THE FORMATION OF STREAMWISE VORTICES IN A 3D STABLY STRATIFIED TEMPORAL MIXING LAYER BY LARGE-EDDY SIMULATION

Denise Maria Varella Martinez, denisevmartinez@yahoo.com.br
Departamento de Matemática, Fundação Universidade Federal do Rio Grande - FURG. Av. Itália, Km 8 - 962001-900, Rio Grande, RS, Brasil

Edith Beatriz Camano Schettini, bcamano@iph.ufrg.br
Instituto de Pesquisas Hidráulicas, Universidade Federal do Rio Grande do Sul - UFRGS. Av. Bento Gonçalves, 9500 - 91501-970 - Porto Alegre, RS, Brasil

Jorge Hugo Silvestrini, jorgehs@pucrs.br
Departamento de Engenharia Mecânica e Mecatrônica, Pontifícia Universidade Católica do Rio Grande do Sul -PUCRS Av. Ipiranga , 6681- Porto Alegre - RS - 90619-900, Brasil

Abstract. *The processes that governs the formation of the streamwise vortices in a mixing layer are still incompletely understood, particularly with regards to effects of stratification. With the objective to contribute on this subject, the present study investigates the behaviour of a 3D stably stratified temporal mixing layer using Large-Eddy Simulation (LES), with the aid of the Filtered Structure Function (FSF) subgrid-scale model. The development of streamwise vortices and its interactions with the secondary Kelvin-Helmholtz (KH) structures are focused. Typical Richardson numbers (Ri) ranging from 0 to 0.2 are considered while the Reynolds number (Re) is kept constant and equal to 2000. The used numerical code solves the complete Navier-Stokes equations in the Boussinesq approximation using LES, for an incompressible fluid in a cubic domain. A sixth-order compact finite difference scheme is used to compute the spatial derivatives, while the integration in time is performed with a third-order low-storage Runge-Kutta method. Initial conditions for the streamwise velocity and density profile are imposed based on typical error functions. A small white noise is used as initial condition for the three components of velocity fluctuations. The secondary KH instability is identified, using LES, for $Re = 2000$ and $Ri \geq 0.05$ when there is a pairing of the simulated vortices. This fact is not found in the current literature for a stably stratified mixing layer. The numerical results show the formation of streamwise vortices for all Richardson numbers simulated. It is verified that the secondary KH instability affects the formation of the streamwise vortices and apparently speed up the transition to turbulence of a strongly stratified mixing layer. The numerical results show a reasonable agreement with experimental literature.*

Keywords:*Streamwise vortices, stably stratified, Large-Eddy Simulation, Kelvin-Helmholtz, secondary KH instability.*

1. INTRODUCTION

The dynamics of a stably stratified mixing layer is a problem of considerable interest in fluid dynamics with applications in both geophysical sciences and engineering. In a stratified mixing layer the transition to turbulence is controlled by competition between the buoyancy and inertial forces. This competition modifies the dynamics of the stably stratified mixing layer and alters the kind of instability that can be developed.

In stratified mixing layers the $3D$ process is more complex than in unstratified mixing layers. This fact is due to the greater number of secondary instabilities that propagate in the flow. The instabilities that may be developed in a $3D$ stably stratified mixing layer could be divided in two groups: one that grows within the vortex core and the other that develops in the region between the cores of the KH vortices. Within the cores two types of instabilities are found: the translative instability, discovered by Pierrehumbert and Widnalli (1982), that does not depend on buoyancy effects and the gravitational convective instability that is driven by buoyancy effects (Schowalter et al., 1994).

The gravitational convective instability is found within unstable regions of the KH core, which consist of heavy and light fluid wrapped in a spiral roll. This makes unstable the sub layers of density generated during the roll-up of Kelvin-Helmholtz vortices. Simultaneously, the KH vortices are unstable to different $3D$ disturbances, where the most unstable mode is characterized by a spanwise oscillation in phase with primary vortices of KH. This instability, called translative, is well known in the literature as being the responsable for the beginning of the three-dimensionality in the unstratified mixing layer and, consequentely, to the formation of streamwise vortices.

The instability that grows in the region between two KH vortices, known as secondary shear instability, was predicted theoretically by Klassen and Peltier (1991), and verified in laboratory experiments by Schowalter et al. (1994). This instability occurs due a streamwise density gradient ($\partial\rho/\partial x$), that correspond to the spanwise component of the baroclinic torque in the Boussinesq approximation. This streamwise density gradient feeds the region between the KH vortices with vorticity and forms thin vorticity layers there, defined as a baroclinic layer by Staquet (1995).

Depending on the stratification degree of the flow (characterized by Richardson number, Ri) and on the imposed initial conditions two different secondary instabilities are eventually found to develop upon the baroclinic layer. One instability originates near in the core regions of the KH vortex, called near-core instability, and propagates towards the baroclinic layer, and another one called secondary Kelvin-Helmholtz instability, that amplifies in the baroclinic layer and forms small secondary vortices of the KH type. The near-core instability, discovered by Staquet (1995), would be located close to the place where the secondary convective instability develops.

The secondary instabilities, above cited, generate $3D$ disturbances that form the streamwise vortices. These vortices are developed after the saturation of the primary KH vortices. The streamwise vortices extend over/under the core of the main vortex forming the upward/downward classical mushroom-like vortices. The processes that governs the formation of the streamwise vortices in a mixing layer are still incompletely understood, particularly with regard to the effects of stratification.

In Martinez et al. $(2005(b))$ and Martinez et al. $(2006(a))$, were investigated, using Direct Numerical Simulation (DNS), the influence of the stable stratification, the spanwise size of the domain and the initial conditions on the development of the streamwise vortices of a stably stratified mixing layer, for $Re = 200$. In these works it was verified that the formation of the streamwise vortices changes due to stratification, the type of the imposed condition and the transversal lenght of the domain.

The DNS technique is excellent to investigate the instabilities that occur in a stably stratified mixing layer. But due to the limited memory capacity of the computers it is only possible to simulate moderated Reynolds number. A way for solving the complete Navier-Stokes equations with the approach of Boussinesq at high Re can be carried out through the Large-Eddy Simulation (LES) technique. In LES of turbulence the flow quantities are decomposed into a large-scale contribution and a small-scale contribution by a spatial filter. The large-scale contributions are explicitly calculated, whereas only the effects of the small-scale contibutions on the large-scale flow are described by a so-called subgrid model.

The present work investigates the transition to turbulence in a $3D$ stably stratified temporal mixing layer, using Large-Eddy Simulation (LES), with the aid of the Filtered Structure Function (FSF) subgrid-scale model. The main objective of this work is to analyze the influence of a stable stratification in the formation of streamwise vortices and to verify its interactions with the secondary KH structures.

2. Governing Equations in the Boussinesq approximation using LES

In LES the large scale motions in the flow are solved, whereas the effect of the small scale motions is modelled by a so-called subgrid model. LES requires less computational effort or can simulated flows at higher Reynolds number than Direct Numerical Simulation (DNS), which attempts to solve all scales present in the flow.

The variables that appears in the equations are separate in a part called large scales or filtered variable, $\overline{f}(x_i,t)$, and in another part called small scales or subgrid scales, $f'(x_i,t)$. Then, with the objective to separate the great ones of the small scales a filter cutoff length Δx is used. The purpose of the formalism LES is to consider a spatial filter $G_{\Delta x}$ of width Δx, which filters out the subgrid scales of wavelength $< \Delta x$. The filtered field is defined as (Lesieur, 1997):

$$\overline{f}(x_i,t) = \int f(y_i,t) G_{\Delta x}(x_i - y_i)\, dy_i. \tag{1}$$

The filter $G_{\Delta x}$ is applied to equations the Navier-Stokes within the Boussinesq approximation for a density-stratified fluid (Lesieur et al., 2005), and using the concept the eddy-viscosity (Boussinesq's hypothesis) to determine the subgrid-scale tensor and the deformation tensor resultant of the filtering process. Thus, the complete Navier-Stokes equations in the Boussinesq approximation using LES are:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial(\overline{u}_i\overline{u}_j)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial \overline{P}^*}{\partial x_i} + \frac{\partial}{\partial x_j}\left((\nu+\nu_t)\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right) + g_i\delta_{i3}\frac{\overline{\rho}}{\rho_0}, \tag{2}$$

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0, \tag{3}$$

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial(\overline{\rho}\,\overline{u}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left((\kappa+\kappa_t)\frac{\partial \overline{\rho}}{\partial x_j}\right), \tag{4}$$

where ν is the kinematic viscosity coefficient, ν_t is eddy-viscosity, κ the molecular diffusivity coefficient, $\kappa_t = \nu_t/Pr_t$ is eddy-diffusivity, Pr_t is the turbulent Prandtl number, considered constant and equal 1, ρ is density or active scalar, u_i is the velocity field, P^* is the modified pressure field.

The eddy-viscosity is calculated with the aid of the Filtered Structure Function (FSF) subgrid-scale model, proposed by Ducros et al. (1996).

2.1 Subgrid-scale model

The modeling of the small scales through an appropriate subgrid scale is extremely complicated, mainly when one uses methods of finite differences in the physical space. The Filtered Structure Function (FSF) model satisfies a correct prediction of the resolved vorticity field when used jointly with pseudo-spectral methods. Model FSF combines the advantages of low computational cost without the problems of excessive dissipation thus making it possible to study transitional flows (Da Silva et al., 2004).

The FSF model is defined by applying a high-pass filter to the velocity field, before calculating the local second-order structure function. The high-pass filter is a Laplacian, discretized by second-order centered finite differences and iterated three times. The Laplacian filter is of the form:

$$L(\overline{u_i}) = \sum_j [\overline{u_i}(\vec{x} - \Delta\vec{x_j}, t) - 2\overline{u_i}(\vec{x}, t) + \overline{u_i}(\vec{x_j} + \Delta\vec{x_j}, t)] \quad (5)$$

where $j = 1, 2, 3$ for free flows.

The final formulation obtained by Ducros et al. (1996), is:

$$\nu_t^{FSF}(\vec{x}, t) = 0.0014 C_k^{-3/2} \Delta x [\overline{F_2}(\vec{x}, \Delta x)]^{1/2}. \quad (6)$$

where

$$\overline{F_2}(\vec{x}, \Delta x) = \langle [L^3(\overline{u_i})(\vec{x} + \vec{r}, t) - (L^3\overline{u_i})(\vec{x}, t)]^2 \rangle_{\|\vec{r}\| = \Delta x}. \quad (7)$$

is the local second-order velocity structure function of the filtered field, which is calculated with a local statistical average of square velocity differences between $\vec{x}$ and the six closet points surrounding $\vec{x}$ on the computataional grid.

3. Numerical Method and Initial Conditions

The temporal mixing layer with periodic conditions in the streamwise (x) and spanwise (y) directions and free-slip boundary condition in the vertical (z) direction is considered.

There are two dimensionless relevant parameters: the Reynolds number $Re = U\delta_i/\nu$ (based on the half velocity difference across the shear layer and on the initial vorticity thickness, defined by $\delta_i = 2U/(du/dz)_{max}$), and the Richardson number $Ri = g\Delta\rho R\delta_i/\rho_0 U^2$ (where $\Delta\rho R$ is density scale and R is the ratio of initial vorticity thickness and the density thickness).

The time is made dimensionless using the advective scale δ_i/U. We choose the units of length, velocity and density such that $\delta_i = 1, U = 1$ and $\Delta\rho = 1/R$. In this manner, $Re = 1/\nu$ and $Ri = g/\rho_0$. The initial conditions are defined in terms of the velocity and density fields as:

$$u(z, t = 0) = U erf\left(\frac{\sqrt{\pi}z}{\delta_i}\right) \quad (8)$$

$$\rho(z, t = 0) = -\frac{1}{R} erf\left(\frac{\sqrt{\pi}Rz}{\delta_i}\right). \quad (9)$$

In the present case, no density fluctuation is superposed upon $\rho(z)$ at $t = 0$. A small white noise is used as initial condition for the three components of velocity fluctuations. The computational domain is taken equal to $L_x = L_y = L_z = 7N\delta_i$ in order to obtain N vortices in the streamwise direction.

Equations (2-4) are solved numerically using a sixth-order compact finite difference scheme (Lele, 1992) to compute the spatial derivatives, while the integration in time is performed with a third-order low-storage Runge-Kutta method (Williamson, 1980). The incompressibility condition, Eq.(3), is ensured with a fractional step method via resolution of the Poisson equation for the pressure (more details about the numerical code can be found in Lardeau et al., 2002, and Silvestrini et al., 2002).

4. Results

The computational parameters for the simulations are given in the Tab.1. A white noise of equal amplitude in the three directions (x, y, z) is added to the basic velocity profile (Eq.8).

In the present work the results of the simulation $3D4V$ are analyzed, with the objective to verify if the flow approaches of the experimental results obtained by Bell and Mehta (1990). Numerical tests were done for $Ri = 0$ and $Re = 2000$ in a cubic domain with extension of four times the fundamental streamwise wavelength ($\lambda_a = 7$) and computational grid of $n_x, n_y, n_z = 192 \times 192 \times 193$ points.

Figure 1 shown the comparison of the Reynolds stress (simulation $3D4V$) with the experimental results of Bell and Mehta. In the numerical results, the vertical coordinate is normalized with the thickness of local vorticity corresponding

Table 1. Physical and numerical parameters -$Re = 2000$

Simulation	Ri	Domain $L_x \times L_y \times L_z$	Grid $n_x \times n_y \times n_z$
$3D4V$	$Ri = 0$	$28 \times 28 \times 28$	$192 \times 192 \times 193$
$3DI$	$Ri = 0$	$14 \times 14 \times 14$	$128 \times 128 \times 129$
$3DII$	$Ri = 0.05$	$14 \times 14 \times 14$	$128 \times 128 \times 129$
$3DIII$	$Ri = 0.1$	$14 \times 14 \times 14$	$128 \times 128 \times 129$
$3DIV$	$Ri = 0.2$	$14 \times 14 \times 14$	$128 \times 128 \times 129$
$3DI192$	$Ri = 0.05$	$14 \times 14 \times 14$	$192 \times 192 \times 193$
$3DII192$	$Ri = 0.1$	$14 \times 14 \times 14$	$192 \times 192 \times 193$

at time $t = 63.4$. In the figure are ploted the experimental data (in blue), corresponding the measure section $x = 250cm$ of the called case untripped, where the boundary layers for the end of the separatriz plate are still laminar. It is observed that it has an acceptable agreement between the obtained results using LES and the experimental results of Bell and Mehta (1990).

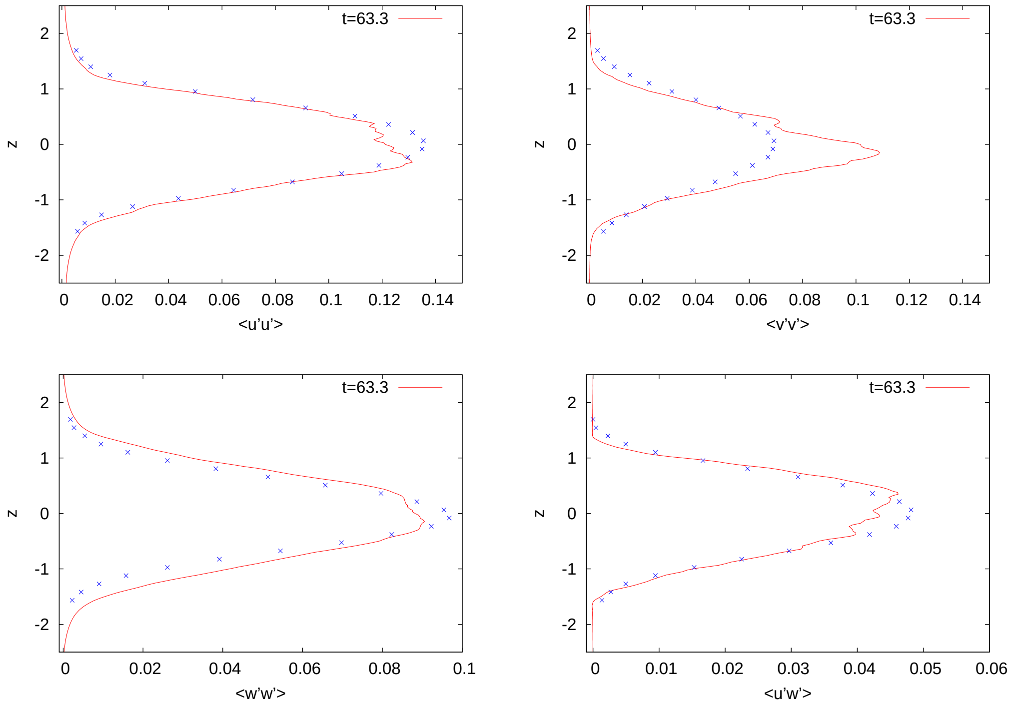


Figure 1. The Reynolds stress. Comparison of the numerical results (red color) with experimental data (blue color).

4.1 Evidence of the secondary Kelvin-Helmholtz instability

In previous studies (Martinez et al., 2004, Martinez et al.,2006(a)), via Direct Numerical Simulation, it was shown the great influence of the stable stratification on the development of the instabilities in a mixing layer. The results showed that high stratification inhibits the pairing process, reduces the buoyancy flux, weakens the vertical motions, decreases the thickness of the mixing layer and affects the formation of streamwise vortices.

The $3D$ process of a stratified mixing layer is associated with the translative instability, the gravitational convective instability and the secondary shear instability. The presence of secondary shear instability in a stratified mixing layer is due to baroclinic vorticity generation, which concentrates the vorticity between the KH vortices, and form the baroclinic layer.

The occurrence of the secondary Kelvin-Helmholtz instability in the baroclinic layer of a stably stratified mixing layer, was demostrated in $2D$ simulations for Reynolds number 500, 1000 and 2000 (Martinez et al., 2005(a); Martinez, 2006; Martinez et al., 2006(b)). Related works show the formation of small vortices of the KH type in the baroclinic layer.

These vortices are similar, in appearance and dynamics, to the primary vortices of KH. In our simulation, could be shown that the secondary KH vortices changes the dynamics of the flow, apparently speed up the transition to turbulence of a strongly stratified mixing layer and modifies the formation of streamwise vortices.

The numerical results (Martinez et al. $2005(a)$) show the development of a jet in the baroclinic layer adjacent to vorticity layers of opposite signs. These layers are created baroclinically by convective motions inside the primary KH vortex due to the formation of a negative vorticity layer generated between two co-rotating positive vortices. The negative vorticity layer unstables the baroclinic layer and forms small vortices of the KH type. The production of negative vorticity in the vortex is rapidly followed by the growth of the secondary KH instability in the baroclinic layer; moreover, this instability does not developed if the negative vorticity is too low compared to the positive one (this instability is referred to as the near-core instability). The intensity of the negative vorticity layer depends on the Richardson and Reynolds numbers and defines occurrence or not of secondary KH structures.

Figure 2 shows streamwise cross-sectional plots of spanwise vorticity at $y = 6.5$ (above) and Q-criterium isosurfaces (below), for simulations $3DI192$ and $3DII192$, respectively .

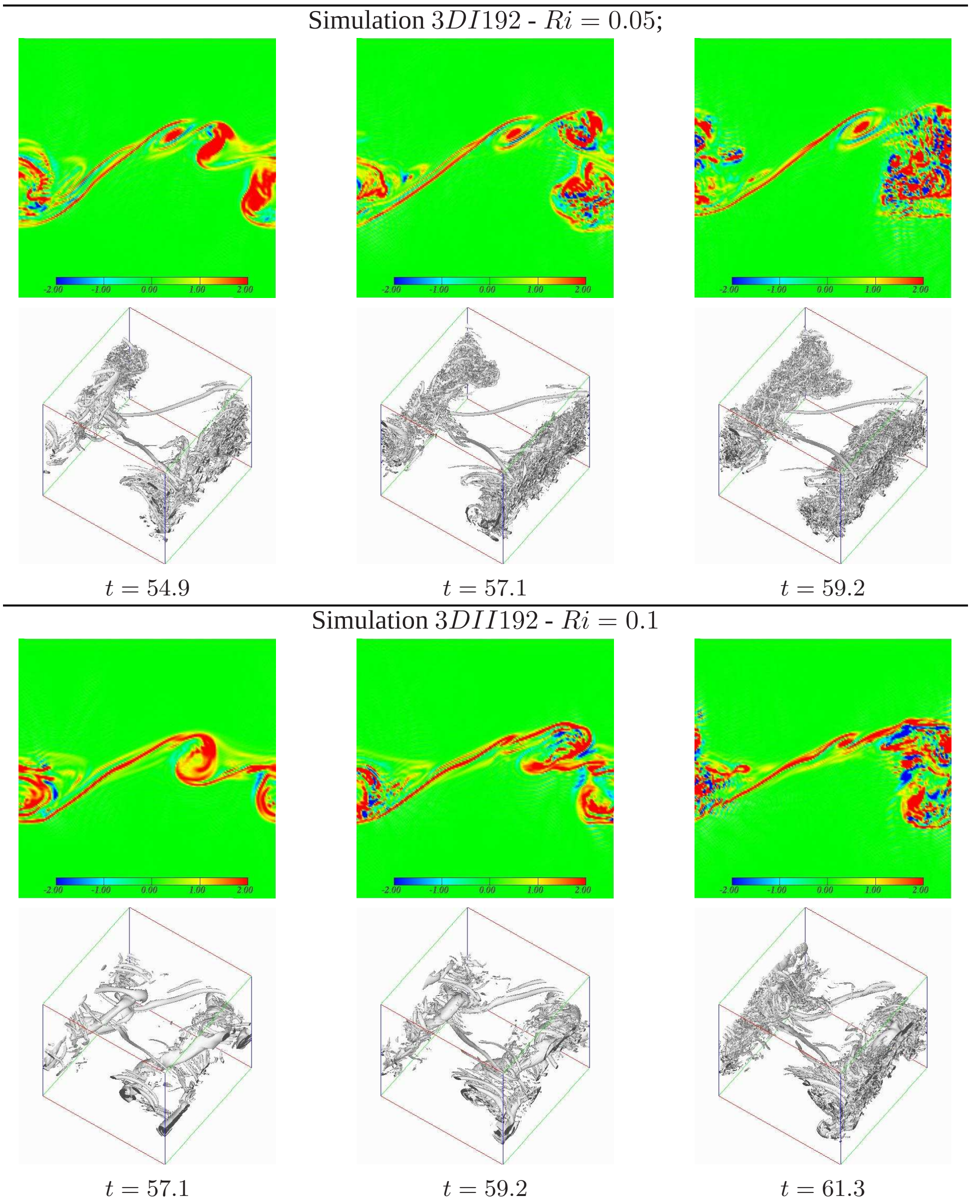


Figure 2. Evidence of the secondary KH instability in the baroclinic layer.

Figure 2 shows that the secondary KH instability appears with lesser intensity for $Ri = 0.05$ that for $Ri = 0.1$. The secondary KH instability does not develop as in the case of strong stratification, possibly because the vorticity of the baroclinic layer and the adjacent layer of negative vorticity are too weak to allow the development of this instability.

In the current simulations is shown that the stratification modifies the behavior of the flow and points evidences of secondary KH instability. The presence of the secondary KH instability clearly is verified in pictures for $Ri = 0.05$ and $Ri = 0.1$.

The secondary vortices of KH appear in flow after the pairing process of the primary vortices of the KH, as in the DNS (Martinez et al., $2006(b)$). However, in the present work, the secondary KH vortices developed in the flow afther streamwise vortices were already formed. This fact seems to indicate that initial condition, Richardson number and Reynolds number are important factors for the development of the secondary KH vortices in the stably stratifed mixing layer.

4.2 Influence of the Richardson number

The Figure 3 shows isosurfaces of criterium Q for simulation in a computational grid of $n_x, n_y, n_z = 128 \times 128 \times 129$ points for $Re = 2000$ and $Ri = 0$,0.05, 0.1 and 0.2, until the instant where the small scales dominate the dynamics of the flow.

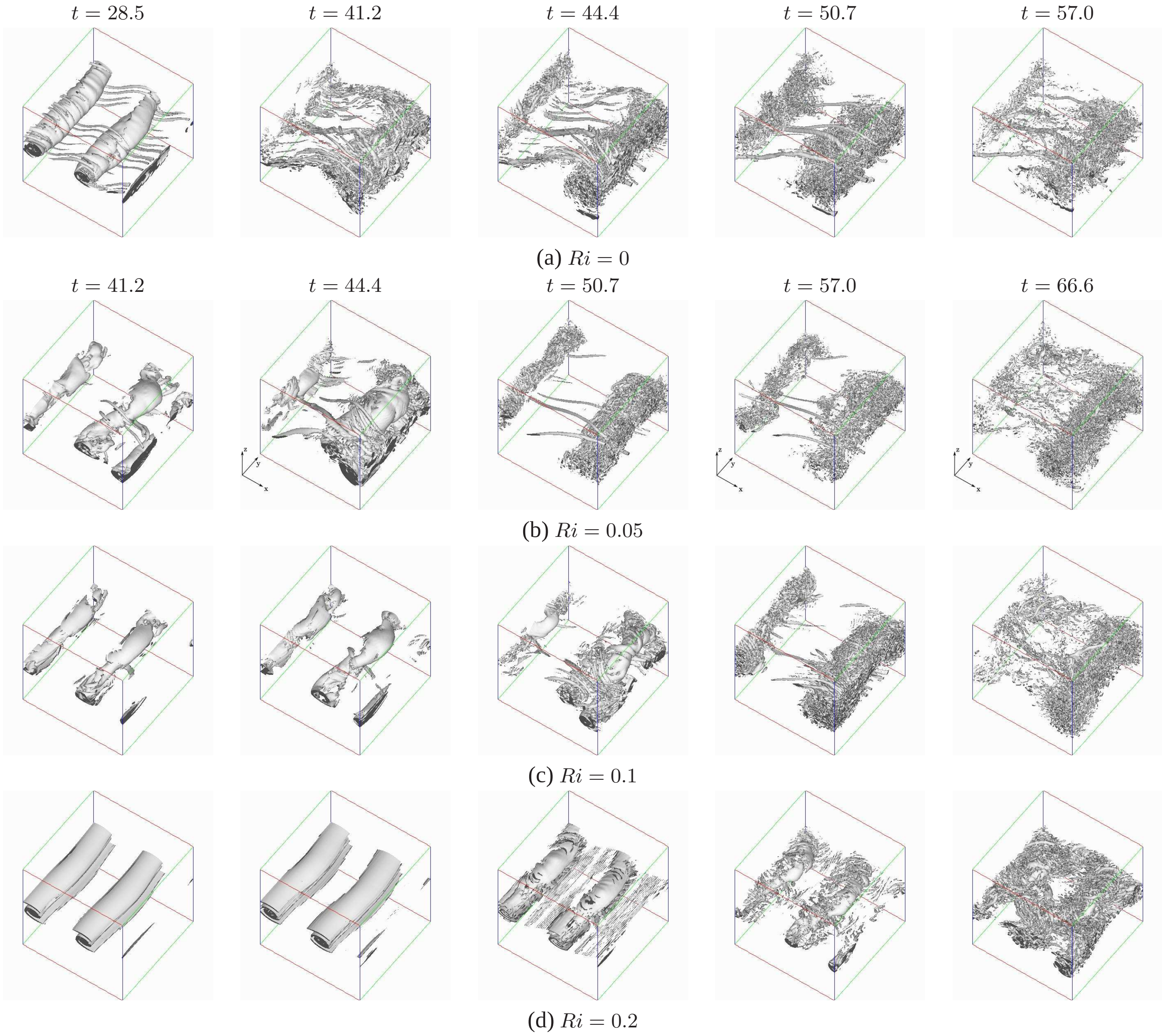
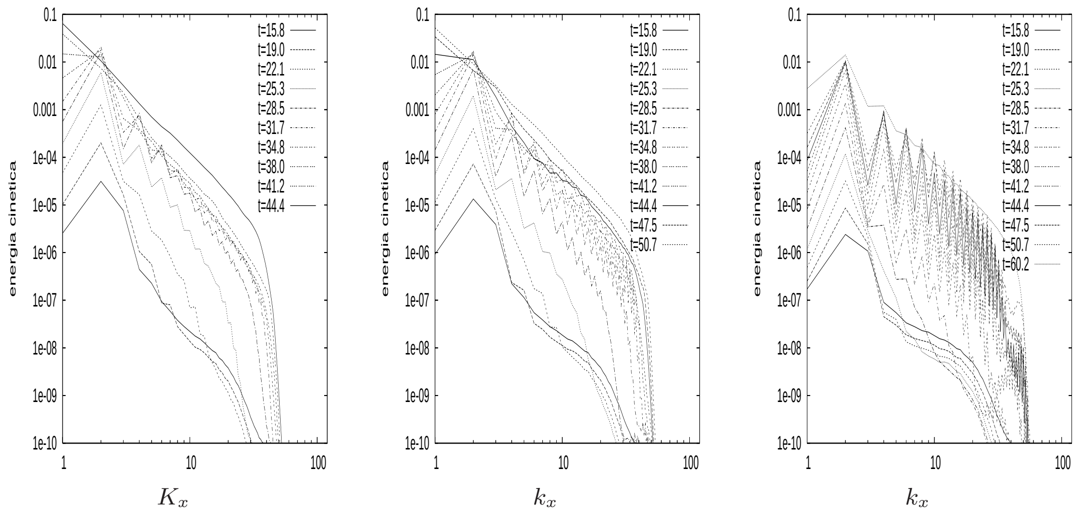


Figure 3. Isosurfaces of criterium Q. Simulation: (a) $3DI$; (b) $3DII$; (c) $3DIII$ and (d) $3DIV$; $Re = 2000$.

In stratififed case pairs of counter-rotating streamwise vortices arise ($t = 28.5$). The streamwise vortices show a high degree of coherence and extend nearly the same vortical intensity over the complete braid region. The picture of the isosurfaces Q for $Ri = 0.05$ and $Ri = 0.1$ ($t = 44.4$) shows the translative instability acting on the resultant vortex of the pairing process and the development of streamwise vortices, while for $Ri = 0.2$ the flow is still two-dimensional. The stratification delays de pairing process and the development of translative instability.

In the Fig.3, it is observed, that in stratified cases ($Ri = 0.05$, $Ri = 0.1$ and $Ri = 0.2$) the secondary shear instability dominates the three-dimensionalization of the flow and the streamwise vortices are seen to be less developed over the complete domain. A strong stable stratification has a stabilizing effect on the growth of the primary KH instability and delays de pairing process. This results in a reduction of the gravitational convective and translative instability. This fact modifies the formation of the streamwise vortices, because the vorticity is more amplified in the region betweem the KH vortices (braid) than within the core. This is due to the streamwise density gradient which decreases the levels of vorticity in the KH vortices while increases in the braid region.

The precise numerical treatment of the turbulence requires that the entire band of scales wich ranges from the energy-carrying to the dissipative motions is resolved in time and space. Figure 4 shows the spectrum of kinetic energy in function of the streamwise wavenumber at different times for $Ri = 0.05$, $Ri = 0.1$ and $Ri = 0.2$ and $Re = 2000$. It is observed that the nonlinear interactions between modes distribute the energy through of the width of spectrum and induce a cascada of energy in direction to small scales. Figure 4 shows that to the measure that the Richardson number increases the restriction of the amplitude of the vertical motions attenuates the energy transference and reduces the intensity of the turbulence. In all cases, Fig.4, the energy spectrum takes its maximum within the low-wavenumber regime, while it shows behavior turbulent in high-wavenumber. This spectrum suggest a mechanism of transference of energy towards dissipative scales. The spectrum reaches a law $E(k_x) \sim k_x^{-3}$, followed of a zone of exponential decline.



(a) $Ri = 0.05$ (b) $Ri = 0.1$ (c) $Ri = 0.2$

Figure 4. Spectrum of kinetic energy in function of the stremwise wave number at different times for simulations:(a) $3DII$; (b) $3DIII$ and (c) $3DIV$.

5. Conclusions

The purpose of the present study was to investigate the influence of the stable stratification in the formation of streamwise vortices in a $3D$ stably stratified mixing layer, for $Re = 2000$, using Large-Eddy Simulation (LES), with the aid of the Filtered Structure Function (FSF) subgrid-scale model. The numerical results showed that the secondary KH vortices appears either weak stratification ($Ri = 0.05$) as for strong stratification ($Ri = 0.1$). The presence of secondary KH vortices modifies the behavior of the flow and the formation of the streamwise vortices.

The secondary vortices of KH appeared in flow after the pairing process of the primary vortices of the KH, as in the DNS (Martinez et al., $2006(b)$). However, in the present work, the secondary KH vortices developed in the flow afther streamwise vortices were already formed. This fact seems to indicate that initial condition, Richardson number and Reynolds number are important factors for the development of the secondary KH vortices in the stably stratifed mixing layer. In the literature was not found evidences of secondary KH vortices in a baroclinic layer of the $3D$ stably stratified temporal mixing layer, simulated through LES.

The high stratification delays the primary Kelvin-Helmholtz instability and pairing process of the KH vortices. This results in a reduction of the gravitational convective and translative instability, and consequently, on the formation of the streamwise vortices. This occur due to the streamwise density gradient which, in stratified cases, decreases the levels of vorticity in the KH vortices while increases in the braid region. The numerical results, for unstratified case, showed a goal agreement with the experimental results of Bell and Metha, 1990.

We can conlude, based on the simulations presented in this work, that the subgrid-scale model used, the Function Filtered Structure, was apt to solve small the scales of the flow without amplifying the effect caused for the stratification.

6. ACKNOWLEDGEMENTS

The authors wish to thank the CAPES-PICDT and CNPQ for sponsoring this project.

7. REFERENCES

Bell, J. H. and Metha, R. D., 1990, "Development of a Two-Stream Mixing Layer from Tripped and Untripped Boundary Layers", AIAA Journal, vol.28, pp.2034-2042.

Ducros, F., Comte, P. and Lesieur, M., 1996, "Large-eddy simulation of transition to turbulence in a boundary layer developing spatially over a flat plate", J. Fluid Mech., vol.326, pp.1-36.

Da Silva, C. B., Pereira, J. C. F., 2004, "The effect of subgrid-scale models on the vortices computed from large-eddy simulations", Phys. of Fluids, vol.16, pp. 4506-4534.

Lardeau, S., Lamballais, E. and Bonnet, J. P., 2002, "Direct Numerical Simulations of a Jet Controlled by Fluid Injection", j. Turbulence, vol.3, pp.1-25.

Lele, S. K., 1992, "Compact finite difference schemes with spectral-like resolution", J. Comp. Phys., vol.103, pp.16-42.

Lesieur, M., 1997,"Turbulence in Fluids", Kluwer Academic Publishers, Netherlands, pp.515.

Lesieur, M., Métais, O. and Comte, P., 2005, "Large-Eddy Simulations of Turbulence", Cambridge University Press, New York, USA, pp.250.

Klaassen, G. P. and Peltier, W. R., 1991, "The influence of stratification on secondary instability in free shear layers",J. Fluid Mech., vol.227, pp.71-106.

Martinez, D. M. V., Schettini, E. B. C. and Silvestrini, J. H., 2004, "Transition to turbulence in a stable stratified temporal mixing layer through Direct Numerical Simulation", Proceedings of the 10th Brazilian Congress of Thermal Sciences and Engineering", ABCM, Rio de Janeiro, Brasil.

Martinez(a), D. M. V., Schettini, E. B. C. and Silvestrini, J. H., 2005(a), "Secondary Kelvin-Helmholtz Instability in a Stably Stratified Temporal Mixing Layer", Proceedings of the 18th Congress of Mechanical and Engineering, Ouro Preto, Brazil.

Martinez(b), D. M. V., Schettini, E. B. C., and Silvestrini, J.H., 2005(b), "Numerical investigation on the formation of streamwise vortices in a stably stratified temporal mixing layer", Proceedings of the Whorkshop on Direct and Large-Eddy Simulation-6 -ERCOFTAC, Poitiers, France, Springer, pp.591-598.

Martinez, D. M. V., 2006, "Transição à Turbulência na Camada de Mistura Estavelmente Estratificada utilizando Simulação Numérica Direta e Simulação de Grandes Escalas", PhD. Thesis, Universidade Federal do Rio Grande do Sul, Brasil, pp.1-152.

Martinez, D. M. V., Schettini, E. B. C. and Silvestrini, J. H., 2006(a), "The influence of Stable Stratification on the Transition to turbulence in a Temporal mixing Layer", J. of Braz. Soc. of Mech. Sci.and Eng., vol. XXVIII, pp. 230-240.

Martinez, D. M. V., Schettini, E. B. C. and Silvestrini, J. H., 2006(b), "Secondary Kelvin-Helmholtz instbility in a two and three- dimensional stably stratified temporal mixing layer", Proceedings of the 5th Escola de Primavera de Transição e Turbulência, ABCM, Rio de Janeiro, Brasil.

Martinez, D. M. V., Schettini, E. B. C. and Silvestrini, J. H., 2006(c),"Secondary Kelvin-Helmholtz instbility in a 3D stably stratified temporal mixing layer by Direct Numerical Simulation", Proceedings of the XV Congreso sobre Métodos Numéricos y sus Aplicaciones, Santa Fé, Argentina.

Staquet, C., 1995, "Two-dimensional secondary instabilities in a strongly stratified shear layer", J. Fluid Mech.,vol.296, pp.73-126.

Showalter, D. G., Van Atta, C. W. and Lasheras, J. C., 1994, "A study of streamwise vortex structure in a stratified shear layer", J. Fluid Mech., vol.281, pp.247-291.

Silvestrini, J. H. and Lamballais, E., 2002, "Direct Numerical Simulations of Wakes with virtual Cylinders", Int. J. Comp. Fluid Dyn., vol.16, pp.305-314.

Pierrehumbert, R. T. and Widnall, S. E.,1982, "The two and three-dimensional instabilities of a spatially periodic shear flows", J. Fluid Mech., vol.114, pp.59-82.

Williamson, J. H., 1980, "Low-storage Runge-Kutta schemes", J. Comp. Phys., vol.35, pp.48.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.