# INVESTIGATION OF CORRECTIONS MODELS FOR SURRUGATE BASED OPTIMIZATION

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***Abstract.*** *In recent years surrogate based otimization (SBO) has been extensively employed for practical engineering design mainly to overcome the drawbacks of the computational cost required for multiple numerical simulations. In this work a physical based surrogate is proposed for optimum design of space truss. To improve convergence corrections to the proposed model are applied. Different forms to represent the corrections are used. Global and local approximations are constructed. A sizing structural otimization (SSO) framework considering a SBO algorithm with different surrogate correction approaches are implemented. The performance of implemented tools are investigated.*

***Keywords***: *Correction, Approximation, Optimization*

## 1. INTRODUCTION

Optimization techniques have been extensively used to obtain economical and reliable structural designs. To apply such techniques to real engineering problems could be an issue as the computational cost required for multiple numerical simulations could be, in certain cases, prohibitive. In the last decade, approximation concepts (Haftka et al, 2004) started to be more extensively applied in optimization procedures as a strategy to overcome such drawback. The approximations methodologies can be divided into two types: functional and physical (Haftka et al, 2004). While in the first an alternate and explicit expression is sought for the objective function and/or the constraints of the problem, the focus of the second is on replacing the original problem by one which is approximately equivalent but which is easier to be solve. The later captures the behavior of the high fidelity model in global scale with reduced computational cost but with lower fidelity (e.q. simplified physics, coarser discretization, looser stopping criteria).

In the present work, a classical model used in the past mainly for behavior prediction and still extensively used today by practical engineers in a predesign stage, is investigated for use as a physical surrogate in sensitivity analysis and optimization of large space truss structures. The structural compliance is the objective function to be considered. The design variables are the cross sectional areas which are grouped for different regions. The initial total volume is kept as a constant.

For the problem described above a grid analogy is used in substitution to the real truss and correction models are used to approximate the difference between the real (truss) and approximate (grid) FE model. Additive and multiplicative are the corrections forms to be investigated. Global and local approaches are considered.

The surrogate model proposed will be used to perform both global and local type of approximation. In the later the approximation of the correction can be constructed using data fitting schemes or Taylor series expansion and it will require a globalization scheme to ensure convergence. To accomplish that an interactive procedure, such as sequential approximated optimization (SAO) (Guinta, 2002), (Afonso, *et al.*, 2005), in which the results of high and low-fidelity analysis have to be checked periodically in the course of the optimization process is implemented. A trust region based method (Guinta, 2002), (Afonso, *et al.*, 2005), (Alexandrov, *et al.*, 1997) is used to update the design variable space for each subproblem (SAO iteration). In this process different levels of consistency between the surrogate and the high-fidelity model are imposed.

A sizing structural optimization which integrates geometric definition, structural analysis, sensitivity analysis and mathematical programming algorithms considering the surrogate models mentioned above is implemented. The sequential quadratic programming algorithm (SQP) (Powell, 1978) is the optimizer of choice. A classical 3D truss example will be studied and comparisons between the different approaches will be conducted.

## 2. PROBLEM DESCRIPTION

### 2.1. High-fidelity model – Space Truss ( $f_{hi}$ )

Consider the structure shown in Figure 1 which is a two-layer space truss with rectangular mesh in both planes, with any node of one plane connected by web members to four adjacent nodes of the other plane. The structural compliance (to be approximated) is given by

$$f_{hi} = \boldsymbol{u}^t(\boldsymbol{x})\boldsymbol{F} \ , \tag{1}$$

where $\boldsymbol{u}(\boldsymbol{x})$ are the displacements, $\boldsymbol{F}$ is the vector of applied loads and $\boldsymbol{x}$ defines the areas of top, and bottom chords as well diagonal members.

This sort of structures involves three-dimensional arrangements of thousands of bars and a large number of degrees of freedom. Direct use of 3D frame analysis for functions/derivatives evaluations may result in a lengthy optimization process. This aspect motivated the study of approximation techniques to shorten the computational times. This is discussed the next few sections.

Figure 1. Brasília exhibition center, Brasília, Brazil.

$a_x, a_y = 1.5m$
$l_x, l_y = 24m$
$d = 1.3m$
$E = 210GPa$

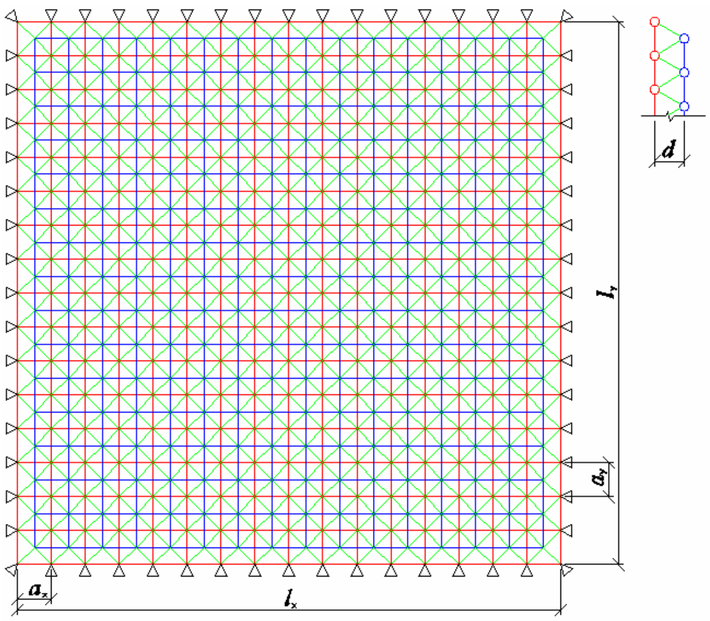


Figure 2. 3D Truss structure.

### 2.2. Low-fidelity model – Grid analogy ( $f_g$ )

For the problem shown in Figure 2, it has been shown that (Flower and Schmidt, 1971) in the interior of the truss the vertical deflection, $w$ satisfies:

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} = q \,, \tag{2}$$

when mesh size goes to zero. This is the differential equation of a plate with zero torsional rigidity. Flexural rigidities $D_x$ and $D_y$ are derived from chord areas considered smeared over grid spacing in the plane of upper and lower chords. Assuming different sizes for top and bottom chords one gets:

$$D_x = \frac{k_x E A_{tx} d^2}{a_x} \qquad D_y = \frac{k_y E A_{ty} d^2}{a_y} \tag{3}$$

$$k_x = \frac{A_{bx}}{A_{bx} + A_{tx}} \qquad k_y = \frac{A_{by}}{A_{by} + A_{ty}} \tag{4}$$

where: $d$ = depth of truss; $a_x$, $a_y$ = chord member spacing in $x$ and $y$ directions; $A_{tx}, A_{bx}$ = cross sectional areas of top and bottom chord members in $x$ direction; $A_{ty}, A_{by}$ = cross sectional areas of top and bottom chord members in $y$ direction (see Figure 2).

Numerical experiments show reasonable agreement of equivalent plate solutions in the case of simple supports on four sides.

It has been shown by Renton (1966) that as grid spacing becomes smaller the behavior of beam grillages can be described by the following differential equation:

$$\frac{EI_x}{b_y} \frac{\partial^4 w}{\partial x^4} + \left( \frac{GJ_x}{b_y} + \frac{GJ_y}{b_x} \right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{EI_y}{b_x} \frac{\partial^4 w}{\partial y^4} = \frac{W}{b_x b_y} \,, \tag{5}$$

where: $EI_x$ and $GJ_x$ = flexural and torsional rigidities of beams of span $b_x$ in $x$ direction; $EI_y$ and $GJ_y$ = flexural and torsional rigidities of beams of span $b_y$ in $y$ direction; $W$ = total load over area $b_x b_y$ (see Fig. 3). Comparing Eq. (2) and (5) it can be seen that a reasonable physical model for a space truss is a grillage whose members have the follow properties:

$$I_x = \frac{b_y}{a_x} k_x A_{tx} d^2 \qquad I_y = \frac{b_x}{a_y} k_y A_{ty} d^2 \qquad J_x = J_y = 0 \tag{6}$$

$b_x, b_y = 6m$
$l_x, l_y = 24m$
$E = 210GPa$

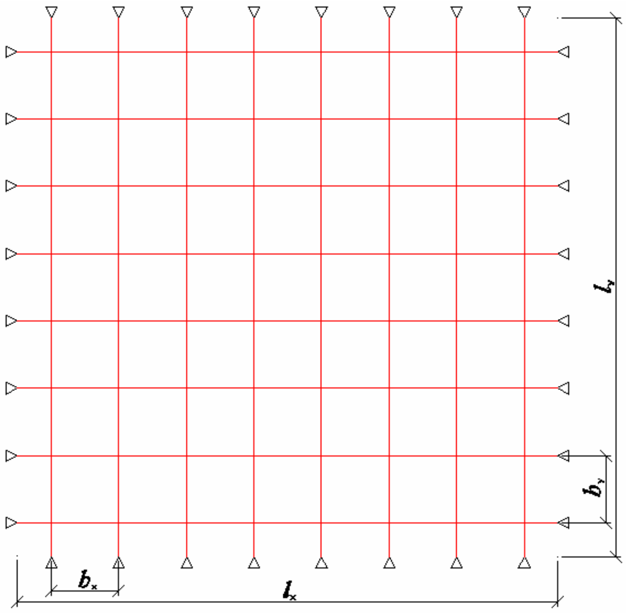


Figure 3. Equivalent grid model.

## 3. MODEL CORRECTIONS

Correction models are proposed to be constructed to ensure matching between the fidelity models. In this work additive and multiplicative correction approaches will be considered. Any of these models can employ Taylor series functions or data fitting polynomials. The surrogate models employed in this work considering both forms of corrections with its respectively gradients are written as follows

$$\hat{f}(x) = f_g(x) + \delta_a(x), \qquad \nabla\hat{f}(x) = \nabla f_g(x) + \nabla\delta_a(x) \tag{7}$$

$$\hat{f}(x) = f_g(x)\delta_m(x), \qquad \nabla\hat{f}(x) = \delta_m(x)\nabla f_g(x) + f_g(x)\nabla\delta_m(x) \tag{8}$$

in which $f_g(x)$ is the function calculated from grillage analysis, $\delta_a(x)$ is the additive applied correction and $\delta_m(x)$ is the multiplicative applied correction. Both forms are evaluated using one of the schemes presented below.

### 3.1. Data fit based corrections

Data fit type approximations typically involve interpolation or regression (polynomial) of a set of data generated from the true model. Interpolation models commonly used as approximation forms are based on techniques known as kriging, a well-known tool in the field of statistics and geostatistics. The main advantage of data fit schemes is that there is no need to compute sensitivities for the high-fidelity model. So for complex problems involving high simulation cost and noisy functions this will be an appropriate choice.

In the present work this strategy will combine high-fidelity (space truss) and low-fidelity (grid) analysis to construct the correction approximated terms of Eqs. (7) and (8).

The sampling considered in this work can be generated considering two classical DOE techniques that are the Rectangular Grid (deterministic) and Latin Hypercube (stochastic) (Guinta and Watson, 1998).

#### 3.1.1. Response surface correction

For general purpose applications, this scheme is suitable to build local approximations only. For an input vector $x$ the approximated correction term of Eqs. (7) and (8) will be a regression model such as

$$\hat{\delta}_a = \hat{\delta}_m = \hat{\delta}(x) = \sum_{i=1}^{N} N_i(x)\beta_i = N(x)\beta, \tag{9}$$

in which the regression model involves a function $N(x)$ with $p$ chosen terms such as

$$N(x) = \lfloor N_1(x)\ldots N_p(x)\rfloor, \tag{10}$$

and the unknown coefficients $\beta$.

*Coefficients Prediction*

Considering a sampling with $m$ design points $S = [x_1 \ldots x_m]$ the polynomial forms of the regression terms build the design matrix $\mathcal{N}$ given by

$$\mathcal{N} = \begin{bmatrix} N_1(x_1)\ldots N_p(x_1) \\ \vdots \\ N_1(x_m)\ldots N_p(x_m) \end{bmatrix}. \tag{11}$$

Considering $\delta(x_i)$ $i = 1\ldots m$ the true error in the $m$ samplings, the unknown coefficients are obtained using a least square method to solve the following equation

$$\delta = \mathcal{N}\beta, \tag{12}$$

in which $\delta$ is a vector of m components and $\mathcal{N}$ is the regression terms computed in the samplings, $\in \Re^{m\times p}$ (with $p < m$).

In this work, a regression model with polynomials of order two is considered. For a two design variable problem $(x_1, x_2)$ this implies:

$$N(x) = \begin{bmatrix} 1 & x_1 & x_2 & x_1 x_2 & x_1^2 & x_2^2 \end{bmatrix}. \tag{13}$$

### 3.1.2. Kriging surface correction

Kriging models [1, 3-6] differ from regression models presented before in the sense that they in general give a global approximation to the response and also can capture oscillatory response trends. Moreover, the sample values are assumed to exhibit spatial correlation with response values modeled via a Gaussian process around each sample location.

The approximated form for this particular model has an additional term, a random function which in general follows a normal Gaussian distribution with zero mean, variance $\sigma^2$ and non-zero covariance. Under consideration of such approach the estimates for $\hat{\delta}(x)$ are obtained by

$$\hat{\delta}(\mathbf{x}) = \delta(\mathbf{x})\hat{\beta} + \mathbf{r}^t(\mathbf{x})\mathbf{R}^{-1}(\Delta - \mathcal{N}\hat{\beta}), \tag{14}$$

where $\hat{\beta}$ is unknown, $\Delta = [\delta_1, \ldots, \delta_m]^T$ i.e. the true error ( $f_{hi} - f_g$ ) at the sampling and $\mathcal{N}$ is a $m \times k$ design matrix defined before Eq. (11). For the particular case of ordinary kriging $\mathcal{N}$ is a column vector of length $m$ filled with ones.

The above equation requires calculation of the correlation vector $\mathbf{r}(\mathbf{x})$, which correlates an untried $\mathbf{x}$ and the $m$ sampled data points, such as

$$\mathbf{r}(\mathbf{x}) = \left[R(\mathbf{x}, \mathbf{x}^{(1)}), R(\mathbf{x}, \mathbf{x}^{(2)}), \ldots, R(\mathbf{x}, \mathbf{x}^{(m)})\right]^T. \tag{15}$$

The values for $\hat{\beta}$ and estimated variance are obtained using generalized least squares as

$$\hat{\beta} = (\mathcal{N}^T \mathbf{R}^{-1} \mathcal{N})^{-1} \mathcal{N}^T \mathbf{R}^{-1} \delta, \tag{16}$$

and the estimate variance is given by

$$\sigma^2 = \frac{(\delta - \hat{\beta}\mathcal{N})^T \mathbf{R}^{-1} (\delta - \hat{\beta}\mathcal{N})}{m}. \tag{17}$$

In Eq. (15) both $\mathbf{r}$ and $\mathbf{R}$ depend on the unknown parameter $\theta_k$. This is found using maximum likelihood estimation which is a $m$ dimensional minimization problem (best guesses as described (Keane and Nair, 2005)).

In kriging models some assessment strategies are required to check a priori if a generated model is adequate. Previous work from the authors (Afonso, *et al.*, 2007) investigated the RMSE (Root Mean Square Error) and PRESS (Predicted Error Sum of Squares) as guidance for selecting the best model from the existing model construction possibilities. For the particular problem addressed here the best kriging model was found considering ordinary kriging, $\theta = \theta^*$, (obtained by maximum likelihood estimation) and LHS sampling distribution as DOE approach. Such kriging form is considered in the present study to build the model correction when interpolation is the choice.

### 3.2. Taylor series based corrections

These classes of functions are valid in the neighborhood or the point at which they are generated. Several orders of local approximations can be constructed. Linear approximation is the form used in this work. Under such consideration the approximated additive ( $\hat{\delta}_a$ ) and multiplicative ( $\hat{\delta}_m$ ) correction terms are respectively written as

$$\hat{\delta}_a(x) = \delta_a(x_0) + \nabla\delta_a(x_0)^T (x - x_0), \tag{18}$$

$$\hat{\delta}_m(x) = \delta_m(x_0) + \nabla\delta_m(x_0)^T (x - x_0), \tag{19}$$

in which

$$\delta_a(x_0) = f_{hi}(x_0) - f_g(x_0), \tag{20}$$

$$\nabla\delta_a(x_0) = \nabla f_{hi}(x_0) - \nabla f_g(x_0), \tag{21}$$

and

$$\delta_m(x_0) = f_{hi}(x_0)/f_g(x_0), \tag{22}$$

$$\nabla\delta_m(x_0) = \frac{1}{f_g(x_0)}\nabla f_{hi}(x_0) - \frac{f_{hi}(x_0)}{f_g^2(x_0)}\nabla f_g(x_0). \tag{23}$$

## 4. SAO SCHEME

As already pointed out, SAO (Guinta and Eldred, 2000) methods are necessary when using local type of approximations forms. Typically, a SAO methodology decomposes the original optimization problem into a sequence of optimization subproblems, confined to a small subregion of the optimization design space. Surrogate functions (low-cost) are created and used by the optimizer. In the present work, either first-order approximations or analytical functions built from data fit schemes over pre-selected sampling of points are used to create explicit approximations for the correction term and its gradients used in the proposed surrogate model Eqs. (7) and (8). The different models are here named for reference SAO_TS (Taylor Series), SAO_RS (Response Surface) and SAO_K (Kriging), respectively.

### 4.1. Mathematical formulation

Typically, a SAO methodology decomposes the original optimization problem into a sequence of optimization subproblems, confined into small subregion of optimization design space. Surrogate functions (low-cost) are created and used by the optimizer. A trust region based method is used to update the design variable space for each subproblem (SAO iteration). Mathematically each subproblem $k$ is defined as

| | | |
|---|---|---|
| Minimize | $\hat{f}^k(x)$ | (24) |
| subject to | $\hat{g}_i^k(x) \le 0,\ i = 1,\ldots,m$ | (25) |
| | $x_l \le x_l^k \le x \le x_u^k \le x_u, \quad k = 0,1,2,\ldots,k_{\max}$ | |
| where | $x_l^k = x_c^k - \Delta^k$ | (26) |
| | $x_u^k = x_c^k + \Delta^k$ | (27) |

In above equations, $\hat{f}^k(x)$ and $\hat{g}^k(x)$ are respectively the approximated (surrogate) objective and constraints functions, $x_c^k$ is the center point of the trust region, $\Delta^k$ is the width of the trust region and $x_l^k$ , $x_u^k$ are respectively the lower and upper bounds of the design variables for a SAO iteration $k$ (Eldred, *et al.*, 2004).

### 4.2. SAO algorithm

Each subproblem described before define a SAO iteration. The total number of iteration is $\mathrm{k}_{\max}$ . Each iteration starts with a trust region $\mathrm{t}^k$ of size $\Delta^k$ and an initial design $x_0^k$ . Solving Eq. (24) an approximate optimum $x_*^k$ is found. A comparison between the high and low fidelity response at $x_*^k$ and $x_0^k$ dictates the next center point $x_c$ and the next trust region size $\Delta$ of iteration $\mathrm{k}+1$. The consistency requirements are imposed before starting iteration $\mathrm{k}+1$. This process continues until convergence is reached. In short, the main steps involved in the computations are:

1. Compute the expensive and/or nonsmooth objective function and constraints at the central point in the subregion,
2. Construct surrogate model in the subregion,
3. Optimize within the subregion using the surrogate objective function and constraints,
4. Compute the true objective function and constraints at the optimum identified in Step 3,
5. Check for convergence,
6. Move/shrink/expand the subregion according to the accuracy of the approximated model compared to the true function and constraint values
7. Impose local consistency, and
8. Check for overall optimization convergence. If it is achieved stop the SAO procedure; otherwise return to Step 3.

### 4.3. Trust Region Update Scheme

To update the trust region size $\Delta^k$ for each optimization subproblem we considered the approach described in reference (Eldred, *et al.*, 2004). A parameter $\rho^k$ controls the trust region size. This parameter measures the accuracy of the surrogate function at $x_*^k$ and is calculated as

$$\rho^k = \min\left(\rho_f^k, \rho_g^k\right), \quad \text{for } \mathrm{k} = 0,1,2,\ldots,\mathrm{k}_{\max} \tag{28}$$

where

$$\rho_f^k = \frac{f_{hi}(x_c^k) - f_{hi}(x_*^k)}{\hat{f}(x_c^k) - \hat{f}(x_*^k)}, \tag{29}$$

and

$$\rho_g^k = \frac{g_{hi}(x_c^k) - g_{hi}(x_*^k)}{\hat{g}(x_c^k) - \hat{g}(x_*^k)}. \tag{30}$$

The next trust region size is updated as follows:

$$\begin{aligned} \Delta^{k+1} &= 0.5\Delta^k \text{, if } \rho^k \le 0, \\ &= 0.5\Delta^k \text{, if } 0 < \rho^k \le 0.25, \\ &= \Delta^k \text{, if if } 0.25 < \rho^k < 0.75 \text{ or } \rho^k > 1.25, \\ &= 2\Delta^k \text{, if if } 0.75 \le \rho^k \le 1.25. \end{aligned} \tag{31}$$

The next iterate $x_c^{k+1}$ is obtained according to

$$\begin{aligned} x_c^{k+1} &= x_*^k \text{, if } \rho^k > 0, \\ &= x_c^k \text{, if } \rho^k \le 0. \end{aligned} \tag{32}$$

In which $f_{hi}$, $g_{hi}$ are the high fidelity objective and constraints functions.

### 4.4. Consistency requirement

In this work $C^0$ and $C^1$ local consistency are imposed in our surrogate models. For data fit schemes, SAO_RS and SAO_K, only zeroth order consistency is enforced. Zeroth and first-order consistency at $x = x_c$ ($\hat{f}(x_c) = f_{hi}(x_c)$, $\nabla\hat{f}(x_c) = \nabla f_{hi}(x_c)$, respectively) can be demonstrated for additive corrections by substituting Eqs. (20) and (21) into Eq. (18), substituting resulting expressions into Eq. (7), ant then evaluating at $x = x_c$. Similarly, consistency for multiplicative corrections can be demonstrated by substituting Eqs. (22) and (23) into Eq. (19), substituting resulting into Eq. (8), and then evaluating at $x = x_c$.

### 4.5. Convergence Criteria

Prior to carry out to the next iterate the SAO procedure checks if the design has converged to a stationary solution. In this situation the following convergence condition have to be met:

- A better design can not be found for the last specified number of iterations (3).
- The design shows minor improvements for the last iteration. This is measured if the following condition is met:

$$\text{tol} = \left| \frac{f_{hi}(x_*^k) - \hat{f}(x_*^k)}{f_{hi}(x_*^k)} \right| \le 10^{-4} \tag{33}$$

## 5. EXAMPLES

A space truss with geometry shown in Figure 2, is considered to analyze the approximation strategies presented here. The number of meshes is 16 in both directions. Cross-section of bottom chord and diagonal bars are 75% of those of the top chord. This models the fact that bars in compression tend to be heavier than those in tension. Two design variables ($x_1, x_2$), are considered for optimization. Cross-section of bars are linked to those as shown in Figure 4. Vertical displacement of nodes in the four sides is set to zero. Enough degree of freedom are additionally constrained to eliminate rigid body motions. A distributed load of 80 kN/m$^2$ is acting on the structure and is applied as concentrated forces at nodes of top chord. The objective is to minimize compliance subjected to volume corresponding to initial design $x_1 = 3.9973$; $x_2 = 3.9973$ cm$^2$. Side constraints imposed are $2 \le x_1, x_2 \le 15$. The grillage model considered is illustrated in Figure 5 where the linkage of the design variables is highlighted. The strategies used to obtain optima are show in Table 1. The global data fitting models employs 25 sampling obtaining by LHS distribution. The time required to obtain the high-fidelity functions at the samplings are 20.77 seconds.

Local data fitting strategies consider six samplings for each SAO iteration confined to the trust region of the particular subproblem. The initial trust region size was 10% of the space design domain.

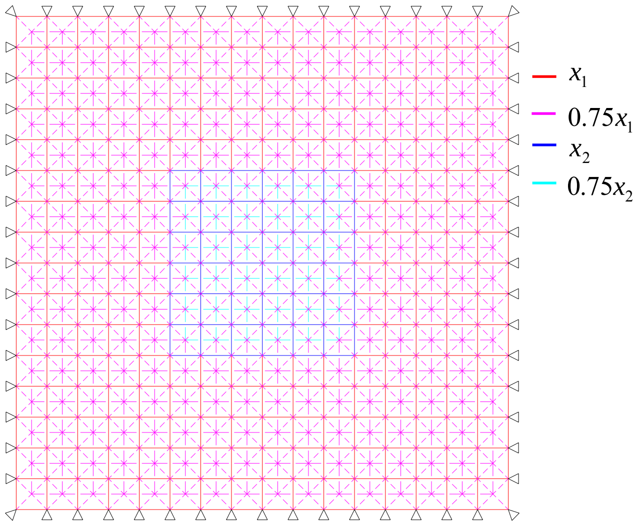


Figure 4. Space truss analyzed: Linkage of the design variables

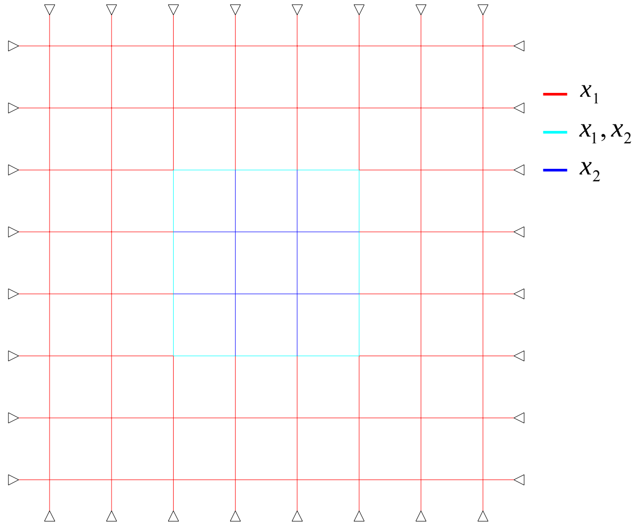


Figure 5. Equivalent grid analyzed: Linkage of design variable

Table 1. Optimization strategies considered

| **Strategy** | **Description** |
|---|---|
| High-fidelity | Full 3D Truss model |
| Global | |
| Grid | Grid analogy without correction |
| Grid_K_A | Grid analogy with additive kriging correction |
| Grid_K_M | Grid analogy with multiplicative kriging correction |
| Local | |
| SAO_TS_A | Grid analogy with additive Taylor series $C^1$ consistency |
| SAO_TS_M | Grid analogy with multiplicative Taylor series $C^1$ consistency |
| SAO_K_A | Grid analogy with additive kriging correction |
| SAO_K_M | Grid analogy with multiplicative kriging correction |
| SAO_RS_A | Grid analogy with additive quadratic response surface correction |
| SAO_RS_M | Grid analogy with multiplicative quadratic response surface correction |

For the study conducted here, the main observations can be made (see Tab. 2):

- All strategies save CPU time with respect to full high-fidelity optimization except for local strategy with data fit.
- Physical surrogate by itself is capable of producing a good solution with 6.3% error on the objective function. However the observed error on $x_2$ is 18.8%. Note that Kriging correction improved substantially the solution.
- $C^1$ consistency of SAO_TS strategies produces the best approximation models. The difference between additive and multiplicative correction alternatives is minor. It is important to emphasize that only four high-fidelity function and gradient evaluations were required for convergence. The maximum error found in the solution is 0.25% in variable $x_2$.
- Data fitting local corrections resulted in higher computational times, but it is an adequate choice in case gradient are not readily available or the function exhibits numerical noise.
- Kriging is the technique of choice for global approximation in engineering problems in general (Guinta and Watson, 1998). It also outperformed quadratic response surface data fitting for local approximation.

Figures 6 and 7 illustrate the high-fidelity (3D FE Truss) surface and its contour plot over the design domain $2 \leq x_1, x_2 \leq 15$. The global Grid_K_M surface and contour are respectively shown in Figs. 8 and 9. As observed the approximated model reproduces very well the general trends of the high-fidelity model.

Figure 10, illustrates the evolution of the SAO trust region based scheme for the best local model constructed (SAO_TS_A). The sequence of solutions (dots in the figure) towards the true optimum solution (x symbol) in only four SAO iterations can be perceived.

Table 2. Optimization results

| Method | $x_1$ (cm$^2$) | error | $x_2$ (cm$^2$) | error | Compliance (kN.cm) | error | HF-eval | Time (s) |
|---|---|---|---|---|---|---|---|---|
| High-fidelity | 3.4142 | - | 11.3833 | - | 850.0917 | - | 7 | 23.73 |
| GLOBAL | | | | | | | | |
| Grid | 3.2452 | 4.95% | 13.5235 | 18.80% | 796.6321 | 6.29% | - | 1.70 |
| Grid_K_A | 3.4301 | 0.47% | 11.1823 | 1.77% | 841.8784 | 0.97% | 25 | 21.94 |
| Grid_K_M | 3.4227 | 0.25% | 11.2749 | 0.95% | 848.0865 | 0.24% | 25 | 21.92 |
| LOCAL | | | | | | | | |
| SAO_Grid | 3.2474 | 4.89% | 13.4963 | 18.56% | 796.6380 | 6.29% | - | 1.81 |
| SAO_TS_A | 3.4165 | 0.07% | 11.3545 | 0.25% | 850.1105 | 0.00% | 4 | 14.66 |
| SAO_TS_M | 3.4168 | 0.08% | 11.3501 | 0.29% | 850.1102 | 0.00% | 4 | 13.72 |
| SAO_K_A | 3.4020 | 0.36% | 11.5382 | 1.36% | 850.1030 | 0.00% | 5x6 | 29.24 |
| SAO_K_M | 3.4484 | 1.00% | 10.9504 | 3.80% | 850.4689 | 0.04% | 5x6 | 29.83 |
| SAO_RS_A | 3.4808 | 1.95% | 10.5401 | 7.41% | 859.7370 | 1.13% | 7x6 | 40.89 |
| SAO_RS_M | 3.4549 | 1.19% | 10.8670 | 4.54% | 847.1564 | 0.35% | 6x6 | 34.82 |

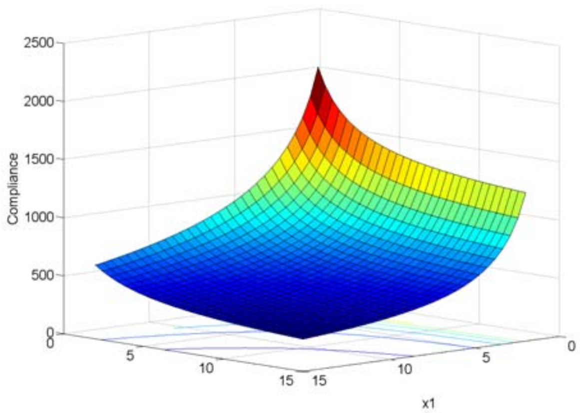


Figure 6. 3D FE truss surface

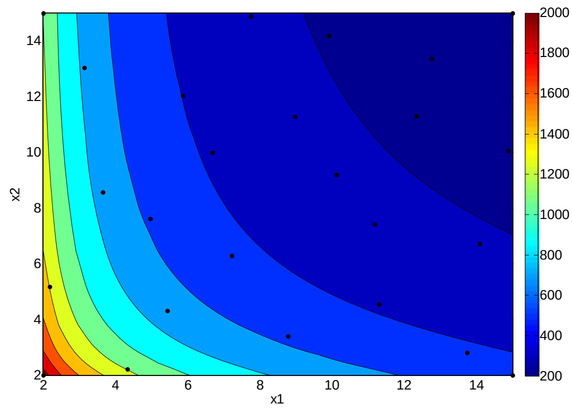


Figure 7. 3D FE truss contour

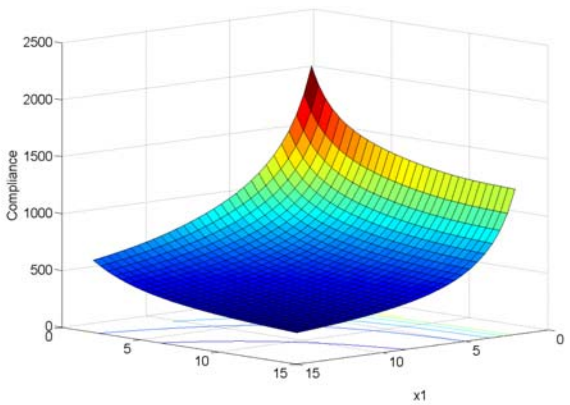


Figure 8. Grid_K_M surface

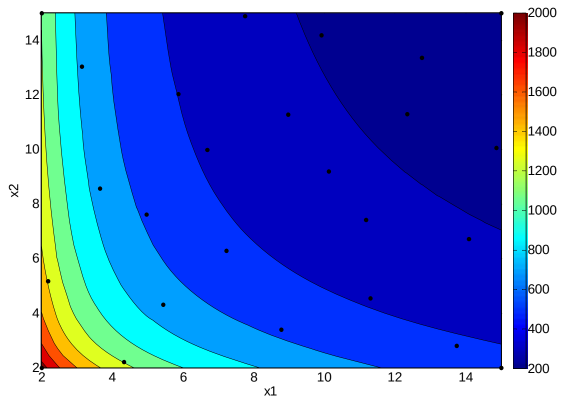


Figure 9. Grid_K_M contour

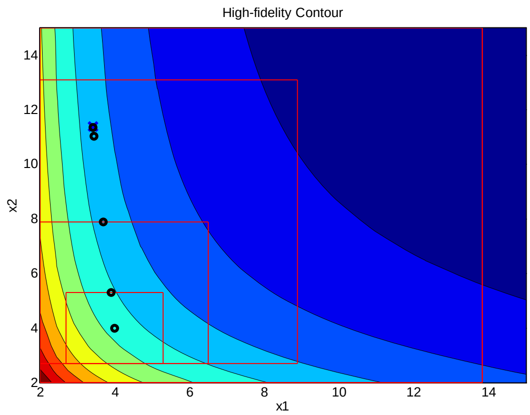


Figure 10. SAO_TS_A evolution over the 3D FE truss contour

## 6. CONCLUSIONS

Different correction schemes were implemented and tested to be used with a physical model as a surrogate of a classical 3D truss problem.

Due to the quality of the proposed physical model (grid analogy) all approximated models with corrections converge to the true solution.

When derivatives are available and unimodal functions (as in the presented case) are involved local TS based corrections are recommended to be used as very few high-fidelity functions evaluations (and its gradients) will be required for convergence.

In the opposite case, when gradients are unavailable or expensive to be obtained and/or noisy functions are involved or when a physical model of good quality is not available local approximations models considering kriging techniques should be the strategy of choice.

## 7. ACKNOWLEDGEMENTS

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