ACOUSTIC SENSITIVITY ANALYSIS OF LAYERED POROELASTIC SYSTEMS

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Abstract. *In general, the insulation acoustic systems must be as light as possible. The poroelastic materials are frequently used in the insulation systems to absorb acoustic energy in the medium and high frequencies domain. To increase the performance of the poroelastic material in the low frequency domain, we can apply the optimization techniques to find the best distribution and shape of the insulation systems. Some optimization techniques like topological, evolutionary, genetic can be apply to find the best shape for a specific design domain. To implement the optimization methods based on descent direction, it is important to have accurate objective function gradients (sensitivities) and constraints with respect to the design variables. In this paper, one analytical sensitivity formulation is proposed, implemented and validate for two poroelastic model descriptions: the fluid equivalent model based on the fluid pressure variable and the coupled mixed formulation based on the structural displacement and fluid pressure variables. The Biot-Allard relations are used to model the material behavior. The cost function considered is absorbing performance of the poroelastic material samples in a Kundt tube. The design variables are elementary densities in the context of a SIMP model (Simple Isotropic Material with Penalization), which represents the homogenization of the acoustic and poroelastic medias. The numeric results of insulation system in several configurations are simulated. The sensitivity relations are obtained by a direct analytical method and by the adjoint method. The numerical relations are validated by comparison with the obtained results by a central finite difference method.*

Keywords: *Sensitivity analysis, poroelasticity, optimum design*

1. INTRODUCTION

The goal of this study is to apply the sensitivity analysis methodologies in design acoustic insulating systems. The insulation system must be as light as possible and the acoustic absorption in the low frequency domain must be increased for certain frequency gaps. The design of insulation system has been an important research topic for many years. More recently, (Béecot and Sgard, 2006), (Gazonas et al., 2006) and (Tsay, 2006) introduced the systematic design methodologies to enhance the sound absorption capability of insulation systems. The sensitivity analysis is the process to determine the first derivatives of the cost function to the design variables. In nonlinear optimization problems, we can use the sensitivity value to adjust the parameter vector in direction to the local minimum. In general, the sensitivity analysis is the more costly step of optimization process. The sensitivity analysis computational cost become larger when the design variables number increase. The sensitivity analysis is a important step in the optimization process. The cost function derivatives must be determine accurately for the convergence guarantee. There are several methods to calculate the sensitivities values. The choice of the method is based on the design variable number, the dependency relations of the cost function with respect to the design variables, the ratio of the requirement of computational efficiency and the human effort required in the computational implementation. Several finite element formulations for sound absorbing materials have been developed in the last thirty years. Coupled fluid-structure models based on the Biot theory have been introduced and improved by (Kang and Bolton, 1995), (Panneton and Atalla, 1996) and (Lamary et al., 2001). In (Atalla et al., 1998), the classical Biot-Allard equations have been rewritten in terms of the solid phase macroscopic displacement vector and interstitial fluid phase macroscopic pressure. In this paper, two symmetric formulations developed by (Panneton and Atalla, 1996) are presented. The first one is the mixed formulation (u,p) and the second one is the equivalent-fluid formulation. For both formulations, we can find a solid phase macroscopic displacement vector u_i and the interstitial fluid macroscopic pressure p as the state variables. For the equivalent-fluid formulation, the solid phase is motionless. The resultant coupled system is similar to the classical Fluid-Structure (u,p) system (Panneton and Atalla, 1997a). A weak solution done by a semi-discrete Finite Element model is used. Finally, a numerical demonstration of the computational implementation is presented and a sensitivity analysis of the topological design variables is showed. In this work, it is used the “SIMP” method (Simple Interpolation Material with Penalization) for the material parametrization model. The SIMP method is a proportional “fictitious material” model where the continuous variable density μ, $0 < \mu \leq 1$ is introduced. The material density is used as topological design variable. Using this technique, the perforated material is introduced as periodic poro-acoustic cell and can be formulated in meso-scale, (Olny, 1999). In this scale, we have the acoustic and porous phases. The new material properties used in the acoustic absorbing material design was tested in (Atalla et al., 2001) and

(Olny and Boutin, 2003). In this paper, we compared the efficiency of the sensitivity analysis applied to poro-acoustic systems. The numerical sensitivities of a layered acoustic insulation device with respect the topology are calculated by three methodologies: direct, adjoint and finite difference. The outline of the rest of the paper is as follows. First of all, in the Section 2, we state the poroacoustic problem for the Biot-Allard model and it is introduced the coupled formulation (u,p). In this context, the finite element method is applied to describe the system. In Section 3, we present the material parametrization model done by the SIMP model. In Section 4, we present the details of sensitivity analysis methods used in this work. In Section 5, the numerical results are presented and the performance of the methods are illustrated. The conclusions are outlined in Section 6.

2. GOVERNING FIELD EQUATIONS

The classical hypothesis for linear acoustic and elastic behavior are assumed (Allard, 1993). In this approach the wave propagation theory for the coupled medium is valid for low frequency range and fully saturated conditions. In this case, all dependent quantities represent small fluctuations around a static reference value and the poroelastic properties (porosity, tortuosity, etc) are continues in the domain.

Two different domains can be found in a porous-acoustic system. For the acoustic phase, the behavior of the fluid is governed by the Helmholtz Equation. For the porous domain, all dependent quantities represent small fluctuations around a static reference value and the porous material properties (porosity, tortuosity, etc) are continues. The State Equations of the coupled formulation (u,p) used for modeling the absorbing material can found in the works (Panneton and Atalla, 1996), (Panneton and Atalla, 1997a) and (Panneton and Atalla, 1997b)).

Based in this works, we have implemented a frequency domain dynamic formulation. For harmonic motion, the coupled system can be written in a weak integral form and the Galerkin method can be applied to find the discrete equivalent system. In the next section, the weak form for the acoustic and poroelastic domains is developed.

2.1 The weak integral form

Using the Galerkin method, taking δp as the admissible virtual variation of the fluid phase pressure field (p), for the acoustic domain, we can find the weak integral form for the acoustic domain as (1).

$$\int_\Omega \frac{1}{\omega^2\rho_0} p_{,i}\delta p_{,i} d\Omega - \int_\Omega \frac{1}{\rho_0 c_0^2} p\delta p d\Omega - \int_\Gamma \frac{1}{\omega^2\rho_0}\frac{\partial p}{\partial n}\delta p d\Gamma = 0 \tag{1}$$

where ω is the frequency, ρ_0 is the mass density of the fluid (air), c_0 is the speed of the sound propagation in the fluid phase.

For the poroelastic domain, taking δu_i as the admissible virtual variation of the solid phase displacement vector (u_i), the weak integral form results in the solid phase Equation (2) and the fluid phase Equation (3).

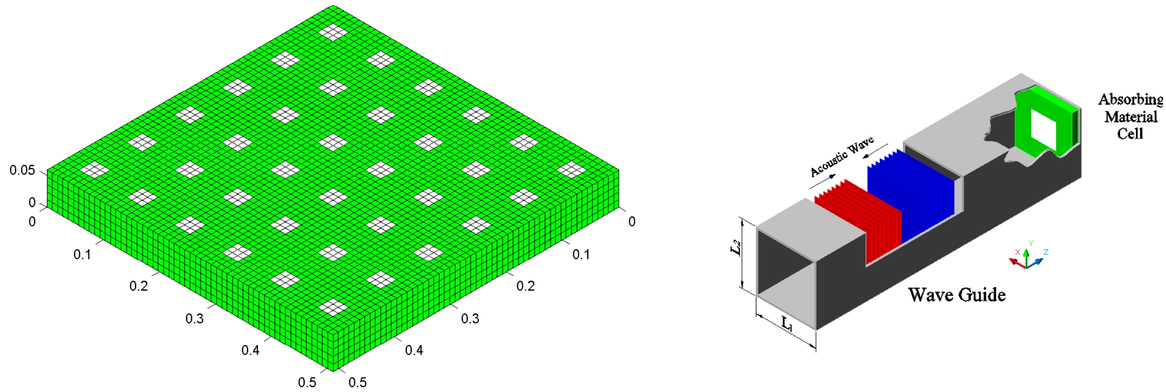
$$\int_\Omega \widehat{\sigma}^s_{ij}\varepsilon^s_{ij}(\delta u_i) d\Omega - \omega^2\int_\Omega \widetilde{\rho} u_i \delta u_i d\Omega - \int_\Omega \widetilde{\gamma} p_{,i}\delta u_i d\Omega - \int_\Gamma \widehat{\sigma}^s_{ij}\cdot n_j \cdot \delta u_i d\Gamma = 0 \tag{2}$$

$$\int_\Omega \frac{h^2}{\omega^2\widetilde{\rho}_{22}} p_{,i}\delta p_{,i} d\Omega - \int_\Omega \frac{h^2}{\widetilde{R}} p\delta p d\Omega - \int_\Omega \widetilde{\gamma} u_i \delta p_{,i} d\Omega + \int_\Gamma \left(\widetilde{\gamma} u_n - \frac{h^2}{\omega^2\widetilde{\rho}_{22}}\frac{\partial p}{\partial n}\right)\delta p d\Gamma = 0 \tag{3}$$

where the Ω and Γ denote the poroelastic domain and its boundary, respectively. The vector n_j is the unitary normal vector and pointing outward the boundary Γ, $\partial p/\partial n$ is the directional derivative of the fluid phase pressure. The term $\widehat{\sigma}^s_{ij}$ represent the elastic linear skeleton stress tensor in vacuum, ε^s_{ij} is elastic strain tensor, $\widetilde{\rho}_{22}$ is the fluid mass coefficient that take into account the fact that the relative flow in the pores is not uniform and $\widetilde{\rho}$ is the complex effective density of the porous domain, $\widetilde{\gamma}$ is a new coupling term and defined in the works (Panneton and Atalla, 1997b) and (Atalla et al., 1998).

The interface conditions of the porous-acoustic system take into account the continuity of fluid normal displacements, continuity of pressures, mass flow conservation and internal forces equilibrium. The mixed formulation (u,p) simplifies the assemblage process of the porous-acoustic systems. Therefore, for coupling among poroelastic and acoustic medias is not necessary calculate any interface matrix, (Debergue et al., 1999).

To solve the problem in a meso-scale, we can analyse one representative cell witch one is repeated to compose a macro insulation panel, show in the Figure (1).



The Macro-Scale The Meso-Scale Detail

Figure 1. Insulation panel in macro scale modeling

Using the system geometric symmetry feature and the periodic system properties, we can adopt the hypothesis of the plane wave excitation and solve the Kundt tube problem like is represented in detail in the Figure (1).

In this conditions, the Poro-Acoustic System is excited by wave guide approach. A waveguide modal superposition technique is used to model the pure acoustic domain. The development of the waveguide modal superposition theory can be found in the works (Atalla et al., 2001) and (Sgard et al., 2005). The Porous-Wave Guide interface condition involving porous coatings can be expressed as an inhomogeneous mixed Dirichlet-Neumann boundary condition, Equation (4).

$$k\frac{\partial p}{\partial n} = Ap + p_0 \tag{4}$$

where p is the acoustic pressure on the interface, k is a coefficient linked to the acoustic media impedance, A is the admittance term of the porous-wave guide interface and p_0 is the incidence pressure amplitude.

The discrete form of dynamic frequency domain equation is done by (5).

$$\begin{bmatrix} [K] - \omega^2 [M] & -[C] \\ -[C]^t & 1/\omega^2 [H] - [Q] + [A] \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2[A]\, p_0 \end{Bmatrix} \tag{5}$$

where $[K]$ is the phase solid stiffness matrix and $[M]$ is the phase solid mass matrix of the Poro-Acoustic material. the matrix $[H]$ is the volumetric matrix of the Poro-Acoustic material domain and $[Q]$ are the phase fluid compressibility matrix. The rectangular matrix $[C]$ is the coupling matrix among the solid and fluid phases in the poroelastic material. In a compact form, the Equation (5) can be rewritten as:

$$[Z_A]\{U\} = \{F\} \tag{6}$$

where $[Z_A]$ is the dynamic matrix and can be expressed as $[Z_A] = [Z] + [A]$. The dynamic matrix of porous phase is $[Z]$ and the contribution of the wave guide is expressed for $[A]$. The system (6) is solved to determine the dynamic response $\{U\}$.

In this formulation, $[Z_A]$ is symmetric and each mode has four degrees of freedom: three displacements and one pressure, done by (Silva and Pavanello, 2003) and (Silva and Pavanello, 2004).

3. MATERIAL PARAMETRIZATION MODEL

To solve a topological optimization problem, it is necessary a mathematic description of the composite material. The composite material is done by two components: the acoustic and poroelastic phases. In this paper, it is used a "SIMP" model, (Bendsoe and Sigmund, 2003), (Bulman et al., 2001) and (Hassani and Hinton, 1998), to describe the fictitious material. The composite material properties values are continuous with respect to the density μ and can be interpolated by acoustic and poroelastic properties values, done by Equation (7).

$$b(\mu) = \mu^p b_p + (1 - \mu^p) b_a \tag{7}$$

where $b(\mu)$ is the interpolated property of the fictitious material, b_a e b_p represent the acoustic and poroelastic phase properties, respectively and p is the penalty constant, (Silva and Pavanello, 2007).

In the Figure (2), it is presented a composite Poro-Acoustic material relations. In this curves, the variation of the different sound propagation speeds in the medium for different density values are presented. The propagation velocity varies among $343m/s$ for the acoustic phase and $198m/s$ for the poroelastic phase. Several values of penalty constant p has been considered.

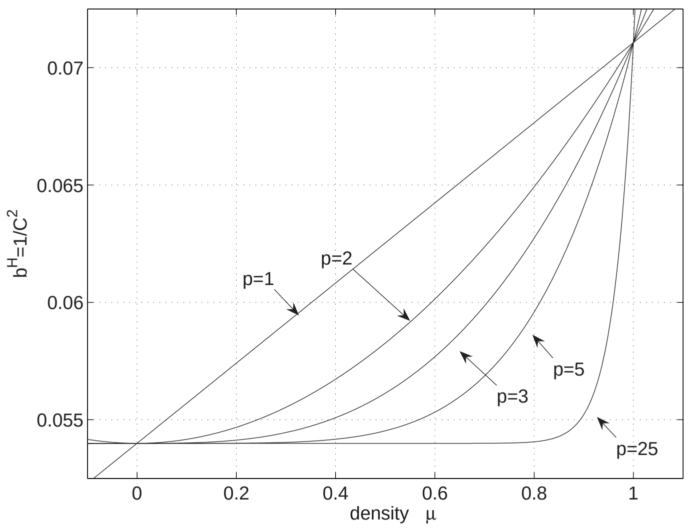


Figure 2. Curves of the Poro-Acoustic material - SIMP Model

Using this material model for describe the intermediary conditions, it is possible to determine the sensitivities of one macroscopic cost function with respect to local material densities. For the Finite Element Model, we have one density by element, which can be changed by optimization process to find the best material. In the next section, we present the methodologies to performance the sensitivity analysis.

4. SENSITIVITY ANALYSIS IN PORO-ACOUSTIC SYSTEMS

In a topological design, the sensitivity of the cost function can be expressed as the gradient function with respect to the elementary design variables μ_k. In the dynamic problems, the sensitivity vector is frequency dependent. For the acoustic absorption maximization problem, we need calculate the absorption function sensitivity with respect to the elementary densities, done by Equation (8).

$$\nabla\alpha\left(\omega,\mu_k\right)=\frac{d\alpha\left(\omega,\mu_k\right)}{d\mu_k} \tag{8}$$

where α represents the acoustic absorption coefficient value.

The acoustic absorption α, using a finite element approximation, can be calculated by Equations (9) and (10).

$$\alpha=\sum_{k=1}^{N}\alpha^k=\frac{\Pi_{diss}}{\Pi_{inc}} \tag{9}$$

where Π_{diss} is the dissipative potential of the poroelastic medium, Π_{inc} is the incidence potential done by acoustic impedance relations. The dissipative potential Π_{diss} can be expressed as:

$$\Pi_{diss}=\frac{1}{2}\,\omega\,\Im\left[\{U\}^t[Z]\{U\}\right] \tag{10}$$

where $[Z]$ is dynamic matrix of the Poro-Acoustic System, the subscript ^{t} is the complex-conjugated operator and $\Im$ represents the imaginary operator, N is the number of elements, α^k is the elementary absorption and $\{U\}$ represents the generalized state variables composed by the nodal structural displacements and the nodal pressures, according to Equation (11).

$$\{U\} = \left\{ \begin{array}{c} u_i \\ p \end{array} \right\} \tag{11}$$

4.1 Methodologies of Design Sensitivity Analysis

The first sensitivity analysis method presented here is the Direct Method. The procedure consist in determine the first derivative of the cost function α with respect to the design variables in a explicit form, according to Equation (12).

$$\frac{d\alpha}{d\mu_k} = \frac{1}{\Pi_{inc}} \frac{d\Pi_{diss}}{d\mu_k} \tag{12}$$

In this case the incidence potential Π_{inc} is constant with respect to elementary density variation μ_k. The response sensitivity with respect to design variables set is done by Equilibrium Equation differentiation, Equation (6), resulting in:

$$\frac{\partial\{U\}}{\partial\mu_k} = -[Z_A]^{-1} \frac{\partial[Z]}{\partial\mu_k}\{U\} \tag{13}$$

The Equation (13) can be used to find the sensitivity of the cost function directly. The linear system must be solve for each design variable sensitivity analysis. Therefore, the direct sensitivity computation in the topological optimization problems become very expensive because we have a big number of design variables.

The vector $\left(\frac{\partial[Z]}{\partial\mu_k}\{U\}\right)$ can be calculated in the element domain, done by Equation (14).

$$\frac{\partial[Z]}{\partial\mu_k}\{U\} = \frac{\partial[Z]_k}{\partial\mu_k}\{U\}_k \tag{14}$$

where $[Z]_k$ represents the elementary dynamic matrix and $\{U\}_k$ is the elementary response.

Substituting the Equation (13) in the Equation (12), it is possible to determine the acoustic absorption sensitivity in a direct approach, according to Equation (15).

$$\frac{d\alpha}{d\mu_k} = \frac{1}{2\Pi_{inc}} \omega \, \Im \left(\frac{\partial\{U\}^t}{\partial\mu_k}[Z]\{U\} + \{U\}^t \frac{\partial[Z]}{\partial\mu_k}\{U\} + \{U\}^t[Z]\frac{\partial\{U\}}{\partial\mu_k} \right) \tag{15}$$

The second approach is the adjoint method. This method is more efficient than the direct method in problems where the number of design variables is larger. In this procedure, the sensitivity response is calculate in an implicit form. The Equation (16) is the total derivative of the cost function α.

$$\frac{d\alpha}{d\mu_k} = \frac{\partial\alpha}{\partial\mu_k} + \frac{\partial\alpha}{\partial\{U\}}\frac{\partial\{U\}}{\partial\mu_k} \tag{16}$$

The Equilibrium Equation (6) can be rewritten here as, Equation (17).

$$[Z_A]\{U\} - \{F\} = \{R\} = 0 \tag{17}$$

For the next step, it is necessary the Equilibrium Equation differentiation, described in the Equation (17), resulting in the Equation (18).

$$\frac{d\{R\}}{d\mu_k} = \frac{\partial\{R\}}{\partial\mu_k} + \left[\frac{\partial\{R\}}{\partial\{U\}}\right]\frac{\partial\{U\}}{\partial\mu_k} = 0 \tag{18}$$

Therefore, we can express the sensitivity response relation, done by Equation (19).

$$\frac{\partial\{U\}}{\partial\mu_k} = -\left[\frac{\partial\{R\}}{\partial\{U\}}\right]^{-1}\frac{\partial\{R\}}{\partial\mu_k} \tag{19}$$

Substituting the Equation (19) in (16), we can determine the acoustic absorption sensitivity by adjoint method. In this method, it is created a adjoint problem to determine the sensitivity response. The adjoint system solution is implicit to the sensitivity analysis, in others words, the response sensitivity analysis is independent of the design variables number and only one adjoint linear system must be solve.

$$\frac{d\alpha}{d\mu_k} = \frac{\partial\alpha}{\partial\mu_k} - \frac{\partial\alpha}{\partial\{U\}}\left[\frac{\partial\{R\}}{\partial\{U\}}\right]^{-1}\frac{\partial\{R\}}{\partial\mu_k} \tag{20}$$

We introduce the adjoint term λ^t to determine the sensitivity analysis by adjoint methodology, Equation (21).

$$\frac{d\alpha}{d\mu_k} = \frac{\partial\alpha}{\partial\mu_k} + \{\lambda\}^t\frac{\partial\{R\}}{\partial\mu_k} \tag{21}$$

The adjoint problem is done by Equation (22).

$$\{\lambda\}^t = -\frac{\partial\alpha}{\partial\{U\}}\left[\frac{\partial\{R\}}{\partial\{U\}}\right]^{-1} \tag{22}$$

Rewritten the Equation (22) in a linear system form, we have:

$$\left[\frac{\partial\{R\}}{\partial\{U\}}\right]^{t}\{\lambda\} = -\frac{\partial\alpha}{\partial\{U\}} \tag{23}$$

where $\partial\alpha/\partial\{U\}$ is the partial derivative of the cost function with respect to the response $\{U\}$ and the partial derivative of the Equilibrium Equation $\{R\} = 0$ with respect to the response $\{U\}$ is done by Equation (24).

$$\left[\frac{\partial\{R\}}{\partial\{U\}}\right] = [Z_A] \tag{24}$$

Based on the Equation (9) and (10), the acoustic absorption partial derivative with respect to the design variables μ_k can be calculated by Equation (25).

$$\frac{\partial\alpha}{\partial\mu_k} = \frac{1}{2\Pi_{inc}}\,\omega\,\Im\left(\{U\}^t\frac{\partial[Z]}{\partial\mu_k}\{U\}\right) \tag{25}$$

The last method used here is the finite difference numerical approach. In general, this method is very expensive because it is necessary calculate the ratio among a finite cost function variation with respect each design variable. It is based in the definition of the derivate, done by Equation (26).

$$\frac{d\alpha}{d\mu_k} = \lim_{\Delta\mu\to 0}\frac{\Delta\alpha}{\Delta\mu_k} \cong \frac{\alpha_{\mu_k} - \alpha_{\mu'_k}}{\mu_k - \mu'_k} \tag{26}$$

where μ_k represents the design variables vector and μ'_k represents the modified design variables vector with a little perturbation value. The absorbing acoustic values for each design variable setting are α_k e α'_k, respectively.

In order to achieve accurate results with the finite difference method, it is necessary to use a little perturbation in the design variables, according to Equation (27).

$$||\mu_k - x'_k|| \to 0 \tag{27}$$

In this work, we use the centering finite difference technique, where the cost function evaluation is done by backward and forward solutions, describe by Equation (28).

$$\frac{d\alpha}{d\mu_k} \cong \frac{\Delta\alpha}{\Delta\mu_k} = \frac{\alpha\left(\mu_k + h\right) - \alpha\left(\mu_k - h\right)}{2h} \tag{28}$$

where h must be enough short to reach a good precision of the numeric method.

Three methods were presented here for sensitivity analysis applied in the Poro-Acoustic designs. In the next section, we present the numerical results to validate these methodologies applied in our proposed material one interpolation law.

5. NUMERICAL RESULTS

In order to compare the three sensitivity analysis methods, one rectangular cell with dimensions of 0.115 x $0.085m$ is used. Three sides of the cell are bonded by a rigid impervious wall. A normal incident plane wave guide of unit amplitude excites the absorbing material on the left side ($x = 0$). The domain is divided into 9 x 9 four-node plane elements. Initially, the finite elements mesh is composed of porous elements ($\mu_k = 1$). The porous domain and the finite element mesh are presented in the Figure (3).

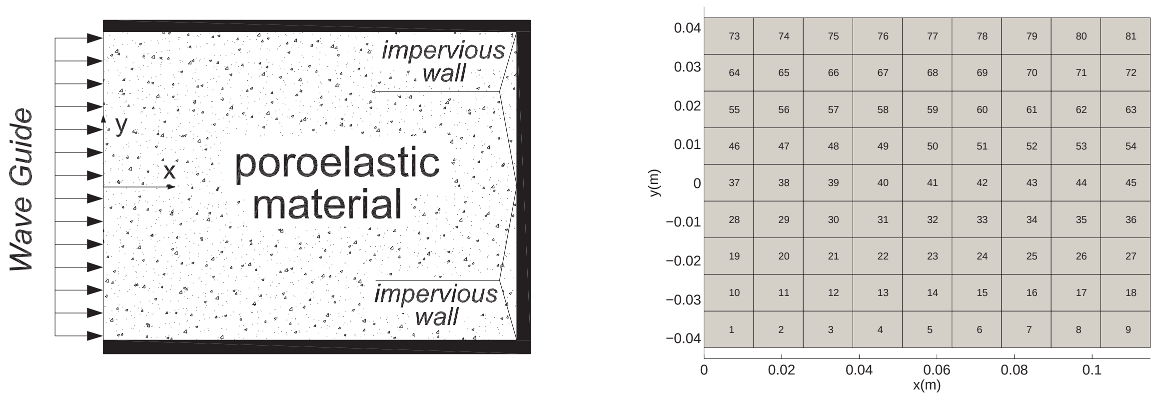


Figure 3. Finite Element Mesh for the Sensitivity Analysis

The material properties used in this simulation are given in Table (1).

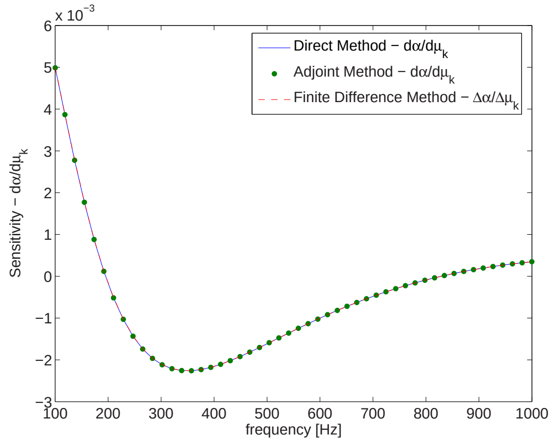
Table 1. Properties of the porous material

Material	h (Porosity)	α_∞ (Tortuosity)	$\sigma[Ns/m^4]$ (Resistivity)	$\Lambda[\mu m]$ (Viscous Length)	$\Lambda'[\mu m]$ (Thermal Length)
Rock-wool	0.94	2.1	135000	49	166

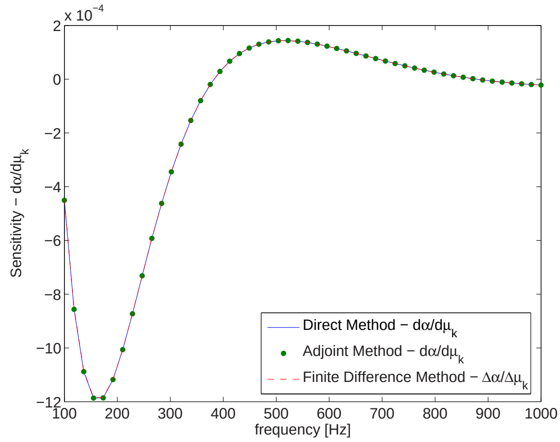
In order to test the proposed implementation, two analysis are done. In the first one, the acoustic absorption sensibility value was determinated with respect to elementary densities μ_k and evaluated in the frequency range (100-1000 Hz). In the second test, the sensitivity of one element is obtained for different values of elementary densities in a specific frequency point.

The numerical results are performance by the three methods presented: direct, adjoint and finite difference techniques. The numerical precision, processing time and efficiency are compared.

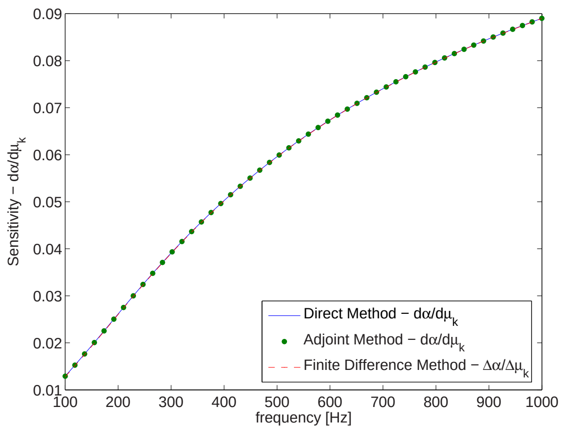
For the elements 5, 7, 10, and 22, the results for the frequency analysis are shown in the Figure (4). In this case, the sensitivity is calculated in a $\mu_k = 1$ condition, i.e., the cell is filled with poroelastic material.



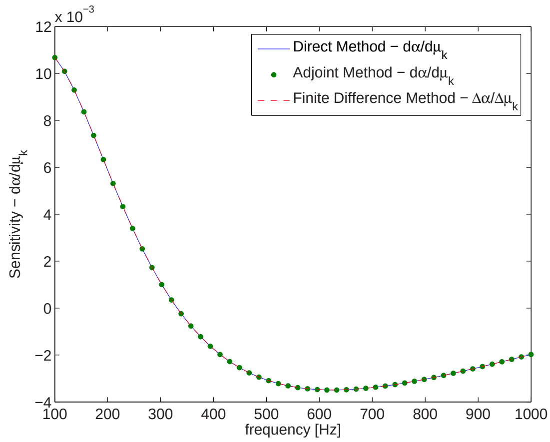
Element 5



Element 7



Element 10

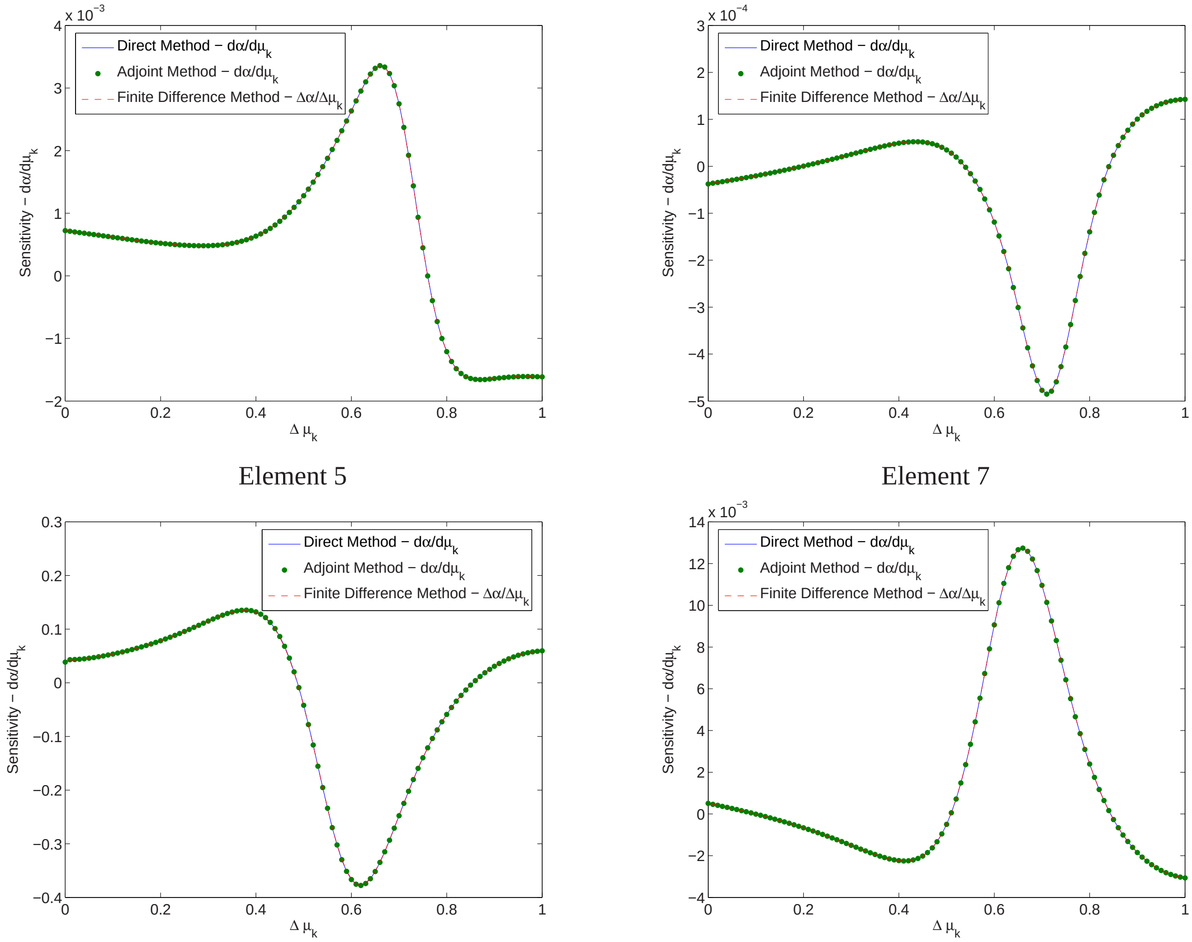


Element 22

Figure 4. Element Sensitivity in function of the frequency value

A good agreement is observed, showed in the curves of the Figure (4), when we compare the results obtained for the three methods. The analytical results obtained by direct and adjoint methods are validated by the numerical results obtained by finite difference numerical method.

The second sensitivity analysis test is performed by design variable variation, i.e., with an elementary density variation μ_k in the frequency 500 Hz. In the Figure (5), the results for the elements 5, 7, 10 e 22 are presented.



Element 5 | Element 7

Element 10 | Element 22

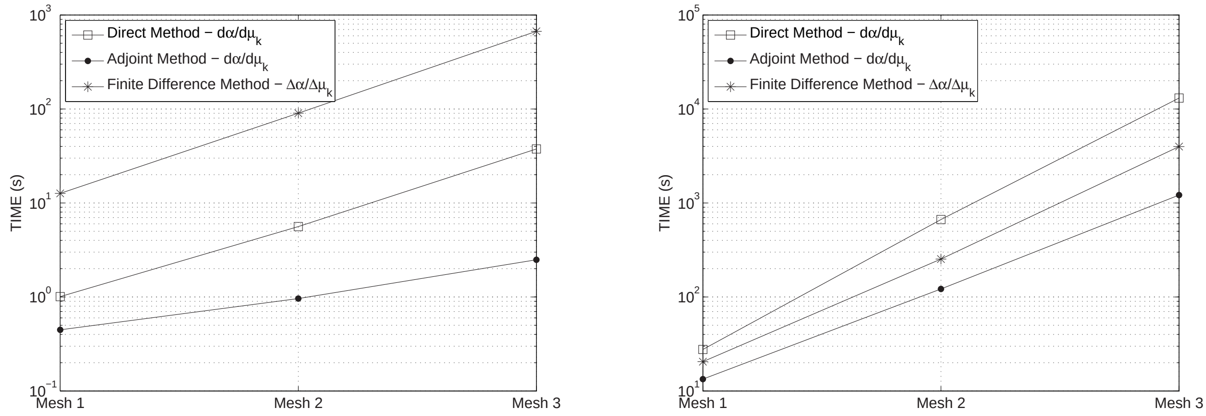
Figure 5. Element Sensitivity in function of the elementary density variation

The performance of the sensitivity analysis methods applied in the Poro-Acoustic Systems have been analyzed. Three different meshes were tested. The first mesh has 5 x 5 linear quadrilateral elements, the second one has 10 x 10 linear quadrilateral elements and the last mesh was analyzed with 15 x 15 linear quadrilateral elements. In the Table (2), the time results, in seconds, for each mesh in the two types of performance test are presented.

Table 2. Sensitivity Methods Performance

Time (s)	Mesh 5x5		Mesh 9x9		Mesh 15x15	
	Test 1	Test 2	Test 1	Test 2	Test 1	Test 2
Direct - Analytical Method	1,009	27,652	5,611	666,327	37,583	13075,588
Adjoint - Analytical Method	0,448	13,375	0,962	121,640	2,489	1213,042
Finite Difference - Numerical Method	12,655	20,508	90,278	253,483	670,000	3983,330

In the Figure (6) the performance times described in the Table (2) are presented. It is possible to see the difference of the performance from the sensitivity analysis methods applied in the Poro-Acoustic problem described in this work.



Performance of the frequency variation test

Performance of the density variation test

Figure 6. Performance of the Sensitivity Analysis Methods

It can be noted that the adjoint method is more efficient than other methods. In all cases, where the design variable number is low, the adjoint method show the better performance. The adjoint method was faster in the two tests presented here: with a frequency and elementary density variations for the three meshes considered in this work.

In general the Finite Difference Method should show inferior performance, but in the elementary density variation analysis the direct method was less efficient than the Finite Difference Method. The computational implementation for the direct method is expensive because it is necessary solve a large number of equations.

6. CONCLUSIONS

In this work, two tests have been performance. The first one evaluate the sensitivity value with respect the frequency in a single porosity condition and the second one evaluate the sensitivity value for each elementary density variation. By computational code implementation analysis, it can explain the poor performance of the direct and finite difference methods. In theses approaches, it is necessary to solve a big number of the global linear systems. The adjoint method is very fast and more efficient than direct and finite difference methods. In general, the numerical methods show low time performance. The sensitivity analysis procedure can effectively used to solve a frequency optimization problem applied in absorbing materials design.

7. ACKNOWLEDGEMENTS

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