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DYNAMICS OF CRACKED THIN-WALLED BEAMS

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Abstract. *In different engineering structures, the fatigue induced cracks represent one of the most dangerous sources of eventual catastrophic failures. Accordingly, the early detection of the crack and the estimation of its gravity are tasks of crucial importance. There are a number of localized methods to detect and analyze the presence of cracks in structures, however many of them have a troublesome implementation, particulary when the crack is located in places with little accessibility. Under these circumstances, the global methods of detection reached an important repercussion, especially those methods based in the measurement of dynamic responses. Actually, in that methodology the experimental registers of dynamical parameters (like frequencies, accelerations, etc) are compared with the ones of a mathematical model of the structure. Thus, one of the most attractive and employed methodologies consists in the determination of the natural frequencies of the structure and the comparison and evaluation with the ones calculated with an appropriate mathematical model. The mathematical model has to incorporate parameters related to the position and size of the cracks. Afterwards, one can characterize the crack (size and position) by solving an inverse problem consisting in the minimization of the difference between the actual and calculated frequency values. The methodology was implemented in a number of theoretical beam models incorporating cracks for metals as well as composite materials. However, according to the authors' knowledge, an appropriate model of thin-walled beams to be implemented together with the crack detection methodology has not been yet developed. In the present paper a finite element of an I-beam incorporating a crack due to fatigue loading is developed. This approach consists in combining concepts of the fracture mechanics theory with a generalized thin-walled beam model developed by the first two authors. This new element is employed to analyze the effect of location and size of the crack for different geometries and boundary conditions.*

Keywords: *Thin-Walled beams, Fatigue cracks, Dynamics*

1. INTRODUCTION

The presence of cracks in vibrating structural members is a matter of crucial importance for engineering designs and for the assessment of the structural life. Consequently, an important amount of scientific research has been devoted to this subject. In a middle nineties work (Dimarogonas, 1996) the state-of-the-art concerning to the methodologies for the analysis of cracked structural members was appropriately reviewed. It is well known that the vibration-based inspection techniques are effective tools for the detection of structural defects and cracks (Adams and Cawley, 1985). The localization and size of cracks in beam structures can be determined, in principle, from variations in the natural frequencies, mode shapes and amplitudes of the forced response (Chen and Chen, 1988; Salawu, 1997). The presence of cracks in a beam structure leads to a local loss of stiffness, or in other words a crack introduces a local flexibility which modifies the dynamic behavior of the structure. Many of the recent works concerning to the dynamic behavior of cracked beam structures were devoted to the analysis of rotors (Darpe et al., 2004; Papadopoulos and Dimarogonas, 1992) and for particular solid and also hollow cross-section beams (Kisa and Brandon, 2000; Zheng and Fan, 2003; Nobile and Viola, 2001). Nevertheless, studies devoted to the behavior of cracked general thin-walled beams, despite of their importance in structural applications, appear to be non existent or at least really scarce. In the knowledge of the authors, the recent work of Gomes Cardoso et al (2005), is one of the first articles to tackle the problem of the dynamics of cracked thin-walled U-beams. However a simplified approach consisting in reducing the area properties to account for the crack was employed.

In this paper an attempt is made to analyze the dynamic behavior of a shear deformable thin-walled cracked I-beam. A finite element is developed in order to introduce the crack in a beam model formulation accounting for shear flexibility due to bending and warping (Cortínez and Piovan, 2002). The element is developed by considering the increase of flexibility due to the crack by means of concepts of fracture mechanics together with a model of shear deformable thin-walled beams for isotropic materials. The beam element is formed by three independent panels: a web and two flanges. The flexibility matrix for each panel is obtained. In one of the flanges a flexibility matrix for the crack is added to the corresponding flange flexibility matrix. Afterwards the stiffness matrix for each panel is obtained from the flexibility matrix and finally the element stiffness matrix is obtained by assembling the three components. The presence of a crack leads to coupled flexural/longitudinal/torsional motions that are normally decoupled. Parametric studies are performed with the present element in order to characterize the effect of size and location of the damage in an I-beam with different parametrical ratios.

2. FINITE ELEMENT MODEL FOR A CRACKED THIN-WALLED BEAM

In Fig.1 one can see the layout of a metallic I-beam composed by three panels: two flanges and a web. The panels of flanges and web are numbered as 1, 2 and 3. Consider a beam segment that has a single crack surface on one of its flanges. In order to describe the cracked finite element for a thin-walled beam accounting for shear flexibility due to bending and warping one has to analyze each panel to obtain the stiffness contribution. However to account for the presence of the crack, the stiffness matrix of the beam element has to be modified. The modified stiffness matrix considers the coupling effects due to the presence of a crack, that is, full longitudinal-bending-twisting coupling. It has to be mentioned that a metallic bisymmetric I-beam without a crack do not exhibit such coupling effects (Cortínez and Piovan, 2002) and a metallic mono-symmetric I-beam without a crack has only bending-twisting coupling (Cortínez et al., 1999). As one can see in Fig. 1,the crack is located at a distance x from the left end of the beam element. In Fig. 2 it is possible to see the flange with a crack and its local forces and displacements.

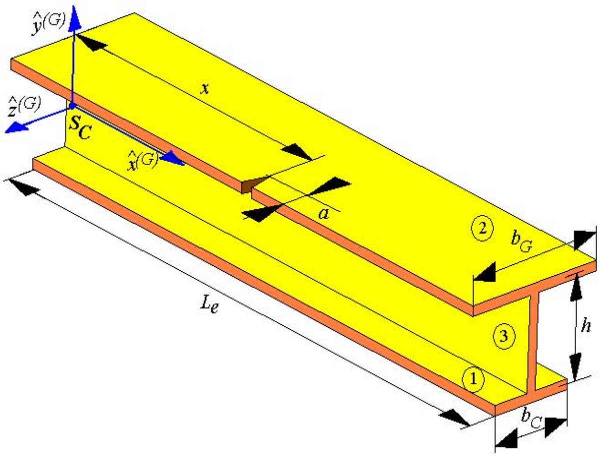

Figure 1. Sketch of the cracked thin-walled I-beam

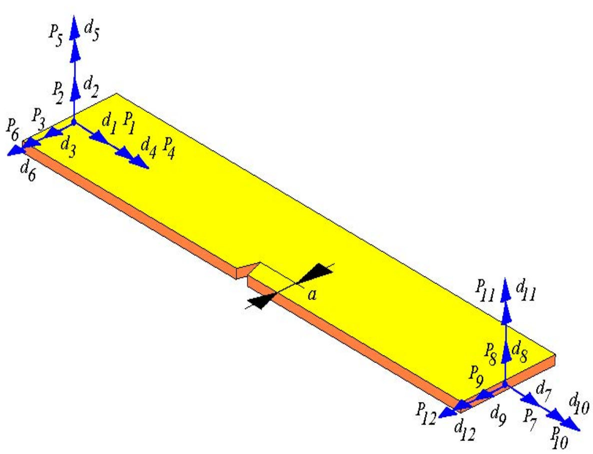

Figure 2. Sketch of a cracked flange, description of local forces and displacements

The flexibility matrix of the cracked flange is first derived. This is done using the Castigliano's theorem:

$$d_i = \frac{\partial U_2^{(0)}}{\partial P_i} + \frac{\partial U_2^{(C)}}{\partial P_i} \tag{1}$$

where $U_2^{(0)}$ is the strain energy of the uncracked flange number 2 and $U_2^{(C)}$ is the strain energy associated with the presence of the crack. These terms of the strain energy can be expressed in the following form:

$$U_2^{(0)} = \frac{1}{2}\int_0^{L_e} \left[\frac{P_1^2}{EA_2} + \frac{P_2^2}{GA_{R2}} + \frac{P_3^2}{GA_{R2}} + \frac{P_4^2}{GI_{02}} + \frac{(P_5 + P_3 x)^2}{EI_{Y2}} + \frac{(P_6 - P_2 x)^2}{EI_{Z2}} \right] L_e dx \tag{2}$$

$$U_2^{(C)} = e\int_0^{a} \frac{(1-\nu)}{E} \left[\left(\sum_{i=1}^{6} K_{Ii}\right)^2 + \left(\sum_{i=1}^{6} K_{IIi}\right)^2 + (1+\nu)\left(\sum_{i=1}^{6} K_{IIIi}\right)^2 \right] da \tag{3}$$

where E and G are the longitudinal and transversal elasticity moduli, ν is the Poisson ratio; A_2, A_{R2}, I_{02}, I_{Y2} and I_{Z2} are respectively the area, reduced area, torsion constant and inertia moments of the flange 2. K_{Ii}, K_{IIi}, and K_{IIIi} are the stress intensity factors for the opening, sliding and shearing modes, respectively. These stress intensity factors can be described in terms of the local forces in the following form:

$$\begin{array}{l} K_I = \left\{ \frac{P_1 F_1}{A_2}, 0, 0, 0, \frac{(P_5 + P_3 x) F_1}{I_{Y2}}, \frac{(P_6 - P_2 x) F_2}{I_{Z2}} \right\} \sqrt{\pi a} \\ K_{II} = \left\{ 0, 0, \frac{P_3 F_{II}}{A_{R2}}, \frac{P_4 b_G F_{II}}{2 I_0}, 0, 0 \right\} \sqrt{\pi a} \\ K_{III} = \left\{ 0, \frac{P_2 F_{III}}{A_{R2}}, 0, \frac{P_4 a F_{III}}{I_0}, 0, 0 \right\} \sqrt{\pi a} \end{array} \tag{4}$$

The functions F_1, F_2, F_{II} and F_{III} are given by:

$$\begin{array}{l} F_1(\xi) = \sqrt{\frac{2Tan[\pi\xi/2]}{\pi\xi}} \frac{0.752 + 2.02\xi + 0.37(1 - Sin[\pi\xi/2])^3}{Cos[\pi\xi/2]} \\ F_2(\xi) = \sqrt{\frac{2Tan[\pi\xi/2]}{\pi\xi}} \frac{0.923 + 0.199(1 - Sin[\pi\xi/2])^4}{Cos[\pi\xi/2]} \\ F_{II}(\xi) = \frac{1.122 - 0.561\xi + 0.085\xi^2 + 0.18\xi^3}{\sqrt{1-\xi}} \\ F_{III}(\xi) = \sqrt{\frac{2Tan[\pi\xi/2]}{\pi\xi}} \end{array} \tag{5}$$

where $\xi = a/b_G$, b_G is the width of the flange.

It is clear that the integration in the Eq. (3) has to be performed in the crack domain. The local displacements d_1 to d_6 of the cracked flange can be obtained from Eq. (1), in terms of the local forces, as:

$$\{d_{1-6}\}_{L2} = \left(\left[F^{(0)}\right]_{L2} + \left[F^{(C)}\right]_{L2} \right) \{P_{1-6}\}_{L2} \tag{6}$$

$\left[F^{(0)}\right]_{L2}$ is the flexibility matrix of the uncracked flange and $\left[F^{(C)}\right]_{L2}$ is the flexibility matrix associated to the presence of crack. Now, taking into account the local equilibrium of the flange one can find the following relationship:

$$\{P_{1-12}\}_{L2} = [T]\{P_{1-6}\}_{L2} \tag{7}$$

where the transformation matrix $[T]$ is given by:

$$[T]^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & L_e \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -L_e & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{8}$$

Now taking into account Eq. (7) together with Eq. (6) and operating in a similar way for the web and the remaining un-cracked flange, it is possible to obtain the local forces in terms of the local displacement for the three panels:

$$\begin{array}{l} \{P_{1-12}\}_{L1} = [T]\left(\left[F^{(0)}\right]_{L1}\right)^{-1}[T]^T\{d_{1-12}\}_{L1} \\ \{P_{1-12}\}_{L2} = [T]\left(\left[F^{(0)}\right]_{L2} + \left[F^{(C)}\right]_{L2}\right)^{-1}[T]^T\{d_{1-12}\}_{L2} \\ \{P_{1-12}\}_{L3} = [T]\left(\left[F^{(0)}\right]_{L3}\right)^{-1}[T]^T\{d_{1-12}\}_{L3} \end{array} \tag{9}$$

Then, the local stiffness matrix for the panels are:

$$\begin{aligned}&\left[K^{(e)}\right]_{L1}=[T]\left(\left[F^{(0)}\right]_{L1}\right)^{-1}[T]^T\\&\left[K^{(e)}\right]_{L2}=[T]\left(\left[F^{(0)}\right]_{L2}+\left[F^{(C)}\right]_{L2}\right)^{-1}[T]^T\\&\left[K^{(e)}\right]_{L3}=[T]\left(\left[F^{(0)}\right]_{L3}\right)^{-1}[T]^T\end{aligned} \tag{10}$$

The local panel displacements $\{d_{1-12}\}_{Lj}$, $j=1,2,3$ are compatible with the global beam-element displacements by means of the following expression:

$$\{d_{1-12}\}_{Lj}=[T_D]_{LjG}\{d_{1-14}\}_G=\begin{bmatrix}[T_d] & [0]\\ [0] & [T_d]\end{bmatrix}\{d_{1-14}\}_G \tag{11}$$

where the global vector of displacements and compatibility matrix are given by:

$$\{d_{1-14}\}_G=\{u_{XC1},u_{YC1},\theta_{Z1},u_{ZC1},\theta_{Y1},\phi_{X1},\theta_{X1},u_{XC2},u_{YC2},\theta_{Z2},u_{ZC2},\theta_{Y2},\phi_{X2},\theta_{X2}\} \tag{12}$$

$$[T_d]=\begin{bmatrix}1&0&-\bar{y}_j&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&1&0&y_j&0\\0&0&0&0&0&1&0\\0&0&0&0&1&0&-y_j\\0&0&1&0&0&0&0\end{bmatrix} \tag{13}$$

In Eq. (12), u_{XC} is the longitudinal displacement, u_{YC} and u_{ZC} are the lateral displacements of the shear center S_C, ϕ_X is the twisting angle (measured from the shear center), θ_Y and θ_Z are bending rotations (measured from the centroid C), and θ_X is a warping intensity variable. Whereas in Eq. (13), y_j, $j=1,2,3$ are the distance from the cross-sectional shear center to the gravity center of the corresponding panel, and on the other hand $\bar{y}_j$, $j=1,2,3$ are the distance from the cross-sectional centroid to the gravity center of the corresponding panel.

Now, considering Eq. (10) and Eq. (11), the stiffness matrix of the cracked beam element can be obtained by means of the assembling expression:

$$\left[K^{(e)}\right]=\sum_{j=1}^{3}[T_D]^T_{LjG}\left[K^{(e)}\right]_{Lj}[T_D]_{LjG} \tag{14}$$

The mass matrix can be obtained by employing into the expression of the kinetic energy, the appropriate shape functions of a beam-element for a consistent integration (Cortínez et al. 1999):

$$\begin{aligned}&\left[M^{(e)}\right]=\int_0^{L_e}\left[\rho A\{f_1\}^T\{f_1\}+\rho A\{f_2\}^T\{f_2\}+\rho I_Z\{f_3\}^T\{f_3\}+\rho I_Y\{f_5\}^T\{f_5\}\right]L_e dx+\\&\int_0^{L_e}\left[\rho C_W\{f_7\}^T\{f_7\}+\rho I_S\{f_6\}^T\{f_6\}+\rho A y_o\left(\{f_6\}^T\{f_4\}+\{f_4\}^T\{f_6\}\right)\right]L_e dx\end{aligned} \tag{15}$$

where

$$\begin{aligned}&\{f_1\}=\{g_{11}(x),0,0,0,0,0,0,g_{12}(x),0,0,0,0,0,0\}\\&\{f_2\}=\{0,g_{21}(x),g_{22}(x),0,0,0,0,0,g_{23}(x),g_{24}(x),0,0,0,0\}\\&\{f_3\}=\{0,g_{31}(x),g_{32}(x),0,0,0,0,0,g_{33}(x),g_{34}(x),0,0,0,0\}\\&\{f_4\}=\{0,0,0,g_{21}(x),g_{22}(x),0,0,0,0,0,g_{23}(x),g_{24}(x),0,0\}\\&\{f_5\}=\{0,0,0,g_{31}(x),g_{32}(x),0,0,0,0,0,g_{33}(x),g_{34}(x),0,0\}\\&\{f_6\}=\{0,0,0,0,0,g_{21}(x),g_{22}(x),0,0,0,0,0,g_{23}(x),g_{24}(x)\}\\&\{f_7\}=\{0,0,0,0,0,g_{31}(x),g_{32}(x),0,0,0,0,0,g_{33}(x),g_{34}(x)\}\end{aligned} \tag{16}$$

The functions $g_{ij}(x)$ verify the static equilibrium equations for a beam element and can be obtained from the open literature (Cortínez and Rossi, 1998). In Eq. (15), y_o is the distance between the shear center and the centroid of the cross-section, ρA is the translational inertia, ρI_Y and ρI_Z are the rotary inertias, ρC_W is the warping inertia and ρI_S is the twisting inertia.

This cracked finite beam-element may be employed assembled together with other uncracked elements in order to model a given thin-walled beam structure. It is clear that the cracked element provides a localized increase of flexibility (or a local loss of stiffness). This effect can be captured with changes in the mode shapes of certain frequencies as it is shown in the following paragraph.

The problem of determining the natural vibration frequencies and the associated mode shapes of a systems leads to the solution of the following eigenvalue problem:

$$[K]\{q\}=\omega^2[M]\{q\} \tag{17}$$

where $[K]$, $[M]$ and $\{q\}$, are the global stiffness matrix, mass matrix and displacements vector, respectively.

3. NUMERICAL STUDIES

A bisymmetric I-beam with a crack in one of its flanges is analyzed. The material properties of the beam are $E = 2.1 \times 10^{11} N/m^2$, $\nu = 0.3$ and $\rho = 7850Kg/m^3$. The cross-sectional properties are $b_G = b_C = h = 0.6m$, the thickness is $e = 0.03m$ and the length is such that $h/L = 0.1$.

In the first example a clamped-clamped beam with a single crack in the upper flange located in the middle of the beam is selected. A model of 41 elements was employed. In Tab. 1 one can see the variation of the first eight frequencies with respect to the Depth of crack. As it is expected, the frequencies of the cracked beam show a slight variation with respect to the case free of crack. Even in the cases with deeper cracks, the difference between the frequencies of the cracked beam with respect to the uncracked beam, can reach values in the order of 2.5%. Although the frequencies of cracked and uncracked beams have a small difference, their corresponding mode shapes differ substantially. Thus in Fig. 3 one can see the first mode shape for the uncracked I-beam, whereas in Fig. 4 one can see what happens with the first mode shape of the beam with a crack of a depth $2a/b_G = 80\%$. The mode shape in both figures is eminently torsional, however in Fig.4 one can see that due to the presence of a crack, the flexural and longitudinal motions are also coupled. It has to be said that in order to show appropriately the coupling effect, in Fig. 4 the flexural and longitudinal motions were magnified ten times from their original values.

Table 1. Variation of the first eight frequencies for a beam with central single crack in the upper flange

Frequency	Depth of crack $2a/b_G$						
Order	0%	5%	10%	20%	40%	60%	80%
1	53,181	53,181	53,180	53,176	53,108	52,802	52,020
2	68,328	68,328	68,328	68,323	68,233	67,854	67,019
3	106,188	106,183	106,167	106,104	105,854	105,504	105,206
4	136,784	136,783	136,780	136,769	136,725	136,652	136,550
5	176,506	176,505	176,502	176,490	176,442	176,362	176,251
6	241,129	241,129	241,129	241,128	241,124	241,117	241,109
7	248,643	248,643	248,641	248,619	248,243	246,599	242,673
8	321,544	321,543	321,542	321,512	321,012	318,926	314,605

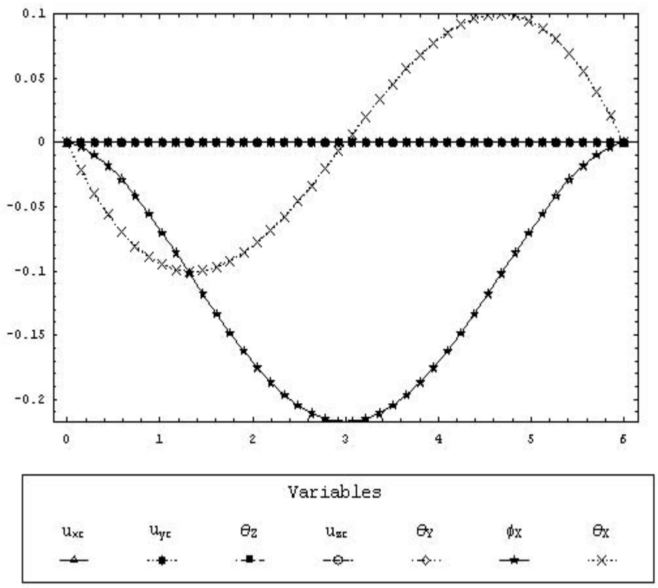

Figure 3. First mode shape for the uncracked beam

As a second example, the same beam but with cracks near the clamped ends is analyzed. The variation of the frequencies with respect to the Depth of crack is depicted. Note that in this case there is a sensible difference in the first frequency that can reach up to ten percent. However the most sensible differences between the dynamic behavior of uncracked and cracked beams can be observed in the mode shapes of Fig. 3 and Fig. 5, respectively.

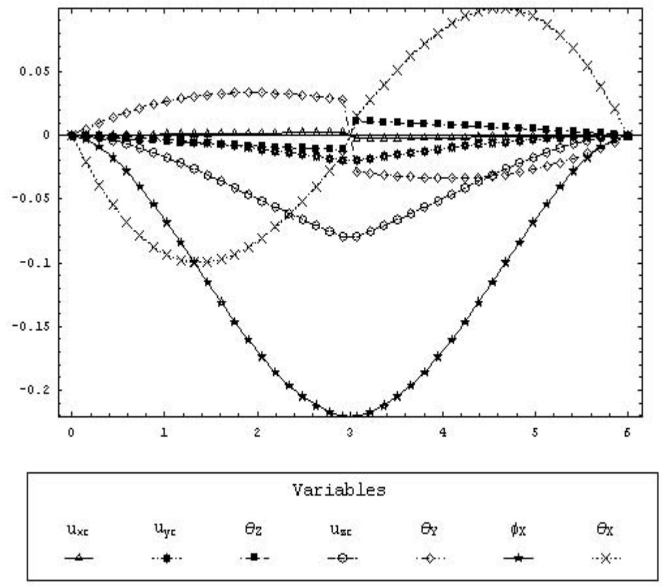

Figure 4. First mode shape for the cracked beam. The crack is located in the middle of the beam

Table 2. Variation of the first eight frequencies for a beam with central single crack in the upper flange

Frequency Order	Depth of crack $2a/b_G$ 0%	5%	10%	20%	40%	60%	80%
1	53,181	53,180	53,177	53,150	52,783	51,225	47,639
2	68,328	68,327	68,324	68,290	67,842	66,349	64,382
3	106,188	106,162	106,084	105,772	104,618	103,369	102,823
4	136,784	136,779	136,765	136,675	135,817	132,776	126,939
5	176,506	176,501	176,484	176,376	175,336	172,234	168,218
6	241,129	241,100	241,011	240,654	239,332	237,932	236,039
7	248,643	248,630	248,588	248,380	246,920	242,771	237,355
8	321,544	321,529	321,480	321,235	319,486	315,042	309,485

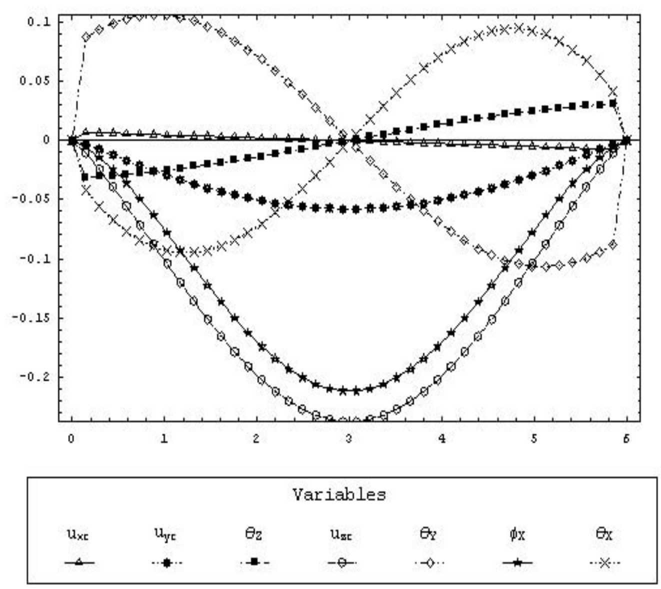

Figure 5. First mode shape for the cracked beam with cracks near the clamped ends

4. CONCLUSIONS

In the present paper a new formulation for a thin-walled I-beam with a fatigue crack was developed. A new finite element that includes the effect of the crack in a shear-deformable beam theory was introduced. The presence of a crack in a thin-walled I-beam model leads to the coupling of bending-longitudinal, bending-twisting and longitudinal-twisting motions. These coupling effects were observed and appropriately enhanced in the mode shapes of the cracked beam. The variations in the mode shapes of cracked beams with respect to the uncracked case, can be a useful tool for detection of cracks.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Adams, R.D. and Cawley P., 1985, "Vibration techniques in nondestructive testing", The Shock and Vibration Bulletin, Part 3, Naval Research Laboratory, Washington DC.

Chen, L.W. and Chen, C.L., 1988, "Vibration and stability of cracked thick rotating blades", Computers and Structures, Vol.28, pp. 67-74.

Cortínez, V.H. and Piovan, M.T., 2002, "Vibration and buckling of composite thin walled beams with shear deformability", Journal of Sound and Vibration, Vol.258(4), pp. 701-723.

Cortínez, V.H. and R.E. Rossi, 1998, "Dinámica de Vigas con Secciones Abiertas de Paredes Delgadas Deformables por Corte Sujetas a un Estado Inicial de Tensiones", Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería, Vol.14(3), pp.293-316.

Cortínez, V.H., Piovan, M.T. and R.E. Rossi, 1999, "Coupled Flexural-torsional vibrations of Timoshenko beams", Journal of Sound and Vibration, Vol.224(2), pp. 375-378.

Darpe, A.K., Gupta, K. and Chawla, A., 2004, "Coupled bending, longitudinal and torsional vibrations of cracked rotor", Journal of Sound and Vibration, Vol.269, pp. 33-60.

Dimarogonas, A.D., 1996, "Vibration of cracked structures: a state of the art review", Engineering Fracture Mechanics, Vol.55, pp. 831-857.

Gomes Cardoso, C., Doz, G.N. and Vidal de Brito, J.L., 2005, "Identificacao de danos em estructuras de viga por meio de suas propiedades dinamicas", Proceedings CILAMCE XXVI, Guarapari, ES, Brasil

Kisa, M. and Brandon, J., 2000, "The effects of closure of cracks on the dynamics of cracked cantilever beams", Journal of Sound and Vibration, Vol.238, pp. 1-18.

Nobile, L. and Viola, E., 2001, "Cracked beam formulation for structural analysis", Paper number 1840 Transactions SMIRT 16, Washington DC.

Papadopoulos, C.A. and Dimarogonas, A.D., 1992, "Coupled vibrations of cracked shafts", Journal of Vibrations and Acoustics, Vol.114, pp. 461-467.

Piovan, M.T. and Cortínez, V.C., 2005, "The transverse shear deformability in dynamics of thin walled composite beams: consistency of different approaches", Journal of Sound and Vibration, Vol.285(3), pp.721-733.

Salawu, O.S., 1997, "Detection of structural damage through changes in frequency: a review", Engineering Structures Vol.19(9), pp. 718-723.

Zheng, D.Y. and Fan, S.C., 2003, "Vibration and stability of cracked hollow-sectional beams", Journal of Sound and Vibration, Vol.267, pp. 933-954.

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