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DESIGN OF COMPOSITE LAMINATE STRUCTURES USING A GENETIC ALGORITHM AND FINITE ELEMENT ANALYSIS

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Abstract. *This paper deals with the design of composite laminate structures using genetic algorithms and finite element analysis. Two examples are presented to show the flexibility of this tool in solving different kind of problems. The first case is a multiobjective problem where total weight and cost of an in-plane loaded plate must be minimized. Objective functions consisting on a weighted sum of these two quantities are adopted to hold both objectives at the same time. The weighting factor used in these function is varied, shifting the optimization emphasis, to obtain a set of optimum solutions. Laminae material, fiber orientation and number of plies are the design variables, which are selected from a set of discrete values. The problem constraints are given by material failure and structure buckling safety factors. The second design case deals with the stiffness maximization of a cylindrical shell under pressure load. Laminated fibers orientations are optimized to enlarge the critical load level and minimize the central structure displacement who are obtained by geometrically non-linear analysis. Additionally to material failure, the number of contiguous plies with same fiber orientation are considered as a design constraint.*

Keywords*: Multiobjective optimization, Genetic algorithms, Laminated composites, Finite elements.*

1. INTRODUCTION

In the recent decades the use of composite laminate materials on structural applications has been growing, requiring great effort on development of analysis and design techniques. The large number of design variables and the complexity of the mechanical behavior are outstanding characteristics of composite material structures design. Such characteristics turn the project much more difficult and laborious than those involving conventional materials.

Optimization methods have been used in the sense of turn the composite material structural design a more systematic and well defined task (Gürdal *et al.*, 1999). Moreover, less dependence with respect to designer sensitivity and maximum material performance can be achieved. Initially, the same method used to optimize conventional materials structures was adopted for composite material structures optimization. This mathematical method works with continuous variables and performs the search for the best solution through de design space based on gradient information. The effort on using such methods met with limited success, since composite laminate design is, in practice, restricted by the manufacturing process that limit the variables to few discrete values. Moreover, typically exist many configurations close to optimal laminate configuration (locally optima regions) that can prevent the gradient based methods to reach the global optimum.

As alternative to gradient based methods, many other techniques were tested, having the genetic algorithm (GA) stand out the others because it perfectly adjusts to the problem characteristics. GAs are probabilistic optimization methods that seek to mimic the biological reproduction and natural selection process through random, but structured, operations. The design variables are coded as genes and grouped together on chromosomes strings that represent an organism (possible solution on the design space), what allow GAs to manipulate discrete variables. Instead of working with just one search point in the design space, GA uses a population of designs that, by reproduction operations, evolve through successive generations. Many search points dispersed in the design space prevent the GA to get stuck in a local optimum area, and avoiding a premature convergence of the process. New possible designs (organisms) are generated by applying genetic operators on existing population organisms (mimicking the natural genetic mechanisms). The evolution of successive generations towards the optimization objectives is achieved by using concept of survival of the fittest (where fittest organisms have more chances to reproduce and continue in the next generation) what mimic the natural selection process. The organism fitness is obtained directly from an objective function, that uses simple structure information. No gradient evaluation is necessary to perform the search by GA.

Although GA has shown to be well adapted to composite laminate structural optimization problems, it requires a large number of analyses in each process, what is a fundamental drawback. In more complex problems, were numerical methods are necessary for the structural analysis, an excessive number of analyses can turn GA impracticable. Many techniques have been developed to minimize these problems. Essentially, the classical GA structure has been adapted to take advantage of composite laminates characteristics. The GA restructuring is done by a new variable codification and by the way new genetic operators act on the gene string.

Two examples of GA application on composite laminate structure optimization are presented in this work. The first one consist on a multiobjective optimization where the total weight and cost of an in-plane loaded plate are minimized. The objective function formulation is based on a technique for multiobjective problems optimization, were the emphasis given to each objective can be adjusted, allowing the GA to obtain the pareto-optimal set. The second example shows the use of GA to optimize a cylindrical shell under pressure load, and with geometrically non-linear behavior. The optimization objective is to maximize the structure stiffness, using critical load and maximum central displacement as parameters to evaluate the feasible solutions. In both examples the whole design space was analyzed in order to prove that the GA is successful in finding global optimum at each case. The GA efficiency is evaluated by the number of analyses required for each optimization procedure and by the apparent reliability.

2. STRUCTURAL ANALYSIS

Real composite material structures optimization problems depends on a reliable structural analysis. Even for simple geometric configurations, the determination of the mechanical behavior is difficult in the case of composite materials. It happens because of the complex mechanisms like coupling between extension, bending and torsion deformations, depending on the stacking sequence. The available closed mathematical formulations introduce much simplification on the analysis or, in many cases, they are not able to predict the structural behavior, mainly for complex geometries. These make necessary to use numerical methods that can predict satisfactorily the structure response for a given design load.

There are many works on the field of numerical simulation of composite material structures using the finite element method (FEM). However, the expressive number of analyses usually required by GA limits the use of FEM as an analysis tool because of the high computational cost for the analysis of each individual design.

In this work, a triangular flat plate and shell element with 18 degrees of freedom called DKT (Discrete Kirchhoff Triangle) is used. This element was developed by Bathe and Ho (1981) for the non-linear analysis of isotropic plates and shells. Additionally to the structure displacements, the analysis tool must be able to determinate the stress components at the composite layers in order to predict material failure. This is a very common constraint adopted in most of optimization problems, and it is used in this work too. The Tsai-Wu failure criterion (Daniel and Iashai, 1994) is used in the failure prediction, evaluated at both faces of each ply at each of the three numerical integration point of each finite element. In geometrically non-linear analysis the material failure is verified at each load step of the interactive solution method, which is stopped if material failure is detected. A safety factor against failure λ_f can be obtained using the Tsai-Wu failure function with the material strength parameter for traction, compression and shearing at each of the principal material axes.

3. GENETIC ALGORITHMS FOR COMPOSITE LAMINATES

This work uses a GA provided of many modifications with respect to the classical GA to adapt it to the specific case of composite structural optimization (Almeida, 2006). In the next sections special genetic operators and strategies are explained. None of the classical GA structure is presented, but the reader can find details about it in Goldberg (1989).

3.1. Composite laminate codification

In GA optimization process, the structure is considered as an organism with its characteristics defined in chromosomes, as occurs in natural organisms. Each stored information is seen as a gene that refers to one of the structure laminate plies. A computational representation of chromosomes is done by a string containing coded information of laminate properties. In this work each laminate is represented by a pair of chromosomes, as it was done by Soremekun et al. (2001). In the first one, called "orientation chromosome", information about fiber orientation of each laminate layer are stored. The second, called "material chromosome", is used to point the layers material properties group (ply thickness, elastic and strength constants). So, a laminate layer is represented by a pair of genes, each in one of the two chromosomes but at the same relative position. The first pair is referred to the outermost layer, being the inner layers referred by the succeeding pairs. As only symmetric laminated are used in this work, just half of the laminate layers are coded in the gene strings, and so the total number of genes in a chromosome is proportional to half of the maximum admissible number of layers in a specific design.

Integers numbers are used to form two gene alphabets, one for each chromosome, that are used to code a composite laminate into two gene strings. The numbers on an alphabet represent the discrete possible values for the design variables. In the orientation gene alphabet, each number represents a predefined stack in a layer, which can contain more than one ply. The orientation genes defines how many plies exists on a layer and how are the plies of these fiber oriented. Similarly to the orientation gene, material alphabet genes define the material properties group that can be assigned to each layer. To allow variations of the total number of layers in the different designs when the optimization process is performed, an empty stack code “0” is used. Specific genetic operators are responsible to add or delete the laminate layers by changing gene codes. The maximum number of layers is limited by the total number of genes in the chromosome.

3.2. Genetic operators for composite laminate optimization

3.2.1. Crossover

Crossover is an essential GA operator, having the fundamental task to generate new organisms (child) in a reproduction process, combining genetic information taken from a pair of organisms (parents) selected from a pre-existing population. The parents selection is a probabilistic process, but greater chance of selection is given to the fittest organisms. The created child will hopefully be better than his predecessors, since his genes were part of organisms with good fitness. The crossover operator used in this work is similar to classical crossover operator with few modifications. A crossover point is randomly determined and the gene strings are split at the same point in both parent. This point must be localized in a non-empty stack gene string region (Soremekun et al., 2001), and both material and orientation chromosomes must be split at the same point. The left part of parent 1 and the right part of parent 2 are combined to form a child. The probability of application of crossover operators is set to 100% in all the examples presented, because it is considered a fundamental operator in GA.

3.2.2. Mutation

The classical GA operator acts over the chromosomal string changing a gene value. It is implemented to each gene, at a small probability, introducing a different value chosen from the gene alphabet. In spite of the randomity of this process, it is possible to incorporate to the mutation operator some knowledge about mechanical response of composite materials when one or more of its characteristics are altered. This may let to a less random process and guide the evolution towards optimization objectives. These modifications lead to new operators called *orientation alteration*, *material alteration*, *ply addition* and *ply deletion*, replacing the classical mutation in the GA (Nagendra *et al*, 1996 and Soremekun et al., 2001).

Orientation alteration and *material alteration* operators are implemented very similarly to classical mutation. The differences are based in the fact that they are independently applied to orientation and material chromosomes, respectively. Different orientation and material operator probabilities (p_{oa} and p_{ma}) may be adopted, which is convenient, since orientation and material chromosome may converge at different velocities in a optimization process. Additionally, these operators are restricted to non-empty stack regions of the gene string.

The variation of the total number of laminate layers is made by *ply addition* and *ply deletion* operators. By a given p_{pa} probability, *ply addition* operator introduces a new layer close to the laminate mid-plane (end of chromosome), and delete an existing empty stack. The new genes are determined randomly by chosen codes at each related gene alphabet. In an opposite way, *ply deletion* take out a last pair of genes of organism chromosome (innermost layer) and adds a pair of empty ply genes at the outermost position. This operator is implemented with a given p_{pd} probability. Both operators manipulate at the innermost laminate layer because it has a little effect on bend-twist performance of the plate or shell, causing no abrupt changes in the design.

3.2.3. Permutation

The main characteristic of permutation operator is the ability to modify laminate stack sequence without changes of the total number of plies with fibers oriented on each permissible direction. This allows GA to change the bending behavior of the laminate without modifying its in-plane mechanical response. The permutation operator implemented in this work is equivalent to the *gene-swap* operator (Nagendra *et al*, 1996 and Soremekun et al., 2001), where two pairs of genes, representing non-empty layers, are chosen randomly and they have their position shifted in the chromosome, resulting on a new staking sequence. Such operation occurs at a given probability p_{per}, usually with a larger value than those corresponding mutation operators probabilities.

3.3. Selection Schemes

There are many ways to obtain the population of successive generations in a GA. In classical algorithms new generations are formed only by children created from an existing population. This process has many drawbacks since there is no warranty of improvement or maintenance of achieved evolution when all population is replaced. To solve this problem new selection schemes were created, being one of them the elitism scheme, which consists in transfer good organisms from old population to a new generation, preserving desirable genetic information. This paper deals with a multiple elitist scheme (Soremekun, 1997).

In the multiple elitist scheme, both parent and child populations of size P are independently ranked from best to worst fitness. These two populations are then combined and ranked together, resulting in a combined population with $2P$ organisms. Then, best Ne individuals of the combined population are transferred to the new generation. The best individuals of child population that have not already been used are taken to fill the remainder of the new generation. The number of top elements (Ne) to be transferred to the new generation is a GA parameter to be adjusted at each application case

4. NUMERICAL APPLICATIONS

In the following sections two examples of GA applied to optimize composite laminate structures are presented. To prove the success of the optimization procedure and to characterize the problem design space, all the possible laminate configurations are previously analyzed. Additionally, to obtain the algorithm reliability and computational cost, N optimizations with the GA are carried out for each example. The apparent reliability (R) is determined by taking the number of optimizations for which the GA finds at least one global optimum (No), divided by the total number of applications of the GA (N). It defines the chances of obtaining the global optimum in a single application of the GA. As the structural analysis employing the FEM is usually the most time consuming task in the optimization procedure, the GA cost is determined by

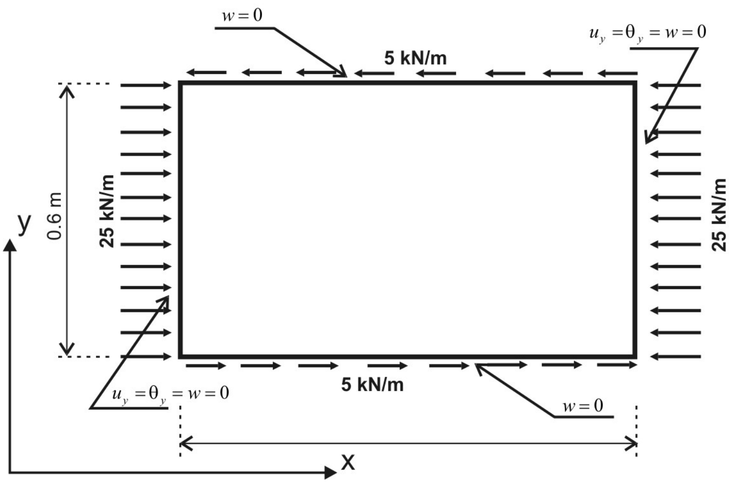
$$An = \frac{\sum_{i=1}^{N} X_g^i P}{N}, \tag{1}$$

where X^i_g is the total number of generations analyzed in the i-th optimization procedure.

The criterion to stop the optimization process, used in both examples, is based on two parameters, the upper limit of the number of generations (N_{LG}) and the maximum number of generations with no improvement of the best design (N_{SD}). Once one of these limits is reached, the optimization process is stopped and the best laminate of the last generation is taken as the optimization result. N_{LG} and N_{SD} are defined in each optimization procedure, depending on the problem complexity.

4.1. Cost and weight minimization of an in-plane loaded composite laminate plate.

In this example the total cost and the weight of an in-plane loaded composite laminate plate are minimized. No instability or material failure is admissible for a feasible design. Plate geometry, load and boundary conditions are presented in Fig. 1. The resultant laminate must be symmetric and it must have from 3 to 12 layers, formed by 2 plies each one, and they are oriented at $0°_2$, $\pm45°$ and $90°_2$. Additionally, each layer can be made of two unidirectional composites, Kevlar-epoxy or Graphite-epoxy, which have different mechanical properties and cost, as it is shown in Fig. 1. The elastic constants are the Young's modulus on fiber direction (E1) and transverse to fiber direction (E2), shear modulus (G12) and the Poisson's ratio (ν12), respectively. Strength parameters for traction and compression for longitudinal and transversal directions are given by F1t, F1c, F2t, and F2c respectively, and F6 is the shear strength. The remainder parameters are the specific weight (ρ), the cost per unit weight (C), and the ply thickness (t). A regular finite element mesh with 432 elements and 247 nodes is used in the structural analysis.



Properties	Kevlar-epoxy	Graphite-epoxy
E1	87.0 GPa	181.0 GPa
E2	5.5 GPa	10.3 GPa
G12	2.2 GPa	7.17 GPa
ν12	0.34	0.28
t	0.18 mm	0.13 mm
ρ	13.5 kN/m³	15.7 kN/m³
C	1.0 uc/N	3.0 uc/N
F1t	1280.0 MPa	1500.0 MPa
F1c	335.0 MPa	1500.0 MPa
F2t	30.0 MPa	40.0 MPa
F2c	158.0 MPa	246.0 MPa
F6	49.0 MPa	68.0 MPa

* Elastic and strength properties from Daniel and Iashai, 1994.

Figure 1. Composite laminate plate under in-plane load

The safety factor for material failure λ_f is given by the Tsai-Wu failure criterion (Daniel and Iashai, 1994), while the safety factor for structural elastic instability λ_b is obtained solving the eigenvalue problem involving linear and non-linear stiffness matrices. A design is considered to be feasible when both λ_f and λ_b are grater or equal to 1.0.

The optimization is made through the manipulation of the total number of layers, fiber orientations and ply material. The numbers 1, 2 3 are used to represent the stacks $0°_2$, ±45° and $90°_2$ in the orientation chromosome, respectively, while the numbers 1 and 2 are used to represent Kevlar-epoxy and Graphite-epoxy material in the material chromosome, respectively. The laminate is represented by a pair of chromosomes with 6 genes each, resulting in 55944 possible designs. A distribution of weight and cost of all the possible feasible designs in the problem is shown in Fig. 2. The points A to F in this figure are the pareto-optimal designs, which must be obtained by the GA, according to the emphasis given to each of the optimization objectives. Details of the pareto-optimal points are presented in Tab. 1.

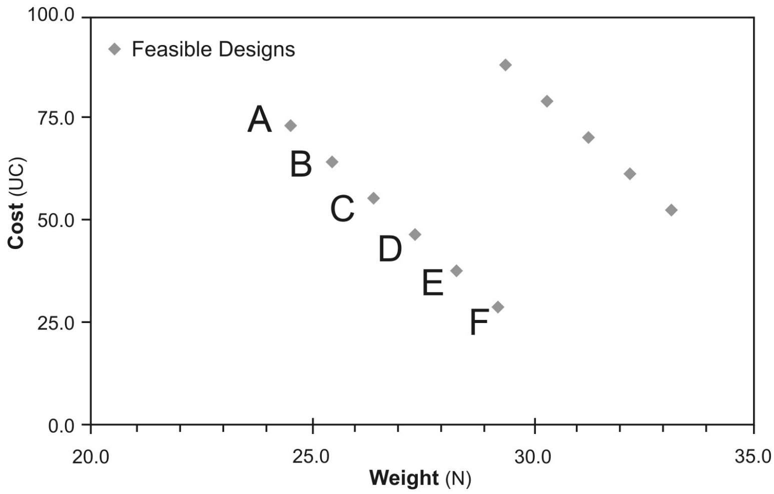


Figure 2. Weight and Cost of feasible designs

Table 1. Pareto-optimal designs.

Optimal design	Laminate	Weight (N)	Cost (uc)	λ_b	λ_f
A	$\left[\pm 45_3^{ke},\ 90_4^{ke}\right]_S$	24.49	73.46	1.30	16.04
B	$\left[\pm 45^{ge},\ \pm 45^{ke},\ 90_4^{ke},\ 0_2^{ke}\right]_S$	25.44	64.62	1.56	17.16
C	$\left[\pm 45^{ge},\ 90_2^{ge},\ 0_2^{ke},\ \pm 45^{ke},\ 90_2^{ke}\right]_S$	26.39	55.77	1.64	18.00
D	$\left[90_2^{ge},\ 0_2^{ge},\ \pm 45^{ge},\ \pm 45_2^{ke}\right]_S$	27.34	46.93	1.50	31.84
E	$\left[90_2^{ge},\ 0_2^{ge},\ \pm 45_2^{ge},\ 0_2^{ke}\right]_S$	28.30	38.09	1.30	30.93
F	$\left[\pm 45^{ge},\ 90_2^{ge},\ 0_2^{ge},\ \pm 45^{ge},\ 0_2^{ge}\right]_S$	29.25	29.25	1.04	55.12

An objective function must be established to determine the designs fitness to be used by the GA in the optimization process. In this example weight and cost are supposed to be simultaneously minimized, what characterizes a multiobjective optimization problem. The objective function formulation must include both parameters and to allow modifications on the emphasis given to each of them. GA can obtain each of the pareto-optimal designs by shifting the objective emphasis adjusting the weighting factor α. The optimization constraints must also be considered in the objective function by penalizing the unfeasible designs and with benefit to those with a large safety factor.

First it was tested a standard objective function formulation for multiobjective optimization, consisting in a linear combination of the two objectives. This attempt was frustrated because the pareto-optimal set is arranged as a line in the wheight-cost plane, as can be seen in Fig. 1. In this case, every value of α leads to one of the extreme points A or F.

The alternative objective function used in this paper for multiobjective optimization is given in by

$$\begin{cases} OBJ = \left(\sqrt{\left[\alpha\left(W^*\right)^2\right]^2 + \left[(1-\alpha)\left(C^*\right)^2\right]^2} \right)^{-1} + 10^{-6}\lambda^*, & if\ \lambda^* \geq 1 \\ OBJ = \left(\lambda^*\right)^2 \left(\sqrt{\left[\alpha\left(W^*\right)^2\right]^2 + \left[(1-\alpha)\left(C^*\right)^2\right]^2} \right)^{-1}, & if\ \lambda^* < 1 \end{cases} \text{, where } \begin{cases} W^* = \dfrac{W - W_{\min}}{W_{\max} - W_{\min}} + 1 \\ C^* = \dfrac{C - C_{\min}}{C_{\max} - C_{\min}} + 1 \end{cases} \quad (2)$$

In these equations W^* e C^* are the normalized total weight and cost of the plate, respectively. They are normalized using the maximum and minimum values that are easily obtained by the extreme combination of materials and number of layers. The structural safety factor λ^* is given by the minimum value between λ_b and λ_f.

Instead of working directly with weight and cost, the implemented formulation uses the square of these variables, already weighted by the factor α. This alternative gives a curved distribution of the optimal designs in the weight-cost plane. The optimal solution is defined as the closer point to the weight-cost axes origin, being this distance given by "$\sqrt{\left[\alpha\left(W^*\right)^2\right]^2 + \left[(1-\alpha)\left(C^*\right)^2\right]^2}$". The safety factor λ^* is used to penalize unfeasible designs and to benefit the feasible ones.

In the optimization process, GA is applied 25 times for each α, which is taken varying from 0.0 to 1.0 with increments equal to 0.1. The GA is used with a population size P=30 and the elitist scheme parameter Ne=4. The genetic operators are used with the following probabilities p_{ao}=4%, p_{am}=2%, p_{pa}=4%, p_{pd}=8% e p_{per}=80%. The criterion parameters to stop the process are N_{LG} = 300 and N_{SD}= 100.

The GA can obtain one of the pareto-optimal designs for every tested α, but not all of the 25 GA applications for each value of α are successful in obtaining one of these points. This happens because there are many designs with the same weight and cost of the optimal design, but with a small safety factor. As a benefit for safety factor was implemented in the objective function, these designs have slightly different fitness. Table 2 shows the apparent reliability (R) and its standard deviation (σ), resulting of AG optimizations at two situations. The first situation considers only the optimal designs presented at Tab. 1., and the second considers as valid solutions all the designs with the same weight and cost of those in Tab. 1. These are called "near optimal design" and are found in every optimization and for all α values, showing the GA efficiency.

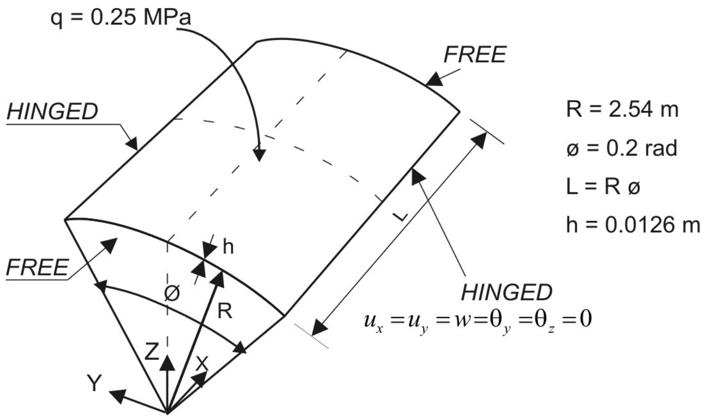
Table 2 also shows for each α the average number of applications of the GA required to convergence, An, and the parameter J, determined by taken An and dividing it by the size of the design space. The GA reveled to be expensive (6.5%<J<9.5%), and it can be attributed to a retardation to stop the process until the maximum number of generations caused by new fitters designs (with same weight and cost but better safety factor), obtained at the end of the optimization procedure.

Table 2. GA optimization results.

α	Pareto-optimal set	Optimal designs		Near optimal designs		An	J
		R	σ	R	σ		
0	A	100%	0.0%	100%	0%	3791	6.8%
0.1	A	100%	0.0%	100%	0%	3646	6.5%
0.2	A	100%	0.0%	100%	0%	3690	6.6%
0.3	B	100%	0.0%	100%	0%	3976	7.1%
0.4	B	100%	0.0%	100%	0%	4320	7.7%
0.5	C	20%	8.0%	100%	0%	4501	8.0%
0.6	D	16%	7.3%	100%	0%	4139	7.4%
0.7	E	20%	8.0%	100%	0%	5291	9.5%
0.8	F	88%	6.5%	100%	0%	4898	8.8%
0.9	F	76%	8.5%	100%	0%	4378	7.8%
1	F	96%	3.9%	100%	0%	4710	8.4%

4.2. Stiffness maximization of a composite laminated shell with geometrically non-linear behavior.

The cylindrical shell under uniform pressure load is shown in Fig .3. In the optimization process, the finite element analysis is carried out taken into account geometrically non-linear effects. It was considered that in the composite laminate material only the fiber angles may assume different discrete values, while all the other parameters remain fixed. Material failure and the number of contiguous plies with the same fiber orientation are considered as design constraints. This last constraint is imposed in order to avoid the failure of the composite material due to matrix rupture.



Unidirectional Glass-epoxy properties	
E1	39.0 GPa
E2	8.6 GPa
G12	3.8 GPa
ν12	0.28
ρ	20.6 kN/m³
F1t	1080.0 MPa
F1c	620.0 MPa
F2t	39.0 MPa
F2c	128.0 MPa
F6	89.0 MPa

* Elastic and strength properties from Daniel and Iashai, 1994.

Figure 3. Cylindrical shell under a uniform pressure load

A finite element mesh for the whole domain having 800 elements and 441 nodes is adopted and the Generalized Displacement Control Method (Yang and Shieh, 1990) is used to solve the non-linear problem. In Fig. 3 are also included material properties (elastic constants, specific weight and strength parameters) corresponding to glass-epoxy. It is considered that the composite material is formed by 14 layers having a fixed total thickness h = 12.6 mm. Each layer is formed by 2 plies that may have fiber orientations such as $0°_2$, ±45° and $90°_2$. Contiguous plies with same fiber orientation are limited to 4 plies. An orientation chromosome with 7 genes is used. Codes 1, 2 and 3 are attributed, respectively, to the laminate sequence $0°_2$, ±45° and $90°_2$.

To evaluate the shell stiffness two parameters, obtained form the structural analysis, are used. The first one is the critical load level (NC_{crit}) determined when the curve pressure x displacement at the central point reaches the first peak. The second parameter is the maximum value of the displacement of the central point ($U_{máx}$) at the end of the load increment or when material failure is observed. Two variables are used in order to consider situations where constraints are satisfied or they are violated. The first variable is the maximum load level acting on the structure without material failure ($NC_{máx}$) and the second variable is an integer number (V_{nlc}) indicating how many times the constraint referred to contiguous plies with the same fiber orientation have been violated. In Fig. 4 values of NC_{crit} and $U_{máx}$ corresponding to situations where the two constraints are not violated (feasible designs) are shown.

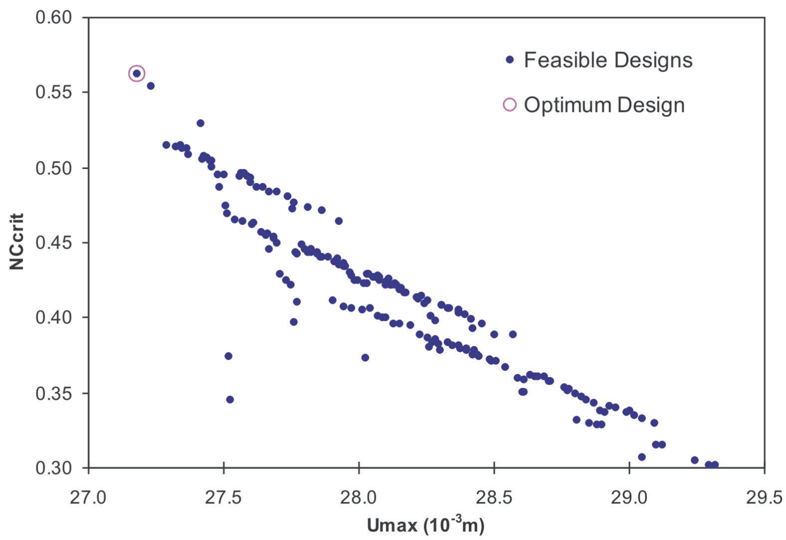


Figure 4. Critical load level x maximum central displacement of feasible designs

The objective here is to maximize the structure stiffness, obtaining the maximum value of NC_{crit} associated to the minimum value of $U_{máx}$. The structure fitness is given in the GA by the following function:

$$FIT = \left(\frac{NC_{crit} \cdot NC^2_{\max}}{U_{\max} \cdot \left(V_{nlc} + 1\right)} \right) \tag{3}$$

In Eq. (3), $NC^2_{máx}$ is used to penalize structural failures before the application of the total load, whereas $(V_{nlc} + 1)$ is employed to penalize configurations where the maximum number of plies with the same fiber orientation is greater than 4. The optimal solution, contained in Fig. 4, is defined by the stacking sequence $\left[(90_4,\ \pm 45)_2,\ 90_2\right]_S$ and the following value of the parameters: $NC_{crit} = 0.563$ and $U_{max} = 27.2 \times 10^{-3}$ m.

In order to evaluate the parameter N_{LG} (limit for the number of generations) and P (number of individuals in the population) with respect to the performance of the GA, 5 values of each of parameter are adopted and 50 optimizations are carried out for each of the 25 combinations of the two parameters. All these combinations of N_{LG} and P are presented in Tab. 3, where Ne and N_{SD}, associated with N_{LG} and P had also been included. The probability of fiber orientation alterations is taken in this case as being p_{ao}=4%, while for the probability of permutation p_{per}=80% is adopted. As the number of layers remains fixed and only glass-epoxy is used, the probabilities $p_{am} = p_{pa} = p_{pd} = 0$.

Table 3. Study of some parameters used in the GA.

Comb	*P (Ne)*	N_{LG} (N_{SD})	*Comb*	*P (Ne)*	N_{LG} (N_{SD})	*Comb*	*P (Ne)*	N_{LG} (N_{SD})
1	50 (8)	300 (100)	11	50 (8)	108 (36)	21	50 (8)	33 (11)
2	30 (5)	300 (100)	12	30 (5)	108 (36)	22	30 (5)	33 (11)
3	18 (3)	300 (100)	13	18 (3)	108 (36)	23	18 (3)	33 (11)
4	10 (1)	300 (100)	14	10 (1)	108 (36)	24	10 (1)	33 (11)
5	6 (1)	300 (100)	15	6 (1)	108 (36)	25	6 (1)	33 (11)
6	50 (8)	180 (60)	16	50 (8)	60 (20)			
7	30 (5)	180 (60)	17	30 (5)	60 (20)			
8	18 (3)	180 (60)	18	18 (3)	60 (20)			
9	10 (1)	180 (60)	19	10 (1)	60 (20)			
10	6 (1)	180 (60)	20	6 (1)	60 (20)			

In Fig. 5, some results of the optimization are presented. Obviously, when values of N_{LG} increase *An* has an higher value, while *R* decreases.

The main goal of this case is reached because the design with the maximum stiffness is obtained for the different combinations of some of its parameters. Additionally, a combination for values of N_{LG} and P giving the minimum number of structural analyses has been also studied. It depends of the required reliability, because the computational cost varies proportionally. Table 4 shows the combination with the lower computational cost for each value of *R* above 90%. In this table, *J* is the relationship between *An* and total number of solutions in the design space.

Table 4. The best combinations according to increasing values of the apparent reliability *R*.

R	*Comb*	*P*	N_{LG}	*An*	*J*
92%	23	18	33	394	18%
94%	21	50	33	896	41%
96%	5	6	300	793	36%
98%	18	18	60	567	26%
100%	13	18	108	893	41%

As can be observed, 18 individuals is the ideal size for the population, appearing three times with the lower values of *An* and the best values of *R*. It is also observed that in combination number 18, with *P*=18, the apparent reliability is *R*=98%, the number of structural analyses is not very high (567) and, for this case, N_{LG} =60.

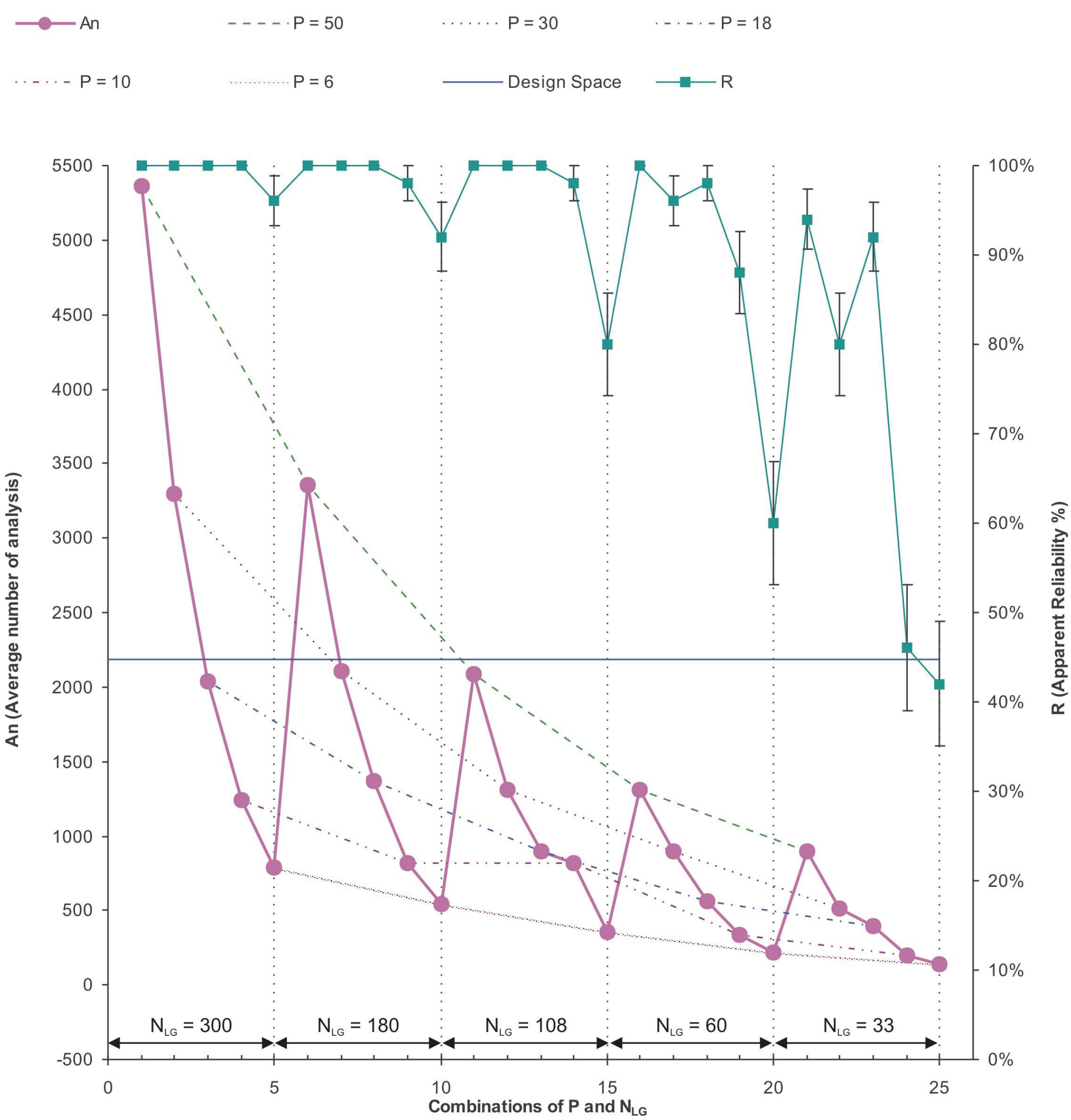


Figure 5. Average value of the number of structural analysis per optimization process and apparent reliability for each combination of the parameters of the GA

4. FINAL REMARKS

The GA was applied successfully to optimize composite laminate structures. In the first example a new objective function was used to find the pareto-optimal set. It was not possible to reach this goal using a conventional objective function. Some difficulties were observed when secondary objectives, such as the maximization of the safety factor, were incorporated, but, in spite of this aspect, the algorithm obtained optimum values for the main objectives (weight and cost). In the second example a cylindrical shell with geometrically non-linear behavior was analyzed. In this problem the influence of different values of P and N_{LG}, taking R and An as parameter, was studied. Using 25 combinations of P and N_{LG} the tendency of R and An was analyzed. Finally, intervals of P and N_{LG} where the GA is more efficient were determined.

5. ACKNOWLEDGEMENT

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