THE INFLUENCE OF SOIL DAMPING MECHANISMS AND GEO-PROFILES ON THE STATIONARY RESPONSE OF 3D RIGID BLOCK FOUNDATIONS

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Abstract. *The article addresses the stationary dynamic response of three-dimensional rigid block foundations interacting with distinct soil profiles. Dynamic compliance matrices for rigid and massless foundations interacting with a half-space, a horizontal layer and a non-horizontal layer are reported. A direct version of the Boundary Element Method is applied to synthesize the soil response. A sample of compliance functions for complex soil profiles are furnished. The soil profile and the foundation arrangement determine the structure of the compliance matrix. For half-spaces and horizontal layers the vertical degree of freedom is uncoupled from the other DOFs. For non-horizontal layers the compliance matrix is full, coupling all foundation DOFs in the plane of analysis. The presence of a layer with finite dimensions introduces resonances, which are noticeable in the compliance matrices and in the frequency response functions of the block foundations. The layer rigid bottom changes dramatically the foundation response, when compared to foundations interacting with homogenous half-spaces. For half-spaces the geometric damping mechanism plays the dominant role. Material damping has a secondary effect in the dynamic response of rigid structures interacting with half-spaces.*

Keywords*: Dynamic soil-structure interaction, Rotor dynamics, Foundation vibration, Boundary Element Method.*

1. INTRODUCTION

The present article addresses the dynamic response of a block foundation interacting with the supporting soil. Harmonic time behavior is considered leading to a stationary response. Both the foundation and the soil model are three-dimensional. The block foundation is considered rigid. The soil is a linear, isotropic and homogeneous media. Material or internal damping is also introduced in the soil by means of the elastic-viscoelastic correspondence principle [Christensen, 1982]. Distinct soil profiles are considered: the homogeneous half-space, a horizontal layer over a rigid bedrock and a non-horizontal layer also over a rigid bottom. As the soil profiles present at least one unlimited dimension, waves that are generated at the soil-block foundation interface propagate without reflection, withdrawing energy from the system. This damping mechanism is known as geometric damping and the mathematical expression describing this mechanism is the Sommerfeld radiation condition [Sommerfeld, 1949, Hall&Olivetto, 2003]. So in the present analysis both damping mechanisms, material and geometric, are present. The internal damping is considered hysteretic with a frequency independent coefficient [Findley, 1989].

The soil dynamic response is given in terms of a dynamic compliance matrix. This dynamic compliance matrix describes the response of a rigid and massless foundation interacting with the prescribed soil profile [Gazetas 1983, Hall&Olivetto 2003]. The compliance matrices for the present article were obtained by a direct version of the Boundary Element Method (DBEM) based on the work of Carrion (2002). Surface and embedded foundations are considered.

Although the complete system, foundation and soil, are three-dimensional, the analysis presented in this article, that is, the excitations and the determined degrees of freedom, are restricted to a plane transversal to the soil horizontal free surface. The equations of motion of the soil-foundation system, presents a structure with several non-diagonal elements. These off-diagonal elements represent a coupling of the system degrees of freedom.

2. EQUATIONS OF MOTION OF THE SOIL-FOUNDATION SYSTEM

Figure 1 shows the plane (*x*-*z*) with a scheme for a rigid foundation embedded in a layer over a rigid bottom. The rigid foundation has dimensions (2*a* x 2*b* x h_F) with mass m_F. The vertical distance between the foundation mass center and the origin of the coordinate system (*x,y,z*) is given by h_G. The external excitation vector $\{F_E\}=\{F_Z \quad F_X \quad F_Y \quad M_Z \quad M_X \quad M_Y\}^T$ is applied at a distance h_B from the foundation mass center. The foundation embedment ratio is *E*. The response of the rigid and massless foundation is given with respect to the origin of the coordinate system, shown in Fig. 2. With respect to this point the vector of the soil excitation is $\{F^S\}=\{F_Z^S \quad F_X^S \quad F_Y^S \quad M_Z^S \quad M_X^S \quad M_Y^S\}^T$ and the vector containing the rigid foundation degrees of freedom is $\{U^S\}=\{U_Z^S \quad U_X^S \quad U_Y^S \quad \Phi_Z^S \quad \Phi_X^S \quad \Phi_Y^S\}^T$.

The equations of motion of the soil-foundation system excited by a circular frequency ω, considering only the degrees of freedom in the (*x-z*) plane, is given by [Sousa, 2007]:

$$\left(-\omega^2\begin{bmatrix} m_F & 0 & 0 \\ 0 & m_F & m_F h_G \\ 0 & m_F h_G & I_{xx}^G + m_F {h_G}^2 \end{bmatrix} + Ga\begin{bmatrix} S_{UzFz} & S_{UzFx} & S_{UzMy} \\ S_{UxFz} & S_{UxFx} & S_{UxMy} \\ S_{\Phi yFz} & S_{\Phi yFx} & S_{\Phi yMy} \end{bmatrix}\right)\begin{Bmatrix} U_z^S \\ U_x^S \\ \Phi_y^S \end{Bmatrix} = \begin{Bmatrix} F_z \\ F_x \\ M_y + (h_B + h_G)F_x \end{Bmatrix} \tag{1}$$

In Eq. (1) G represents the shear modulus of the soil and a the half-width of the foundation. The terms S_{ij} are the elements of the soil compliance matrix. The synthesis of these functions will be addressed in the next section.

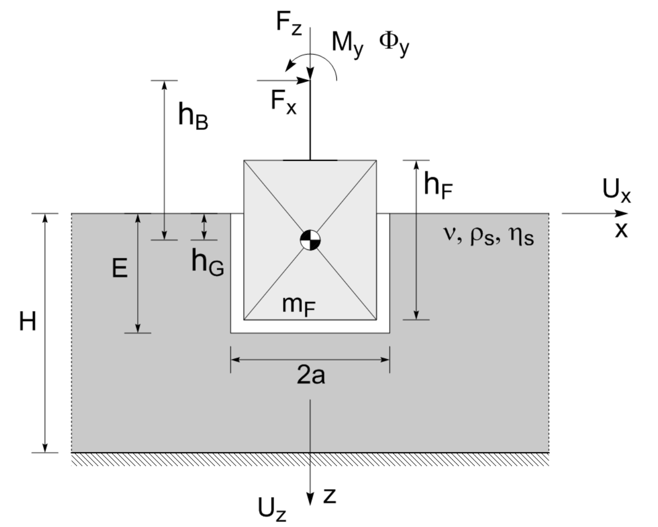


Figure 1: Scheme for a rigid block foundation embedded in an horizontal layer resting on a rigid bottom

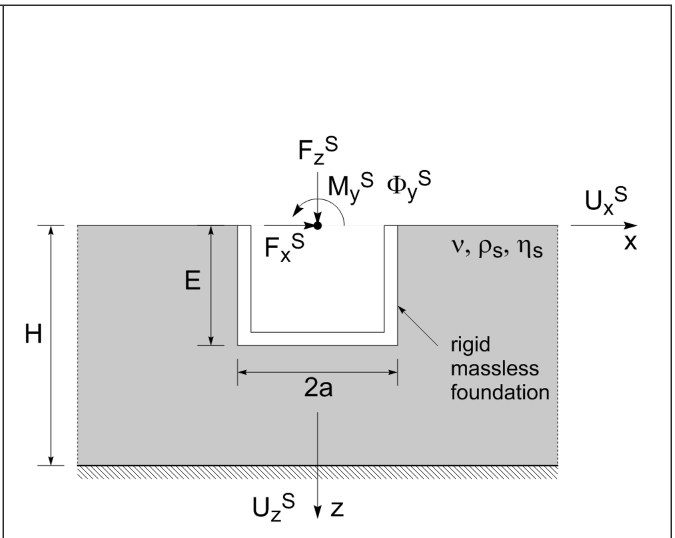


Figure 2: Excitations and degrees of freedom for a rigid and massless foundation

3. BOUNDARY ELEMENT SYNTHESIS OF THE RIGID FOUNDATION COMPLIANCE MATRIX

This section reports the synthesis of the dynamic compliance matrices for rigid and massless foundations. Inertia properties of the block foundations are incorporated through Eq. (1). The direct version of the Boundary Element Method (DBEM) is used to model and solve the stationary dynamic soil-structure interaction problem [Carrion 2002, Dominguez 1995]. The soil, discretized by the BEM, leads to the system of linear algebraic equations, that in matrix form may be expressed as:

$$H_{ij}\,u_j = G_{ij}\,t_j \tag{2}$$

In Eq. (2) H_{ij} and G_{ij} are the influence matrices resulting from the numerical integration over the area of each Boundary Element of the fundamental solutions t_{ij}^* e u_{ij}^* multiplied by the interpolation functions and the proper Jacobian [Carrion 2002, Dominguez 1995]. Dividing the discretized soil boundary into nodes pertaining to the soil foundation interface $\{u_f\}$ and the remaining nodes $\{u_s\}$, the matrix Eq. (2) may be expanded to yield:

$$\begin{bmatrix} H_{ff} & H_{fs} \\ H_{sf} & H_{ss} \end{bmatrix}\begin{Bmatrix} u_f \\ u_s \end{Bmatrix} = \begin{bmatrix} G_{ff} & G_{fs} \\ G_{sf} & G_{ss} \end{bmatrix}\begin{Bmatrix} t_f \\ t_s \end{Bmatrix} \tag{3}$$

After the matrices H_{ij} and G_{ij} have been synthesized, rigid body kinematics compatibility restrictions $[CC]$ may be applied between the nodes of the soil foundation interface $\{u_f\}$ and the vector of the rigid foundation degrees of freedom $\{U^S\}$. Analogously, equilibrium equations $[EQ]$ may be applied between the tractions at the nodes of the soil foundation interface $\{t_f\}$ and the vector of the external excitation $\{F^S\}$ leading to [Carrion, 2002]:

$$\{u_f\} = [CC]\{U^S\} \text{ and } \{F^S\} = [EQ]\{t_f\} \tag{4}$$

To synthesize the rigid and massless foundation dynamic compliance matrix, additional boundary conditions must be prescribed. Usually it is assumed that the tractions at the soil free surface vanish $\{t_s\} = \{0\}$. Under these assumptions, Eqs. (3) and (4) may be combined to yield:

$$\begin{bmatrix} [H_{ff}][CC] & [H_{fs}] & [-G_{ff}] \\ [H_{sf}][CC] & [H_{ss}] & [-G_{sf}] \\ [0] & [0] & [EQ] \end{bmatrix} = \begin{Bmatrix} \{U^S\} \\ \{u_s\} \\ \{t_f\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{F^S\} \end{Bmatrix} \tag{5}$$

Equation (5) may be used to synthesize a stationary frequency dependent compliance matrix $[N(\omega)]$ for the rigid foundation, relating the foundation degrees of freedom (DOF) $\{U^S\}$ to the vector containing external forces applied at the foundation $\{F^S\}$:

$$\{U^S\} = \frac{1}{Ga}[N(\omega)]\{F^S\} \tag{6}$$

The rows of matrix $[N]$ may be obtained from the solution $\{U^S\}$ of Eq. (5) for a sequence of unit values for the distinct components of the load vector $\{F^S\}$. This procedure furnishes the structure as well as the frequency content of the compliance matrix elements. Equation (7) furnishes the resulting compliance matrix [N] and the impedance [S] = [N]$^{-1}$ for the excitations and degrees of freedom in the plane (x-z):

$$\begin{Bmatrix} U_Z^S \\ U_x^S \\ \Phi_y^S \end{Bmatrix} = \frac{1}{Ga} \begin{bmatrix} N_{UzFz} & N_{UzFx} & N_{UzMy} \\ N_{UxFz} & N_{UxFx} & N_{UxMy} \\ N_{\Phi yFz} & N_{\Phi yFx} & N_{\Phi yMy} \end{bmatrix} \begin{Bmatrix} F_Z^S \\ F_x^S \\ M_y^S \end{Bmatrix} \text{ and } \begin{Bmatrix} F_Z^S \\ F_x^S \\ M_y^S \end{Bmatrix} = Ga \begin{bmatrix} S_{UzFz} & S_{UzFx} & S_{UzMy} \\ S_{UxFz} & S_{UxFx} & S_{UxMy} \\ S_{\Phi yFz} & S_{\Phi yFx} & S_{\Phi yMy} \end{bmatrix} \begin{Bmatrix} U_Z^S \\ U_x^S \\ \Phi_y^S \end{Bmatrix} \tag{7}$$

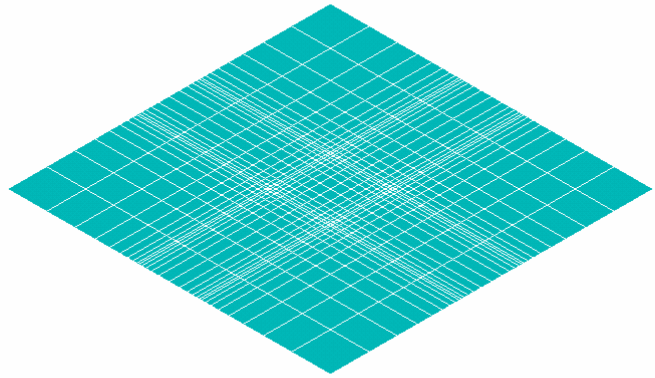
The structure of the compliance matrix (7) depends on the soil profile and on the geometry of the foundation. It can be shown that for surface foundations ($E = 0$) and smooth contact conditions at the soil-foundation interface, the compliance and the impedance matrix are diagonal, presenting no coupling of the foundation DOFs.

4. STRUCTURE AND FREQUENCY CONTENT OF THE COMPLIANCE MATRICES

In this section both the structure of the impedance matrix and the frequency content of its elements will be addressed. Three distinct soil profiles are described, namely, the half-space, a layer over horizontal rigid bottom and a layer over inclined rigid bedrock. Constant rectangular elements were used throughout the present BE study. Spatially homogeneous and non-homogeneous meshes are used.

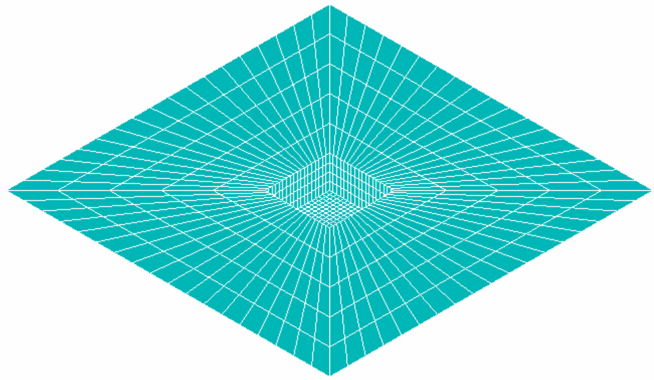
4.1. Half-Space. Initially, the compliance matrices for the half-space are reported. Surface and embedded foundations are considered. For surface foundations E=0 and for the embedded foundations $E = a$. For all simulations considered the soil free surface was discretized within the range $x \leq |5a|$ and $y \leq |5a|$. The number of elements at the soil foundation interface is given by n_f and the number of elements for the remaining of the mesh is n_s. The meshes for the half-space soil profile are shown in Figs. 3 and 4. The structure of the compliance matrix $[N]$ for both surface or embedded foundations interacting with the half-space is given by:

$$\begin{Bmatrix} U_Z^S \\ U_x^S \\ \Phi_y^S \end{Bmatrix} = \frac{1}{Ga} \begin{bmatrix} N_{UzFz} & 0 & 0 \\ 0 & N_{UxFx} & N_{UxMy} \\ 0 & N_{\Phi yFx} & N_{\Phi yMy} \end{bmatrix} \begin{Bmatrix} F_Z^S \\ F_x^S \\ M_y^S \end{Bmatrix} \tag{8}$$



$n_f = 144$, $n_s = 784$

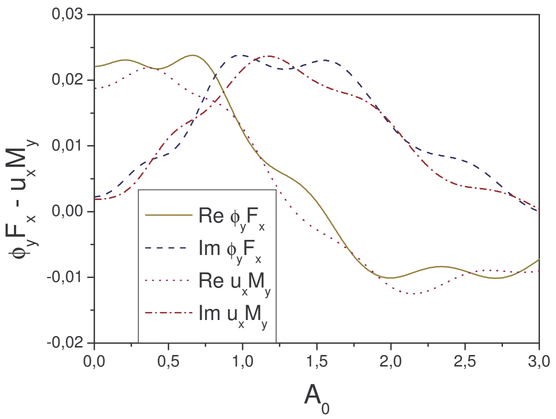
Figure 3: Half-space mesh for surface foundation



$n_f = 612$, $n_s = 972$

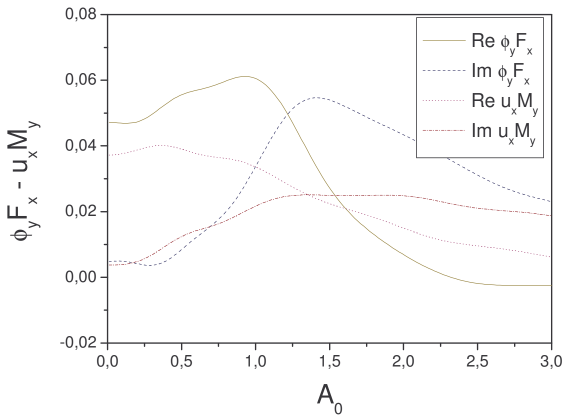
Figure 4 Half-space mesh for embedded foundation

The real and imaginary part of the off-diagonal compliance functions for the surface and for the embedded foundations are furnished in Figs. 5 and 6, respectively. The constitutive parameters used in the numerical synthesis are a = 1m, G = 1N/m^2, ν = 0.25, ρ_s = 1kg/m^3 and the hysteretic damping coefficient η = 0.05. The dimensionless frequency parameter A_0 is defined as $A_0 = \omega a / c_s$ with c_s being the shear wave velocity of the elastic soil.



d) N_{UxMy} , $N_{\Phi yFx}$

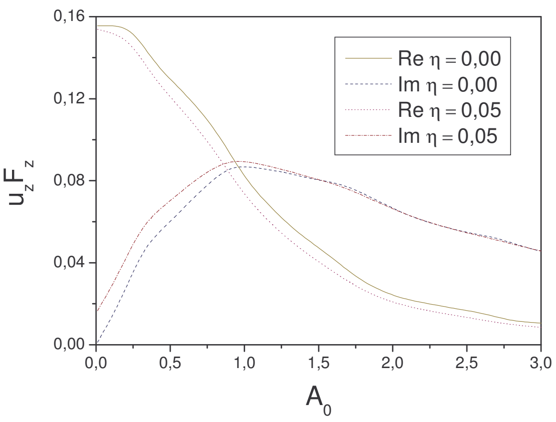
Figure 5: Off-diagonal compliance functions for the rigid surface foundation over the half-space



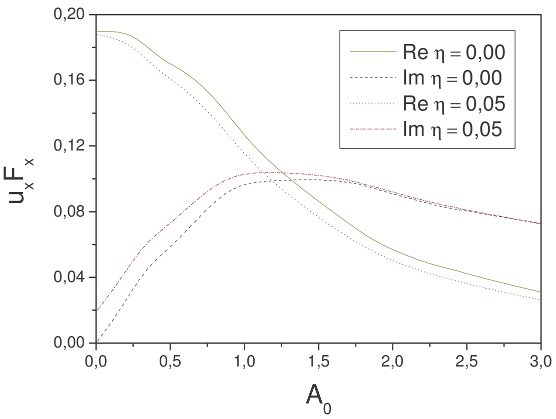
d) N_{UxMy} and $N_{\Phi yFx}$

Figure 6: Off-diagonal compliance functions for the rigid foundation embedded in the half-space

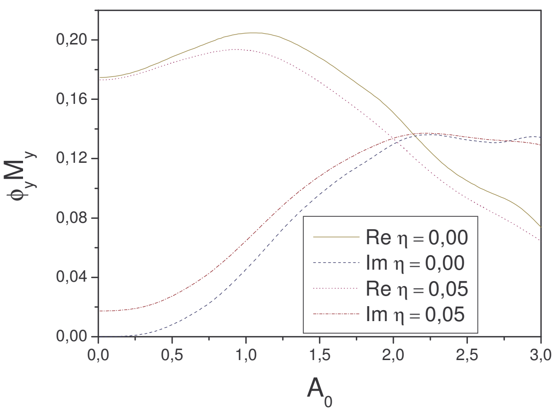
The diagonal elements of the same foundation arrangement are given, respectively, in Figs 7 and 8 for surface and embedded foundations. In both cases are used hysteretic damping coefficient η = 0.00 and η = 0.05.It can be clearly recognized that the embedment increases the rigidity (impedance of the rigid foundation).



a) N_{UzFz}

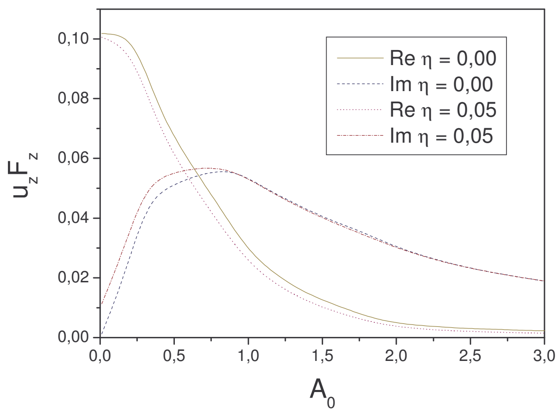


b) N_{UxFx}

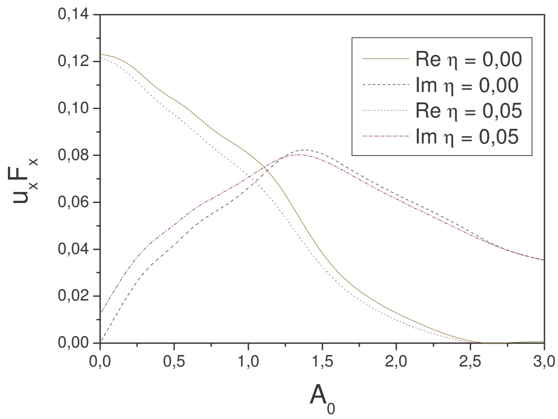


c) $N_{\Phi yMy}$

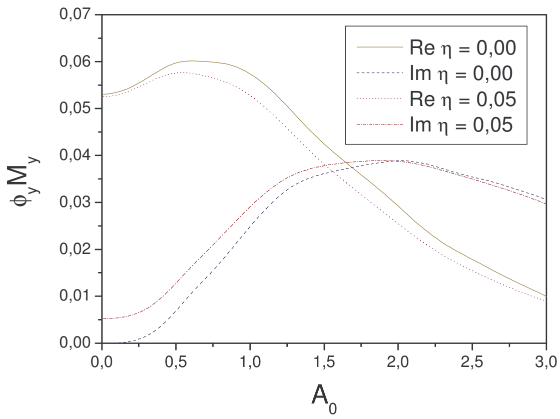
Figure 7: Compliance functions for the rigid surface foundation over the half-space



a) N_{UzFz}



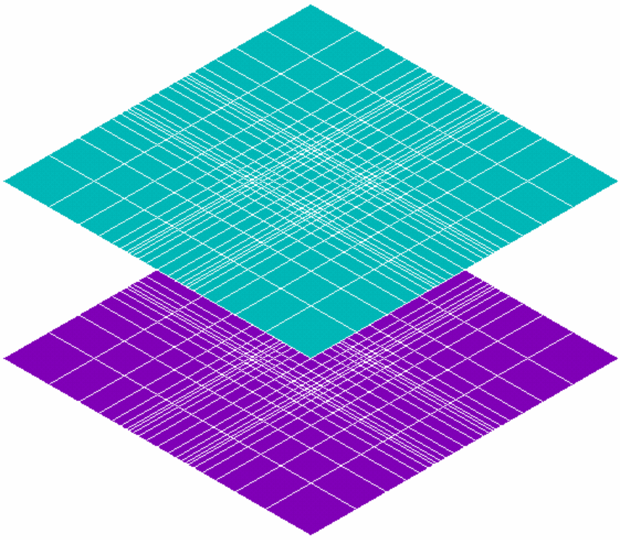
b) N_{UxFx}



c) $N_{\Phi yMy}$

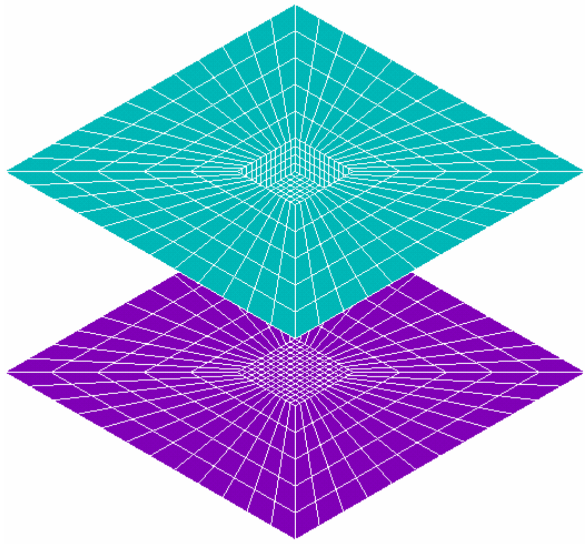
Figure 8: Compliance functions for the rigid foundation embedded in the half-space

4.2. Soil Layer over horizontal rigid bottom. The second set of compliance functions synthesized for the present study is a horizontal layer with depth $H = 5a$ (see Fig. 1). The present stydy shows that the structure of the compliance matrices for the structures over or embedded in the layer are similar to that exhibited in Eq. (8). Figs. 9 and 10 show the meshes used to obtain the compliances (and/or the rigidities).



$n_f = 100$, $n_s = 868$

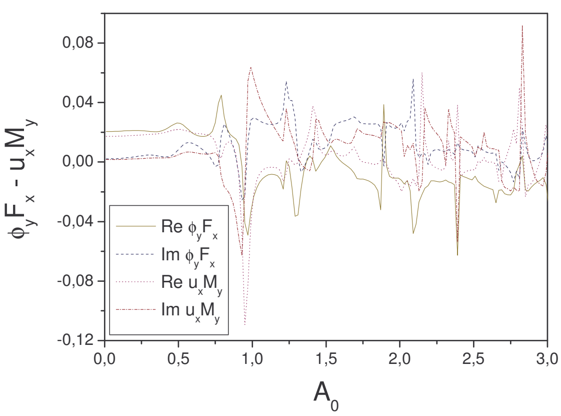
Figure 9: Mesh for surface foundation over a layer with rigid horizontal bottom



$n_f = 336$, $n_s = 624$

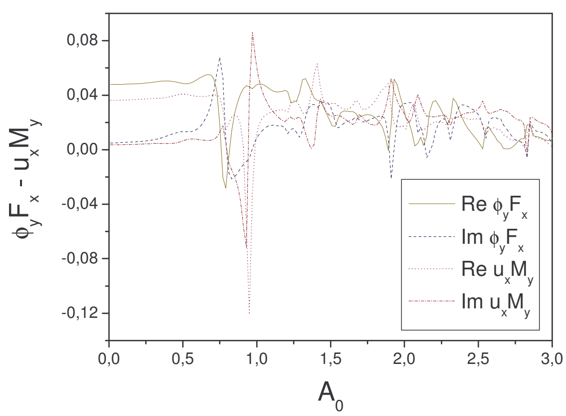
Figure 10 Mesh for foundation embedded in a layer with rigid horizontal bottom

The real and imaginary parts of the off-diagonal compliance functions for the foundation on the surface and embedded in the horizontal layer can be seen in Figs. 11 and 12 respectively. Analogously, the diagonal elements of these compliance matrices are given in Figs. 13 and 14, for the surface and embedded foundations. Comparing the results presented in Figs. 5 and 6 for the half-space with Figs. 13 and 14 obtained for the horizontal layer, it is clear that the limited dimension of the layer depth causes resonances, which are clearly visible in the compliances.



d) $N_{UxMy} = N_{\Phi yFx}$

Figure 11: Off-diagonal compliance functions for the rigid surface foundation over a layer on horizontal bottom

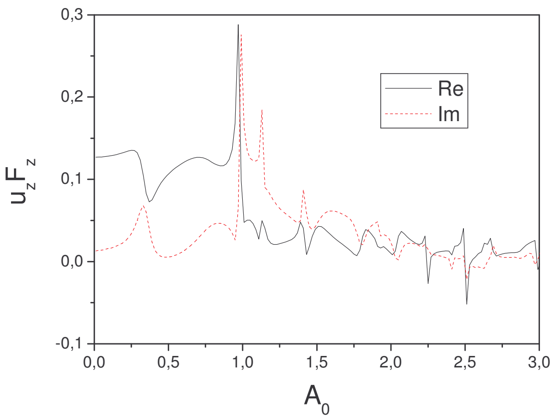


d) N_{UxMy} and $N_{\Phi yFx}$

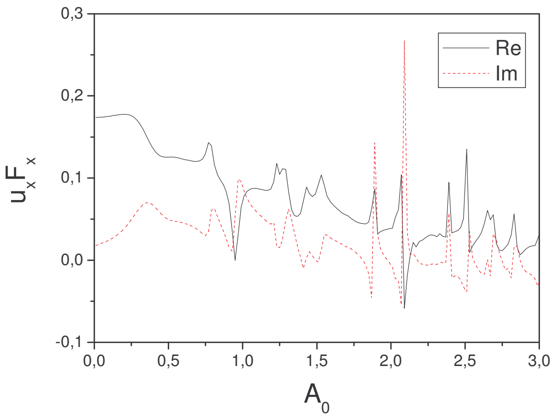
Figure 12: Off-diagonal compliance functions for the rigid foundation embedded in a layer on horizontal bottom

4.3. Soil Layer over a non-horizontal rigid bottom. One of the important issues being addressed in the current study is the dynamic response of foundations interacting with layers that do not present a horizontal rigid bottom. Figure 15 shows, schematically a layer resting on an inclined bottom. The inclination is parallel to the *y*-axis. The initial layer depth is $H_1 = 5a$ and the final depth is $H_2 = 6a$. The inclination length is $L_c = 2a$. The remaining parameters are those used in the previous calculations. Figure 16 presents the BE mesh for the embedded foundation interacting with the non-horizontal soil layer. A similar mesh has also been created for the surface foundation.

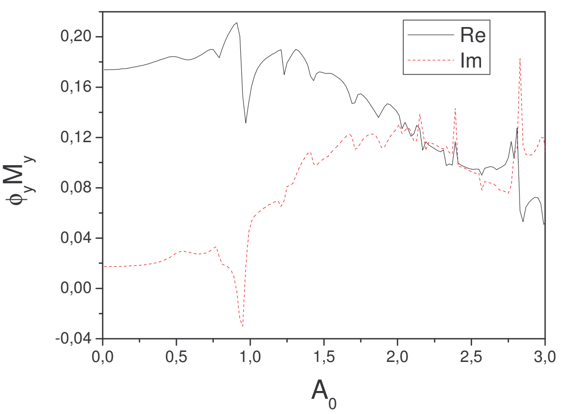
The inclined bottom changes dramatically the structure of the compliance matrix. For this case the structure is equal to the one presented in Eq. (7). This matrix with a full structure expresses the fact that all foundation DOFs in the plane (*x*-*z*) are coupled. Figure 17 shows the elements of the first column of the compliance matrix for a foundation embedded in a non-horizontal soil layer.



a) N_{UzFz}

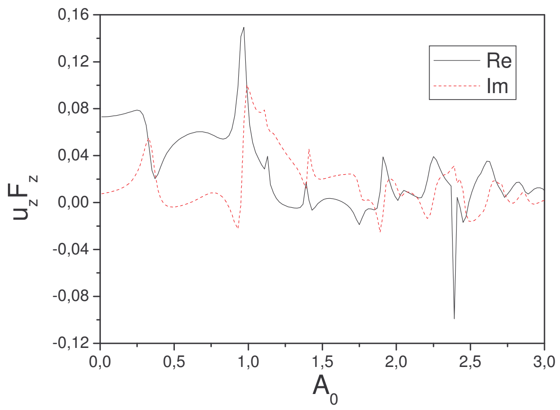


b) N_{UxFx}

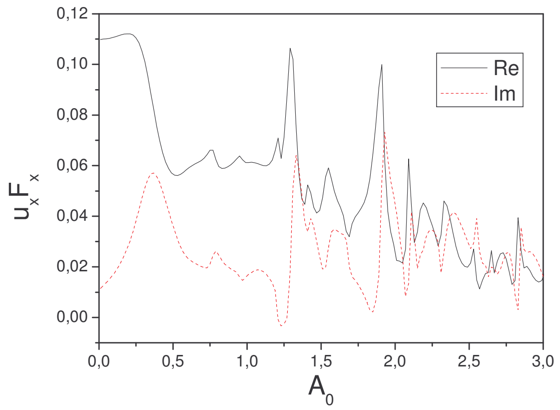


c) $N_{\Phi yMy}$

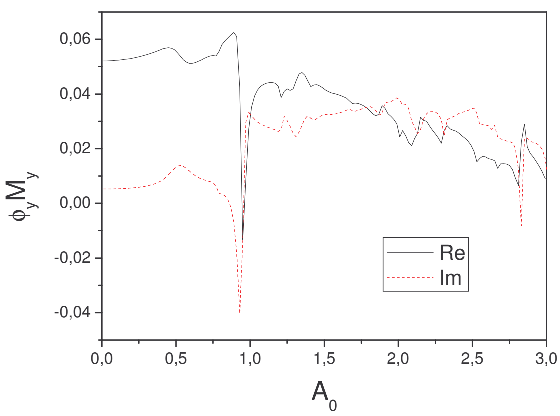
Figure 13: Compliance functions for the rigid surface foundation over a layer on horizontal bottom



a) N_{UzFz}

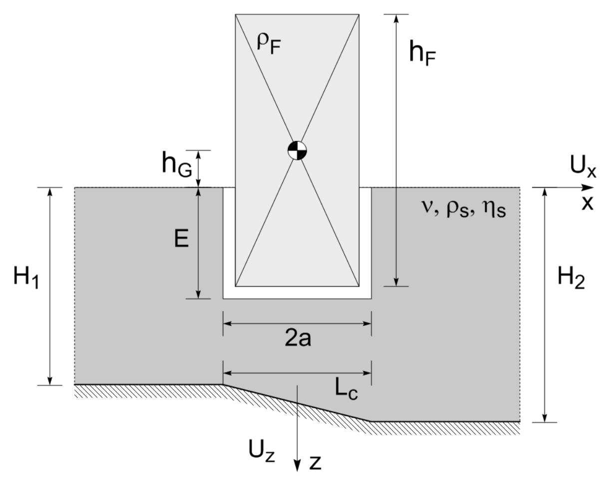


b) N_{UxFx}



c) $N_{\Phi yMy}$

Figure 14: Compliance functions for the rigid foundation embedded in a layer on horizontal bottom



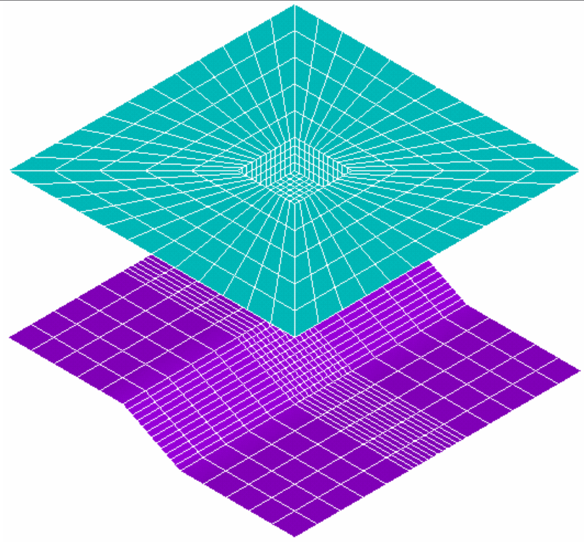
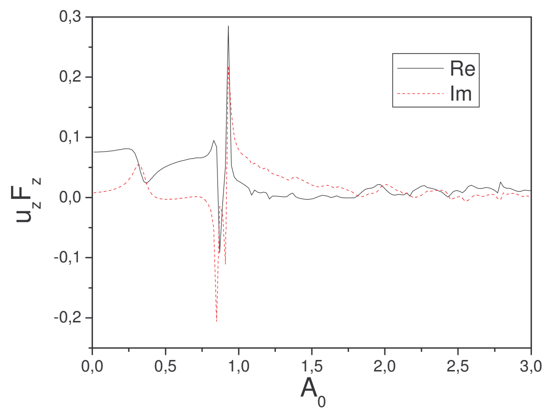
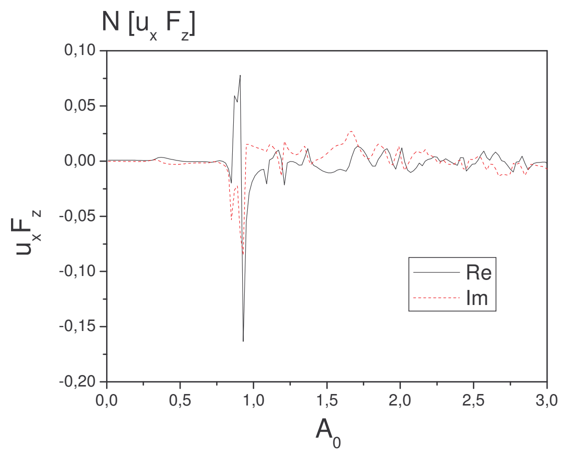


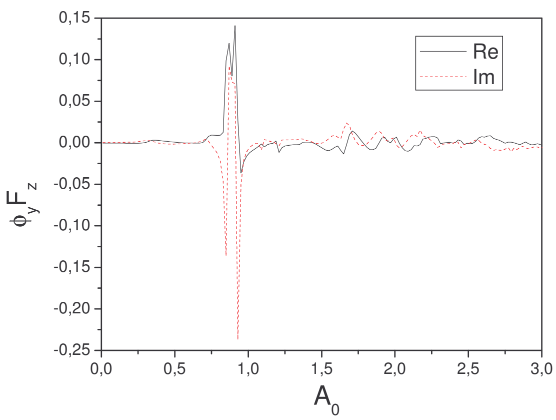
Figure 15: Scheme for foundation interacting with a non-horizontal soil layer.	Figure 16: BE mesh for embedded foundation in a non-horizontal soil layer.



a) N_{UzFz}



b) N_{UxFz}



c) $N_{\Phi yFz}$

Figure 17: Vertical compliance functions for a foundation embedded within a non-horizontal soil layer

5. STATIONARY DYNAMIC RESPONSE OF ROTORS AND FOUNDATION INTERACTING WITH SOIL

In this section a sample of numerical results for the dynamic response of the rigid block foundation is presented.

5.1. Surface block foundation on the half-space.

Figure 18 shows the vertical response U_Z^S of a square block foundation (2*a* x 2*a*) resting on the surface of a viscoelastic half-space, *E = 0*. The block foundation is excited vertically, $F_Z^S = 1$ [N]. The compliance functions for this foundation are given in Figs. 5 and 7a to 7c. It is assumed that the whole foundation mass shrinks to a single plane, coincident to the rigid surface foundation plane. That is: $h_B = h_G = 0$ (see Fig. 1). The foundation mass is equal 1.25 times the soil mass with the same volume [Sousa, 2007]. To access the role of the geometric and material damping, initially the foundation response is determined by modelling the soil as a simple static spring. The value of the static vertical spring is equal to the static value of the real component of N_{UzFz} taken from Fig. 7a. In the sequence, complex vertical compliance functions N_{UzFz} for distinct value of the soil-damping coefficient η_S are considered: $\eta_S = (0.0; 0.01; 0.05; 0.20)$. The response in figure 18 shows clearly that when no geometric damping is present (static spring case), the foundation experiences a large displacement resonance. On the other hand the introduction of the soil dynamic response by means of the complex compliance functions, damps the system in a very strong way. The role of the internal damping coefficient is almost negligible compared to the role of the geometric damping. If this behavior will be reproduced in other soil profiles, is an issue that deserves further investigation.

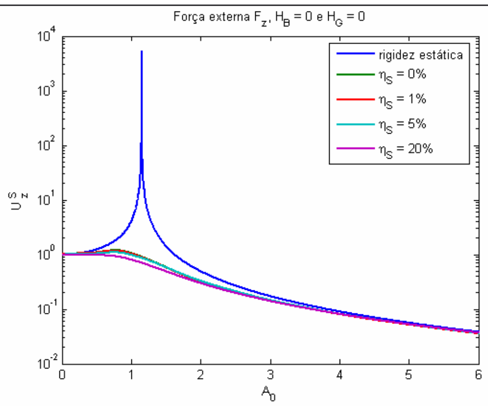


Figure 18: Vertical response of a block foundation over half-space considering distinct damping mechanisms

5.2. Embedded foundation on a horizontal layer .

Figure 19a shows the horizontal response U_X^S of an embedded foundation (*E = a*) due to a horizontal excitation F_X^S for the 3 distinct types of soil profiles, the half space, the horizontal layer and the non-horizontal layer. Figure 19b shows the rotation DOF Φ_Y^S due to the same horizontal excitation. The foundation has height $h_F = 3a$. The foundation density ρ_F is 1.25 times that of the soil ρ_S. It is very clear that the finite layers present resonances that strongly affect the rigid block response. The system dynamics became much more complex in the presence of layers of finite depth.

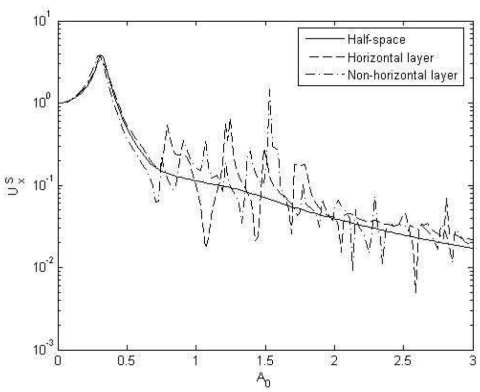


Figure 19a: Horizontal response of embedded foundations on 3 distinct soil profiles

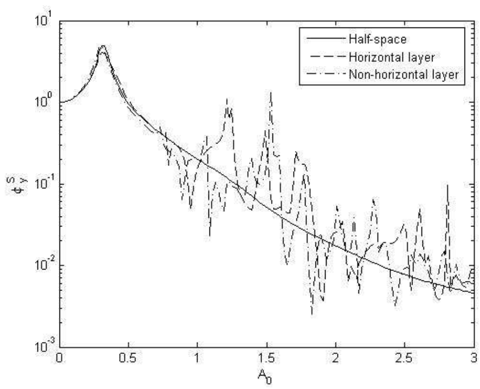


Figure 19b: Rocking response of embedded foundations on 3 distinct soil profiles

5.3. Embedded foundation on a non-horizontal layer .

Figure 20 shows the three DOFs that are excited by a vertical force applied at the foundation block embedded in a layer over non-horizontal rigid bottom. As previously mentioned, the non-horizontal layer produces a fully populated compliance matrix, in which all DOFs are coupled, or are excited by any applied external force. This is a behavior that has not been reported previously.

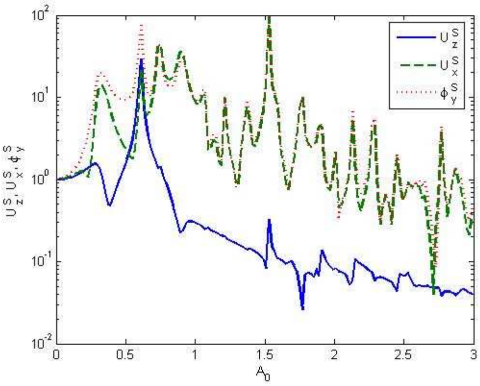


Figure 20: Response of embedded rigid block in a non-horizontal layer over rigid bottom.

6. CONCLUSIONS

The article addresses the stationary dynamic response of three-dimensional rigid block foundations interacting with distinct soil profiles. Dynamic compliance matrices for rigid and massless foundations interacting with a half-space, a horizontal layer and a non-horizontal layer are reported. A direct version of the Boundary Element Method is applied to synthesize the soil response. A sample of compliance functions for complex soil profiles is furnished. The soil profile and the foundation arrangement determine the structure of the compliance matrix. For half-spaces and horizontal layers the vertical degree of freedom is uncoupled from the other DOFs. For non-horizontal layers the compliance matrix is fully populated, coupling all foundation DOFs in the plane of analysis. The presence of a layer with finite dimensions introduces resonances, which are noticeable in the compliance matrices and in the frequency response functions of the block foundations. The layer rigid bottom changes dramatically the foundation response, when compared to foundations interacting with homogenous half-spaces. For half-spaces the geometric damping mechanism plays a dominant role. Material damping has a secondary effect in the dynamic response of rigid structures interacting with half-spaces.

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