UNIFIED FORMULATION FOR HEAT AND MASS TRANSFER IN ROTARY REGENERATORS

L. A. Sphaier, ***lasphaier@mec.uff.br*** **or** ***lasphaier@vm.uff.br***
Laboratório de Mecânica Teórica e Aplicada, Departamento de Engenharia Mecânica, Universidade Federal Fluminense, Rua Passo da Pátria 156, bloco E, sala 216, Niterói, RJ, 24210-240, Brazil.

Abstract. *Heat and mass regenerators have been employed for some time in different applications such as dehumidification, desiccant air-conditioning systems, and energy recovery. For the sake of better understanding the transport mechanisms involved in these exchangers, unified formulation for coupled heat and mass transfer in the presence of physical adsorption occurring within rotary regenerators is presented. The proposed mathematical model is capable of simulating the transport phenomena that take place within desiccant and enthalpy wheels, as well as in stationary devices. A consistent normalization scheme based on physically meaningful parameters is provided. The formulation is properly validated and results of a test-case problem while varying some of the dimensionless parameters is presented.*

Keywords: *Rotary Regenerator, Desiccant Wheel, Enthalpy Wheel, Physical Adsorption*

Nomenclature

A_s	surface area
c, c_p	specific heats
C	sensible heat capacity rate
C_r^*, C^*	sensible heat capacity ratios
Bi	Biot number
$\mathcal{D}$	mass-diffusivity
D^H	hydraulic diameter
f_s	sorbent mass fraction in matrix
Fi, Fo	Fick and Fourier numbers
h	convective transfer coefficient
i	specific enthalpy
$\tilde{\imath}$	specific enthalpy on dry basis
i_{vap}	latent heat of vaporization
i_{sor}	heat of sorption
$i_{v,\Delta T}$	interface heat of sorbate transfer
k	thermal conductivity
L	regenerator length
Le	Lewis number
m, $\dot{m}$	mass and mass flow rate
N	number of revolutions
$\mathrm{N_{tu}}$	number of transfer units
ΔR_f	sorbent matrix thickness
T	temperature
$u_,$	bulk stream velocity in channels

V	volumetric capacity rate
V^*_r, V^*	volumetric capacity ratios
W	concentration in adsorbed phase
Y	concentration in gas phases
Greek symbols	
α	thermal diffusivity
ϵ_f	porosity of sorbent matrix
ε	effectiveness
ρ	specific mass
ψ_r	psychrometric ratio
τ_I, τ_II	duration of periods in sections I and II
$\tau_{dw,}$	dwell time
τ_G, τ_S	tortuosities
ϕ	relative humidity
Subscripts	
f	porous sorbent felt
s, g	solid and gas-phases in matrix
da	dry air
df	dry sorbent felt
ls	saturated liquid sorbate
vs	saturated sorbate vapor
ref	reference value
$p.s.$	at process stream interface
Superscripts	
h	sensible heat transfer
m	mass transfer
i	enthalpy transfer
$*$	dimensionless quantity
$^-$	time-averaged value
$\star$	dry reference quantity

1. Introduction

A proper understanding of the coupled heat and mass transfer occurring in rotary regenerators is a fundamental factor for designing effective exchangers. Several investigations related to the simulation of heat and mass transfer in regenerators have been conducted. Particularly interesting among applications of regenerators are desiccant dehumidifiers, (Charoensupaya and Worek, 1988a,b; Gao et al., 2005; Jurinak and Mitchell, 1984; Majumdar, 1998; Majumdar and Worek, 1989; Mathiprakasam and Lavan, 1980; Mei and Lavan, 1983; Nia et al., 2006; Roy and Gidaspow, 1974; Zheng and Worek, 1993), and enthalpy exchangers, (Klein et al., 1990; Simonson and Besant, 1997, 1999; Stiesch et al., 1995). Although considerable research has been individually devoted to either desiccant and enthalpy wheels, unified formulations valid for both types of exchangers were only available as one-dimensional forms (Banks, 1985; Holmberg, 1979; Maclaine-Cross and Banks, 1972; Van den Bulck et al., 1985), which do not take into account diffusional effects in the adsorbent material. An attempt to unify the formulations for rotary regenerators considering diffusional effects was presented by Niu and Zhang (2002a,b); however, among other limitations, the adopted normalization scheme was rather inadequate, as dimensionless parameters presenting little physical significance were employed. In fact, this is a problem that can be seen in various other investigations. More recently, a complete unified formulation including gas-side and solid-side resistances to both heat and mass transfer as local diffusional processes, as well as a proper normalization scheme was presented (Sphaier and Worek, 2004). Although a coherent normalization scheme was employed, some improvements to the mathematical model, especially regarding the dimensionless analysis were afterwards proposed. Hence, the purpose of the current contribution is to communicate the improved mathematical model and normalization scheme, which in addition is presented using a more concise notation. Besides the amended formulation, validation results and previously unpublished test-case results are presented.

2. Mathematical modeling of transport problem

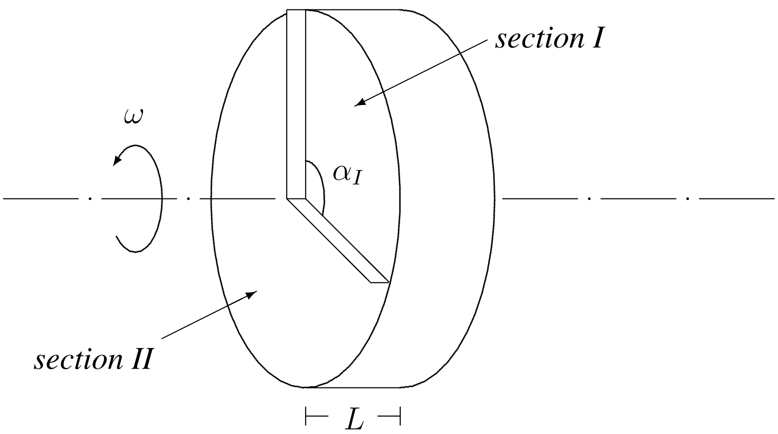

Figure 1. Rotary regenerator.

Figure 1 illustrates a typical rotary regenerative exchanger. The rotor is cyclically exposed to two process streams. These streams are separately fed to the regenerator through two ducts, thereby dividing the regenerator into two sections. The rotor is composed of numerous channels, parallel to the rotation axis, with relatively small cross-sectional areas. Because of the fixed sectioning, at every instant a fixed fraction of the channels is subjected to inlet-stream I, while inlet-stream II is delivered to the remaining fraction. Consequently, a flow-channel is said to be operating either in *period I* or *period II*, according to its current location. Regardless of the operating period, the fluid in channels will be simply referred to as *process stream*.

Despite the rotation, the channels are analyzed as stationary, by choosing a proper reference coordinate system, fixed to a representative channel. The walls of each channel are composed of a porous sorbent felt as displayed in Fig. 2. Due to the actual geometry of these channels, an exact, three-dimensional, representation of the system would be too complex, requiring a prohibitive computational effort for obtaining excessively detailed information. Then again, a one-dimensional description is overly simplified, since it cannot include accurate information about the heat and mass diffusion occurring within the porous material. A balance between these two extremes leads to a formulation having a feasible computational solution time, yet without compromising the accuracy of the description. In this formulation the cross-sectional diffusional effects in felt are considered in only one direction — along the porous material's thickness.

The porous material is modeled as a homogeneous medium composed of a solid portion and pores, in which both gas and adsorbed liquid phases coexist. In order to facilitate the analysis, a subscripting scheme indicating the considered phase is employed. Starting with the solid portion of the material, the subscript s refers to quantities in this solid phase. Within pores, g is employed. For quantities within the porous material with no required phase distinction, a simple f subscript is used. In the process stream, a single-phase is present, and for the sake of simplicity, no subscript is employed

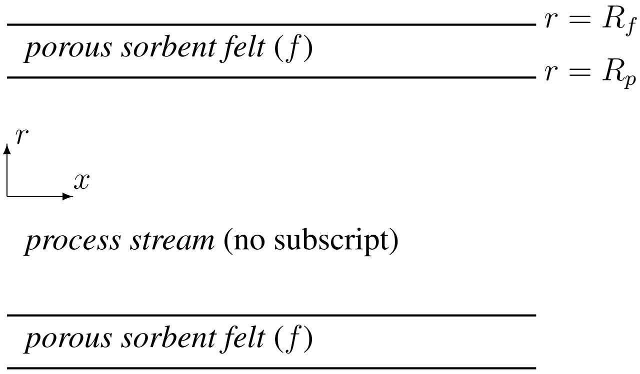

Figure 2. Single flow-channel structure.

for this phase. Besides this subscripting scheme, the sorbate concentrations are described in terms of variables employing mass ratios on dry-bases: Y (mass of sorbate vapor to that of the remaining components in the gas-mixture) and W (mass of liquid sorbate in the adsorbed phase to that of dry sorbent). Furthermore, to allow the porous felt to be composed of both adsorbent and inert materials, a quantity representing the mass fraction of actual sorbent in the porous material is defined: f_s.

2.1 Dimensionless parameters

Regenerative exchanger dimensionless groups for are defined following (Shah, 1981), and some of the parameters obtained are similar those in (Simonson and Besant, 1999). Nevertheless, additional parameters relevant to heat and mass diffusion are introduced due to the presence of these phenomena in the felt. Classical dimensionless parameters present in heat and mass transfer analyses are defined as follows:

$$\mathrm{Bi}_f^{\mathsf{h}} = \frac{h^{\mathsf{h}}\,\Delta R_f}{k_f^\star}, \qquad \mathrm{Bi}_g^{\mathsf{m}} = \frac{h^{\mathsf{m}}\,\Delta R_f}{\mathcal{D}_g^\star}, \qquad \mathrm{Bi}_s^{\mathsf{m}} = \frac{h^{\mathsf{m}}\,\Delta R_f}{\mathcal{D}_s^\star}, \tag{1}$$

$$\mathrm{Fo}_f = \frac{\alpha_f^\star\,\tau}{\Delta R_f^2}, \qquad \mathrm{Fi}_g = \frac{\mathcal{D}_g^\star\,\tau}{\Delta R_f^2}, \qquad \mathrm{Fi}_s = \frac{\mathcal{D}_s^\star\,\tau}{\Delta R_f^2}, \tag{2}$$

$$\mathrm{Le} = \frac{\alpha^\star}{\mathcal{D}^\star}, \qquad \mathrm{Le}_g = \frac{\alpha_f^\star}{\mathcal{D}_g^\star}, \qquad \mathrm{Le}_s = \frac{\alpha_f^\star}{\mathcal{D}_s^\star}, \tag{3}$$

$$\mathrm{Nu} = \frac{h^{\mathsf{h}} D^H}{k^\star}, \qquad \mathrm{Sh} = \frac{h^{\mathsf{m}} D^H}{\mathcal{D}^\star}, \qquad \psi_r = \frac{h^{\mathsf{h}}}{h^{\mathsf{m}}\rho^\star c_p^\star} = \mathrm{Le}\frac{\mathrm{Nu}}{\mathrm{Sh}} = \frac{\mathrm{N}_{\mathsf{tu}}^{\mathsf{h}}}{\mathrm{N}_{\mathsf{tu}}^{\mathsf{m}}} = \frac{\mathrm{N}_{\mathrm{tu,o}}^{\mathsf{h}}}{\mathrm{N}_{\mathrm{tu,o}}^{\mathsf{m}}}. \tag{4}$$

where τ is the period of one process, *i.e.* $\tau = \tau_{\mathrm{I}}$ or $\tau = \tau_{\mathrm{II}}$ according to the regenerator section.

The dimensionless parameters related to regenerative exchange are defined as:

$$\mathrm{N}_{\mathrm{tu,I}}^{\mathsf{h}} = \frac{(h^{\mathsf{h}}A_s)|_{\mathrm{I}}}{\mathrm{C_I}}, \qquad \mathrm{N}_{\mathrm{tu,II}}^{\mathsf{h}} = \frac{(h^{\mathsf{h}}A_s)|_{\mathrm{II}}}{\mathrm{C_{II}}}, \qquad \mathrm{N}_{\mathrm{tu,o}}^{\mathsf{h}} = \frac{1}{\mathrm{C_{min}}}\left[\frac{1}{(h^{\mathsf{h}}A_s)|_{\mathrm{I}}} + \frac{1}{(h^{\mathsf{h}}A_s)|_{\mathrm{II}}}\right]^{-1}, \tag{5}$$

$$\mathrm{N}_{\mathrm{tu,I}}^{\mathsf{m}} = \frac{(h^{\mathsf{m}}A_s)|_{\mathrm{I}}}{\mathrm{V_I}}, \qquad \mathrm{N}_{\mathrm{tu,II}}^{\mathsf{m}} = \frac{(h^{\mathsf{m}}A_s)|_{\mathrm{II}}}{\mathrm{V_{II}}}, \qquad \mathrm{N}_{\mathrm{tu,o}}^{\mathsf{m}} = \frac{1}{\mathrm{V_{min}}}\left[\frac{1}{(h^{\mathsf{m}}A_s)|_{\mathrm{I}}} + \frac{1}{(h^{\mathsf{m}}A_s)|_{\mathrm{II}}}\right]^{-1}. \tag{6}$$

$$\mathrm{C}^* = \frac{\mathrm{C_{min}}}{\mathrm{C_{max}}} = \frac{\mathrm{V_{min}}}{\mathrm{V_{max}}} = \mathrm{V}^*, \qquad \mathrm{C}_{\mathrm{r}}^* = \frac{\mathrm{C}_{\mathrm{r},f}}{\mathrm{C_{min}}}, \qquad \mathrm{V}_{\mathrm{r}}^* = \frac{\mathrm{V}_{\mathrm{r},f}}{\mathrm{V_{min}}}, \tag{7}$$

$$(h^{\mathsf{h}}A_s)^* \;=\; \frac{(h^{\mathsf{h}}A_s)\text{ on the }\mathrm{C}_{\min}\text{ side}}{(h^{\mathsf{h}}A_s)\text{ on the }\mathrm{C}_{\max}\text{ side}}, \qquad (h^{\mathsf{m}}A_s)^* \;=\; \frac{(h^{\mathsf{m}}A_s)\text{ on the }\mathrm{V}_{\min}\text{ side}}{(h^{\mathsf{m}}A_s)\text{ on the }\mathrm{V}_{\max}\text{ side}}. \tag{8}$$

$$\tau^*_{dw} \;=\; \frac{\tau_{dw}}{\tau} \;=\; \frac{L}{u_{,}\,\tau}. \tag{9}$$

where,

$$\mathrm{C}_{\mathrm{I}} \;=\; (\dot m^\star\, c^\star_p)|_{\mathrm{I}}, \qquad \mathrm{C}_{\mathrm{II}} \;=\; (\dot m^\star\, c^\star_p)|_{\mathrm{II}}, \qquad \mathrm{C}_{\mathrm{r},f} \;=\; \frac{(m^\star_f\, c^\star_f)}{\tau_{\mathrm{I}} + \tau_{\mathrm{II}}}, \tag{10}$$

$$\mathrm{V}_{\mathrm{I}} \;=\; \frac{\mathrm{C}_{\mathrm{I}}}{\rho^\star\, c^\star_p}, \qquad \mathrm{V}_{\mathrm{II}} \;=\; \frac{\mathrm{C}_{\mathrm{II}}}{\rho^\star\, c^\star_p}, \qquad \mathrm{V}_{\mathrm{r},f} \;=\; \frac{\mathrm{C}_{\mathrm{r},f}}{\rho^\star_f\, c^\star_f} \tag{11}$$

2.2 Dimensionless governing equations

After introducing the following dimensionless quantities:

$$x^* \;=\; \frac{x}{L}, \qquad r^* \;=\; \frac{r - R_p}{\Delta R_f}, \qquad \nabla_* \;=\; \left(\frac{\Delta R_f}{L}\frac{\partial}{\partial x^*}, \frac{\partial}{\partial r^*}\right), \tag{12}$$

$$T^* \;=\; \frac{T \;-\; T_{\mathrm{ref}}}{\Delta T_{\mathrm{ref}}}, \qquad T^*_f \;=\; \frac{T_f \;-\; T_{\mathrm{ref}}}{\Delta T_{\mathrm{ref}}}, \tag{13}$$

$$Y^* \;=\; \frac{Y - Y_{\mathrm{ref}}}{\Delta Y_{\mathrm{ref}}}, \qquad Y^*_f \;=\; \frac{Y_f - Y_{\mathrm{ref}}}{\Delta Y_{\mathrm{ref}}}, \qquad W^*_f \;=\; \frac{W_f}{W^{\max}_f}, \tag{14}$$

$$t^* \;=\; \frac{t \;-\; N\,(\tau_{\mathrm{I}} + \tau_{\mathrm{II}})}{\tau_{\mathrm{I}}}, \qquad \text{for section I}, \tag{15}$$

$$t^* \;=\; \frac{t \;-\; \tau_{\mathrm{I}} - \; N\,(\tau_{\mathrm{I}} + \tau_{\mathrm{II}})}{\tau_{\mathrm{II}}}, \qquad \text{for section II}. \tag{16}$$

The following normalized equations are obtained:

$$(1 - \epsilon_f)\, f_s\, \Omega \frac{\partial W^*_f}{\partial t^*} \;+\; \epsilon_f\, \frac{\partial Y^*_f}{\partial t^*} \;=\; \mathrm{Fi}_s\, f_s\, \Omega \nabla_* \cdot (\delta_s \nabla_* W^*_f) \;+\; \mathrm{Fi}_g\, \nabla_* \cdot (\delta_g \nabla_* Y^*_f), \tag{17}$$

$$\chi_f\, \frac{\partial T^*_f}{\partial t^*} \;=\; \mathrm{Fo}_f\, \nabla_* \cdot (\kappa_f\, \nabla_* T^*_f) \;+\; f_s\, \Omega \left((1 - \epsilon_f)\, \frac{\partial W^*_f}{\partial t^*} \;-\; \mathrm{Fi}_s\, \nabla_* \cdot (\delta_s \nabla_* W^*_f)\right)\, i^*_{sor}, \tag{18}$$

$$\tau^*_{dw}\, \frac{\partial Y^*}{\partial t^*} \;+\; \frac{\partial Y^*}{\partial x^*} \;=\; \mathrm{N}^{\mathsf{m}}_{\mathsf{tu}}\, (Y^*_f|_{p.s.} - Y^*), \tag{19}$$

$$\chi\left(\tau^*_{dw}\, \frac{\partial T^*}{\partial t^*} \;+\; \frac{\partial T^*}{\partial x^*}\right) \;=\; \mathrm{N}^{\mathsf{h}}_{\mathsf{tu}}\, (T^*_f|_{p.s.} - T^*), \tag{20}$$

where, the parameters $\mathrm{N}^{\mathsf{m}}_{\mathsf{tu}}$, $\mathrm{N}^{\mathsf{h}}_{\mathsf{tu}}$, Fo_f, Fi_g, Fi_s and τ^*_{dw} can assume different values for each section.

At the interface $r^* = 0$, the following boundary conditions apply:

$$-\, f_s\, \Omega\, \frac{\delta_s}{\mathrm{Bi}^{\mathsf{m}}_s}\frac{\partial W^*_f}{\partial r^*} \;-\; \frac{\delta_g}{\mathrm{Bi}^{\mathsf{m}}_g}\frac{\partial Y^*_f}{\partial r^*} \;=\; (Y^* \;-\; Y^*_f), \tag{21}$$

$$-\kappa_f \frac{\partial T_f^*}{\partial r^*} = \mathrm{Bi}_f^{\mathsf{h}}\,(T^* - T_f^*) + \frac{\mathrm{Bi}_g^{\mathsf{m}}}{\mathrm{Le}_g}\,(Y^* - Y_f^*)\,i_{v,\Delta T}^* - \frac{f_s\,\Omega}{\mathrm{Le}_s}\,\delta_s \frac{\partial W_f^*}{\partial r^*}\,i_{sor}^*, \tag{22}$$

whereas at the other boundaries one finds:

$$\left(\frac{\partial Y_f^*}{\partial r^*}\right)_{r^*=1} = \left(\frac{\partial T_f^*}{\partial r^*}\right)_{r^*=1} = 0, \quad \left(\frac{\partial Y_f^*}{\partial x^*}\right)_{x^*=0} = \left(\frac{\partial T_f^*}{\partial x^*}\right)_{x^*=0} = 0, \quad \left(\frac{\partial Y_f^*}{\partial x^*}\right)_{x^*=1} = \left(\frac{\partial T_f^*}{\partial x^*}\right)_{x^*=1} = 0. \tag{23}$$

The periodicity of the problem appears in the inlet conditions:

$$\begin{aligned} Y^*(0,t^*) &= Y_{in,\mathrm{I}}^*(t^*), \quad T^*(0,t^*) = T_{in,\mathrm{I}}^*(t^*), \quad \text{for section I},\\ Y^*(0,t^*) &= Y_{in,\mathrm{II}}^*(t^*), \quad T^*(0,t^*) = T_{in,\mathrm{II}}^*(t^*), \quad \text{for section II}. \end{aligned} \tag{24}$$

valid for $N = 1, 2, \ldots$, where N is the number of cycles. Moreover, considering a counterflow arrangement, the following change of variable is applied at the end of each process:

$$x_{next}^* = 1 - x_{current}^*. \tag{25}$$

The additional symbols that appear in the normalized equations are the dimensionless forms of i_{sor}, $i_{v,\Delta T}$ and $W_f^{\max}$, and dimensionless coefficients that account for variable properties:

$$i_{sor}^* = \frac{i_{sor}\,\rho_g^\star\,\Delta Y_{\mathrm{ref}}}{\rho_f^\star\,c_f^\star\,\Delta T_{\mathrm{ref}}}, \qquad i_{v,\Delta T}^* = \frac{i_{v,\Delta T}\,\rho_g^\star\,\Delta Y_{\mathrm{ref}}}{\rho_f^\star\,c_f^\star\,\Delta T_{\mathrm{ref}}}, \qquad \Omega = \frac{\rho_s^\star\,W_f^{\max}}{\rho_g^\star\,\Delta Y_{\mathrm{ref}}}, \tag{26}$$

$$\kappa_f = \frac{k_f}{k_f^\star}, \qquad \delta_g = \frac{\epsilon_f}{\tau_G}\frac{\mathcal{D}_g}{\mathcal{D}_g^\star}, \qquad \delta_s = \frac{1-\epsilon_f}{\tau_S}\frac{\mathcal{D}_s}{\mathcal{D}_s^\star}, \qquad \chi = \frac{\rho\,c_p}{\rho^\star c_p^\star}, \qquad \chi_f = \frac{\rho_f\,c_f}{\rho_f^\star\,c_f^\star}. \tag{27}$$

This set of equations can be used to simulate a relatively broad class of transport problems, from coupled periodic heat and mass transfer that occurs within rotary or fixed-bed exchanges to simple, decoupled, sensible heat transfer or isothermal mass transfer in similar devices. Further information regarding the derivation and normalization of the previous equations can be found in the studies (Sphaier and Worek, 2006a) and (Sphaier and Worek, 2006b).

3. Results

The previously presented formulation was solved using a combination of numerical methods, described in (Sphaier and Worek, 2007). In order to validate the current mathematical model, initially, simulated effectiveness results for a sensible heat transfer regenerator are compared to those presented by Kays and London (1964). The enthalpy transfer effectiveness, together with the mass-transfer effectiveness, are defined as:

$$\varepsilon^{\mathsf{m}} = \frac{\mathrm{C_I}}{\mathrm{C_{min}}}\,\frac{\bar{Y}_{out,\mathrm{I}} - \bar{Y}_{in,\mathrm{I}}}{\bar{Y}_{in,\mathrm{II}}^* - \bar{Y}_{in,\mathrm{I}}}, \qquad \varepsilon^{\mathsf{i}} = \frac{\mathrm{C_I}}{\mathrm{C_{min}}}\,\frac{\bar{\bar{i}}_{\mathrm{I},out} - \bar{\bar{i}}_{\mathrm{I},in}}{\bar{\bar{i}}_{\mathrm{II},in} - \bar{\bar{i}}_{\mathrm{I},in}}, \tag{28}$$

Table 1 displays the calculated enthalpy effectiveness (current) together with selected cases for periodic-flow exchangers from (Kays and London, 1964) (previous). By observing Tab. 1, one notices very good agreement.

Data from the current formulation were also compared with that experimentally obtained by Brillhart (1997), who performed measurements of periodic heat and mass transfer in a desiccant sample, consisting of a corrugated paper matrix coated with sorbent material. The Nusselt number was obtained for sinusoidal channels from (Niu and Zhang, 2002c), and the psychrometric ratio was obtained from (Wang and Chang, 1998). The equilibrium relation is given by $f_s\,W_f = W_f^{\max}\,\phi_g^{\beta}$, where the constants $W_f^{\max}$ and β are also presented in Tab. 2. The remaining thermo-physical properties of the

Table 1. Comparison with effectiveness for heat transfer regenerator.

$\mathrm{N}^{\mathsf{h}}_{\mathrm{tu,o}}$	$\mathrm{C}^*_{\mathrm{r}} = 1.0$, $\mathrm{C} = 1$		$\mathrm{C}^*_{\mathrm{r}} = 2.0$, $\mathrm{C} = 1$		$\mathrm{C}^*_{\mathrm{r}} = 1.0$, $\mathrm{C}^* = 0.7$	
	previous	current	previous	current	previous	current
2.0	0.601	0.6009	0.649	0.6491	0.644	0.6438
3.0	0.667	0.6673	0.728	0.7280	0.716	0.7156
4.0	0.709	0.7088	0.776	0.7760	0.759	0.7590
5.0	0.738	0.7377	0.809	0.8085	0.789	0.7887
10.0	0.811	0.8116	0.866	0.8660	0.860	0.8605
100.0	0.939	0.9395	0.984	0.9839	N/A	0.9600
$\mathrm{N}^{\mathsf{h}}_{\mathrm{tu,o}}$	$\mathrm{C}^*_{\mathrm{r}} = 5.0$, $\mathrm{C} = 1$		$\mathrm{C}^*_{\mathrm{r}} = 10.0$, $\mathrm{C} = 1$		$\mathrm{C}^*_{\mathrm{r}} = 1.0$, $\mathrm{C}^* = 0.5$	
	previous	current	previous	current	previous	current
2.0	0.664	0.6639	0.666	0.6661	0.669	0.6690
3.0	0.746	0.7464	0.749	0.7492	0.740	0.7402
4.0	0.796	0.7959	0.799	0.7991	0.782	0.7817
5.0	0.829	0.8289	0.832	0.8322	0.809	0.8089
10.0	0.904	0.9045	0.908	0.9079	0.872	0.8716
100.0	0.989	0.9889	0.989	0.9898	N/A	0.9600

Table 2. Input data used in experimental validation.

$c_{p_{da}}$	1007 J/kg·°C	k_{da}	26.3×10^{-3} W/m·°C	ρ_{da}	1.1614 kg/m^3
$c_{p_{vs}}$	1872 J/kg·°C	k_{vs}	19.6×10^{-3} W/m·°C	$\mathcal{D}^\star$	2.94×10^{-5} m^2/s
c_{ls}	4180 J/kg·°C	k_l	613×10^{-3} W/m·°C	$\mathcal{D}^\star_f$	3.43×10^{-6} m^2/s
c_{df}	1340 J/kg·°C	k_{df}	0.011 W/m·°C	ρ_{df}	930 kg/m^3
ϵ_f	0.30	τ_G	3.0	τ_S	3.0
$W_f^{\max}$	0.234 kg/kg	β	0.748	L	202.3 mm
T_{in}^{I}	34.5°C	T_{in}^{II}	120°C	Nu	2.135
Y_{in}^{I}	14.5 g/kg	Y_{in}^{II}	18.5 g/kg	$2\,\Delta R_f$	0.25 mm

tested sample are unavailable in (Brillhart, 1997); however, since the sample consists of paper coated with sorbent, the thermo-physical properties of paper were used. Figure 3 shows temperature and humidity ratio obtained with the current formulation, together with those reported by Brillhart (1997). As can be seen, the results present a very reasonable agreement with the experimental data, with higher discrepancies occurring for the Y-curve at the initial region (period I). Nevertheless, the experimental uncertainties are always elevated in the beginning of an adsorption period, which thereby justifies the high discrepancy obtained in that portion of the cycle.

Figure 3. Experimental validation: outlet values of temperature T and concentration Y with time t.

Next, Tab. 3 presents illustrative results for combined heat and mass transfer with $(h^{\mathsf{h}} A_s)^* = (h^{\mathsf{m}} A_s)^* = \mathrm{C}^* = \mathrm{V}^* = 1$, $\tau_{dw}^{*\,\max} = 0.001$, $K_f = 10^{-3}$, and with low resistances to heat and mass diffusion in the felt ($\mathrm{Bi}_f^{\mathsf{h}} = \mathrm{Bi}_g^{\mathsf{m}} = \mathrm{Bi}_s^{\mathsf{m}} = 0.1$). Also, a small sorbent mass fraction is selected ($f_s = 1\%$), together with a linear isotherm ($W_f = W_f^{\max}\,\phi_g$) and a relatively low heat of sorption ($i_{sor} = 1.2\,i_{vap}$). In addition, the Lewis approximation, which assumes that $\psi_r = 1$, is considered;

as a result, the N_{tu}s for heat and mass transfer have equal values. The presented results, demonstrate that, for the enthalpy effectiveness, the same behavior observed in periodic heat exchangers (Kays and London, 1964; Shah, 1981) is again seen here. However, in general, the effectiveness values are lower than the well established values presented in these references. This occurs because of the low values for the mass effectiveness, mainly resulting from the low sorbent fraction used in this test-case. Analyzing the values for the mass effectiveness, a different behavior is observed. For lower matrix capacity ratios, the mass effectiveness decreases with increasing N_{tu}s; nevertheless, the values of ε^{m} are minimal for this range of C_r^*, such that the absolute change in performance is also small. As C_r^* is increased, the mass effectiveness assumes the same behavior observed in the energy effectiveness ε^{i}.

Table 3. Results for periodic heat and mass transfer exchanger.

$N_{tu,o}^{\mathsf{h}}$	C_r^*							
	1		2		5		10	
	ε^{m}	ε^{i}	ε^{m}	ε^{i}	ε^{m}	ε^{i}	ε^{m}	ε^{i}
3	0.09842	0.31101	0.28985	0.44605	0.61256	0.65655	0.70275	0.71461
5	0.08780	0.33135	0.28373	0.47116	0.67406	0.72675	0.78429	0.79797
10	0.07806	0.35102	0.27888	0.49390	0.73849	0.79610	0.86573	0.87858
50	0.07342	0.36501	0.28096	0.51296	0.80713	0.86518	0.95662	0.96304

4. Summary and conclusions

A unified mathematical formulation valid for both desiccant and enthalpy wheels, as well as for other periodic exchangers, was presented. The equations include the thermal and mass diffusion in the felt as localized phenomena, thereby enabling the formulation to properly simulate these transport mechanisms. Relevant dimensionless groups with physical significance were defined, and a fully normalized system of governing partial differential equations was obtained. For the sake of illustrating the proposed formulation, simulation results of a test-case with periodic combined heat and mass transfer with adsorption were presented, showing the behavior of a periodic exchanger varying the number of transfer units and the capacity ratio. Finally, as this investigation was aimed at presenting a mathematical formulation and proper dimensionless parameters, there is a clear need for conducting parametric analyses for studying the impact of the characteristic parameters on regenerator operation. In spite of the need for further studies, the information herein presented is a relevant contribution to the realm of heat and mass transfer in regenerators.

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