TRAJECTORY OF HIGH SPEED GROUND VEHICLES IN PREDEFINED TRACKS USING OPTIMIZATION TECHNIQUES

Mauro Speranza Neto, D.Sc.
GSV-DEM-ECA/PUC-Rio
msn@mec.puc-rio.br

Danny Hernán Zambrano Carrera, M.Sc.
GSV-DEM-ECA/PUC-Rio
hernanz@mec.puc-rio.br

Abstract. *High speed competition vehicles are required to cover a determined number of laps in a closed trajectory circuit in a time that is the least possible, in the limits of the governing dynamic and driving characteristics of these vehicles. Optimization is a methodology that can be used in order to simulate trajectories and driving techniques used by the competition drivers and to investigate the effects of several parameters in limit conditions of car stability. In this work it is first presented a vehicle model considering the sufficient characteristics for trajectory analysis, influenced by pertinent geometric and physical parameters. In continuation, the problem of the optimal trajectory is defined using optimization procedures, in order to determine how a vehicle will follow the path, considering as an objective function the time to follow it, that must be the minimum, and having as constraints the vehicle dynamic conditions and the path geometry, implementing routines that are used with the Matlab's Optimization Toolbox. Finally the behavior of the vehicle is presented, represented by the model developed previously in a trajectory control loop, in such a way to compare the resulting behavior with the one predicted by the optimization procedure.*

***Keywords**: Vehicle Dynamics, Ground Vehicles Path Optimization, Optimization Techniques*

1. INTRODUCTION

A ground vehicle is a complex system, in which several sub-systems interact and the driver has to notice, besides its behavior regarding to the answers of the steering and powertrain systems, and also the influence of the ambient where the vehicle is moving to control its attitudes and path.

The main element of this complex system is its capacity of generation and control the accelerations in any direction (traction, braking and turns). In a competition car to obtain the maximum performance in any operation condition means to control correctly these variables to travel a circuit in the smallest possible time assisting to the dynamic restrictions of the vehicle and consequently its capacity to generate acceleration in the several directions.

In this work it is shown as classic optimization techniques, together with a simplified model of the vehicle dynamics, can be adopted to determine the ideal path - of minimum time - associated the a competition vehicle in a pre-defined track, using an objective function and constraints to define such problem adequately, using an appropriate method implemented in the existent routines of MATLAB Optimization Toolbox, together with its dynamic models solver Simulink. There is a few number of works (Casanova et al, 2000, Ramanata, 1998, and Velenis and Tsiotra, 2004) that uses similar approach to solve this problem, but none of then mix the classical optimization techniques with a estimated behavior of the vehicles accelerations and a simplified vehicle model, like we are introducing in this paper.

2. VEHICLE MODEL

For the case in subject was considered enough to represent the vehicle as a particle, with its mass concentrated in the Mass Center. The analysis is accomplished supposing the existence of a local frame embarked in the vehicle and the variables transformation for the global frame, in way to obtain its path. The mass point model describes the longitudinal movement through the equation of the vehicle acceleration related to its place in x axis, given by

$$v_x = v_{x0} + \int a_x \, dt \tag{1}$$

where, v_x is the longitudinal velocity respect to time; v_0 is the initial velocity; a_x is the longitudinal acceleration respect to time. Neglecting the local lateral velocity, was considered, in this point mass model, the vehicle rotational movement through a relation between lateral acceleration a_y, the longitudinal velocity and the yaw angular velocity ω_z, given by

$$\omega_z = \frac{a_y}{v_x} \Rightarrow \theta = \int \omega_z \, dt \tag{2}$$

where, θ represents the vehicle attitude (*yaw angle*).

From Eq.' s 1 and 2 is possible to accomplish a coordinates transformation to express the speed of the vehicle in global variables to obtain, soon afterwards, by a time integration, its position, or its path, in agreement with

$$V_x = v_x \cdot \cos\theta \;\; and \;\; V_y = v_x \cdot sen\theta \tag{3}$$

$$X = \int V_x \, dt \quad and \quad Y = \int V_y \, dt \tag{4}$$

where, V_x is the vehicle velocity in the **X** axis, V_y is the vehicle velocity in the **Y** axis, and ***X*** and ***Y*** are coordinates of the vehicle position, all of them in the global frame. Fig. 1 shows the model variables, where *x* represents the distance traveled along the path.

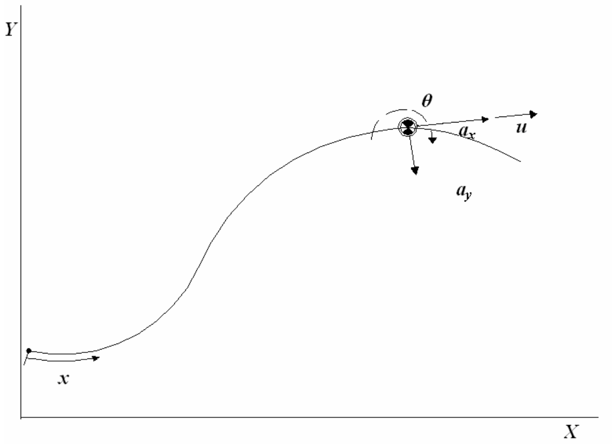

Figure 1. Mass point model variables.

3. ACCELERATION FORMS

The input variables for the dynamic model described above are the vehicle lateral and longitudinal accelerations. Now a day through data acquisition systems, that makes use of embarked transducers, it can be obtained information regarding the main variables associated to the dynamic of the vehicle, in real time, that are treated and analyzed for better understanding its behavior. It can be verified from these data that a certain "pattern" exists for the functions acceleration x time. This pattern will be adopted in the treatment of the optimization problem, simplifying a lot the approach. It is considered that the behavior of the accelerations, with respect to its form, is known, without varying significantly from track for track, or even from vehicle to vehicle, or still from driver to driver. Fig. 2 illustrates Formula 1 car data acquisition system information for the longitudinal speed, and lateral and longitudinal accelerations in a certain track, plotted in relation to its position in the track.

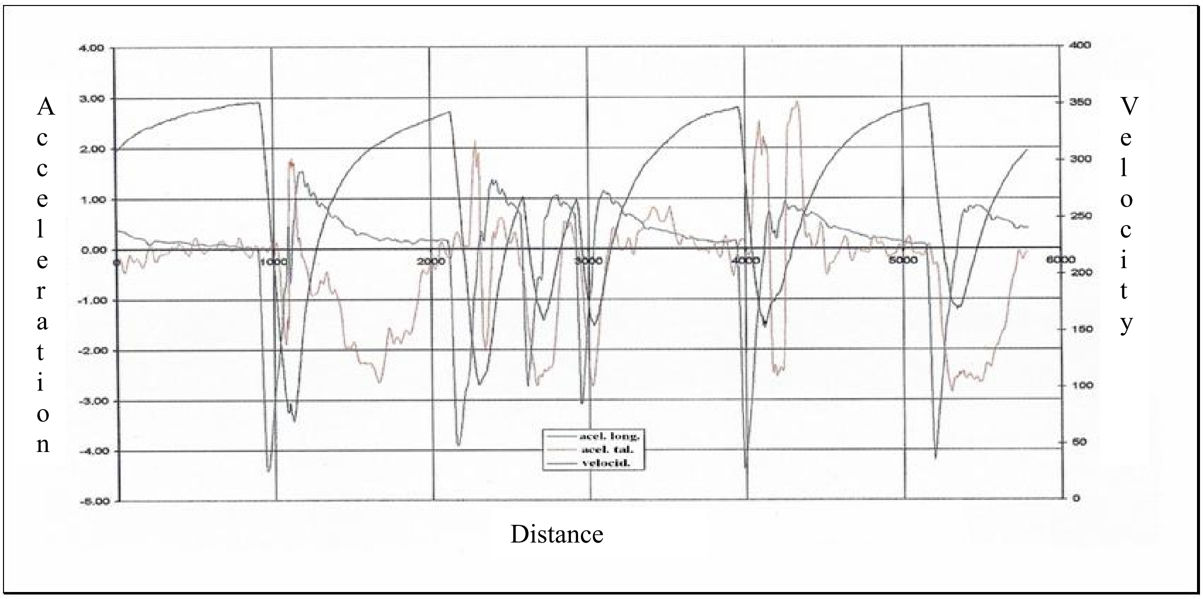

Figure 2. Formula 1 car data aquisition.

So, for the characterization of the **positive (traction) longitudinal acceleration**, it is considered that when the drivers works to the throttle, the vehicle reaches initially a high acceleration, and, as the speed increases, its derivative decreases until arriving to the maximum speed, when the acceleration is null. To represent in a better way possible this behavior, an exponential function is defined as,

$$a_x = a_0^t \cdot e^{\left(\frac{x-x_0^t}{x_f^t - x_0^t}\right)\cdot\left[\ln\left(\frac{a_f}{a_o^t}\right)\right]} \quad (5)$$

where $a_0^t > a_f$ are the initial and final accelerations respectively, with a_0^t limited by the car maximum traction capacity, and a_f is so small as desired, x it is the distance traveled along the track, x_0^t it is the position of the vehicle in the track when the traction action begins, and x_f^t it is its final position in the traction process. It is considered in all analyses $a_f = cte = 0.001\, m/s^2$.

For the **negative (braking) longitudinal acceleration**, when the brake pedal is activated, it is considered that, initially, the vehicle reaches a high slowing down acceleration, and, in agreement with the decrease of the speed, the slowing down acceleration also decreases until arriving to a minimum speed in the which the slowing down acceleration is null (when the drive relieves the brake pedal, and besides its velocity can be also null). To represent in a better way possible this behavior another exponential function is defined, similar to the previous, but with values always negative, given by

$$a_x = -a_0^f \cdot e^{\left(\frac{x-x_0^f}{x_f^f - x_0^f}\right)\cdot\left[\ln\left(\frac{a_f}{a_o^f}\right)\right]} \quad (6)$$

where $a_0^f > a_f$ are the initial and final braking accelerations respectively, with a_0^f limited by the car maximum braking capacity, and a_f is so small as desired, x it is the distance traveled along the track, x_0^f it is the position of the vehicle in the track when the braking action begins, and x_f^f it is its final position in the braking process. Fig. 3 illustrate the supposed behavior for the acceleration profiles in traction and in braking, that act in an acceptable way, when compared with the shown in Fig. 2.

In a turn can be defined 3 stages: one of approach or entrance, in which there is the transition of the braking to the turn, associated to a transient of this curve; a stage in which the vehicle is accomplishing the curve with constant speed, associated to the stead state during the turn; and the stage of the curve exit, in which there is the transition between the lateral acceleration and the full throttle, also associated to a transient. Remembers that the accelerations in any direction in these stages are limited by the capacity of vehicle accelerations generation, characterized by its Friction Circle (Carrera, 2006).

In way to describe such stages, an acceleration profile was considered for each one of them. For the transitory movement associate to the lateral acceleration in the curve entrance, it is defined an exponential function again, given by

$$a_y = a_n^e\left[1 - e^{-4\left(\frac{x-x_0^{ee}}{x_f^{ee}-x_0^{ee}}\right)}\right] \quad (7)$$

where a_n^e is the vehicle lateral acceleration in the stead state curve stage,for a left turn, x is the traveled distance , x_0^{ee} it is the position of the vehicle in the track when the left turn entrance action begins, and x_f^{ee} is the final position in this stage. During the turn stead state stage the lateral acceleration and the speed, by assumption, are maintained constant. Notice that $a_n^e > 0$ for left turns and $a_n^d < 0$ for right turns, according to the convection employed by the vehicle local frame, and the parameters corresponding to the lateral accelerations should have their upper indices substituted from *e* to *d*.

The representation of the lateral acceleration in the turn exit is very similar to the previous, but now it should leave from an initial acceleration different from zero to a null final value, whose is possible using one more time an exponential function,

$$a_y = a_n^e \cdot e^{-4\left(\frac{x-x_0^{se}}{x_f^{se}-x_o^{se}}\right)} \quad (8)$$

where a_n^e is the vehicle lateral acceleration in the stead state curve stage, for a left turn, x is the traveled distance , x_0^{se} it is the position of the vehicle in the track when the left turn exit action begins, and x_f^{se} is the final position in this stage. Fig. 4 illustrates the assumed behavior for the turn's lateral accelerations.

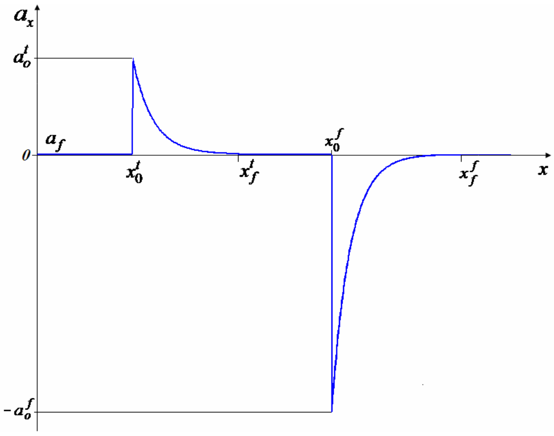

Figure 3. Traction and braking accelerations.

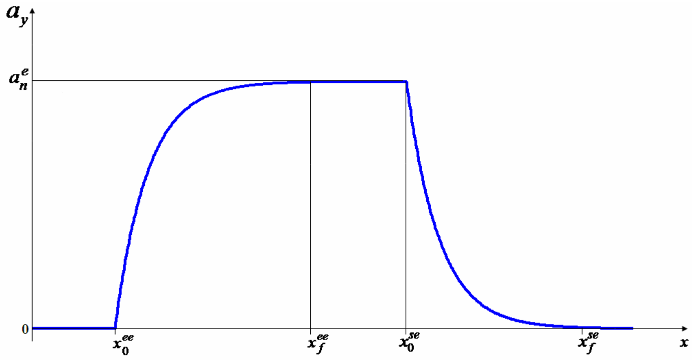

Figure 4. Lateral accelerations in a left turn.

The variables that characterize the behavior of the vehicle in any situation are defined. They form a group of 4 dependent functions of 3 parameters each that will vary depending on the turn type and/or track considered for the analysis. Fig. 5 illustrates a track with a path in which the different stages above mentioned are characterized. Two stages of longitudinal acceleration exists: one of braking (from x_1 to x_2) and another of traction (from x_5 to x_8). Two stages of variable lateral acceleration: one in the curve entrance (from x_3 to x_4) with growing lateral acceleration and another of curve exit (from x_6 to x_7) with decreasing lateral acceleration. There is a stage (between x_4 and x_5) of constants lateral acceleration and speed. Notice the possibility of intersection of the stages and, eventually the inexistence of some stage with pure longitudinal positive acceleration (or slowing down acceleration), when, for instance, the exit of a curve is immediately followed by the entrance in another curve, without a straight line, or a rectilinear path, traveled among them.

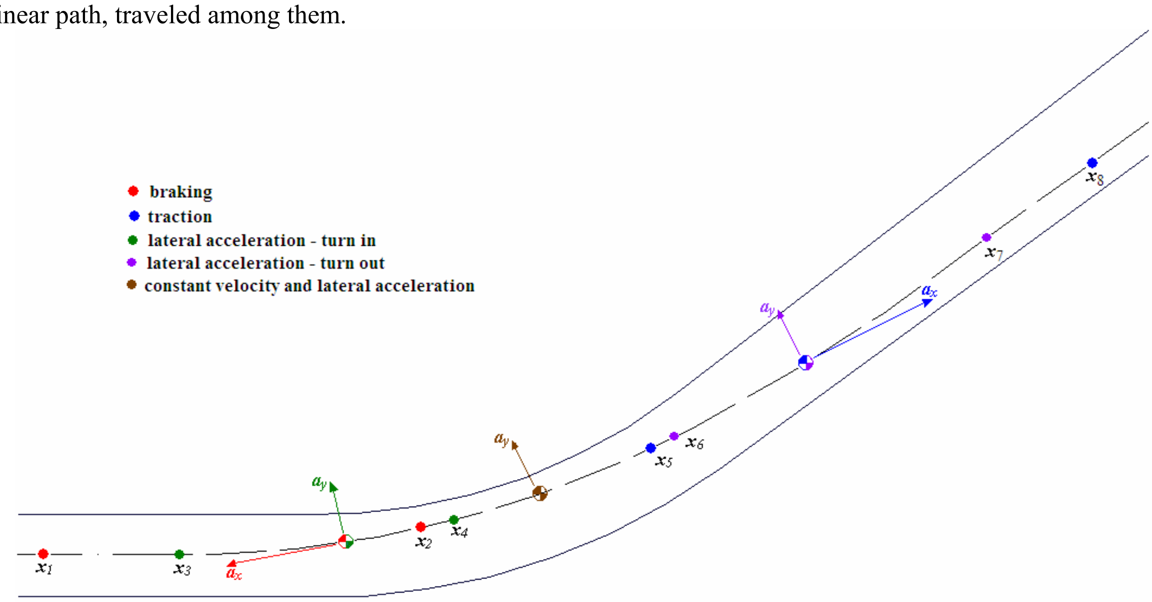

Figure 5. Track with different stage accelerations.

4. OPTIMIZATION PROBLEM

In way to determine the ideal path that supplies the minimum time in a pre-defined track, we can settle down an optimization problem with constraints, in which the design variables are the coefficients that define the accelerations (lateral and longitudinal), that, as commented previously, in the case of a competition vehicle, traveling a track, has profiles with known characteristics. The constraints of this problem are usually three. One of geometric origin: the vehicle should stay inside of the limits of the track (Fig.6), with some tolerance; and other two of physical origin: the accelerations can not cross the established limit for the vehicle Friction Circle (Fig. 7) (due to limitation of the tire-ground contact), and the speed can not be larger than the maximum acceptable (due to limitation of the motor power). So this optimization problem with inequality constraints is defined by

$$\begin{cases} Minimum\ t(\vec{\varsigma}) \\ with\ \vec{\varsigma} = (a_0^t, a_0^f, a_n^e, a_n^d, x_0^t, x_f^t, x_0^f, x_f^f, x_0^{ee}, x_f^{ee}, x_0^{se}, x_f^{se}, x_0^{ed}, x_f^{ed}, x_0^{sd}, x_f^{sd}) \\ Constraint\ to \\ v_x \le 325\ \ km/h \\ \begin{cases} \sqrt{a_x^2 + a_y^2} \le 5g \\ -5g \le a_y \le 5g \\ -5g \le a_x \le 2g \end{cases} \\ y_{\inf.} \le y \le y_{\sup.} \end{cases} \tag{9}$$

The ***time t*** should be obtained in function of the distance traveled by the vehicle along the pre-defined track. The ***objective function time*** therefore it will be minimized varying the accelerations parameters, as defined by Eq, 's 5 to 8. It is adopted as constraint a maximum speed of 325 km/h, common in competition vehicles. In agreement with Fig. 7 the intersession of the three inequalities supplies the constraints in the accelerations - longitudinal and lateral - assuring that these variations stays inside of the Friction Circle, being ***g*** the gravity acceleration. The third constraint is the most complex, because the conditions establishing that the vehicle stays inside of the track acts for each point of longitudinal displacement ***x***, and any associated coordinated ***y*** is among $\boldsymbol{y_{inf.}}$ and $\boldsymbol{y_{sup}}$ of the track. As shown in Fig. 6 ***x*** is the position in the ***X*** axis of the global frame, ***y*** is the position in the ***Y*** axis of the global frame, $\boldsymbol{y_{inf}}$ is the inferior limit of the track and $\boldsymbol{y_{sup}}$ is the superior limit of the track in the ***Y*** axis of the global frame for the position ***x***. Remembers that the dynamic model of the vehicle, Eq.' s 1 to 3, establish relations among the variables associated to the optimization problem, and the path of minimum time will be obtained by the solution of this model, as well as the value of the objective function. It is observed that in the implementation, to reduce the number of design variables involved, the slowing down and curve entrance stages were equaled, and also those of turn exit and acceleration, quite reasonable condition for the situation in subject. Fig. 8 describes the optimization procedure implemented in *Matlab*, using *Simulink* for the solution of the vehicle dynamic model and consequent generation of the optimal path.

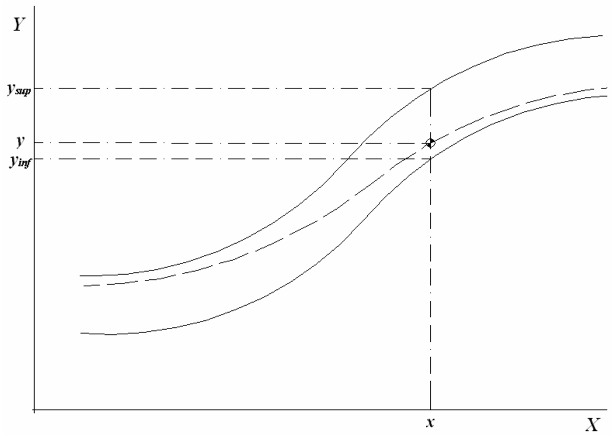

Figure 6. Geometric constraint: race track.

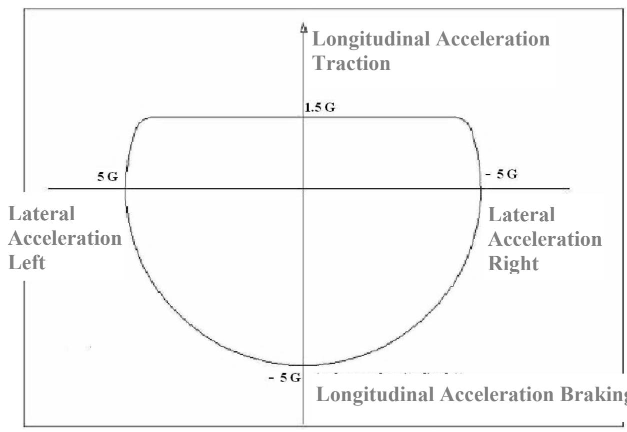

Figure 7. Physical constraint: friction circle.

5. CASE STUDIES

Next are presented some results found in the treatment of typical cases that consists of open tracks, generated by combinations of straight line and curves, in which different levels of acceleration can be imposed to the vehicle in the way to obtain the minimum time.

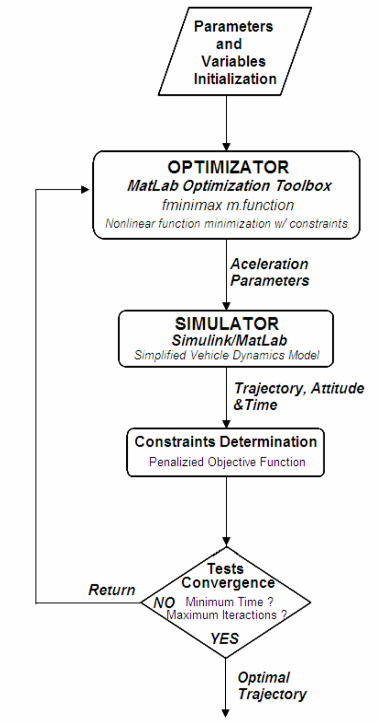

Figure 8. Optimization procedure.

5.1. Constant Velocity in a Simple Turn

In way to evaluate the result obtained by the developed methodology, it was considered a first case supposing a constant speed, equal to 180 km/h (50 m/s), and a simple track was defined: a 50 m straight stage, followed by a 100 m radius and 45 degrees arc turn, and another 100 m straight line. Fig.'s 9 and 10 illustrate the results of the dynamic model for the values of parameters found by the optimization. The obtained path is very similar to the analytically obtained, and its time is of 4.069 seconds, value of the objective function minimum. The maximum lateral acceleration is 37.167 m/s^2 (3.72g); the begin point of the lateral acceleration is at 29.576 m from the starting point, and the place where the maximum acceleration is reached is at 60.695 m from the departure. The point of the curve exit (beginning of the lateral acceleration slowing down) is at 95.472 m of the initial point; the place the where the lateral acceleration is completely null is at 124.233 m from the departure; and finally the total distance traveled by the vehicle is at 203.459 meters. It is noticed that the minimum radius obtained is at 67.262 meters and it happens when the lateral acceleration is maximum. It is verified that the vehicle does not need all its capacity of lateral acceleration to accomplish this path in the considered speed. It can be observed that the curvature radius in the entrance in the curve is smaller than that in the curve exit; it is also relevant to notice that the stage where the lateral acceleration is constant (in the middle of the curve) presents a larger extension than in the ends, in which there is variation of the acceleration.

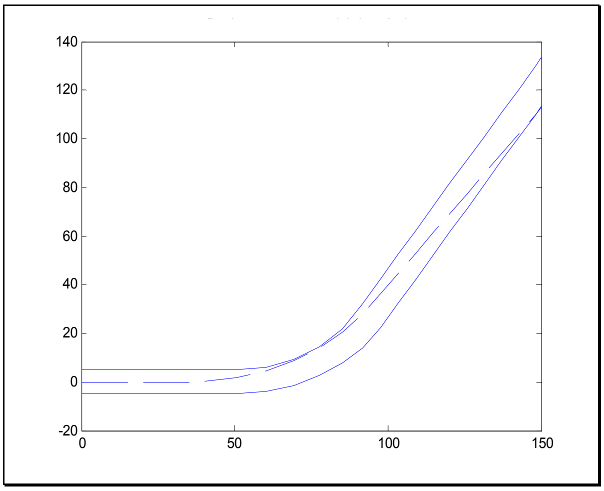

Figure 9. Vehicle displacement.

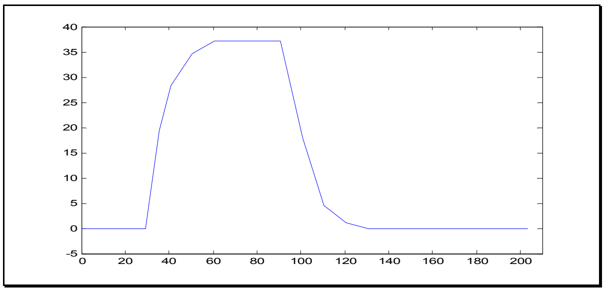

Figure 10. Lateral acceleration x displacement.

5.2. Variable Velocity in a Simple Turn

Now the optimal path is analyzed for variable speed, considering the same track adopted in the previous case for comparison. The initial speed is 50 m/s (180 km/h). In this case, the minimum time found by the optimization is 3.838 seconds, much smaller than that for constant speed. The maximum lateral acceleration is 35 m/s^2, also smaller than the previous. The point for the beginning of the lateral acceleration is at 30.346 m, indicating that in this situation this variable begins more close to the curve, and the point where it reaches its maximum is at 50.135 m, which compared to the previous case indicates that it get to arrive before to this value, meaning that the transient in the entrance of the curve is smaller. The point where the lateral acceleration begins to decrease meets to 82.7 m, then the exit of the curve begins before for constant speed, and it is null in 203.353 m, meaning that the stage where the lateral acceleration decreases is longer than in the previous case. The total distance traveled by the vehicle is 2003.353 m, smaller when compared with the case of constant speed. This indicates that a softer curve is made. The longitudinal acceleration is maximum in 15 m/s^2, happening during the whole exit of the curve. Fig.'s 11 up to 15 illustrate the results of the simulation for the optimal parameters. It is observed that the slowing down acceleration in the entrance of the curve is minimum, 0.11 m/s^2, because the speed is relatively small and do not need to reduce it to accomplish the curve. The initial speed was 50 m/s (180 km/h) and the speed to do the curve 49.964 m/s (179.87 km/h). The minimum curvature radius is 71.326 m, larger than found for constant speed, meaning once again that the curve is softer. The maximum speed of the vehicle when traveling the stage (after the longitudinal acceleration) is 57.044 m/s (205.36 km/h). In Fig. 16, the two paths obtained by the optimization procedure for constant speed (red) and for variable speed (blue) are shown. In the second case it is verified that it is softer and the attitude of the vehicle at the end of the track (exit angle) indicates a better use of the track.

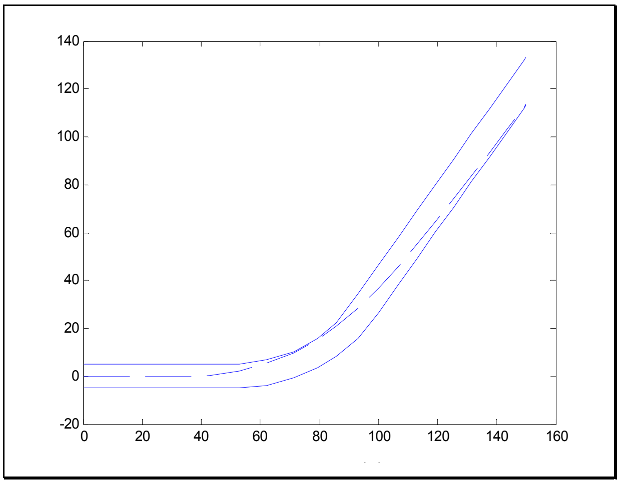

Figure 11. Vehicle displacement.

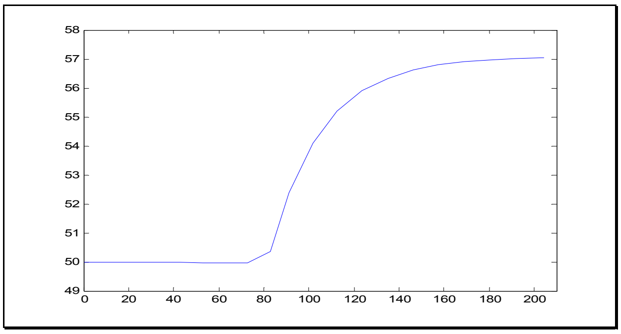

Figure 12. Velocity x displacement.

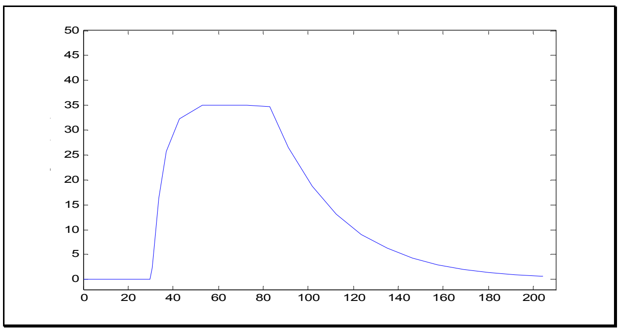

Figure 13. Lateral acceleration x displacement

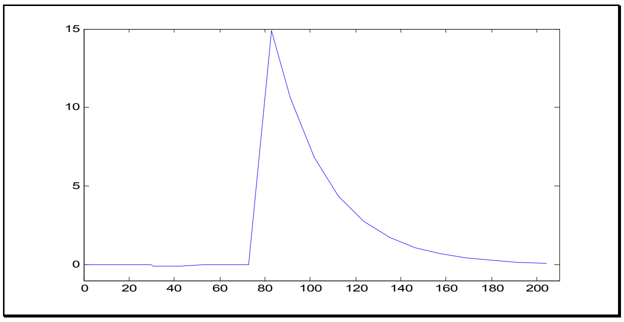

Figure 14. Longitudinal acceleration x displacement

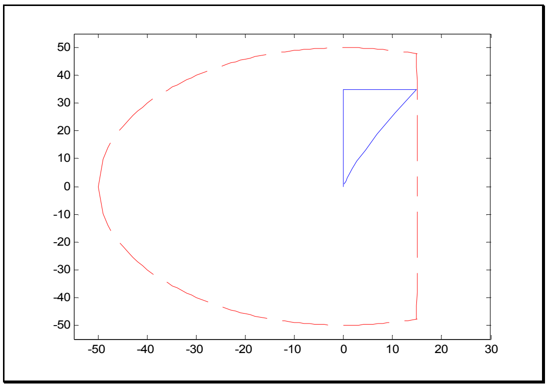

Figure 15. GG diagram (blue) and friction circle (red).

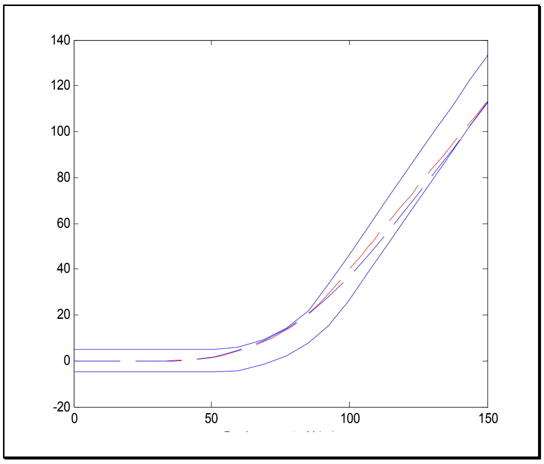

Figure 16. Optimal paths comparation.

5.3. Two Consecutive Turns

In this general case the vehicle begins its movement in the central point of the track, with speed of 80 m/s (288 km/h), in a succession of a straight line, with two curves, one for left and other for the right, with one more straight line soon afterwards, as shown in Fig. 17. The minimum time obtained is of 7.674 seconds, and the results found are presented in Fig.'s 17 to 21. The maximum lateral acceleration in the first curve is 49.190 m/s^2 and in the second curve is 48.565 m/s^2. The point where the curve entrance begins happens at 77.0780 meters from the departure. The place where the lateral acceleration in the first curve is maximum is at 128.222 meters of traveled distance. The point where the exit of the first curve begins happens at 179.979 m from the departure, indicating that is almost 52 meters from the place where the speed is constant and the lateral acceleration is maximum. The point where the vehicle passes from one curve to another happens at 253.494 meters of the traveled distance. In this place the vehicle finishes leaving the first curve and enters immediately on the second, indicating that it slows down longitudinally and the lateral acceleration will be increased until its maximum value. The place where the maximum lateral acceleration happens in the second curve is at 346.875 meters of the traveled distance. The exit of the second curve happens at 425.508 meters. From this point the vehicle accelerates longitudinally and its lateral acceleration decreases until if it reaches the null value. In the second curve almost 79 meters are traveled with constant speed and maximum lateral acceleration. The final point of the movement is at 547.203 meters of the starting point and this is the total distance traveled for vehicle to arrive in its objective. In Fig. 18 it is verified that the minimum speed during the entrance in the first curve is 74.843 m/s, and later the vehicle accelerates during the exit of the first curve up to 75.745 m/s, and soon afterwards it slows down to enter on second curve with the speed of 65.318 m/s. Finally when leaving the second curve the vehicle accelerates and the final speed is approximately 69.720 m/s. The minimum radius in the first curve is +113.872 meters, and the positive sign indicates a left curve. In the second curve the minimum instantaneous radius is -87.850 meters, and the negative sign indicates that the curve is to the right. From Fig. 17, we can see that the attitude exit angle is very close of the direction of the track, and then it can be considered that the vehicle leaves the curve stages in the direction of the track and it goes straight ahead in a straight line. In the GG Diagram shown in Fig. 21, it can be noticed two different polygons: that one on the positive side of lateral accelerations, corresponding to the movement in the first curve; and the other polygon, corresponding to the second curve. It can be seen that in the first curve the vehicle has smaller longitudinal acceleration that in the other curve, but larger longitudinal slowing down is applied in the first curve than on second. It is noticed that in the entrance of the first curve the vehicle slows down with 47.296 m/s^2, and during whole movement the accelerations are inside the acceptable described by the Friction Circle, as shown by the GG Diagram. In the exit of the first curve the vehicle accelerates with 3.818m /s^2, and in this movement with longitudinal and lateral accelerations had a maximum acceleration of 49.340 m/s^2, that is inside of the Friction Circle too. In the entrance of the second curve the vehicle slows down with 44.168 m/s^2, and in the exit of this curve accelerates with 11.675 m/s^2. In this movement a maximum acceleration of 49.950 m/s^2 is reached, indicating that in this track is used the maximum acceleration that the vehicle can support. It is verified that the accelerations and speed profiles are very close to the obtained by the data acquisition system, as shown in Fig. 2. The dynamic behavior of the vehicle is explained in the following way: in the entrance of any curve will be prioritized the lateral acceleration putting the vehicle into the turn direction, to generate a possible path that resembles a "soft" circle arc, that satisfies the constraints imposed initially, slowing down its velocity. In the exit of the curve it will be prioritized the longitudinal acceleration, since it increases the speed directly and it minimizes the time, slowing down the lateral acceleration, because its maximum value was already defined in the entrance of the curve. In the center of the curve the speed and the angular acceleration are constant, as it had been supposed in the development of the models here used.

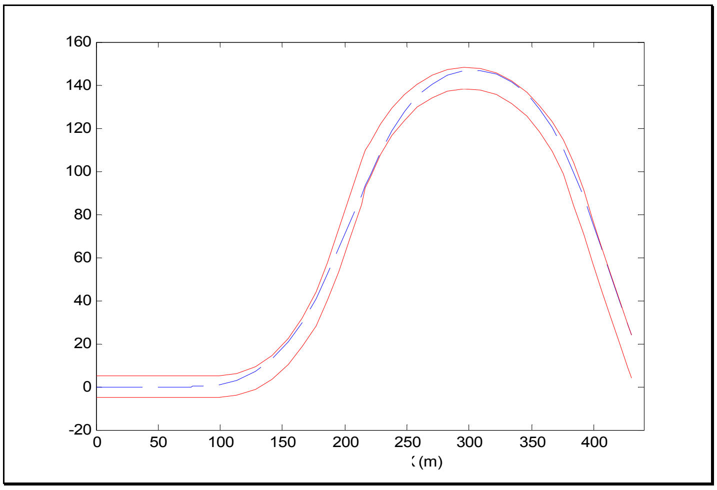

Figure 17. Vehicle displacement.

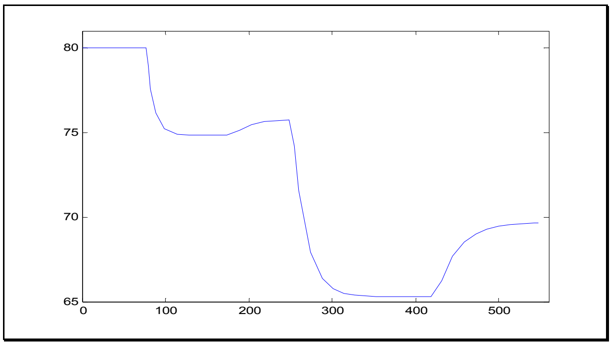

Figure 18. Velocity x displacement.

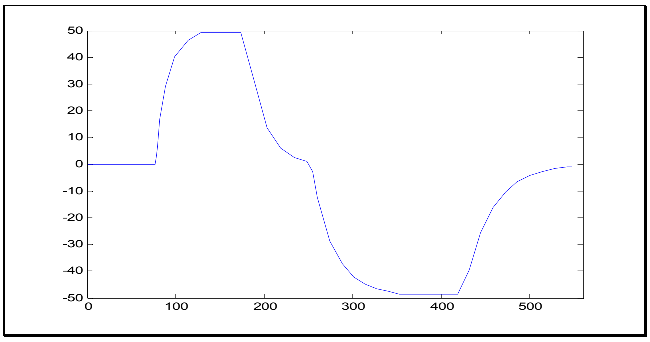

Figure 19. Lateral acceleration x displacement.

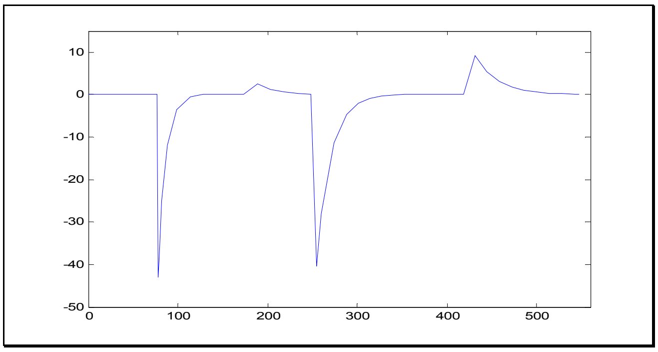

Figure 20. Longitudinal acceleration x displacement

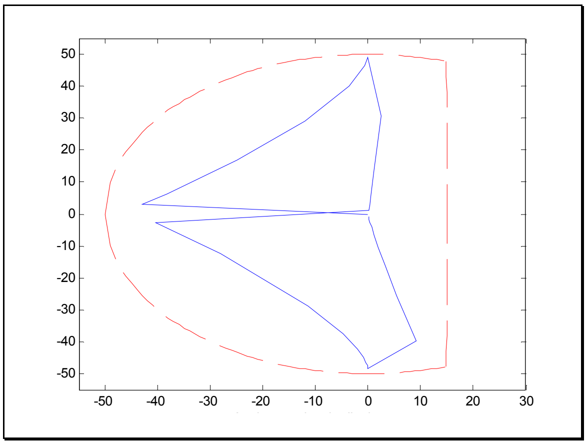

Figure 21. GG diagram (blue) and friction circle (red).

6. FINAL COMMENTS

The procedure of the optimization problem resolution is the main innovation presented in this work, whose objective was the development of a methodology for the ideal path determination, using the optimization and simulation tools found in *Matlab*. It was necessary, initially, to use a very simplified vehicle model, just considering a mass point, in way to make possible the treatment through classic optimization methodologies, without the need of adopting optimal control techniques. However the main physical characteristics of the competition vehicles, influenced by the longitudinal and lateral accelerations, are represented in this model. Behaviors were considered for those variables from real data obtained by data acquisition systems, approximating such accelerations for functions with profile that reproduce them, inside of an acceptable error. The solution given through the adoption of the mass point model was quite creative and opportune, deserving to be more explored and gotten better, including certain (dynamic) effects found in the real behavior of the competition car, without increasing too much the complexity of the representation. In a general way, the found results were quite satisfactory. It was shown that the vehicle dynamics and the optimization procedures can be treated together seeking to obtain results that allow to establish the improvement of the competition vehicles performance in different situations and handling conditions. A next immediate step in the treatment of the optimization problem is the use of a simplified model of the vehicle, but including the effects of the yaw angular speed and lateral speed, associated to the lateral displacement (side slip), in way to turn it closer of the reality, but without increasing its complexity so much, to maintain the unique characteristics of the mass point model concerns to the simplicity,. An item that was not explored in this work, but it is treaty in Hey and Speranza Neto (2007), it is relative to the control of the dynamic model. The approach of this subject, incorporate to the here presented, it will help a lot in the simulation of the vehicle inside of the track, making possible to establish the maneuvers that the driver should do to follow the optimal path. In the present article it is considered the application of the lateral acceleration with some level of longitudinal acceleration, in future works the application will be analyzed in an independent way, that is, the passage from longitudinal acceleration as independent of the lateral acceleration. It could have interception among them, but naturally obtained by the optimization procedure. It is also foreseen interesting to compare the results found by the treatment given in this work to those real ones, obtained from the measured behavior of the competition vehicles, in way to verify if the optimal path is being followed by the drivers, or correctly obtained by the presented procedure. Another foreseen step is the use of constructive and physical parameters of a specific competition vehicle, to work based on those values, without approximations and averages, and with this to validate the results of the employed model and to improve the vehicle performance.

7. REFERENCES

Carrera, D.H.Z., Determinação da Trajetória de Veículos Terrestres a Alta Velocidade em Traçados Pré-Definidos Através de Técnicas de Otimização, Dissertação de Mestrado, Departamento de Engenharia Mecânica, PUC-Rio, 2006.

Casanova, D., Sharp, R. S. and Symonds, P., "Minimum Time Manoeuvring: The Significance of Yaw Inertia", Vehicle System Dynamic, vol. 34, pp. 77-115, 2000.

Charles, C. MacAdam, "Application of an optimal preview control for simulation of closed loop automobile driving", IEEE Transactions on systems, Man, and Cybernetics, vol. SMC-11, N° 6, 1981.

Hatwal, H., Mikulcik, E.C., "An optimal control approach to the path tracking problem for an automobile", the University of Calgary, Calgary, Alberta, Canada, 1986.

Hey, F. and Speranza Neto, M., "Behavior of Ground Vehicles in Closed Paths Using Linear and Nonlinear Dynamics Models in a Control Loop, submetido ao 19º. COBEM, ABCM, 2007

Velenis, E. and Tsiotras, P., "Minimum Time vs Maximum Exit Velocity Path Optimization During Cornering", Georgia Institute of Technology School of aerospace Engineering, Atlanta, GA, USA, 2004.

Velenis, E. and Tsiotras, P., "Optimal velocity profile generation for given acceleration limits; the half-car model case", Georgia Institute of Technology School of aerospace Engineering, Atlanta, GA, USA, 2004.

Ramanata Peeronon, "Optimal Vehicle Path Generator using Optimization Methods", Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 1998.

The MathWorks, "Otimization Toolbox for user with Matlab", version 3, 2005.

8. RESPONSIBILITY NOTICE