# GENETIC ALGORITHMS APPLIED TO ACCIDENT RECONSTRUCTION OF GROUND VEHICLES 

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Abstract. One of the most interesting and challenging applications of the engineering is the solution of the inverse problems, or in other words, those problems which the answer, or the consequence is known, but the cause is not, or the conditions that took such result are the unknowns, mainly when the problem involves a great number of variables and parameters. In this kind of problems we find the scientific approach of the ground vehicle accident reconstruction. Another theme directly related it is the collisions analysis, as in the context of an accident reconstruction, or in the context of the vehicle crashworthiness, associated to it structural integrity and their occupants' passive safety. In this paper is discussed the application of the genetic algorithms, for the treatment of the inverse problem in collisions of ground vehicles, using models of rigid or flexible vehicles. We are studying how the optimization algorithms can supply the group of variables and parameters that more probably would take to the final condition, inside of the imposed restrictions, starting from the vehicles position after a collision. Also using the same optimization procedures, and considering restrictions associated to the problem, we are evaluating the possibility of how to determine the rigidity and plasticity characteristics of the vehicles structures, starting from it deformed condition after a collision. This procedure can also supply information about the impact severity. The preliminary results reached are presented. In spite of being a natural way for the treatment of this kind of problem, there are still few authors that adopt the optimization techniques for it solution and interpretation. This is another effort in the sense of gives a scientific approach to the accident reconstruction and collisions analysis problems, protected and developed by the PUC-Rio Vehicular Systems Group.

Keywords: Vehicle Dynamics, Accidents Reconstruction, Optimization Techniques, Genetic Algorithms.

## 1. INTRODUCTION

In collision accidents involving one or more ground vehicles, it is usually necessary to determine the conditions before the crash, in way to find the responsible agents and the possible causes. The reasons for this search go from juridical to the financial aspects, going through the explanation of technical failures that can drive to futures improvements in the vehicles. In this context, the application of appropriate procedures for the treatment of this inverse problem in engineering (Speranza Neto et. al, 2003), using the scientific method, brings a significant contribution in the sense to turn such analyses more precise and less dependent of mistaken suppositions. That approach seeks to reconstitute the chronological sequence of the events, starting from the final configuration of the accident, by substituting the try-and-error methods by scientific procedures based on the physics laws and formal mathematical methodologies.

Due the complex dynamics involved in the collision of two vehicles, even if limited to the plane, the determination of the inverse problem is very difficult. The used methodology consists of determining the behavior of the vehicles, path and attitude, starting from random initial values or through a specialist's experience, and obtain by try-and-error the initial values that better are adapted to the situation that created the final values and the associated restrictions. It is observed that the motivation of this work is not to substitute the specialist, but to aid him in the generation of the parameters values precedents to the collision. The specialist's experience helps, among other things, to define the universe of possible solutions.

The use of optimization methods in inverse engineering, more specifically in the cases of ground vehicles accidents reconstruction, can be found in the literature (Moser and Steffan, 1998), however with few details, mainly in what concerns to the procedure. The purpose of this work is to define how the genetic algorithms are applied to this problem, using recommendations found in other studies (Kost and Werner, 1994; Pohlheim and Hunt, 1995).

## 2. COLLISION MODELS

It was used for the treatment of the collisions problem of ground vehicles the models presented by Abdulmassih (2003) and Martins (2005), that can be adopted for cases of crash of a vehicle against an obstacle or two vehicles amongst themselves. These models of instantaneous collision and the movement pos-collision works in the plane, with rigid vehicles of three degrees of freedom, taking into account the losses of energy in the collision due to the plastic deformations, the friction and the interpenetration among the vehicles, and the losses for dynamic friction between the tires and the ground until the complete stop of the vehicles.

The collision model basically makes a linear transformation of the input variables - speeds and positions of the vehicles before the collision - through a matrix that contains information about the geometry of the problem, the mass and the moment of inertia of each vehicle, the restitution and interpenetration coefficients, that results in the speeds and positions soon after the collision. Those will be then the inputs for a pos-collision dynamic model that determines the movement of the vehicles to the stop instant, based in their inertial characteristics and in the contact of the tires with the ground, considering the system without any human control.

### 2.1. Collisions Phases

In every collision necessarily happen three distinguishable stages, normally modeled separately: the pre-crash, the crash and the pos-crash. The pre-crash involves the precedent events to the collision and has direct influence on the values of the relevant crash parameters. In the case of collision of ground vehicles, in this stage the variables usually present smaller variation in comparison with the other ones, given the restrictions imposed by the driver's behavior. The crash is characterized by the contrast between the high amplitude of the forces and energy involved, and an interval of minimum time to consider. In the study here presented, such interval of time will be approximated by zero (instantaneous collision).

The pos-crash is constituted by the events soon afterwards to the crash, ending with the stop of the vehicles. This stage can be characterized by the absence of the human influence, what takes a situation of dynamics of vehicles in extreme conditions of forces, under the prevalence of the dynamic friction between the tires and the ground, due to the sliding of the surfaces in contact.

### 2.2. Pre-Collision Model

The modeling of the events precedent to the crash was analyzed and it was evaluated that would not be of interest for the study that we want to present here. It complexity is a challenge approached in several works, that are based on models of human behavior in collisions. The available publications on the theme don't include the due adaptation we needs in the work here described, so we consider to model the problem starting from the conditions immediately previous to the crash. In this situation it is not necessary a model of the human driver, since it is admitted that his reaction, even if there is, it is minimum compared to the speed of the events, the involved forces and his interaction with the vehicle after the collision. In Speranza Neto and Spinola (2005) it is presented an initial study for the human modeling in the command of the vehicles and his influence and interaction with it dynamics, in way to characterize the pre-crash stage completely.

### 2.3. Instantaneous Collision Model

The crash model used here was adapted from the work of Abdulmassih (2003), in way to simplify the optimization program algorithm to be implemented. This model has as input the information of the probleme geometry, the mass, the moment of inertia and speeds of each vehicle before to the impact, the restitution and interpenetration coefficients. Through a linear transformation are obtained as output the speeds and positions soon after the collision.

In the referred work, the direct or inverse problems were described in the collision frame (Figure 1), defined with it $x$ axis perpendicular to the collision area, the $y$ axis along that area, and it origin located in the medium point of the area. Such frame was established based on the hypothesis of a vehicles interception plane area, reached immediately after the crash, and maintained during the collision (Genta, 1997 and Mcmillan, 1983).

The determination of the characteristic geometric parameters of the collision problem in this frame, as well as the definition of the speeds of the vehicles before the crash, was usually a complicated task and subjects to mistakes, unless if it is treated of an accident with specific characteristics. Also the interpretation of the results of the model was not an easy task, once the pos-collision translational speeds, and consequently the movements of the vehicles, should be analyzed in a specific frame for each case and position of the vehicle.

Considering the infinitesimal duration of the crash, applied the conditions of conservation of linear and angular momentum, a linear algebraic equations system is obtained, relating the components of the speeds of the vehicles before (index 1) and after (index 2) the accident, that organized in the matrix form is given by

$$
\left[\begin{array}{cccccc}
m_{A} & 0 & m_{B} & 0 & 0 & 0  \tag{1}\\
0 & m_{A} & 0 & m_{B} & 0 & 0 \\
m_{A}\left(y_{A}-\lambda x_{A}\right) & 0 & 0 & 0 & -J_{A} & 0 \\
0 & 0 & m_{B}\left(y_{B}-\lambda x_{B}\right) & 0 & 0 & -J_{B} \\
e & 0 & -e & 0 & e y_{A} & -e y_{B} \\
\lambda & -1 & 0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
v_{A x 1} \\
v_{A y 1} \\
v_{B x 1} \\
v_{B y 1} \\
\omega_{A 1} \\
\omega_{B 1}
\end{array}\right]=\left[\begin{array}{ccccc}
m_{A} & 0 & m_{B} & 0 & 0 \\
0 & m_{A} & 0 & m_{B} & 0 \\
0 \\
m_{A}\left(y_{A}-\lambda x_{A}\right) & 0 & 0 & 0 & -J_{A} \\
0 & 0 & m_{B}\left(y_{B}-\lambda x_{B}\right) & 0 & 0 \\
-1 & 0 & 1 & 0 & -y_{B} \\
\lambda & -1 & 0 & 0 & 0 \\
y_{B} \\
0
\end{array}\right]\left[\begin{array}{c}
v_{A x 2} \\
v_{A y 2} \\
v_{B x 2} \\
v_{B y 2} \\
\omega_{A 2} \\
\omega_{B 2}
\end{array}\right]
$$

where,
$m_{A}$ and $m_{B}$ are the masses of the vehicles ( kg );
$J_{A}$ and $J_{B}$ are the moments of inertia of the vehicles in relation to the vertical axis that pass through the Mass Center ( CM or $\mathrm{G}_{\mathrm{A} / \mathrm{B}}$ ) $\left(\mathrm{kg} \mathrm{m}^{2}\right)$;
$\left(x_{A}, y_{A}\right)$ and $\left(x_{B}, y_{B}\right)$ are the coordinates of the position of the mass centers of the vehicles in relation to the collision frame(m);
$e$ is the restitution coefficient, defined in agreement with the deformation characteristics associated to the collision, so that $e=0$ perfectly plastic collision; $0<e<1$ inelastic collision; $e=1$ perfectly elastic collision, associated to the relative speed normal to the interception area;
$\lambda$ is a parameter that characterizes a combined effect of friction and interpenetration of the vehicles, associated to the relative speed along the interception area, for which exist limit values that can be used (Macmillan, 1983; Brach, 2005); $\left(v_{A x l}, v_{A y l}\right)$ and $\left(v_{B x x}, v_{B y l}\right)$ are the components of the absolute speeds of the CM of the vehicles $A$ and $B$ before the crash (index 1) in the collision frame ( $\mathrm{m} / \mathrm{s}$ );
$\left(v_{A x 2}, v_{A y 2}\right)$ and $\left(v_{B x 2}, v_{B y 2}\right)$ are the components of the absolute speeds of the CM of the vehicles $A$ and $B$ after the crash (index 2) in the collision frame ( $\mathrm{m} / \mathrm{s}$ );
$\omega_{A I}$ and $\omega_{B I}$ are the angular (around the vertical axis) speeds of the vehicles $A$ and $B$ before the collision ( $\mathrm{rad} / \mathrm{s}$ ); and $\omega_{A 2}$ and $\omega_{B 2}$ are the angular (around the vertical axis) speeds of the vehicles $A$ and $B$ after the collision (rad/s).

The solution of this model makes possible to find the variables after the crash, given the conditions before the collision, and the inverse result is obtained defining the conditions immediately after the crash, to determine the variables before the collision. The collision frame can be established from the geometry of the vehicles and the area reached during the crash. The Figure 2 presents the 4 reference systems employed in the treatment of the problem.


Figure 1. Generic Oblique Collision: Collision Frame.


Figure 2. Reference Systems: Collision, Local and Global.

With the angles of the collision reference systems in relation to the local systems of the vehicles, it should be obtained the position of the respective Mass Centers (CM) in the local frame, as well as the projections of the absolute speeds of the CM. So the CM located in the origin of the local frame, $x_{A}=0$ and $y_{A}=0$, and the projection of the absolute speed of the CM in the local frame, in the collision frame are, respectively, given by
$\left[\begin{array}{l}x_{A} \\ y_{A}\end{array}\right]=\left[\begin{array}{cc}-\cos \phi_{A}^{C} & -\operatorname{sen} \phi_{A}^{C} \\ \operatorname{sen} \phi_{A}^{C} & -\cos \phi_{A}^{C}\end{array}\right]\left[\begin{array}{l}x_{m} \\ y_{m}\end{array}\right]$ and $\left[\begin{array}{l}v_{A x} \\ v_{A y}\end{array}\right]=\left[\begin{array}{cc}\cos \phi_{A}^{C} & \operatorname{sen} \phi_{A}^{C} \\ -\operatorname{sen} \phi_{A}^{C} & \cos \phi_{A}^{C}\end{array}\right]\left[\begin{array}{l}u_{A} \\ v_{A}\end{array}\right]$
After the solution of the collision problem, for the representation of the CM pos-collision translation speeds $\left(v_{A x 2}\right.$, $\left.v_{A y 2}\right)$ and ( $v_{B x 2}, v_{B y 2}$ ), in the local frame of the vehicles it is enough to apply the inverse transformation
$\left[\begin{array}{c}u_{A 2} \\ v_{A 2}\end{array}\right]=\left[\begin{array}{cc}\cos \phi_{A}^{C} & -\operatorname{sen} \phi_{A}^{C} \\ \operatorname{sen} \phi_{A}^{C} & \cos \phi_{A}^{C}\end{array}\right]\left[\begin{array}{l}v_{A x 2} \\ v_{A y 2}\end{array}\right]$ and $\left[\begin{array}{l}u_{B 2} \\ v_{B 2}\end{array}\right]=\left[\begin{array}{cc}\cos \phi_{B}^{C} & -\operatorname{sen} \phi_{B}^{C} \\ \operatorname{sen} \phi_{B}^{C} & \cos \phi_{B}^{C}\end{array}\right]\left[\begin{array}{l}v_{B x 2} \\ v_{B y 2}\end{array}\right]$
In the cases of accidents problems it is usually known, besides the final location of the stopped vehicles, their approximate positions in the road in the instant of the crash, that it can be established in relation to a global frame. Knowing the position of the origin of the collision frame in relation to the global frame, ( $p_{X}, p_{Y}$ ), the angles of the vehicles $\left(\phi_{A}{ }^{G}\right.$ and $\phi_{B}{ }^{G}$ ), the positions of the CM in that reference system the pos-collision problem can be correctly treated. So a coordinate's transformation should be proceeded in order to locate the reference system of each vehicle in relation to the global frame. The Figure 3 presents the necessary operations for the positioning of the CM and initial
orientation of the vehicles, and from the mass centers positions in the collision frame, in the instant immediately before to the crash, their positions in the global reference system are given by

$$
\left[\begin{array}{c}
X_{C M A}  \tag{4}\\
Y_{C M A}
\end{array}\right]=\left[\begin{array}{c}
p_{X} \\
p_{Y}
\end{array}\right]+\left[\begin{array}{cc}
\cos \left(\phi_{A}^{C}+\phi_{A}^{G}\right) & -\operatorname{sen}\left(\phi_{A}^{C}+\phi_{A}^{G}\right) \\
\operatorname{sen}\left(\phi_{A}^{C}+\phi_{A}^{G}\right) & \cos \left(\phi_{A}^{C}+\phi_{A}^{G}\right)
\end{array}\right]\left[\begin{array}{c}
x_{A} \\
y_{A}
\end{array}\right] \text { and }\left[\begin{array}{c}
X_{C M B} \\
Y_{C M B}
\end{array}\right]=\left[\begin{array}{c}
p_{X} \\
p_{Y}
\end{array}\right]+\left[\begin{array}{cc}
\cos \left(\phi_{B}^{C}+\phi_{B}^{G}\right) & -\operatorname{sen}\left(\phi_{B}^{C}+\phi_{B}^{G}\right) \\
\operatorname{sen}\left(\phi_{B}^{C}+\phi_{B}^{G}\right) & \cos \left(\phi_{B}^{C}+\phi_{B}^{G}\right)
\end{array}\right]\left[\begin{array}{c}
x_{B} \\
y_{B}
\end{array}\right]
$$

that will be initial conditions for definition of the path of the vehicles in the pos-collision. For the determination of the vehicles attitude the initial angles will be $\phi_{A}{ }^{G}$ and $\phi_{B}{ }^{G}$. The presented equations, that describe the collision model, starting from information of the geometry, characteristics and properties of the vehicles, and of the geometry of the crash among them, were implemented in MatLab functions, described in full detail in Martins (2005).

### 2.4. Pos-Collision Model

It was developed a generic model to represent the plane dynamics of a rigid vehicle for analysis of it movements in the pos-collision condition, since the instant soon after the crash until it complete stop. The Figure 4 display the global $(X, Y)$ and local $(x, y)$ frames, adopted for the treatment of the dynamic problem of a vehicle in the plane, and the relevant geometric parameters for it analysis: the front and rear tracks ( $b d$ and $b t$ ) and the axes distances ( $l d$ and $l t$ ) to the mass center (CM). The wheelbase is given by $l=l d+l t$. In way to correctly treat the behavior of the vehicle after the collision, should be adopted a model that represents their movements using the simplified Newton-Euler equations, for displacements in the plane, given by

$$
\left\{\begin{array}{l}
\dot{u}=-\frac{\mu g}{4}\left(u_{D D, x}+u_{D E, x}+u_{T D, x}+u_{T E, x}\right)-\frac{b_{A y}}{m} u+v \omega  \tag{5}\\
\dot{v}=-\frac{\mu g}{4}\left(u_{D D, y}+u_{D E, y}+u_{T D, y}+u_{T E, y}\right)-\frac{b_{A y}}{m} v-u \omega \\
\dot{\omega}=-\frac{\mu g}{4 r_{z z}^{2}}\left(\left(u_{D D, x}-u_{D E, x}\right) \frac{b_{d}}{2}+\left(u_{T D, x}-u_{T E, x}\right) \frac{b_{t}}{2}+\left(u_{D D, y}+u_{D E, y}\right) l_{d}-\left(u_{T D, y}+u_{T E, y}\right) l_{t}\right)-\frac{b_{A z}}{J_{z z}} \omega
\end{array}\right.
$$



Figure 3. Position and Orientation of the Vehicle in Global Frame.


Figure 4. Global and Local Reference Frames of a Ground Vehicle.
where,
$u$ is the local (along $x$ ) longitudinal speed ( $\mathrm{m} / \mathrm{s}$ );
$v$ is the local (along $y$ ) lateral speed ( $\mathrm{m} / \mathrm{s}$ );
$\omega$ is the angular speed (around z , perpendicular to the movement plane, passing through the CM$)(\mathrm{rad} / \mathrm{s})$;
$m$ and $J_{z z}$ are, respectively, the mass ( kg ) and the moment of inertia around $\mathrm{z}(\mathrm{kg} . \mathrm{m} 2)$; and
the forces $(\mathrm{N})$ and the moments $(\mathrm{Nm})$ applied to the vehicle, projected in the axes of the local reference system, are obtained from the associated speeds in a certain condition and from rotation of the vehicle around the vertical axis, and from their components $\left(\mathrm{V}_{\mathrm{IJ}, \mathrm{x}} \mathrm{V}_{\mathrm{IJ}, \mathrm{y}}\right)$, that supply the direction of the instantaneous speed, in agreement with the unitary vector
$\vec{u}_{I J}=\frac{\vec{V}_{I J}}{\left|\vec{V}_{I J}\right|}=\frac{1}{\sqrt{V_{I J, x}{ }^{2}+V_{I J, y}{ }^{2}}}\left[\begin{array}{c}V_{I J, x} \\ V_{I J, y} \\ 0\end{array}\right]=\left[\begin{array}{c}u_{I J, x} \\ u_{I J, y} \\ 0\end{array}\right]$
where the indexes $\mathrm{IJ}=\mathrm{DD}, \mathrm{DE}, \mathrm{TD}$, TE identify the wheel. Supposing that the wheels are not steered, considering that all are locked, dragged by the vehicle, the friction force, without taking into account the distribution of instantaneous load in each tire, or even the static distribution, is given approximately by
$\vec{F}_{I J} \cong-\frac{\mu m g}{4} \vec{u}_{I J}$
where $\mu(\cong 0,7)$ is the coefficient of dynamic friction, and the forces and aerodynamic moments, considering the relatively low speeds that vehicles have after a collision are given approximately by
$F_{A x} \cong-\left(\frac{1}{2} \rho C_{x} S u_{0}\right) u=-b_{A x} u$
$F_{A y} \cong-\left(\frac{1}{2} \rho C_{y} S v_{0}\right) v=-b_{A y} v$
$M_{A z} \cong-\left(\frac{1}{2} \rho C_{M z} S l \omega_{0}\right) \omega=-b_{A z} \omega$
where, according to Genta (1997),
$\rho \cong 1,20 \mathrm{~kg} / \mathrm{m} 3$ is the air density;
$C_{x}$ is the longitudinal drag coefficient ( $\approx 0,3$ for medium/small passenger cars);
$S$ is the frontal area ( $\approx 2,0 \mathrm{~m}^{2}$ for medium/small vehicles);
$C_{y}$ is the lateral drag coefficient ( $\approx 0,8$ for medium/small vehicles);
$C_{M z}$ is the coefficient of yaw moment around $z(\approx 0,2$ for medium/small vehicles); and
$l$ is a characteristic length ( m ), that for the case of the yaw movement, it is usually adopted the wheelbase of the vehicle.
It is noticed that the coefficients $b_{A i}$ characterize an equivalent effect of viscous damping, decreasing the system oscillations. It is observed that $\mu g$ is the maximum capacity of the vehicle slowing down that just depends on the friction of the tires with the ground. Reminding finally that $J_{z z}=r_{z z}{ }^{2} \mathrm{~m}$, in the which $r_{z z}$ is the gyration radius around Z , so $\mu g / r_{z z}{ }^{2}$ is the maximum capacity of angular slowing down of the vehicle, that depends on it geometry and mass distribution. As we normally are interested in the mass center translation in relation to the global inertial frame, then it should proceed a change of coordinates, given by

$$
\left[\begin{array}{c}
\dot{X}  \tag{9}\\
\dot{Y}
\end{array}\right]=\left[\begin{array}{cc}
\cos \psi & -\operatorname{sen} \psi \\
\operatorname{sen} \psi & \cos \psi
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right]
$$

with
$\psi=\psi_{0}+\int\left(\omega+\omega_{0}\right) d t$
and

$$
\begin{align*}
X & =X_{0}+\int \dot{X} d t=\int\left(\left(u+u_{0}\right) \cos \psi-\left(v+v_{0} \operatorname{sen} \psi\right)\right) d t \\
Y & \left.=Y_{0}+\int \dot{Y} d t=\int\left(\left(u+u_{0}\right) \operatorname{sen} \psi+\left(v+v_{0}\right) \cos \psi\right)\right) d t \tag{11}
\end{align*}
$$

where,
$\dot{X}$ e $\dot{Y}$ are the absolute translation speeds ( $\mathrm{m} / \mathrm{s}$ ) in the global frame;
$\psi$ is the yaw angle (rad);
$\psi_{0}, X_{0}$, e $Y_{0}$ are, respectively, the initial conditions of orientation (rad) and position (m) of the vehicle;
$\omega_{0}, \dot{X}_{0}$ e $\dot{Y}_{0}$, are, respectively, the initial conditions of angular ( $\mathrm{rad} / \mathrm{s}$ ) and linear ( $\mathrm{m} / \mathrm{s}$ ) speeds, in the global frame; and $u_{0}$ e $v_{0}$, the initial translation speeds ( $\mathrm{m} / \mathrm{s}$ ) in the local frame.

Remembers that the friction forces, and therefore also the moments generated by them, exists while there is movement, this meaning that in the differential equations above, all the terms of the right will turn null when the translation and rotation speeds are null, in other words, when the vehicle stops. The Figures 5 and 6 shows the Simulink/Matlab block diagrams used for two vehicles pos-collision model implementation, in which the stop condition, associated to the command STOP is determined when both vehicles reach all their null speeds.


Figure 5. Pos-collision Model of a Vehicle.


Figure 6. Pos-collision Model for 2 Vehicles with Stop Test.

## 3. GENETIC ALGORITHMS

The Genetic Algorithm is characterized by the generation of random values according to different statistical distributions and the evolution of those values, what feels in way similar to the alive beings' evolution in agreement with Darwin's Evolutionary Theory. As in the nature, in some cases of engineering they are many the factors that define an "individual's" aptitude. Therefore the crossing of the most capable with those with less survival chance is also important, for larger diversity of information to be had and therefore larger chances of arriving to the "most capable individual."

The Figure 10 illustrates the procedure for application of Genetic Algorithm to the accidents reconstruction inverse problem, using the models of instantaneous crash and pos-collision described previously, that in this figure meet in the call Simulator, Figure 10(e). When inserting the problem of accidents reconstruction in an otimizator, an important factor to be defined is the evaluation function, because this will be the responsible connection for "addressing" the algorithm for the best individuals. As better an individual's evaluation, larger probability he will have to pass their information for the following generations. Therefore, the evaluation function opted was the square root of the sum of the squares of the distances among the obtained and expected positions of the vehicles, taken the position, after the stop, of two points of each vehicle (preferably that are opposed geometrically) and measuring the distances regarding the respective expected positions, as can see in the Figure 10(f).

Another challenge was to define which will be the variables that the algorithm should work in search of the solution. The more factors to optimizes exists, wider it will be the universe of search of the problem and therefore more slowly will the convergence to the solution. In other hand, if the factors are too much arrested the limit not to be necessarily suitable with the reality of the events, it won't more probably be arrived to a valid solution. For these reasons it was defined the algorithm inputs (genes), shown in Figure 10 (a) and (b)), as: initial speeds; initial positions; initial attitudes; place of the collision; collided parts of the vehicles.

The limit and/or the average and probabilistic distribution of these factors will be given by the user's/specialist entrance. With that, in spite of the great number of parameters to be optimized, the search universe will be limited. The more real information the analyst has, it will be waiting a faster convergence of the algorithm. Define the geometry of the problem maybe has been the largest challenge. As the genetic algorithm is based on a stochastic process, the positions of the vehicles cannot be defined in global coordinates. This impossibility is owed to the fact that initially the otimizator generates independent values amongst themselves. So, even if limits were imposed for each one of the variables location of each vehicle, they would be however great chances of generation of situations where the vehicles would not collide, or they would collide from way to not to respect the contact areas presupposed. A solution found for this problem was to define the initial position of each vehicle through three parameters: the angle formed among the system of local coordinates of the vehicle, with origin in the geometric center of the same, and the system of global coordinates; and the other parameters would be the collision place and the collided part of each vehicle (Figure 10 (c)). In agreement with the model of instantaneous collision described previously, the angle that each vehicle forms with the system of global coordinates is obtained starting from the definitions of the affected areas in the collision. With that, the genetic algorithm will generate initial values for the collided parts and to the collision place, respecting the established limits, without creating therefore disable solutions a priori.

Through these informations, it is possible to know the global position of each one of the vehicles to each variation of the parameters for the algorithm. It is enough to rotate them for the angle, to consider the collided parts and to anchor them in the collision (Figure 10 (d) place). It fit to stand out that each one of the characteristics above (collided part and
collision place) will be random varied by the GA, among the stipulated limit, without disable solutions are generated. It is important to detach that the procedure of determination of the global position will be all made in the simulation program, therefore to the part of the algorithm otimizator.


Figure 10. GA Combined with the Collisions Simulation Program.

## 4. CASES STUDY

With the intention of generating collision sceneries to evaluate the otimizator, were defined reasonable values for the input variables and the collision corresponding to such values was simulated. The final positions obtained were imposed as objective for the algorithm. Naturally the limit of the otimizator were initially established in way to contain the values imposed. Next will be shown two cases: oblique lateral collision and oblique back collision.

### 4.1. Oblique Lateral Collision

An oblique lateral collision happens when the center of the collided area of at least one of the vehicles is in its lateral one, as sketched in the Figure 11, which the geometry of the collision is shown. For the optimization the following values were used for the algorithm parameters:

- Size of the population: 100
- Numbers of unaffected individuals for generation: 5
- Crossing fraction: $95 \%$
- Crossing function: middleman (it respects the limit inferior and superior)
- Stop criterion: value of the function of smaller evaluation than $100(\mathrm{~mm})$ or 20 generations

The relevant results obtained are in the Figure 12. The value of the evaluation function in this case was of approximately $99,2 \mathrm{~mm}$. Starting from the configuration in the Figure 11, applied the model of instantaneous crahs, it was obtained the initial speeds for the model pos-crash, by the procedure described previously. It can be noticed that the vehicles approach satisfactory form of the expected positions, as it indicates the value of the evaluation function. In the Figure 14 the animation of the path of two vehicles is had in the pos-collision situation. The vehicles clear blue and pink represent the waited final position of the vehicles blue and red respectively. The last picture of the Figure 14 is enlarged in the Figure 13.


Figure 11. Geometry of the Lateral Collision.


Figure 12. Function of Evaluation of the Average and Better Individual to each Generation.


Figure 13. Final of the Simulation: Overlap of the Expected and Obtained Positions.

### 4.2. Oblique Back Collision

An oblique back collision happens when the center of the collided area of at least one of the vehicles is in its rear, as sketched in the Figure 15 in the moment immediately precedent to the crash. The presented configuration could be due to an abrupt braking of the front (outstanding in blue) vehicle. The value of the evaluation function in this case was of approximately $829,3 \mathrm{~mm}$, and the optimization process is quite slow, and the algorithm only ceased for having reached the established maximum limit for the number of generations. In the Figure 17, the animation of the path of the two vehicles is had in the pos-collision situation. The color conventions and form of the vehicles adopted in the previous example were maintained. It can be noticed the difference of positions among the expected and obtained ones, as it indicates the value of the evaluation function. In spite of the results they have not been as good as the previous ones, they are enough for the evaluation of the respective accident, not only considering the final positioning of the vehicles, as shown in the last picture of the Figure 17, but mainly the difficulty of reaching them through a "manual" try-anderror procedure, as it is usually (or it was) done, without the aid of an otimizator.


Figure 15. Geometry of the Back Collision.


Figure 16. Function of Evaluation of the Average and Better Individual to each Generation.


Figura 14. Simulation of the Path of two Vehicles after an Oblique Lateral Collision.

## 5. FINAL CONSIDERATIONS

The next stages in the development of this approach in the accidents reconstruction problems will be the inclusion of models of deformable vehicles, based on Carvalho (2004) and Menezes (2007) works; the formalization of the implementation of the procedure of solution of the inverse problem, using the Genetic Algorithms and techniques of classic optimization; and the introduction of the pre-collision model (Speranza Neto and Spinola, 2005), including the human being's behavior. It should also be begun soon the treatment of problems that involves three-dimensional movements of the vehicles, that will allow to treat, in a complete way, among others, the accidents with rollovers.


Figure 17. Simulation of the Path of two Vehicles after an Oblique Back Collision.

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