedded in a half-space,  $E = a$ . Now the rotor external damping coefficient is changed from the purely elastic case ( $d_E = 0.0$ ) to a large damping value ( $d_E = 0.2$ ). For this study the internal rotor damping coefficient is set equal to zero  $d_I = 0.0$ . The remaining parameters are equal to those of the previous analysis. These figures show very clearly that the external rotor damping causes a strong influence on the resonance which can be associated to or controlled by the rotor properties. This effect is quite opposite to the influence of the geometric damping, which affected primarily the lower resonance controlled by the foundation inertia.

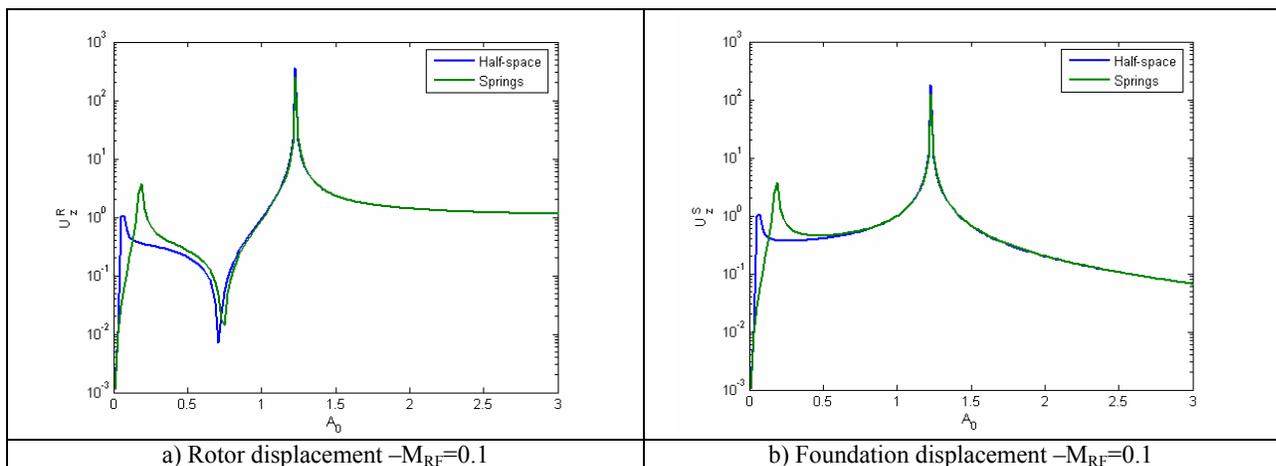


Figure 6: Vertical rotor and embedded foundation response- effect of the geometric damping

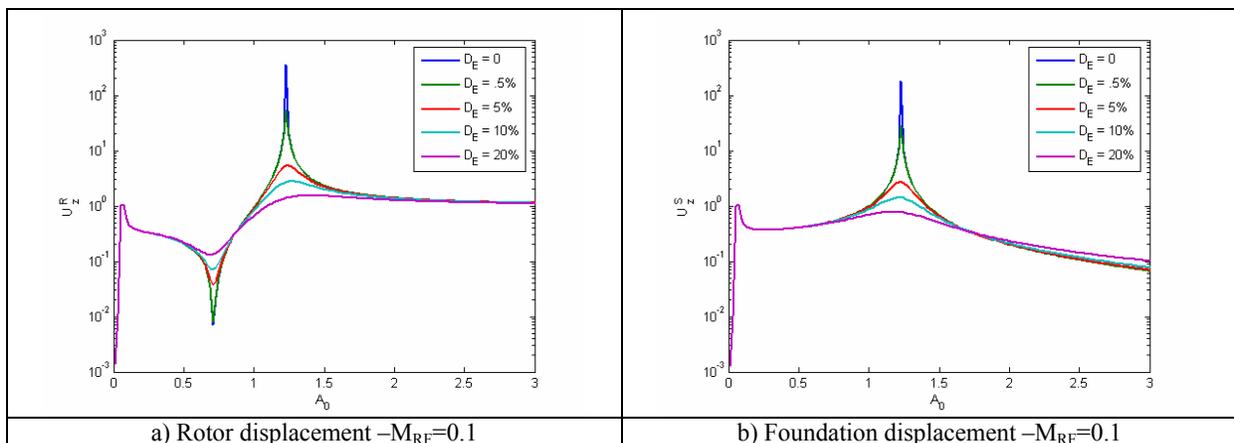


Figure 7: Vertical response of a rotor and embedded foundation response- effect of the rotor external damping

### 5.3. Influence of the soil internal damping coefficient.

In this section the influence of the soil internal or material damping coefficient is addressed. The soil profile is a horizontal layer with constant depth  $H = 5a$  (see figure 2b). The vertical compliance function for this soil profile was given in figura 5a. For this analysis the rotor internal and external damping coefficients are set equal to zero,  $d_E = d_I = 0$ . The parameters used in the calculations are:  $\rho_F / \rho = 2.5$ ,  $m_R / m_F = 0.1$ ,  $h_F = 2a$ ,  $h_G = 0$  and  $h_B = 0$ . The absolute value of the vertical displacement for rotor and foundation over this soil profile, for distinct values of the soils material or internal damping coefficient  $\eta$ , are given in figures 8a and 8b. .

The influence of the internal soil mechanism is not able to displace significantly the resonances present in the response. But the internal soil damping coefficient has a marked influence on the rotor response, being even able to suppress some resonances. The effect of the internal damping on the foundation dynamic is to smooth significantly the response, almost eliminating the many peaks associated to the wave reflections at the fixed layer base. So it is clear that every damping mechanism act in a different form on the system response.

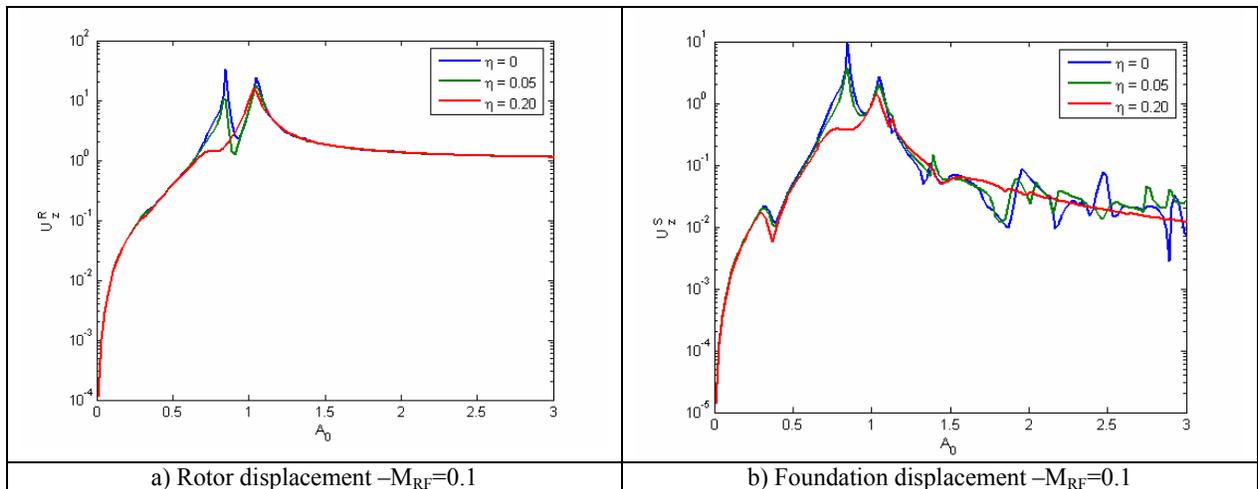


Figure 8: Vertical response of a rotor and foundation embedded in a horizontal layer for distinct values of the soil damping coefficient  $\eta$

### 5.4. Influence of non-horizontal soil profile.

The last numerical study in the present article discusses the role of the soil profile and the embedment ratio on the system response. A non-horizontal soil profile is considered, see figure 2c.

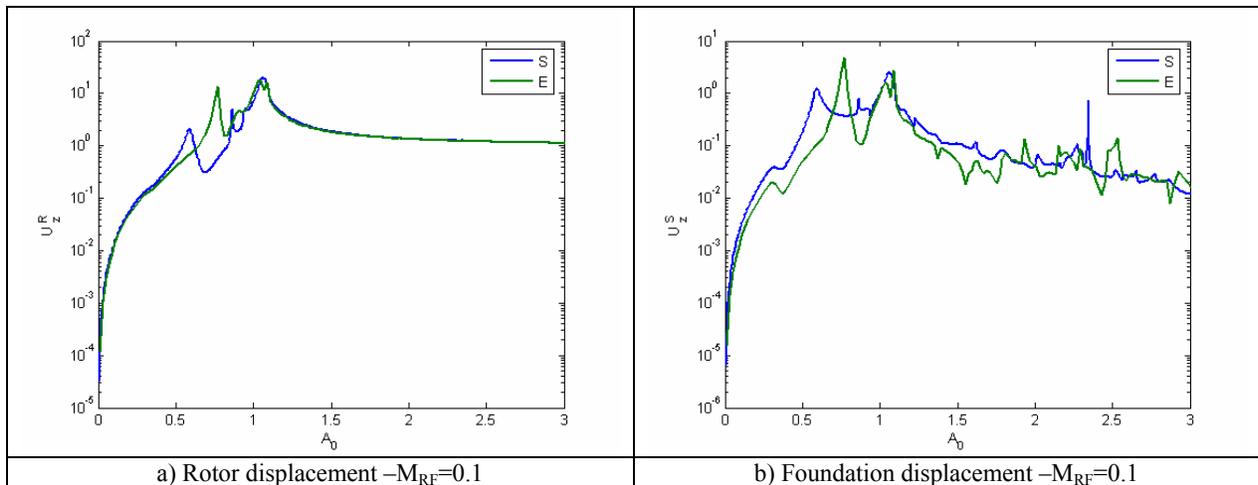


Figure 9: Vertical response of a rotor for a surface (S) and a foundation embedded (E) in a non-horizontal layer

The geometric parameters are:  $H_1 = 5a$ ,  $H_2 = 6a$ ,  $L_C = 2a$ . The rotor-foundation-soil parameters are:  $\rho_f / \rho = 2.5$ ,  $m_R / m_F = 0.1$ ,  $h_F = 2a$ ,  $h_G = 0.5a$ ,  $h_B = 0$  and  $\eta = 0.05$ . There is no rotor damping, internal or external,  $d_E = d_I = 0$ . The vertical compliance functions for this non-horizontal layer is given in figure 5b.

Figures 9a and 9b investigate the role of the foundation embedment ratio on the response. The vertical response for the surface foundation is indicated by the letter (S), while the response of the embedded foundation is indicated by (E). An analysis reveals that embedment may displace the resonances of the rotor and foundation degrees of freedom. Furthermore, the layer with an inclined bottom induces a much more complex dynamic behavior, mainly due to the complex wave propagation and reflection phenomena and due to the fully populated character of the resulting soil compliance/impedance matrix

## 6. CONCLUDING REMARKS

The present article discussed some significant issues relating the dynamic response of a rotor-foundations-soil system. The stationary response of rotor and foundation were addressed. The system presents 3 damping mechanisms, namely the soil geometric damping, the soil internal or material damping and the external damping of the rotor. The article also addressed the role of the soil profile on the system response. The soil response included surface and embedded foundations. The foundations could interact with a half-space or with a layer over rigid bottom. The bottom of the layer could be horizontal and non-horizontal. The soil profile has an influence on the structure of the rigid foundation compliance matrix. Elastic and hysteretic viscoelastic soil domains were addressed. The soil response was determined by means of a direct version of the Boundary Element Method (DBEM).

Through a series of numerical simulations it could be shown that every damping mechanism plays a distinct role on the dynamic response of the rotor-foundation-soil system. Soil geometric damping tend to influence the lower resonances, related to the foundation dynamics. The soil internal damping mechanism does not displace significantly the system resonances, but it can smooth peaks arising from the limited soil profiles. The external rotor damping tends to smooth monotonically the resonance associated or controlled by the rotor.

The article shows only a few case studies. But it shows clearly a set of numerical tools which allow the analyst to perform engineering analysis of complex systems including rotors, foundations and complex soil structures and profiles.

## 7. ACKNOWLEDGEMENTS

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## 9. RESPONSIBILITY NOTICE

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