# PERFORMANCE EVALUATION OF A MODEL-BASED GAS TURBINE ESTIMATOR SYSTEM

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**Abstract.** *This work concerns the performance evaluation of a nonlinear model-based estimator for a single-spool gas turbine engine under realistic operation conditions, namely which exhibit parametric variations and strong measurement noise. Since several gas turbine control techniques, available in the literature, assumes critical variables disponibility for feedback loop and performance evaluation, the analysis of model-based estimator seems relevant considering diferent operation conditions. An Extended Kalman Filter (EKF) is applied to estimate quantities such as pressure, temperature, angular speed and thrust to implement several control techniques (e.g. PI speed control, Nonlinear Predictive Control and LQG Control). The performance evaluation is based on the following features: transient behaviour, static accuracy, robustness to noise, ability to handle parametric variations and design simplicity.*
***Keywords***: *Gas turbine model, Nonlinear estimator, Extended Kalman Filter, Performance evaluation.*

## 1. INTRODUCTION

One of the most useful prime movers in aircraft and electric power generation systems is the gas turbine. As it can be seen in Fig. 1, the main parts of a gas turbine include the inlet duct, compressor, combustor chamber, turbine and nozzle or gas deflector. The physical relations between these components are fixed by the structure of the engine. The operation of the gas turbines can be described as follows: the air is drawn into the engine through the inlet duct by the compressor, which compresses and deliver the air mass into the combustion chamber (Cohen, Rogers and Saravanamuttoo, 1996). Within the combustion chamber the air is mixed with fuel and the mixture ignited, producing a rise in temperature and hence an expansion of the gases. These gases are exhausted through the engine nozzle or the engine gas-deflector, but first they pass through the turbine, which is designed to extract sufficient energy from them to keep the compressor rotating, so that the engine is self-sustaining.

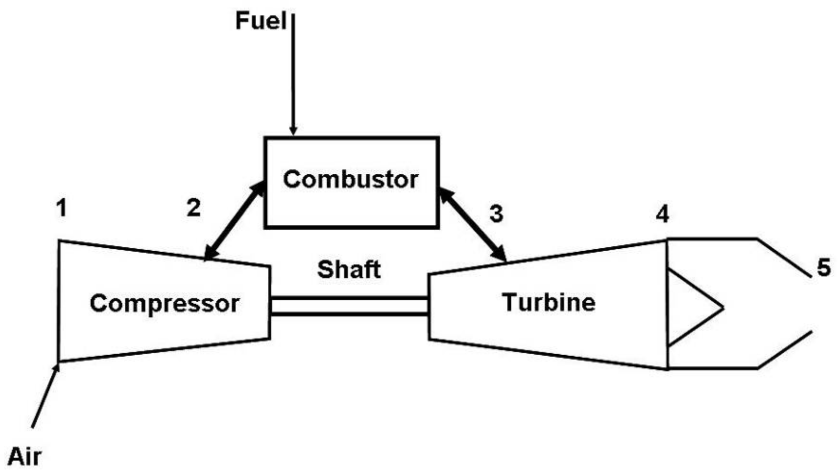


Figure 1. Turbine components diagram

With the development of modern control theory and digital computer technology, computer has become an important tool for designing advanced aero-engine control systems. The application of engine simulator by means of computer has significantly reduced the development cost. It is a first step in designing an engine control system to establish the engine model which can describe the engine steady and transient characteristics accurately (Carrera and Hemerly, 2006).

One of the main control objectives for a gas turbine engine is to achieve rapidly the required thrust change while ensuring stable operation of the compressor and keeping the turbine inlet temperature below the safety limit (Qi and Maccallum, 1992). To achieve this performance objective, it is necessary that the engine's thrust, compressor surge margin and turbine inlet temperature be monitored at all instants of time. In practice, however, these performance variables can rarely be directly measured. In this situation, to control these variables, one solution is to use the available measurements from the engine in conjunction with a mathematical model to estimate these unmeasured variables (Astrom K.J. and Wittenmark B., 1989). In this work, a Kalman filter based estimator is presented aiming at the dynamical behavior analysis of the turbine thrust variable. A low-power gas turbine with fixed nozzle area is considered, therefore, the engine is controlled only by the fuel flow variable.

An overview of the nonlinear thermodynamic model, the turbine and compressor subsystems will be presented in section 2. A summary of a single shaft turbojet engine thrust estimation algorithm is presented in section 3. The numerical evaluation of the proposed estimator, under several operation conditions, is presented in sections 4. Finally, the main conclusions of the work are presented in Section 5.

## 2. NONLINEAR MODEL OF THE GAS TURBINE

In order to get a low order dynamic model, suitable for control purposes, the following simplifying modeling assumptions are made:

a) Constant chemical properties are assumed in each main part of the gas turbine: specific heat at constant pressure and volume, specific gas constant and adiabatic exponent.
b) Heat loss (transmission, conduction and radiation) is neglected.
c) For each module, compressor and turbine, a constant mass flow rate is assumed.
c) In the inlet duct a constant pressure loss coefficient ($\sigma_I$ ) is assumed.
d) In the combustion chamber: a constant pressure loss coefficient ($\sigma_{Comb}$) is assumed, a constant efficiency of combustion ($\eta_{comb}$) is assumed and the enthalpy of fuel is neglected.
e) In the gas-deflector a constant pressure loss coefficient ($\sigma_N$) is assumed.

The gas turbine nonlinear thermodynamic model used for the observer project is presented in equations (1)-(3), Ailer P., Szederkenyi G. and Hangos K. M.( 2002.).

$$\frac{dP_2}{dt} = \frac{R\,T_2}{v_2}\left(\dot{M}_2 - \dot{M}_3\right) \tag{1}$$

$$\frac{dT_3}{dt} = \frac{1}{c_{vmed}\,m_{Comb}}\left(\dot{M}_2 c_{pair} T_2 - \dot{M}_3 c_{pgas} T_3 + Q\eta_{comb}\dot{M}_f + c_{vmed} T_3 (\dot{M}_2 + \dot{M}_f - \dot{M}_3)\right) \tag{2}$$

$$\frac{dn}{dt} = \frac{1}{4\pi^2 I\,n}\left(\dot{M}_3 c_{pgas}(T_3 - T_4)\eta_{mech} - \dot{M}_2 c_{pair}(T_2 - T_1) - 2\pi\frac{3}{50} n M_L\right) \tag{3}$$

where:

| | |
|---|---|
| $P_2$: Compressor discharge pressure [Pa]; | $R$: Specific gas constant [J/kg K]; |
| $T_3$: Turbine inlet temperature [K]; | $T_2$: Compressor exit temperature [K]; |
| $n$: Shaft Speed [rad/sec]; | $v_2$: Combustor volume [m$^3$]; |
| $\dot{M}_1$ : Compressor inlet mass flow [kg/s]; | $\dot{M}_f$ : Fuel mass flow [kg/s]; |
| $\dot{M}_3$ : Turbine inlet mass flow [kg/s]; | $Q$: Heat released by the burning fuel [J]; |
| $c_{pair}$: Air specific heat at constant pressure [J/kg K]; | $c_{pgas}$: Gas specific heat at constant pressure [J/kg K]; |
| $\eta_{comb}$: Combustor efficiency [--]; | $\eta_{mech}$: Mechanical efficiency [--]; |
| $I$: Shaft inertial moment [kg m$^2$]; | $M_L$: Load inertial moment [kg m$^2$]; |
| $c_{vmed}$: mix heat at constant volume [J/kg K]; | $m_{Comb}$: Accumulated mass at combustor chamber [kg] |

An important feature of the turbine model is the component descriptions in the form of thermodynamic performance maps (Cohen G., Rogers H. and Saravanamuttoo H, 1996). Through these maps is possible to determine the torque absorbed or generated by the turbine components. Integration of the net shaft torque with respect to time defines the variations in rotor speed. Once pressure ratios are also required to access the map data, is assumed that flows can be accumulated in volumes separating the major components. The accumulation of flows provides pressure time varying signals that are feedback to access the maps.

In this work, the constitutive relations defined in (4)-(10) complete the nonlinear gas turbine model.

$$P_3 = \sigma_{Comb} P_2 \tag{4}$$

$$P_4 = \frac{P_1}{\sigma_I \sigma_N} \tag{5}$$

$$m_{comb} = \frac{P_3 v_2}{R T_3} \tag{6}$$

$$T_2 = T_1 \left[ 1 + \frac{1}{\eta_C} \left( \left( \frac{P_2}{P_1} \right)^{\frac{\kappa_{air}-1}{\kappa_{air}}} - 1 \right) \right] \tag{7}$$

$$T_4 = T_3 \left[ 1 - \eta_T \left( 1 - \left( \frac{P_4}{P_3} \right)^{\frac{\kappa_{gas}-1}{\kappa_{gas}}} \right) \right] \tag{8}$$

$$\dot{M}_2 = q(\lambda_1) \frac{P_1}{\sqrt{T_1}} \tag{9}$$

$$\dot{M}_3 = q(\lambda_3) \frac{P_3}{\sqrt{T_3}} \tag{10}$$

where:

$P_1$: Compressor inlet pressure [Pa];
$P_4$: Turbine discharge pressure [Pa];
$T_1$: Compressor inlet temperature [K];
$\kappa_{air}$: Air adiabatic Exponent [--];
$\eta_C$: Compressor efficiency [--];

$P_3$: Turbine inlet pressure [Pa];
$R$: Specific gas constant [J/kg K];
$T_4$: Turbine exit temperature [K];
$\kappa_{gas}$: Gas adiabatic Exponent [--];
$\eta_T$: Compressor efficiency [--]

It must be noted that the constitutive relations in (7)-(10) represents performance map approximations for a constrained interval of the dimensionless mass flow rates (Cohen G., Rogers H. and Saravanamuttoo H, 1996). In equation (9) $q(\lambda_1)$ represents a function of the corrected number of revolutions and compressor pressure ratio, and in equation (10) $q(\lambda_3)$ represents a function of the dimensionless velocity and the turbine pressure ratio. In (Ailer P., Szederkenyi G. and Hangos K. M., 2000) two approximation expressions for $q(\lambda_1)$ and $q(\lambda_3)$, based in a minimum distance norm optimization procedure, are presented. The approximations, presented in equations (11)-(14) and table 1, consider measured data from a low power gas turbine installed in the aircraft department of the Technology Budapest University.

$$q(\lambda_1) = a_1 \frac{n}{\sqrt{\frac{T_1}{288.15}}} \frac{P_2}{P_1} + a_2 \frac{n}{\sqrt{\frac{T_1}{288.15}}} + a_3 \frac{P_2}{P_1} + a_4 \tag{11}$$

$$q(\lambda_3) = c_1 \frac{n}{\sqrt{T_3}} \frac{P_3}{P_4} + c_2 \frac{n}{\sqrt{T_3}} + c_3 \frac{P_3}{P_4} + c_4 \tag{12}$$

$$\eta_C = b_1 \frac{n}{\sqrt{\frac{T_1}{288.15}}} q(\lambda_1) + b_2 \frac{n}{\sqrt{\frac{T_1}{288.15}}} + b_3 q(\lambda_1) + b_4 \tag{13}$$

$$\eta_T = d_1 \frac{n}{\sqrt{T_3}} \frac{P_3}{P_4} + d_2 \frac{n}{\sqrt{T_3}} + d_3 \frac{P_3}{P_4} + d_4 \tag{14}$$

Table 1. Constants for the minimum distance norm approximation.

| i | $a_i$ | $c_i$ | $b_i$ | $d_i$ |
|---|---|---|---|---|
| 1 | 0.00035319 | -0.03248 | -0.0005957 | 0.144 |
| 2 | 0.0011097 | 0.0018218 | 0.00028848 | 0.0021314 |
| 3 | -0.4611 | 0.047843 | 0.5265 | -0.19685 |
| 4 | 0.16635 | 0.16026 | 0.42051 | 1.07 |
| norm | 0.026003 | 0.01534 | 0.054047 | 0.11549 |

The range of operation of the nonlinear model is defined in expressions (15) and (16).

$$101334 \le P_3 \le 357894; 650 \le n \le 833.33; 0.00367 \le \dot{M}_f \le 0.027 \quad (15)$$

$$60000 \le P_1 \le 110000; 243.15 \le T_1 \le 308.15; 0 \le M_L \le 0.027 \quad (16)$$

### 3. KALMAN FILTER BASED THRUST ESTIMATOR

Consider the gas turbine state space nonlinear continuous model presented in (17)-(18).

$$\dot{x} = f(x,u) \quad (17)$$

$$y = h(x) \quad (18)$$

where the state vector consider the dynamic and constitutive relations described in equations (1)-(10) and the engine developed thrust (*F*), more precisely,

$$\dot{x} = [P_2 \ T_3 \ n \ F_T] \quad (19)$$

$$y = [T_4 \ P_3 \ n] \quad (20)$$

$$u = \dot{M}_f \quad (21)$$

The thrust engine variable ($F_T$) can be calculated through the velocity increase caused by the engine, $v_e - v_o$, and the pressure difference, $P_4 - P_1$, at the nozzle exit area $A_e$ (Cohen G., Rogers H. and Saravanamuttoo H., 1996). More precisely, the thrust equation is presented in (22).

$$F_T = \dot{M}_4 v_4 - \dot{M}_1 v_1 + (P_4 - P_1)A_e \quad (22)$$

Using equation (5) and assuming a constant speed increase, $\Delta v = v_4 - v_1$, the thrust value can be defined as

$$F_T = \dot{M}_4 v_4 - \dot{M}_1 v_1 + A_e P_1 \left( \frac{1 - \sigma_I \sigma_N}{\sigma_I \sigma_N} \right) \quad (23)$$

It must be noted that the first term in (23) is normally the dominating term.

A deterministic observer based on the extended Kalman filter (EKF) is proposed in this work (Brown R. G. and Hwang P. Y. C, 1982). The observer measurement update equations are presented in (24)-(26).

$$\hat{x}_{k=1} = \hat{x}_{k+1/k} + K_{k+1} e_{k+1} \quad (24)$$

$$K_{k+1} = \left(P_{k+1/k} H_{k+1}^T\right)\left(H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1}\right)^{-1} \quad (25)$$

$$P_{k+1} = P_{k+1/k} - K_{k+1}\left(H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1}\right) K_{k+1}^T \quad (26)$$

The time updates expressions in (27)-(28)

$$\hat{x}_{k+1/k} = f(\hat{x}_k, u_k) \quad (27)$$

$$P_{k+1/k} = Q_k + F_k P_k F_k^T \quad (28)$$

The error variable is defined as

$$e_{k+1} = y_{k+1} - H_{k+1} \hat{x}_{k+1/k} \quad (29)$$

and the dynamic matrix as

$$F_{kij} = \left. \frac{\partial f_i(x_k, u_k)}{\partial x_{jk}} \right|_{x_k = \hat{x}_k} \quad (30)$$

Note that when the Kalman filter is used as a nonlinear observer for deterministic systems, the noise covariance matrices, Q and $R_{k+1}$ can be arbitrarily selected. For example, a proper selection can shape the system convergence characteristics.

The state vector estimate used in the deterministic Extended Kalman observer is presented in equation (31).

$$\hat{x}(k) = \left[\hat{P}_2(k) \quad \hat{T}_3(k) \quad \hat{n}(k) \ \hat{T}_4(k)\right] \quad (31)$$

In order to guarantee observability conditions the observation process and the input vector are defined in (32) and (33) respectively.

$$y(k) = [T_4(k)\ P_3(k)\ \ n(k)] \qquad (32)$$

$$u(k) = \left[ M_f(k)\ \ M_L(k)\ \ \hat{\dot{M}}_1(k)\ \ \hat{\dot{M}}_2(k)\ \ \hat{T}_2(k) \right] \qquad (33)$$

where the constitutive relations in (7)-(10) represent the performance map approximations for a constrained interval of the dimensionless mass flow rates and provide $\dot{M}_1$, $\dot{M}_2$ *and* $T_2$ estimates.

## 4. SIMULATION RESULTS

The example in this section considers the gas turbine operating without control loops. The simulations are based on the following assumptions: a) the model is the exact representation of the engine, i.e. parametrically and structurally correct; b) the same fuel flow step signal for the engine and the estimator algorithm. On the other hand, the estimator assumes initial states values different from the real ones and is initialized twice: at t = 0s (in transient state operation) and at t = 1.5s (in steady state operation). The parameters used in the filter are: initial covariance error matrix: $P(0) = diag(10^{-3}, 1, 1, 1)$, state covariance noise matrix: $Q = diag(10^{-3}, 10^{-3}, 10^{-3}, 10^{-3})$ and measurement covariance noise matrix: $N = diag(10^{-3}, 10^{-3}, 10^{-3})$. In figures 2 and 3, the estimative and real values are presented under several measurement noise.

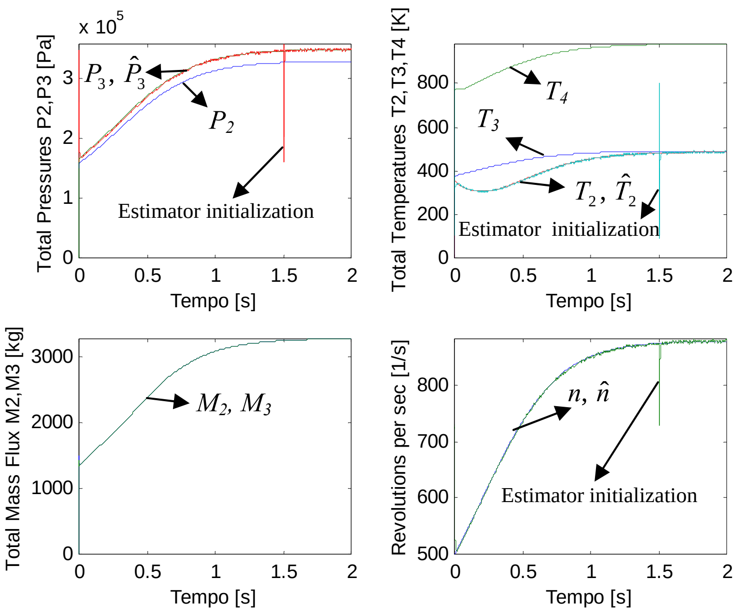


Figure 2. Performance of the EKF observer without control

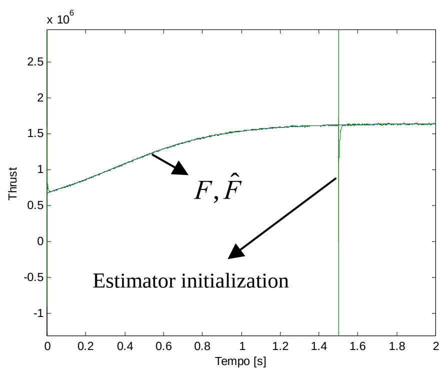


Figure 3. Performance of the EKF observer without control (Thrust variable)

As figures 2 and 3 shows, the observer performance is suitable when observability conditions are satisfied, i.e., when a sufficient number of the engine variables are available. It is also seen that the engine and the model variables converges to the same steady state values, thus an observer based closed loop control can be guaranteed. The estimation algorithm presents suitable performance under several measurement noise and even in steady state conditions (where minimum persistence of excitation is presented).

It should be stressed that the computed thrust and shaft speed variables could be used for feedback control. In future works a detailed observability analysis will be performed in several operation conditions in order to satisfy control applications.

## 5. CONCLUSIONS

This work dealt with the determinations and evaluation of a gas turbine nonlinear state space model. A deterministic observer based on the extended Kalman filter is proposed and evaluated aiming at a thrust feedback control strategy.

The state space gas turbine model considered is suitable for adaptive control schemes. Future works will address the design of an observer based thrust controller subject to more realistic constraints. Furthermore, several gas turbine parameters (e.g. shaft inertia moment, turbine, chamber and compressor volumes) should be identified through the extended Kalman observer.

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## 8. RESPONSIBILITY NOTICE