OPTIMUM DESIGN OF FINITE POROUS DISTRIBUTION WITHIN MATERIALS

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Abstract. *The work reported in this paper explores the development of materials, represented by a finite distribution of mesostructures, that are optimized for specific and desirable applications. Such mesostructures or cells, which may be seen as an elementary representative volume (ERV) containing pores, are fully characterized by a given set of parameters, defining a set of design variables. These variables are optimized in order to minimize the mean compliance of the structure and are constrained to a volume and size constraints. The structure is considered to be composed as an assemblage of ERVs, placed side by side, forming a rectangular grid of ERVs. In order to atest the proposed procedure, we considered for simplicity each mesostructure to be characterized by a single parameter defined as the relative density of the ERV. Moreover, tests were performed by considering various different geometry types of mesostructures. Here, we consider each ERV to be modelled by a super element, each uniquely characterized by it's relative density. Also, we have considered the problems to be restricted to plane stress or strain conditions.*

Keywords: *material design, computational solid mechanics, structural optimization*

1. INTRODUCTION

The use of composite porous materials is rapidly growing in many fields of engineering applications in response to a certain tendency to find new materials with higher strength by weight ratio or, sometimes, to design a material in order to meet a special technologic purpose. Very often, these materials are composed by an assemblage of mesostructures, denoted here as an Elementary Representative Volume (ERV), as illustrated in Figure 1.

Many investigations have been carried out regarding the effect of void or pore distribution inside compact, i.e. fully dense, materials aiming at the reduction of weight, the minimization of the mean compliance or the maximization of the ratio of strength by weight. As a result, many different technologies have been developed for the engineering of porous materials, where the shape, size and/or distribution of pores are optimally designed. Moreover, recent technologies have made the production of such materials economically viable. Among these technologies is the Reflexive Materials Technology, see Owens (1997, 2001), which produces materials with cubic voids embedded in a compact solid matrix, distributed in an optimally predefined form. This process, denoted by Rapid Manufacturing Technology, is able to produce materials whose relative density ranges from 0.5 to 0.95. Other important manufacturing processes for porous materials, see Ashby *et al* (2000), are: The Norsk-Hydro and Cymat processes that are capable of producing foams of aluminum alloys, with a relative density ranging from 0.03 to 1.0, by the injection of gas into the melted alloys; The gas-metal Eutectic solidification process, known also as the GASAR process; and the modified GASAR method, developed by Schapovalov (1993), are capable of producing spherical pores and elongated pores by a controlled gas injection process. Fast prototyping techniques, among others, may also be employed, as shown by Takano and Zako (2000) for the production of functionally graded porous materials.

The objective of this paper is to propose a procedure for the optimal design of engineered porous materials, represented by a finite distribution of mesostructures, which are optimized for specific and desirable applications. Such mesostructures or cells may be seen as an elementary representative volume (ERV) containing pores, which are fully characterized by a set of parameters, defining a set of design variables. The material is considered to be composed as an assemblage of several ERVs, placed side by side, forming a rectangular grid of ERV cells. The selected design variables may represent the size, shape and/or distribution of pores in the material and are optimized in order to minimize the mean compliance of the structure, subjected to a volume and side constraints.

Eshelby's classic theory (1957) for the stress field around a single inclusion is the base for several methods, which have been developed to analyze the response of a composite material. The Mori-Tanaka (1973) method proposes the mutual interaction of the neighboring inclusions. Using these theories to obtain mechanical properties of composite materials has led to the work carried out by Demakos (1999), who evaluated the averaged elastic properties of composites, whose inclusions form a prismatic network interacting with the compact matrix. This approach and other derivations are restricted to composite materials with a periodic pattern, formed by equal size inclusions or pores, repeated in a periodic way through out the entire domain. A mathematical counterpart to these engineering methods,

named as the homogenization theory was proposed in order to handle, in an asymptotic way, composite materials formed by composites with a periodic microstructure pattern, formed by an assemblage of very small microstructures, when compared with the macroscopic dimensions of the overall structure. The homogenization theory assumes a periodic media and requires that the scale length, related to the heterogeneity's, to be much smaller than the composite macroscopic scales. The material is then modeled as an effective homogeneous media, having a set of effective properties. This composite material approach was widely used in topology optimization problems, see Bendsoe and Kikuchi (1988), leading to the determination of the optimal layout of structures, under various loading conditions.

Here, the motivation, due to the availability of new cost effective technology to engineer porous materials, is to consider the optimal design of structures composed as a finite assemblage of mesostructures, placed side by side, forming a rectangular grid, whose cells are characterized by a set of design variables that uniquely defines the distribution, size and shape of pores/voids in a compact material. Notice that, unlike the topology optimization problem, the main interest here is to consider a relatively small finite number of mesostructures to be optimized. However, some of the ideas employed by the topology optimization method, proposed by Kikuchi and Bendsoe (1988), will be employed. Also, some problems, closely related to topology optimization problems, will be considered, allowing us to do some comparison and validation of the proposed method. Two types of structural optimization problems are categorized in the literature: geometric optimization (size and shape) and topology optimization, Gea (1996). The proposed procedure is a true geometric/parametric optimization where the layout of the structure is predefined, whereas in topology optimization the aim is to find the optimum layout of the structure, either by adding or removing parts, or by making holes Haftka (2001).

Thus, by using some of the ideas of the composite material approach to topology optimization methods, a new procedure is proposed for the optimal design of porous materials. The discretization of the domain is performed by the Galerkin Finite Element method, where each mesostructure is modeled as a super element, whose mesh is uniquely characterized by a set of parameters, defining the size and/or shape of the pores. Here, these parameters are optimized in order to minimize the mean compliance of the structure, subjected to a volume and a set of bound constraints, associated with each of the design variables. Figure 1 shows the discretization of the structure's domain in ERVs, each of which are characterized by the design variables *a*, *b* and θ. The design variables, may be represented in this case by ρ_j^k, $k = 1..q$ and $j = 1,..n$, where ρ_j^k represents the *k-th* parameter of the *j-th* RVE, *q* represents the total number of parameters necessary to define the RVE and *n* is the total number of RVEs employed in the discretization of the body domain. At this point, we define: $\boldsymbol{\Omega}$ to be the body domain; $\overline{\mathbf{b}}$ to be a prescribed body force; Γ_u and Γ_t to be the parts of the boundary subjected to a prescribed displacement $\mathbf{u} = \overline{\mathbf{u}}$ and traction $\sigma\mathbf{n} = \overline{\mathbf{t}}$ respectively, with $\partial\Omega = \Gamma_u \cup \Gamma_t$ and $\Gamma_u \cap \Gamma_t = \varnothing$.

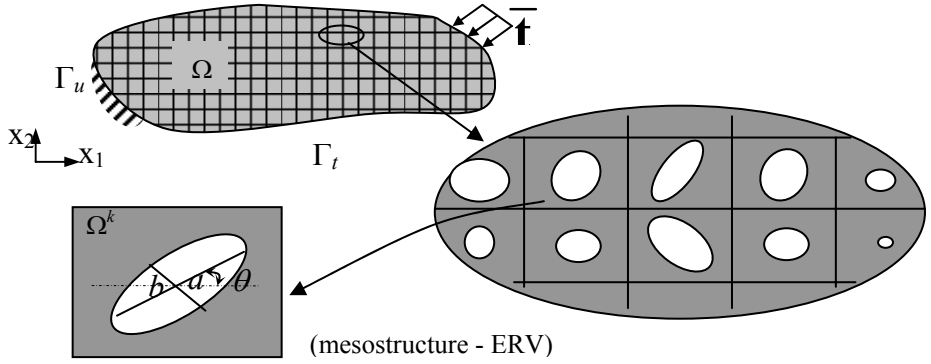


Figure 1 - Macro domain Ω divided by mesostructures (ERV), showing a patch with a hypothetic porosity distribution. In detail, an ERV with an elliptic pore defined by three design parameters (*a*, *b*, θ).

2. THEORETICAL DEVELOPMENT

The design variables $\rho \in R^{n+q}$ are optimized according to an objective or merit function and satisfy a set of constraints. With the aim of validating the proposed procedure, some closely related topology optimization problems will be solved allowing us to do some qualitative comparisons with well known topology optimization results. From this point of view, we consider the objective function to be the mean compliance and the constraints to be given by a volume and side constraints.

2.1. Definition of the Problem

The proposed porous material design problem is formulated as: Determine the relative density $\rho^* \in R^{q+n}$ that is a solution of:

$$\min_{\rho} \; l(\mathbf{u}(\rho)) \tag{1}$$

Subjected to:

$$\frac{1}{\Omega}\sum_{j=1}^{n_{RVE}} \int_{\Omega_j} \varphi\left(\rho_j\right) d\Omega = \alpha \quad \left(\text{Effective volume constraint}\right), \tag{2}$$

and

$$\left(\rho_j^{\mathrm{k}}\right)_{\inf} \le \rho_j^k \le \left(\rho_j^k\right)_{sup}, \quad k = 1...q, \; j = 1...n \quad \left(\text{Side constraints}\right), \tag{3}$$

in which $\left(\rho_j^{\mathrm{k}}\right)_{\inf}$ and $\left(\rho_j^{\mathrm{k}}\right)_{\sup}$ are respectively the lower and upper bound of the *k-th* parameter of the *j-th* RVE, Ω is the volume of the fully dense body, $\varphi\left(\rho_j\right)$ is the relative density of the *j-th* RVE and α is a prescribed volume fraction of the body, $\alpha \in (0,1)$. The displacement field $\mathbf{u}(\rho)$ is the solution of the following boundary value problem: Find $\mathbf{u} \in H$ so that

$$a_\rho(\mathbf{u}, \mathbf{v}) = l(\mathbf{v}), \quad \forall\, \mathbf{v} \in H \tag{4}$$

where

$$a_\rho(\mathbf{u}, \mathbf{v}) = \int_\Omega \mathbf{D}\varepsilon(\mathbf{u}) \cdot \varepsilon(\mathbf{v})\, d\Omega \quad \text{and} \quad l(\mathbf{v}) = \int_\Omega \overline{\mathbf{b}} \cdot \mathbf{v}\, d\Omega + \int_{\Gamma_t} \overline{\mathbf{t}} \cdot \mathbf{v}\, d\Gamma, \tag{5}$$

with $H = \left\{ \mathbf{u}(\mathbf{x}) \in H^1(\Omega) \,\middle|\, \mathbf{u}(\mathbf{x}) = \mathbf{0} \; at \; \mathbf{x} \in \Gamma_u \right\}$ and $\mathbf{D}$ denoting the fourth order elasticity tensor of the fully compact material. Physically, the above minimization problem aims to find the stiffest structure for a given prescribed amount of material. This objective function determines the mean compliance of the structure.

2.2. Discretization of the structure by the finite element method (super element approach)

The ERVs are defined by a set of particular design parameters $\boldsymbol{\rho}_j$ and they comprise the macrostruture. Now, the basic idea is to discretize the macrostructure in finite elements so that each ERV can be modeled by a super element, defined geometrically by its boundary $\partial\Omega^{se}$. Physically, a super element is one of more parts into which the macrostruture is partitioned and represented by a substructure having nodes on its boundary (master d.o.f.) and interior nodes whose degrees of freedom (slave d.o.f.) are a function of the master degree of freedom. A plane domain divided into 84 triangular finite elements is shown in Figure 2, in which there are square pores (voids). Figure 3 shows the same domain, now divided into 7 super elements, which use boundary nodes for their definition.

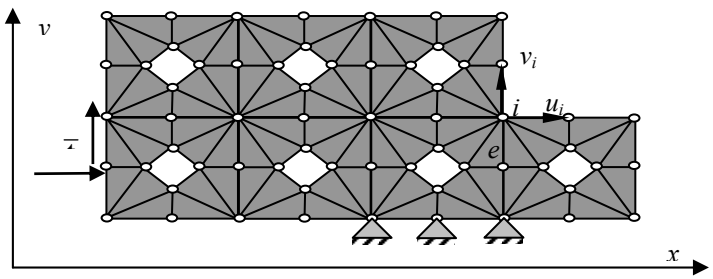


Figure 2 - Plane domain, divided into triangular finite elements, whose cells present a square void.

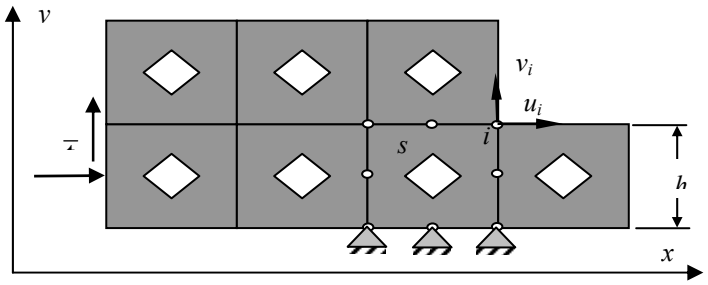


Figure 3 - Plane domain, divided into 7 super elements

After the division of the macrostructure into elements, the following procedure is performed:

1. Evaluation of the super element stiffness matrix $[\mathbf{K}^{se}]$ and effective nodal load vector, by applying the static condensation procedure for the elimination of the slave degrees of freedom.
2. Determination of the global equilibrium equation $[\mathbf{K}]\{\mathbf{U}\} = \{\mathbf{F}\}$, by performing the assemblage of the super elements stiffness matrix and effective nodal load vector
3. Solution for {**U**} (master d.o.f. – degee of freedom), and extraction from {**U**}, for each super element, the associated slave degrees of freedom for pos-processing, such as the determination of the stress field.

The super element stiffness matrix and effective nodal load vector may be computed by considering the following equilibrium equation, for a given ERV,

$$\begin{Bmatrix} \overline{\mathbf{r}}^e \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} \overline{\mathbf{f}}^e \\ \overline{\overline{\mathbf{f}}}^e \end{Bmatrix} = \begin{bmatrix} \overline{\mathbf{K}}^e & \hat{\mathbf{K}}^e \\ \hat{\mathbf{K}}^{e\mathrm{T}} & \overline{\overline{\mathbf{K}}}^e \end{bmatrix} \begin{Bmatrix} \overline{\mathbf{U}}^e \\ \overline{\overline{\mathbf{U}}}^e \end{Bmatrix}. \tag{6}$$

Here, $\overline{\mathbf{U}}^e$ contains the master d.o.f. associated with the ERVs boundary nodes, $\overline{\overline{\mathbf{U}}}^e$ contains the slave d.o.f. that will be condensate, $\overline{\mathbf{r}}^e$ is the reaction nodal load vector due to the interaction with the neighboring elements and $\overline{\mathbf{f}}^e$ and $\overline{\overline{\mathbf{f}}}^e$ are the vectors of nodal forces due to the external loading conditions. The solution of this system of equations gives:

$$\overline{\overline{\mathbf{U}}}^e = -\left(\overline{\overline{\mathbf{K}}}^e\right)^{-1}\left(\hat{\mathbf{K}}^{e\mathrm{T}}\overline{\mathbf{U}}^e - \overline{\overline{\mathbf{f}}}^e\right) \text{ and } \overline{\mathbf{r}}^e = \overline{\mathbf{K}}^e\overline{\mathbf{U}}^e - \hat{\mathbf{K}}^e\left(\overline{\overline{\mathbf{K}}}^e\right)^{-1}\left(\hat{\mathbf{K}}^{e\mathrm{T}}\overline{\mathbf{U}}^e - \overline{\overline{\mathbf{f}}}^e\right) - \overline{\mathbf{f}}^e \tag{7}$$

that may be written as

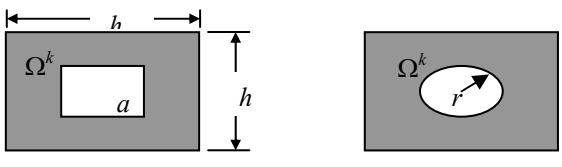
$$\left\{\overline{\mathbf{r}}^e + \mathbf{f}^{se}_{eff}\right\} = \left[\mathbf{K}^{se}\right]\left\{\overline{\mathbf{U}}^e\right\}, \tag{8}$$

which allows the identification of the super element stiffness matrix and effective nodal load vector, associated with the applied external loads, given respectively by

$$\mathbf{K}^{se} = \left(\overline{\mathbf{K}}^e - \hat{\mathbf{K}}^e\left(\overline{\overline{\mathbf{K}}}^e\right)^{-1}\hat{\mathbf{K}}^{e\mathrm{T}}\right) \text{ and } \mathbf{f}^{se}_{eff} = \overline{\mathbf{f}}^e - \hat{\mathbf{K}}^e\left(\overline{\overline{\mathbf{K}}}^e\right)^{-1}\overline{\overline{\mathbf{f}}}^e. \tag{9}$$

2.3. Uni-parametric optimization problem

For simplicity, we consider the ERV to be completely characterized by a single parameter, i.e. q=1 and $\boldsymbol{\rho} \in R^n$ and restrict ourselves to the following two types of one-parameter ERVs, as shown in Fig. 4.



(a) (b)

Figure 4 - Two types of quadratic ERVs with side h: (a) centered square void, defined by side a; (b) centered circular void, defined by radius r.

Moreover, for convenience, we consider the parameter to be given by the relative density of the ERV. In fact, for each cell, there is a one to one relation between $\rho^k \in [0,1]$, denoting now the relative density of the *k-th* ERV and the one parameters *a* and *r* of Fig. 4.

Also, the minimization of the mean compliance of the structure may be formulated as: Find $\boldsymbol{\rho}^* \in R^n$ that solves

$$\begin{cases} \underset{\boldsymbol{\rho}\in\mathbf{X}}{\text{Minimize}}\ f(\boldsymbol{\rho},\mathbf{U}(\boldsymbol{\rho})) = \mathbf{F}^{\mathrm{T}}\mathbf{U}(\boldsymbol{\rho}) \\ \text{subjected to:} \\ \qquad g(\boldsymbol{\rho}) = \sum_{k=1}^{n} \dfrac{\mathrm{A}^k}{\mathrm{A}_T}\left(\rho^k - \alpha\right) \le 0 \end{cases} \tag{10}$$

where $\mathbf{U}(\boldsymbol{\rho})$ is the solution of

$$\left[\mathbf{K}(\boldsymbol{\rho})\right]\{\mathbf{U}(\boldsymbol{\rho})\} = \{\mathbf{F}\}. \tag{11}$$

Here, $\mathbf{U}(\boldsymbol{\rho})$ is the global nodal displacement vector containing the master d.o.f. of the structure, $\mathbf{F}$ is the load vector, $\mathbf{K}(\boldsymbol{\rho})$ is the stiffness matrix of the structure obtained as an assemblage of super elements, A^k is the area of super element *k* and α is a prescribed maximum volume fraction constraint. In addition, $\mathbf{X} = \left\{\boldsymbol{\rho} \in R^n \middle| \rho_i^{\inf} \le \rho_i \le \rho_i^{\sup}, i = 1,n\right\}$, *n* is the total number of super elements in the finite element mesh, $f(\mathbf{u}(\boldsymbol{\rho})) = \beta_0^{-1} f_\rho(\mathbf{u}(\boldsymbol{\rho}))$, with $\beta_0 = \mathbf{F}^{\mathrm{T}}\mathbf{U}(\boldsymbol{\rho}_0)$, is the non-dimensional objective function, for some given initial relative density vector $\boldsymbol{\rho}_0$ and $\mathrm{A}_T = \sum_{k=1}^{n} \mathrm{A}^k$.

In order to solve the optimization problem, we apply the Augmented Lagrangian method. As a result, we formulate the problem as a sequence of box constrained optimization problem. The general procedure may be summarized as:

- Set k=0, λ^k =0, error=1.0, ε^k and *tol*.
- While error>tol

(i) Solve the bound constrained minimization problem

$$\min \chi(\boldsymbol{\rho},\lambda^k,\varepsilon^k),\ \forall \boldsymbol{\rho} \in \mathbf{X} \tag{12}$$

where

$$\chi(\boldsymbol{\rho},\lambda^k,\varepsilon^k) = f(\boldsymbol{\rho}) + \left(2\varepsilon^k\right)^{-1} \Psi(g(\boldsymbol{\rho}),\lambda^k,\varepsilon^k) \tag{13}$$

with

$$\Psi(g(\boldsymbol{\rho}),\varepsilon^k,\lambda^k) = \begin{cases} g(\boldsymbol{\rho})\left[g(\boldsymbol{\rho}) + 2\varepsilon^k\lambda^k\right], \text{ if } g(\boldsymbol{\rho}) \ge -\varepsilon^k\lambda^k \\ -(\varepsilon^k\lambda^k)^2, \text{ if } g(\boldsymbol{\rho}) < -\varepsilon^k\lambda^k \end{cases} \tag{14}$$

Denote the solution by $\boldsymbol{\rho}^k$.

(ii) Update of the Lagrangian multiplier

$$\lambda^{k+1} = \max\left\{0, \lambda^k + \frac{1}{\varepsilon^k} g(\boldsymbol{\rho}^k)\right\} \tag{15}$$

(iii) Compute the error

$$error = \frac{\left|\lambda^{k+1} - \lambda^k\right|}{\max\{1, \lambda^{k+1}\}}, \tag{16}$$

(iv) update penalty parameter

$$\varepsilon^{k+1} = \begin{cases} \gamma\varepsilon^k, \ if\ \ \varepsilon^{k+1} < \varepsilon_{crit}, \text{ for some } \gamma \in (0,1) \\ \varepsilon_{crit}, \text{ otherwise} \end{cases} \tag{17}$$

- End

The bound constrained optimization problem is solved by a memory less projected quasi-Newton method, Costa and Alves (2003).

2.5. Interpolation of the stiffness matrix of the super element, as a function of ρ, by means of cubic splines

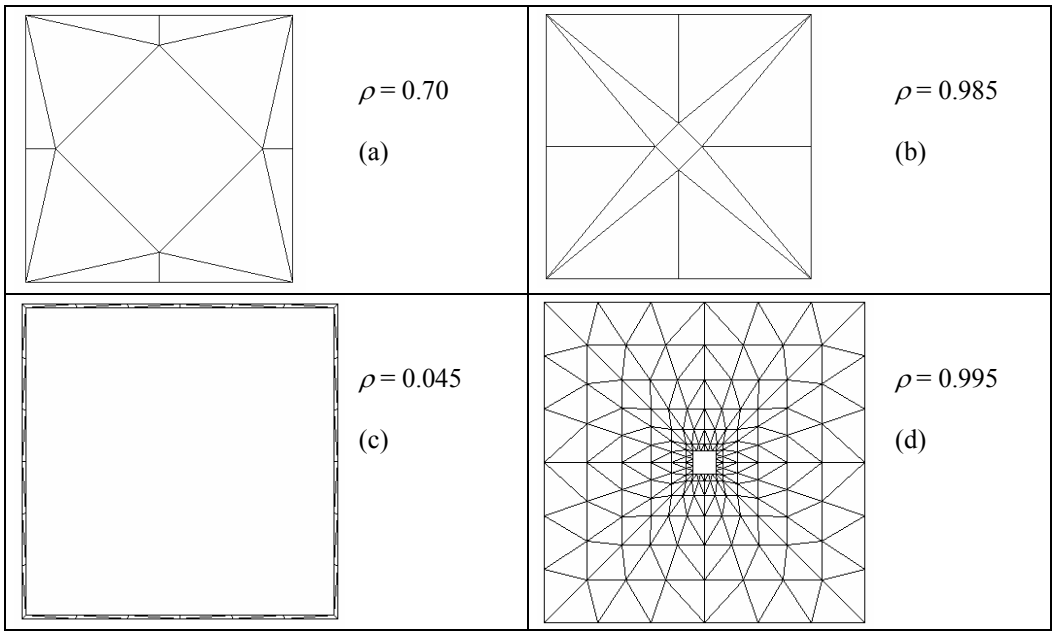
The computation of the super elements stiffness matrix $\mathbf{K}^{se}(\boldsymbol{\rho})$ and the effective nodal load vector $\mathbf{f}_{eff}^{se}(\rho)$ necessary for the determination of the sensitivity response of the structure may be easily calculated numerically. However, if the models size become large, the calculation of the numeric derivative by finite difference methods may become unfeasible. One way to circumvent this problem is to approximate $\mathbf{K}^{se}(\rho)$ and $\mathbf{f}_{eff}^{se}(\rho)$ by an interpolation procedure. This procedure is the same used by Bendsøe and Kikuchi (1988) to solve the elements of homogenized constitutive matrix in the homogenization method. Here, we make use of a cubic Spline interpolation proceedure, where $\mathbf{K}^{se}(\rho)$ and $\mathbf{f}_{eff}^{se}(\rho)$, are determined for a given set of discrete values, i.e., $\left\{\left(\rho_l, \mathbf{K}^{se}(\rho_l)\right)\right\}_{l=0}^{N}$ and $\left\{\left(\rho_l, \mathbf{f}_{eff}^{se}(\rho_l)\right)\right\}_{l=0}^{N}$ and interpolated in each interval $[\rho_l, \rho_{l+1}]$ by cubic splines polynomial.

2.6. Procedure to solve the finite distribution of uniparametric pores problem in materials

Any domain Ω is described by a mesh of super element with side *h*. Each super element, defined from uniparametric ERVs, is described by an internal variable ρ (relative density), whose value is related to the radius *r* in a circular pore, or to the side *a* in a square pore, assigning any value in the interval $\left[\boldsymbol{\rho}^{\inf}, \boldsymbol{\rho}^{\sup}\right]$. Figure 5 illustrates 3 configurations of discretized plane ERVs by using meshes of triangular finite-elements, each one assigning two different values of relative density. After the slave d.o.f. of each ERV have been condensed, the super element will be defined only in term of the master degrees of freedom. Usually, the larger the number of nodes on the super element boundary, the higher the accuracy that can be reached to describe the internal void, as well as assuring a mesh with less-distorted elements, mainly close to the upper and lower bounds of the design variables.

The amount of boundary nodes depends on the pore complexity and on its parameterized size. Usually, for large pores (low relative density) it is necessary to have a large number of boundary nodes, while for small pores (high relative density, near 1) a large number of internal nodes are required. However, the number of boundary nodes for super element configuration must be the same for all ERVs.

In super element design procedure, we must avoid the abrupt difference between the sizes of finite element sides that make up the super element internal mesh. That could specially happen, close to lower and upper bounds of geometric parameters set, which characterizes the pore. So, the maximum pore size must be restricted in order to avoid using finite elements too distorted. Close to relative density maximum limit, it is easier to avoid the problem, using a good super element internal mesh.



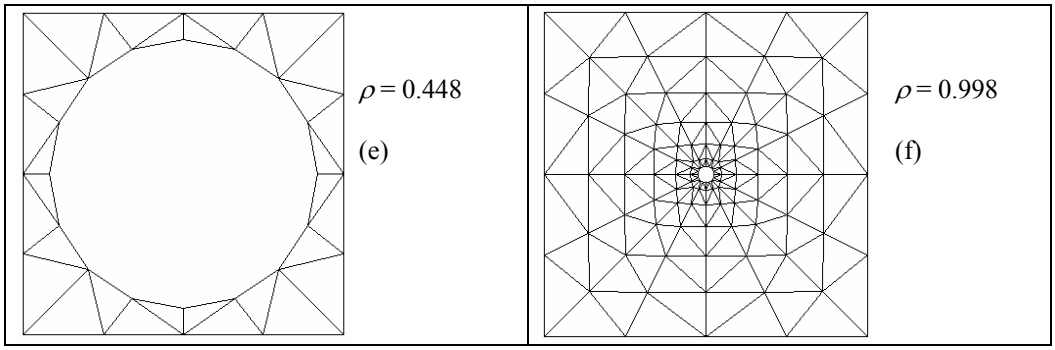


Figure 5 - Discretized EVRs for different configurations and relative densities ρ.

In this work we have implemented several cell models, which are listed in table 1.

Table 1 - Types of ERVs models used to make up the macrostructure.

Model	**Pore configuration**	**Relative density**	**Number of boundary nodes**	**Relative density Interval**
Stiff09	square, $\theta = 45º$	$\rho = 1-(a/h)^2$	8	0.7 to 0.985
Stiff10	square, $\theta = 0º$	$\rho = 1-(a/h)^2$	24	0.1 to 0.995
Stiff11	square, $\theta = 0º$	$\rho = 1-(a/h)^2$	24	0.1 to 0.995 and 1.0
Stiff12	round	$\rho = 1-\pi(r/h)^2$	16	0.45 to 0.995
Stiff13	round	$\rho = 1-\pi(r/h)^2$	16	0.45 to 0.995 and 1.0

2.7. Numeric examples

Here, we consider a beam called MBB in plane stress or plane strain, as shown in figure 6, subject to a distributed load *p*. The domain Ω is divided into a set of super elements, Ω^{se}, that represent the mesostructure's cells, which have pores, as shown in figure 7. The relative density of the ERVs can vary from $\rho^{\inf}$ to an upper density limit close to 1.

Figure 6 - Geometry for the problem being considered.

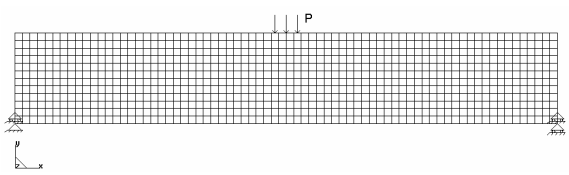


Figure 7 - Macrostructure composed by 864 super elements (72 x 12).

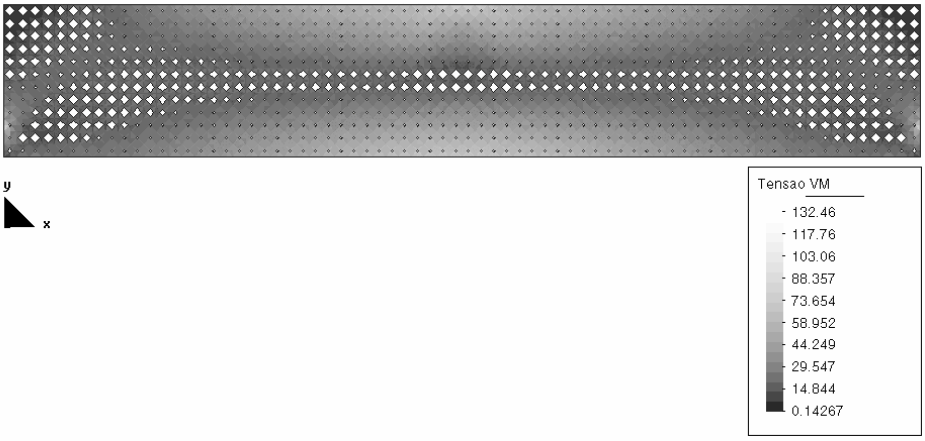


Figure 8 - Von Mises stress and porous distribution of the domain, for a prescribed volume fraction α=0.90.

The von Mises stress distribution in Fig.8 was obtained by using an ERV configuration, denoted here as *stiff9* (table 1), with a square pore having an angle θ= 45º with respect to the x-axis, for a prescribed volume fraction of α=0.90. It is possible to observe that lowest stress values are associated to positions where the super element densities are lower.

Using an ERV configuration with a round pore (*stiff13*), whose relative density can vary from 0.45 to 1.0, the relative density distribution was generated and illustrated in Figures 9 to 11 for a prescribe volume fraction of 0.7, 0.8 and 0.9 respectively, using a model with 864 super elements.

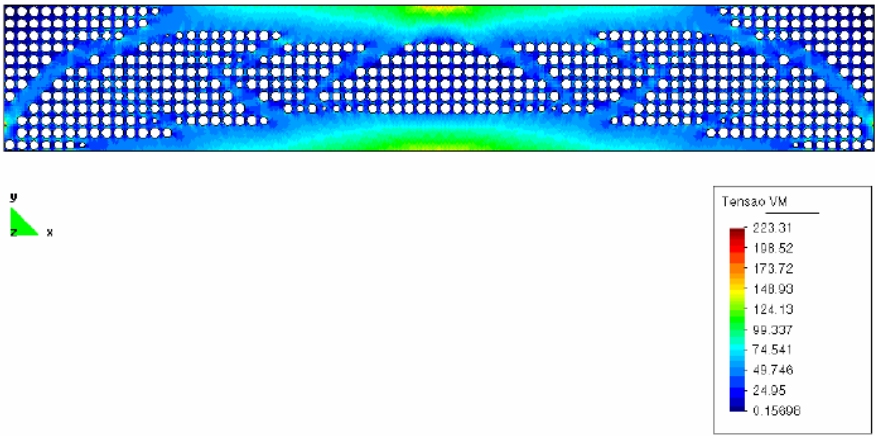


Figure 9 - Von Mises stress field, resulting from porosity distribution on domain, by super element optimization procedure, to a volume fraction α=0.70.

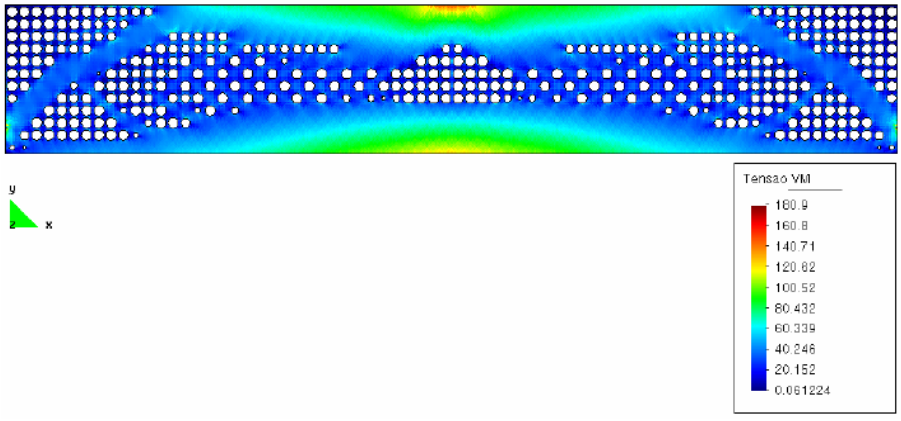


Figure 10 - Von Mises stress field, resulting from porosity distribution on domain, by super element optimization procedure, to a volume fraction α=0.80.

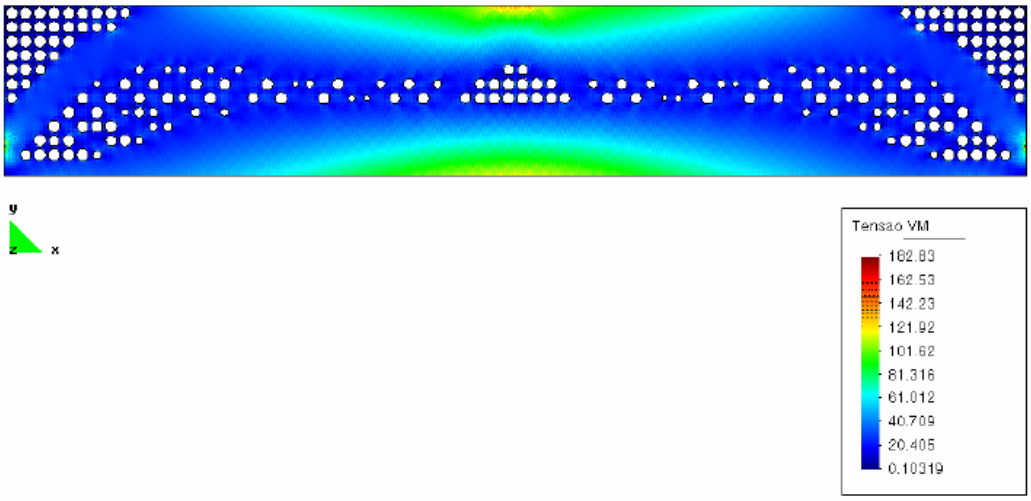


Figure 11 - Von Mises stress field, resulting from porosity distribution on domain, by super element optimization procedure, to a volume fraction α=0.90.

By the proposed procedure it is also possible to obtain results with topology characteristics, or more accurately, quasi-topology, because lower density cannot be zero, but a value in density order near 0.1, depending on the super element configuration. Figure 12, shows the Von Mises distribution on a beam for a prescribed volume fraction of α=0.65. The domain is composed into 864 super elements with densities varying from 0.1 to 0.995 (*stiff10*). The hollow spaces in the optimal quasi-topology configurations are there because of the pores inside the low density ERVs.

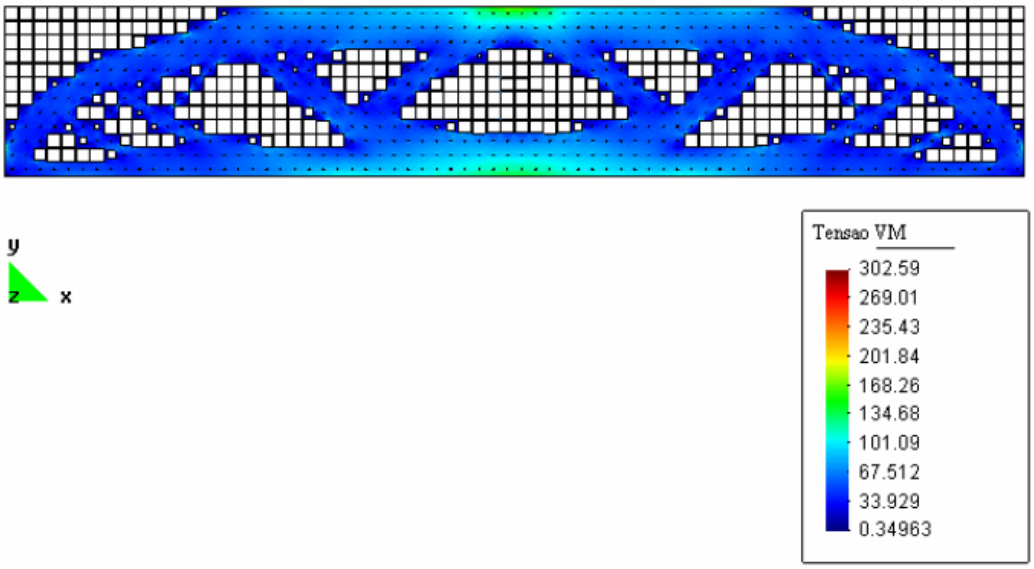


Figure 12 - Von Mises stress field, resulting from porosity distribution on domain, by super element optimization procedure in a quasi-topological configuration, to a volume fraction α=0.65.

2.8. Conclusions

A procedure was proposed for the optimum design of finite porous distributions within materials, i.e., to design materials with optimized mesostructures, for specific and desirable applications.

The algorithm development used some less usual techniques in optimization area, i.e., super elements to describe mesostructures and interpolation by cubic splines to find the super element stiffness matrix and its derivatives in order to perform sensitivity analysis. Also, the proposition of this paper differs from other authors, since the domain is characterized by a finite amount of pores.

With the super element method, we can work with a quasi-topological model, where super element can have a very low density in regions where stresses are low and they can have high density where stresses are high with the aim of minimizing the mean compliance of the structure.

Although, in this paper, the new procedure was not applied to general cases, such approach can be extended easily to more complex cell geometries, different objective functions and loading conditions.

That new procedure can easily be coupled to finite elements commercial packages, so that there could be a future profitable possibility for the application of the proposed methodology, described in this paper.

3. ACKNOWLEDGEMENTS

The support of the CNPq is gratefully acknowledged.

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