BEHAVIOR OF GROUND VEHICLES IN CLOSED PATHS USING LINEAR AND NONLINEAR DYNAMIC MODELS IN A CONTROL LOOP

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Abstract. *In this paper are presented the first results of a research work that has been done by the PUC-Rio Vehicular Systems Group aiming to establish a methodology to reproduce the ground vehicles control by humans. The vehicle dynamic model, the path conditions and the control strategies are discussed. We are aiming to employ this methodology to optimize the vehicles behavior in several operation conditions and emergency situations, accidents reconstruction and collisions analysis, among others. The main results here concerns to the representation of a closed path and the use of classical control strategies to follow such trajectory, comparing the behaviors of a linear and a nonlinear dynamic models, with several degrees of freedom, using physical constraints, such as steering angles and lateral acceleration, among others, to define the control elements parameters.*

Keywords*: Vehicle Dynamics, Ground Vehicles Path Control, Close Path Control.*

1. INTRODUCTION

To reach the desired behavior of a vehicle of any type, nothing more natural and obvious than the application of control techniques, since the steering command of those systems is inherently a closed loop. The main problem is to adapt certain methodologies or strategies, even if classic and consecrated, to the specific characteristics of a certain vehicle, or even to the desired objective and employed variables to control it.

Seeking to know and dominate the strategies of linear and non-linear control applied to the vehicles, a research work is being accomplished by GSV/PUC-Rio to study the influence of the elements of performance (the human being's dynamics, steering system, among others) in the behavior of the vehicles in closed loop (Speranza and Spinola, 2005). That procedure is made by following pre-defined tracks, trying to determine the parameter's adjustment of the control elements to best follow the paths, for any conditions of speeds and/or vehicles.

In this work the behavior of a vehicle described by a dynamic linear model of four degrees of freedom will be analyzed. Proportional and proportional double derivate (PDD) controllers will be tested with the objective of reaching satisfactory results in following the pre-defined paths. The objective is not only to minimize the error and reach a good dynamic behavior but make the controller represent the human behavior when driving a car as analyzed in recent work (Speranza and Spinola, 2005). That system will be simulated in two conditions: the first described by a loop completely linear, where the sines and cosines of the frame change are linearized; and another one described by a non linear control loop, where the sines and cosines of the change of the local frame to the global frame are included. The controllers designed for the linear control loop are tested when applied to the non linear loop.

The computer implementation of the control system is made in Simulink/MatLab. In order to make the generation of data and the analysis of results easier, it was, also, implemented specific MatLab programs to define the tracks and the desired plans (pre-processor), and also to animate the vehicle traveling along the chosen path (pos-processor), besides graphic plots with all the relevant variables: steering wheel angle, longitudinal and lateral speeds, longitudinal and lateral accelerations, among others (Hey, 2007). The present article describes the control loop and the analysis of classic controllers used to control the vehicle, the results found until the moment are discussed and suggestions for future works are presented.

2. LINEAR MODEL OF FOUR DEGREES OF FREEDOM

The dynamic model, that represents the lateral dynamics of the vehicle deduced in recent work (Spinola, 2003) makes possible the determination of the lateral speed (v), the yaw angle (θ), the yaw speed (ω) and the global displacement of the mass center in the direction y (Y). The model has as input the steering wheels angle (δ_f), it is described as:

$$\begin{bmatrix} \dot{v} \\ \dot{\theta} \\ \dot{\omega} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{-2(C_f + C_r)}{m_{tot} u} & 0 & \frac{-2(aC_f - bC_r) - m_{tot} u^2}{m_{tot} u} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-2(aC_f - bC_r)}{I_{yaw} u} & 0 & \frac{-2(a^2 C_f + b^2 C_r)}{I_{yaw} u} & 0 \\ -1 & -u & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ \theta \\ \omega \\ Y \end{bmatrix} + \begin{bmatrix} \frac{2C_f}{m_{tot}} \\ 0 \\ \frac{2aC_f}{I_{yaw}} \\ 0 \end{bmatrix} \delta_f \tag{1}$$

In a way to facilitate the analysis by the root locus and the determination of the controller's parameters, the model in the state form described by Eq. 1 is transcribed to the transfer function form, relating the steering angle δ_f with the variable Y, that represents the position of the center of mass of the vehicle in the axis y of the inertial frame. The transfer function that represents the system for a constant speed of 20 m/s, in agreement with the parameters presented on Table 1, typical of a popular vehicle of medium load, is given by:

$$\frac{Y(s)}{\delta_f(s)} = \frac{26.76.s^2 + 63.04.s + 1036}{s^4 + 5.018.s^3 + 6.492.s^2} \tag{2}$$

Table 1: Linear model parameters.

Parameters	**Symbol Unit**	**Value**
Lateral stiffness of the tires	C_f , C_r [N/rad]	20.000
Total mass of the car	m_{tot} [kg]	1.495
Longitudinal speed	u [m/s]	20
Distance between C.G. and front axle	a [m]	1.203
Distance between C.G. and rear axle	b [m]	1.217
Moment of inertia	I_{yaw} [kg.m^2]	2.500
Maximum steering angle	$\delta_{máx}$ [°]	30

3. CONTROL STRATEGY

Two control strategies are presented in this work for the problem of tracking a pre-defined path. In the first, the variable Y, resulting from Eq. 1, is considered as being the observed variable. The error generated in the control loop is resulted of the difference between the instantaneous global position Y of the vehicle and the position y of the pre-defined path, given the instantaneous global position X of the vehicle (Speranza and Spinola, 2005). That position X is resulted from the integration of the constant longitudinal velocity of the vehicle. When the car is in a curve, the error is calculated from the radial difference, as displays Fig. 1.

In the second strategy, besides observing the error in relation to the center of the track, it is also used the error of orientation of the vehicle in relation to track. During a curve, as displays Fig. 2, the angle of yaw of the vehicle (θ) is compared with the tangent angle of the track in that instant (θ). The relative error to the center of the track and the yaw error are mediated and then added, for after that enter the controller as a single sign (Hey, 2007). A generic representation of the control loop used in this work can be seen in Fig. 3.

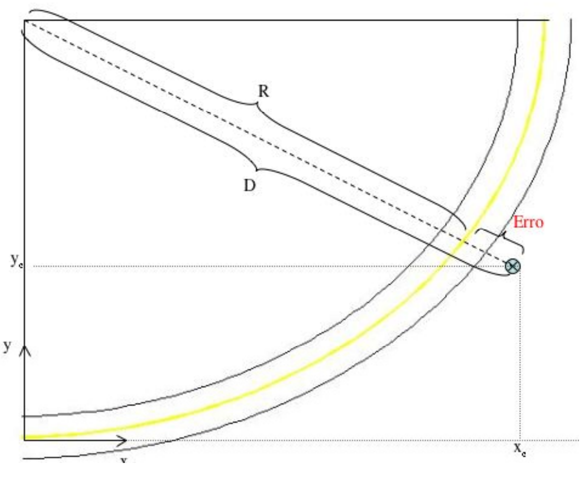

Figure 1. Error to the center of the track.

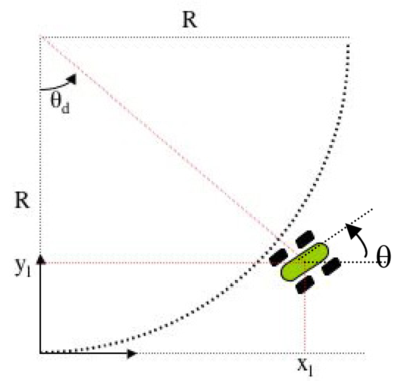

Figure 2. Desired yaw angle.

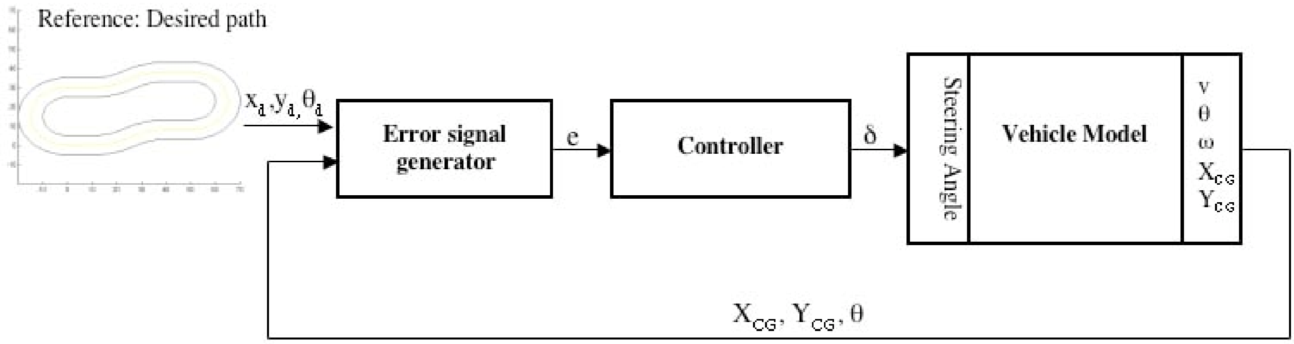

Figure 3. Control loop representation.

It is presented now some results of the accomplished analyses, with the objective of compare and discuss the difference between the two control techniques applied and the behavior of the vehicle. The control loop is the same, changing only the applied controller. The values of 10 m/s and 20 m/s were considered to also evaluate the influence of the longitudinal speed of the vehicle that is supposed constant along the path in this model.

4. CONTROL THROUGH THE CENTER OF THE TRAJECTORY

Using the control strategy that takes into account only the positioning error in relation to the center of the track, the simulation was made for a low speed (10 m/s) in a path that represents a lane change. The used controller was a proportional one with gain adjusted in 10. Fig. 4 represents the displacement of its center of gravity in relation to track. The reference path is the center of the track. It is possible to observe that at this speed, the controller purely proportional makes the car travel the desired path with a quite small error, as it is possible to observe in Fig. 5, that shows the acceleration along the track, the profile of the steering wheels, and the loop error.

The behavior of the steering angle is oscillatory but it is not absurd, while the limit of steering angle of the wheels that is around 30 degrees is kept. The oscillatory profile of the steering takes to lateral accelerations that approach 0,5g' s, acceptable for this vehicle type.

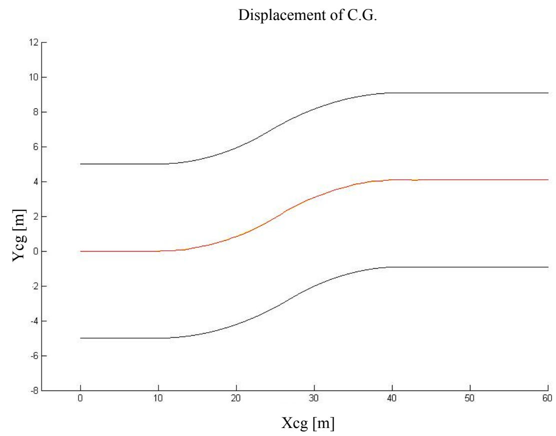

Figure 4. Displacement of the C.G. Proportional gain set to 10.

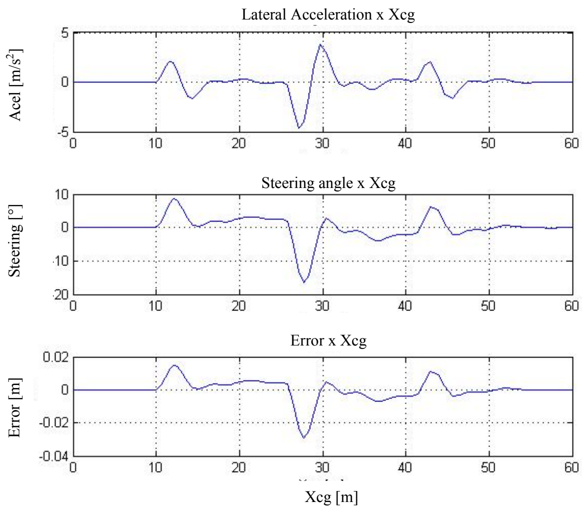

Figure 5. Lateral acceleration, steering and error.

The limitation of the proportional controller is reached when the speed is increased to 20m/s. At that speed, despite the vehicle to travel the path in a satisfactory way, as seen in Fig. 6, the error is larger and the steering angle, as well as the lateral acceleration reaches values (about 4,5 g's) that don't represent the reality, as seen in Fig. 7.

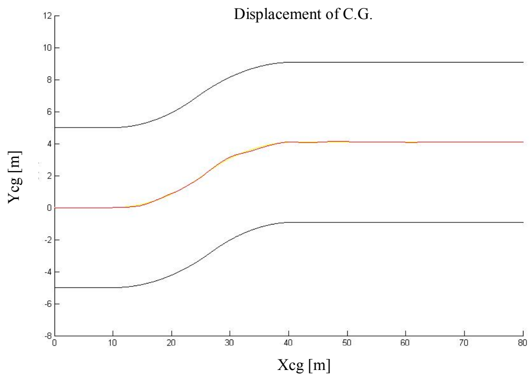

Figure 6. Displacement of the C.G. Longitudinal speed set to 20m/s.

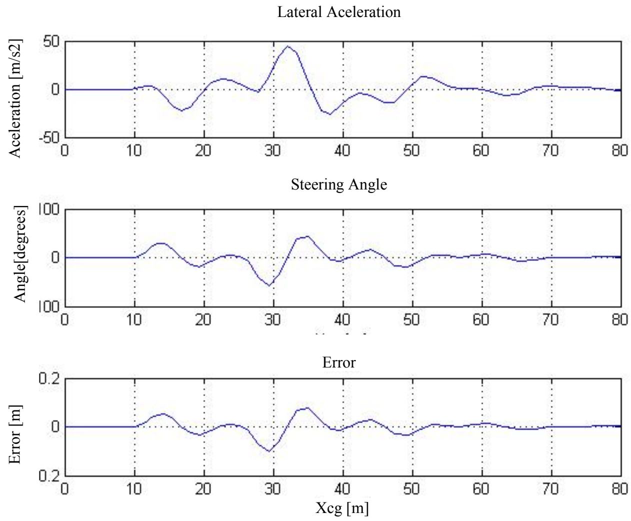

Figure 7. Lateral acceleration, steering and error.

To improve the controller's performance in those conditions, reduce the oscillations and the amplitude of the control sign, two zeros were added in agreement with the study of the root locus of the system. The analysis showed for a proportional controller, that predominant dynamics of the system is oscillatory, as seen in Fig. 8. When PDD (proportional double derivative) with the zeros properly positioned was used, the predominant dynamics is no longer oscillatory with the correct adjustment of the gain. Those zeros have derivative effect, so the controller can work not only with the amplitude of the error but also with its variation tax along the time. That characteristic can reduce the oscillations as seen in Fig. 9.

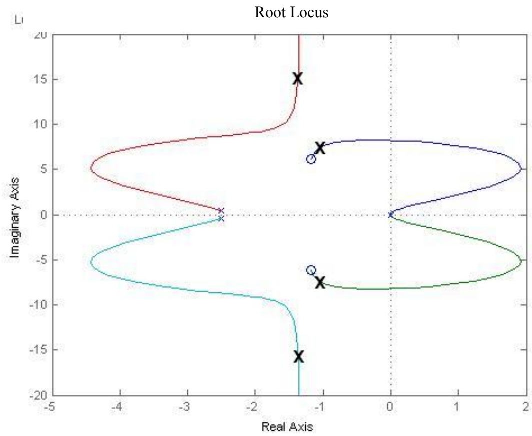

Figure 8. Root locus for proportional controller with gain set to 10 (20m/s).

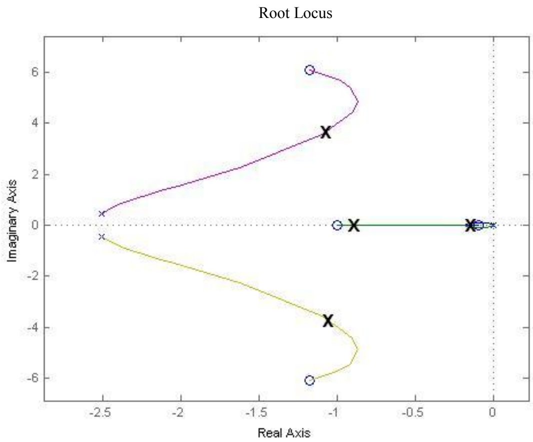

Figure 9. Root locus for PDD controller with gain set to 10 (20m/s).

It was made the same simulations viewed in Fig. 4 and Fig. 6, with the difference that now the used controller is a PDD. Fig. 10 displays the displacement of the center of gravity of the vehicle along the track with a speed of 10 m/s. A first comparison that can be done in relation to Fig. 4 shows the error a little larger, even so the result are satisfactory. Still more satisfactory and representative is the behavior of the steering angle, that doesn't present the oscillations viewed in Fig. 5 and presents values well lower, as seen in Fig. 11. As already seen in Fig. 10, the error is larger than in the case of the purely proportional controller, but the profile of the steering angle is well softer, taking to lower values of lateral acceleration.

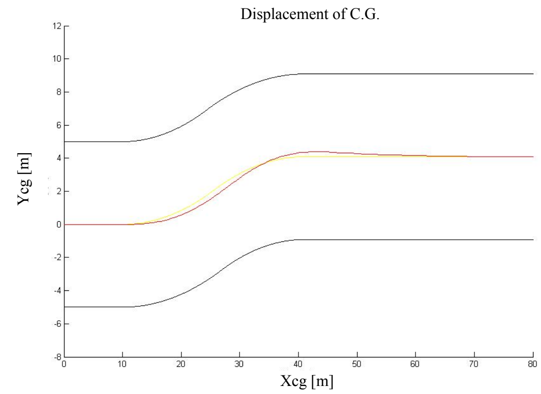

Figure 10. Displacement of the C.G. (10m/s). PDD.

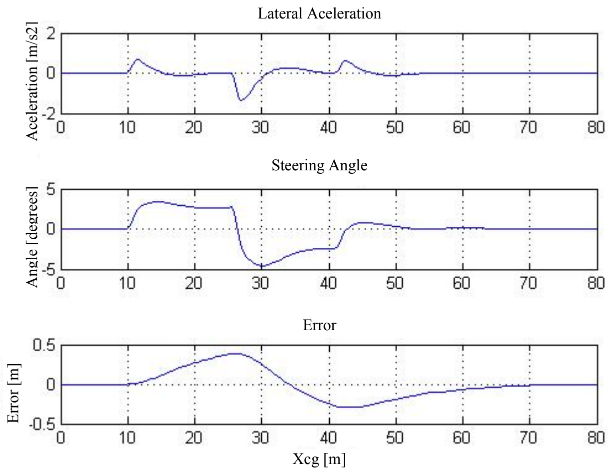

Figure 11. Lateral acceleration, steering and error (10m/s). PDD.

Now it is analyzed the performance of PDD for a speed of 20m/s. Fig. 12. shows the displacement of C.G. of the vehicle along the track. In the same way, when the proportional controller's performance is compared with the PDD, it is noticed that this last one takes a larger tracking error, but in other hand the oscillation and the high values for the steering angle decrease, as seen in Fig. 13. The steering angle presents a profile well softer and reasonable value, even if a little high. It's important observe the result improvement between the two controllers. Future works can focus in those controllers optimization to reach a desired result.

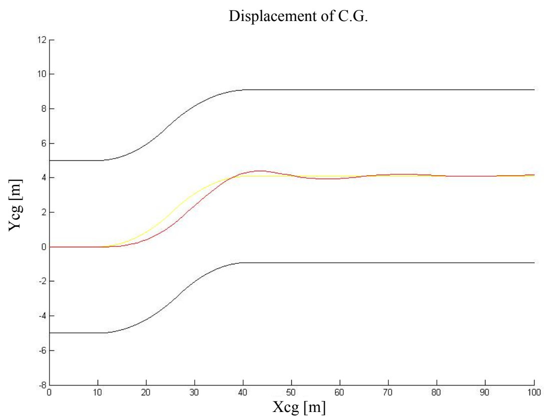

Figure 12. Displacement of the C.G. (20m/s). PDD.

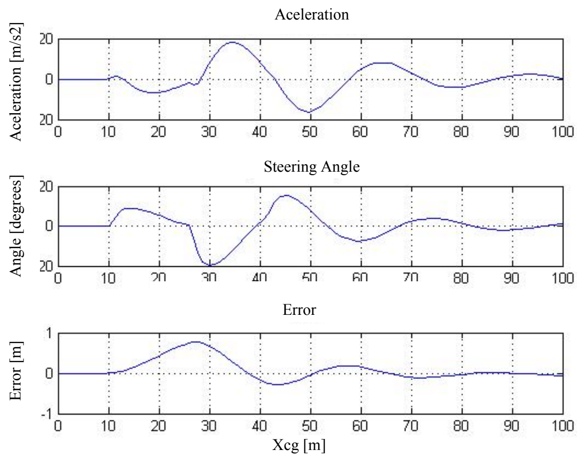

Figure 13. Lateral acceleration, steering and error (20m/s). PDD.

The most important conclusion that is taken from these results is that the PDD controller leads to a softer profile and keeps the steering angle inside the limit of 30 degrees, with the correct adjustment of the gain. The result, comparatively, is a little worse, in other words, for a same gain adjustment, the proportional controller presents less error, but even so the vehicle stays in the track and the error is going to zero after the maneuver. As the steering angle is inside of the acceptable limits, it's still possible to increase the gain to obtain a better result.

5. CONTROL THROUGH THE CENTER OF THE TRAJECTORY AND THE ATTITUDE

Now the yaw angle of the vehicle is considered as an input variable in the control loop. The yaw angle is compared with the tangent angle of the track, which is the reference. It is made a mediated sum between the error of the center of gravity of the vehicle and the error of the yaw angle. The resultant error is made multiplying one of the errors for a value x, smaller than zero, and the other one for (1-*k*). That sum generates a resulting error that is the controller's input. In that case it is important to emphasize the creation of one more adjustment variable that is the consideration (*k*) between the errors described previously. It was seen that the mediation has great influence in the final result and a more detailed study can lead to a better adjustment, in this work the consideration of 0.8 was used for the displacement error against 0.2 of the yaw error, being $k = 0.8$.

Fig. 14 shows the behavior of the vehicle on a speed of 10m/s and with a controller of proportional type with gain adjusted in 10, as in the previous section. Comparing Fig. 4 with Fig. 14 it's seen that the result is a little worse when the information of the yaw angle was introduced. Even so it can be noticed by Fig. 15 that the result as a whole is better, when Fig. 15 is compared with Fig. 5 it's observed that the steering is smaller, in spite of the oscillations, the error also decreased. It is important to emphasize that the error in Fig. 5 is not the same error of Fig. 15. The last one is a resulting sum of the tracking error and yaw error. That resultant reflects a commitment between the position of the vehicle in relation to desired path and its orientation.

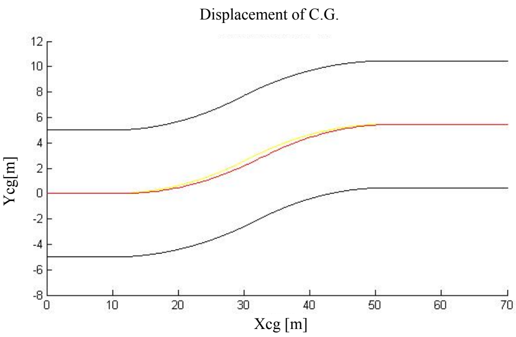

Figure 14. Displacement of the C.G. (10m/s). P.

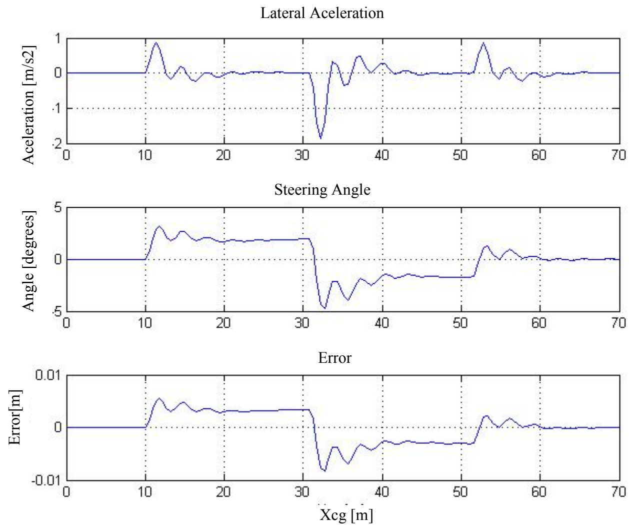

Figure 15. Lateral acceleration, steering and error (10m/s). P.

Fig.16 shows the same simulation of Fig. 14 with the difference that the speed is 20 m/s now. While in the simulation using the first control strategy (only C.G. position error considered) the result for a speed of 20m/s was not so representative, in this case, using the second strategy, is possible to observe a better result than the one seen in Fig. 7, mainly when the steering angle is compared.

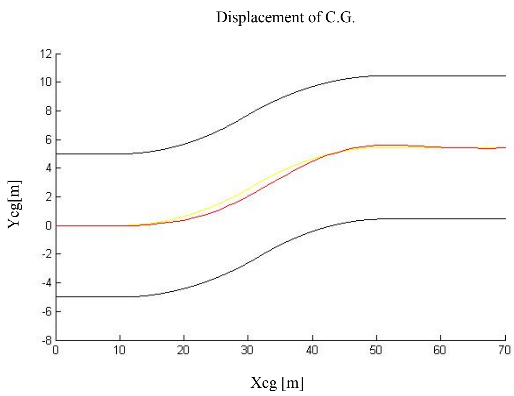

Figure 16. Displacement of the C.G. (20m/s). P.

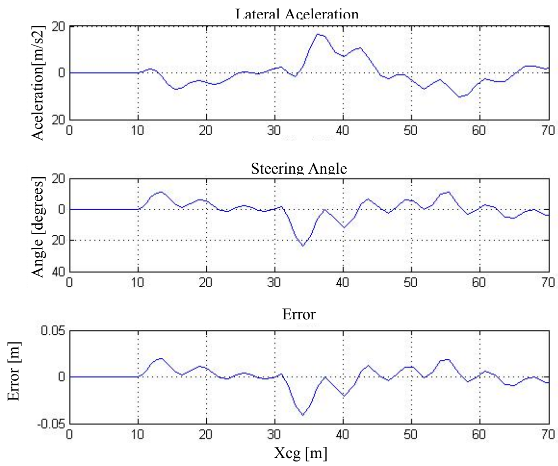

Figure 17. Lateral acceleration, steering and error (20m/s). P.

As observed in the first strategy, there are oscillations resulting from the controller's choice but that can be minimized with the use of PDD. In that way, the same controller PDD of the previous case was used to repeat the simulations, so the influence of the yaw angle in the resulting error can be compared. Fig. 18 shows the displacement of CG of the vehicle along the track for a speed of 10m/s. The first thing that can be noticed is the error in relation to the center of the track, but as said previously, the commitment now is not only to get right the middle of the track, but to do that with the correct orientation of the vehicle. The general result is highly dependent of the consideration given for the two variables (track error and yaw error). Comparing Fig. 19 with Fig. 11 it's possible to see that the steering angle is smaller, as well as the lateral acceleration.

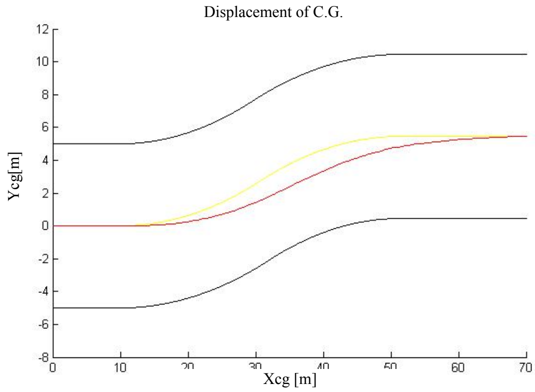

Figure 18. Displacement of the C.G. (10m/s). PDD.

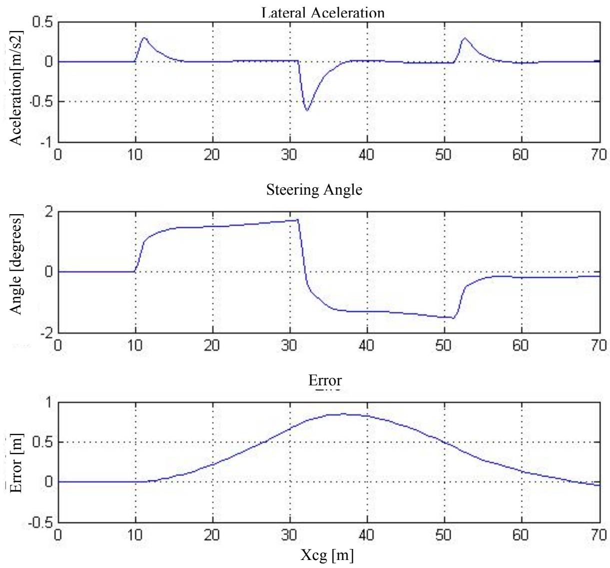

Figure 19. Lateral acceleration, steering and error (10m/s). PDD.

Fig. 20 and Fig. 21 show the same test done with a speed of 20 m/s. Making a comparison between Fig. 20 and Fig. 12, and between Fig 13 and Fig. 21, it can be seen that the second strategy leads to smaller values and softer profile of steering, as well as smaller values of lateral acceleration. The error cannot be compared directly, but for the proposed commitment, the error of the second strategy is smaller.

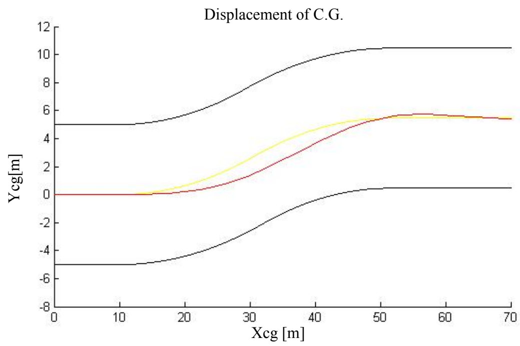

Figure 20. Displacement of the C.G. (20m/s). PDD.

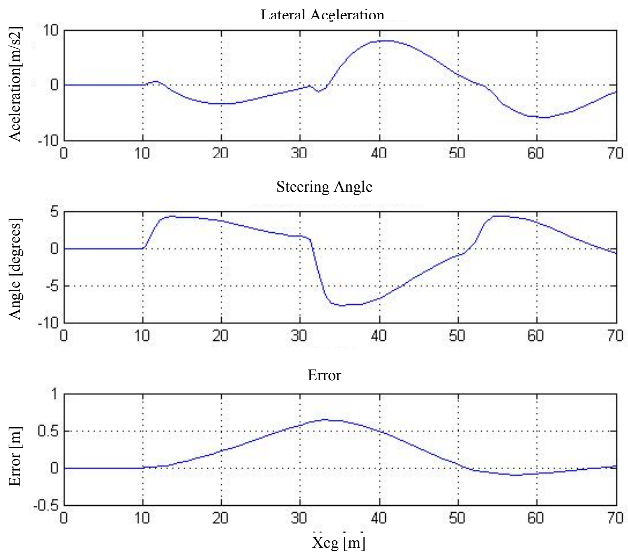

Figure 21. Lateral acceleration, steering and error (20m/s). PDD.

6. NON LINEAR CONTROL LOOP

Until now the control loop composed by the model of the vehicle, the controller and the error generator was completely linear, making possible the analysis of poles and zeros by the root locus plane. The tracks, given as reference, were composed of soft curves. The linearization of sines and cosines limited the arcs of the curves in 20 degrees, maintaining the model in its linear limit.

From now on it is considered the change of the local frame of the vehicle to the global one as being a non-linear relationship that involves sines and cosines. The objective is to analyze the PDD behavior, designed for the linear model when used in the non-linear case, considering the first and second control strategies. The model in subject, employing the same variables of Eq. 1, is given by the system of equations:

$$\begin{cases} \dot{v} = \dfrac{-2(C_f + C_r)}{m_{tot}u}.v - \dfrac{2(a.C_f - b.C_r) + m_{tot}.u^2}{m_{tot}.u}.w + \dfrac{2.C_f}{m_{tot}}.\delta_f \\ \dot{\theta} = w \\ \dot{w} = \dfrac{-2(aC_f - bC_r)}{I_{yaw}u}.v - \dfrac{2(a^2C_f + b^2C_r)}{I_{yaw}u}.w + \dfrac{2.a.C_f}{I_{yaw}}.\delta_f \\ \dot{X} = u.\cos\theta - v.sen\theta \\ \dot{Y} = -u.sen\theta - v.\cos\theta \end{cases} \qquad (3)$$

The first test represents a lane change, as in the linear case, but that demands an angle of deflection larger than 20 degrees. It can be seen in Fig. 24 and Fig. 25 that the behavior of the vehicle resembles the linear case with the difference that the steering is a little larger due the curve is more accentuated. The lateral acceleration still presents values a little high for the considered speed. It is observed that all the simulations, from now on, were done using a speed of 20m/s.

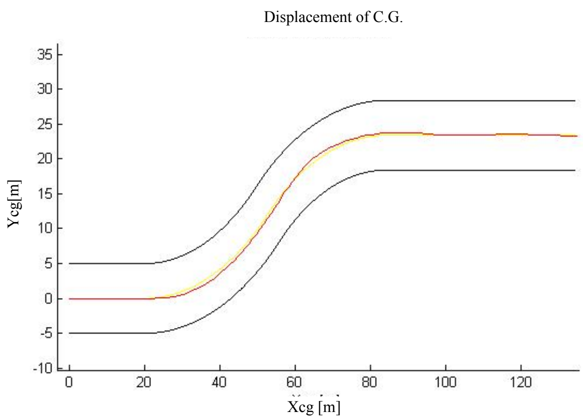

Figure 24. Displacement of the C.G. (20m/s). First control strategy.

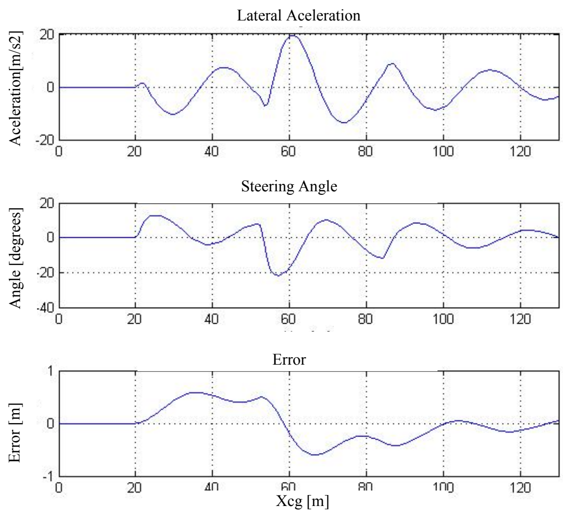

Figure 25. Lateral acceleration, steering and error (20m/s). First control strategy.

Now it is taken as reference a closed circuit that contains the maneuver of lane change seen in the first part of the work and curves of 180 degrees. It can be noticed by the analysis of Fig. 26 and Fig. 27 that the behavior of the steering is quite reasonable and coherent with the displacement of the center of gravity, except for some few oscillations and the fact that the steering angle oscillates a lot and presents very high values, making the result no representative in a real situation. Fig. 27 can verify the difference between the linear model and the non-linear one. There are a lot of oscillations in the non-linear model and the steering angle assumes high values, as well as the lateral acceleration. That shows that the controller's design for the linear model is not ideal when used in the non-linear case.

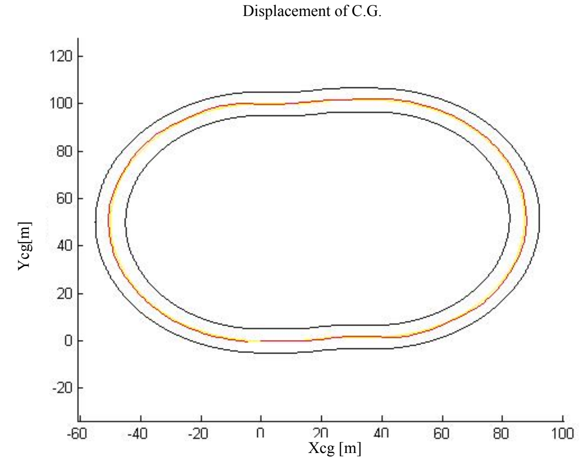

Figure 26. Displacement of the C.G. (20m/s). First control strategy. PDD.

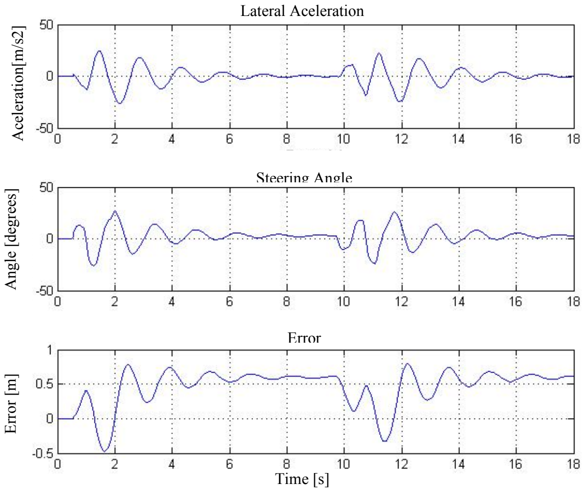

Figure 27. Lateral acceleration, steering and error (20m/s). First control strategy. PDD.

It's presented now the same conditions imposed in Fig. 26, however with the use of the second control technique. It is possible to notice that the results of Fig. 28 and Fig. 29 are better than the one of Fig. 27. The steering angle has smaller values and the result doesn't oscillate so much, showing once again that the second technique, in spite of presenting a larger error in the position of C.G. along the track, returns more coherent values for the variable that control the car.

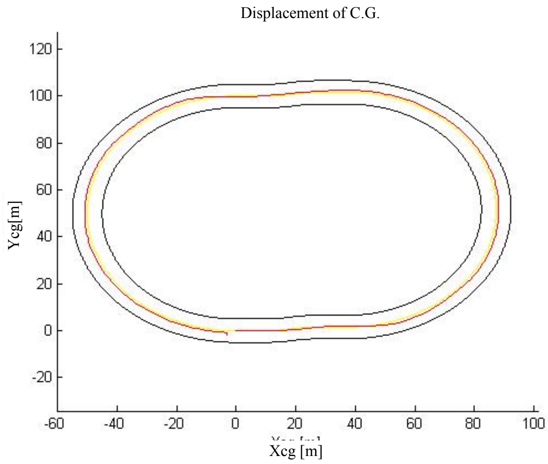

Figure 28. Displacement of the C.G. (20m/s). Second control strategy. PDD

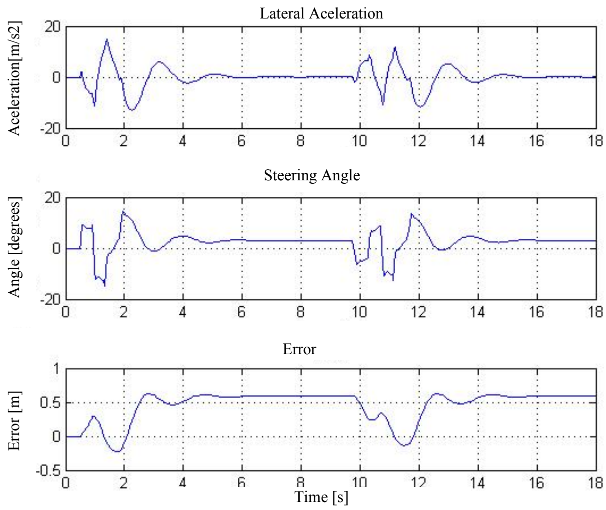

Figure 29. Lateral acceleration, steering and error (u=20m/s). Second control strategy. PDD.

Finally the results for a more sinuous track are presented. It is possible to notice the worsening of the results in Fig. 30 and Fig. 31, when the circuit has more accentuated curves. When the control should be made in a path that imposes more severe conditions on the vehicle is noticed that the steering values arise and the behavior worsens considerably. That shows that the controller designed for the linear model doesn't present a good performance when it is applied to the non linear model, being clearly dependent of the path that is intended to track, fact that that was not taken into account when the controller was designed.

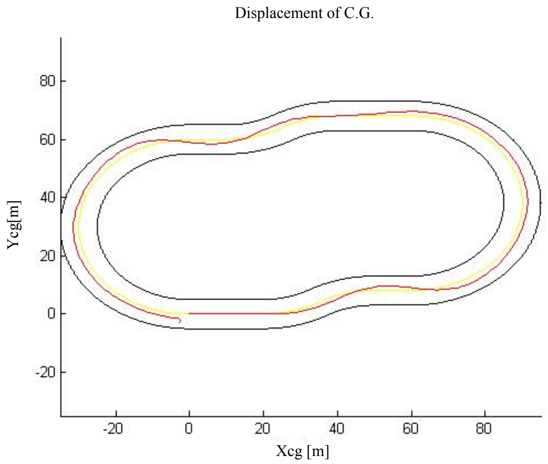

Figure 30. Displacement of the C.G. (20m/s). Second control strategy. PDD

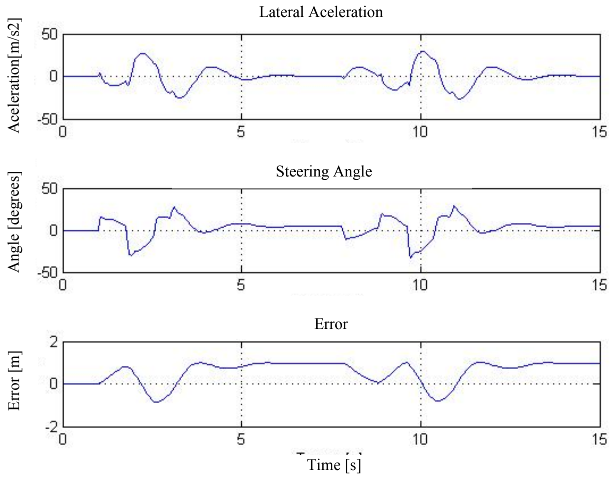

Figure 31. Lateral acceleration, steering and error (20m/s). Second control strategy. PDD

7. FINAL COMMENTS

On the linear case, the results of the classic control were very good considering the displacement of the mass center, but the behavior of the steering angle was oscillatory and non representative using the proportional controller. When a proportional double derivative (PDD) controller was used, the results became better, the steering angle profile became softer and more representative. It was noticed that the longitudinal velocity has great influence on the final result also.

When the yaw error was considered, the results got even better, that's because even with a larger displacement error, the whole error was shorter, in other words, the compromise between C.G. position and yaw angle of the vehicle is taken into account, making the car follow the track with the right orientation.

It was seen in this work that a controller designed to a linear loop can be used on the respective non-linear loop, but it has limitations, mainly when the longitudinal speed is big. The controller can keep the vehicle inside the closed track and the steering angle is not absurd, but as we expected the linear controller design works better with the linear model. Future works can focus on the controller's optimization to make them less dependent of the chosen track.

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