A COMPARATIVE ANALYSIS OF FAILURES CRITERIA APPLIED TO LONG BONES

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Abstract. *Bone is a special category of material, first because is a living tissue, in addition its mechanical properties changes remarkably with position, also its mechanical properties under compression differ from the tensile ones. The proposition of failures criteria to model bone tissue pass by the utilization of well establish criteria, as the ones based in maximum shear stress or based in distortion energy developed for ductile metals, or the ones for brittle materials as the ones based in maximum strain. As bones are remarkably anisotropic, some authors have suggested the utilization of criteria for composite materials. A simple model of a human femur is presented, loaded by a representative combination of axial, torsional and flexural loads. The most representative criteria were analyzed. A couple of more representative failure criteria are used to estimate the factor of safety and the angle of initial fracture at external surface of bone model.*

Keywords: *failure criteria, long bones, modeling*

1. INTRODUCTION

Bone is a material that is non-homogeneous, porous, anisotropic and is continuously remodeled as living tissue. There are two types of bone, the trabecular with pores (50% to 95%) interconnected and filled with marrow and the cortical with pores (5 to 10 %) and Haversian canals. The cortical bone surrounds trabecular bone, forming a external shell. Bones can grow, self-repair and be continuously remodeled by specialized cells that produce bone, the osteoblasts and cells that remove bone, the osteoclasts (Doblaré, 2004). These characteristics difficult the proposition of a specific failure criterion. Many authors as (Keyak, 2000) made attempts to use criteria of failure originally proposed to model behavior of metals (ductile and brittle) or composite materials to model bone behavior. One way of checking the applicability of various criteria to long bones is to compare results of experimental data of referred literature with the results of utilization of a numerical approach as finite element method (FEM). Another way is to use an analytic approach. In this work, after analyzing several failure criteria, a stress analysis of an analytic simple model of a human femur is utilized to make a comparative analysis of the applications of each of literature most successful failures criteria, to estimate factor of safety and initial angle of fracture at external surface of bone model.

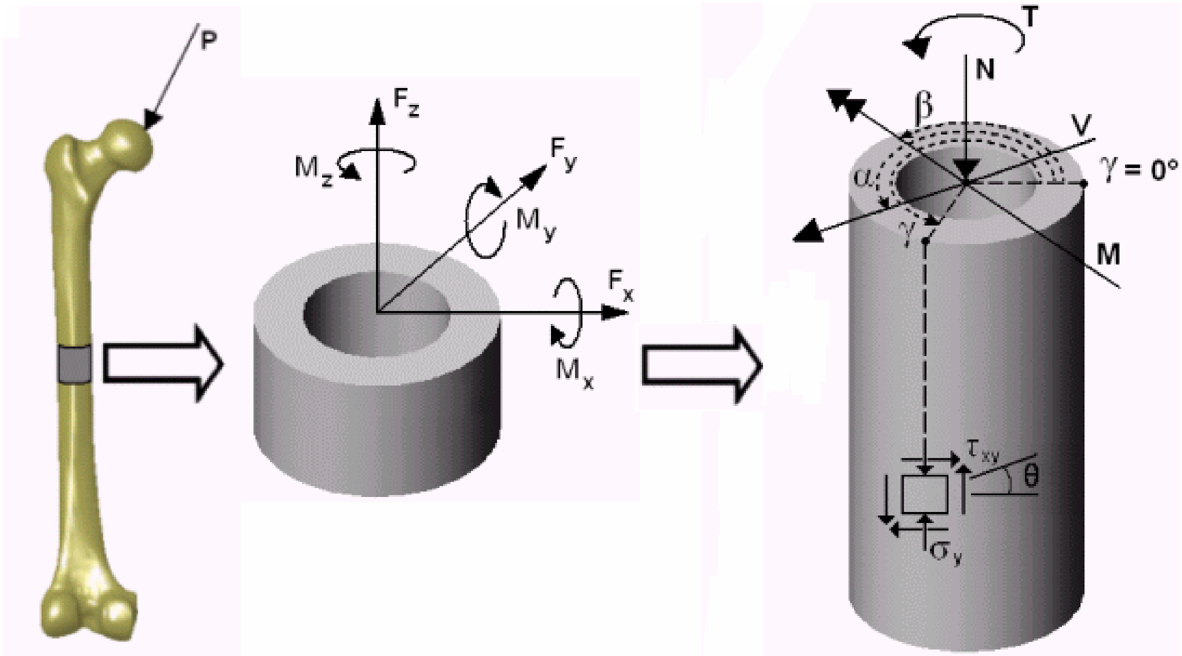
2. STRESS ANALYSIS

The stress analysis of a complex structure like a human long bone, as a femur, can be done through many ways, as numerical, experimental and analytic approaches. The numerical approach is probably the most popular nowadays mainly in function of the wide availability of computers and the utilization of specialized softwares, as the ones based in FEM. The experimental approach for another hand has an unquestionable advantage of working with real data. The implementation of former approaches can be very expensive to mention one disadvantage. Although an analytic approach tend to produce only simplified models of a more complex reality, it has the advantage of showing explicitly the relationship between main variables.

A simple analytic model of human femur is proposed with the main objective of use it to cover every stage of stress analysis from the definition of geometry, transversal section, loading, constrains and materials; up to the final achievement of, for example, the initial angle of failure at external surface of bone model. To carry out this objective several simplification had to be made in this simple femur model as, considering the transversal section as an uniform hollow circle (only considering the effect of cortical bone), considering only a static force *P* acting at the head of femur (not considering the effect of tendons positioned sideways), considering that the femur is simple supported and its material failure behavior can be modeled by one of the proposed failure criterion.

The steps covered by the stress analysis of a point at external surface of bone model can be resumed as: The internal loads F_x, F_y and F_z are obtained by the utilization of direction cosines at *P*. The internal moments M_x, M_y and M_z are obtained by the utilization of definition of moment: $M = \Delta \times P$. These loads are regrouped as *N*, *V*, *M* and *T* and used to calculate stress components (normal and shear). Mohr circle is then utilized to obtain principal stresses that are finally applied in a failure criterion.

Figure 1 shows the representation of a force P acting at the head of femur. In a generic section the effect of P is equivalent to three forces components F_x,F_y and F_z and three moments components M_x , M_y and M_z . Also is equivalent to forces: axial N and shear V and to moments: bending M and torsional T.



(a) (b) (c)

Figure 1: (a) A force P acting at the head of femur, (b) three forces components F_x,F_y and F_z and three moments components M_x , M_y and M_z and (c) forces N and V , moments M and T and angles α ,β , γ and θ .

where, α is the angle with the direction V, β is the angle of neutral axis M and γ is the angle that determine the point of interest, all of them referred to X axis (positive at γ = 0°). θ is a angle at bone model external surface.

The relationship between F_x,F_y , F_z and N, V and between M_x ,M_y , M_z and M , T are presented:

$$N = F_z \tag{1}$$

$$V = \sqrt{F_x^2 + F_y^2} \tag{2}$$

where,

$$F_x = \cos\phi_x P \ , \ F_y = \cos\phi_y P \ , F_z = \cos\phi_z P \text{ and } \alpha = \arctan\left(\frac{F_y}{F_x}\right)$$

$$M = \sqrt{M_x^2 + M_y^2} \tag{3}$$

$$T = M_z \tag{4}$$

where,

$$M_x = (\Delta_y F_z - \Delta_z F_y) \ , \ M_y = (\Delta_z F_x - \Delta_x F_z) , \ M_z = (\Delta_x F_y - \Delta_y F_x) \text{ and } \beta = \arctan\left(\frac{M_y}{M_x}\right)$$

where, Δ_x,Δ_y and Δ_z are respectively the components of position vector Δ that begins at the center of a generic section and ends at the point of contact of force P.

In this introductory model the transversal section of bone is modeled as a hollow circle. The axial stress σ_N expression is:

$$\sigma_N = \frac{N}{A} \tag{5}$$

The bending stress σ_F expression is:

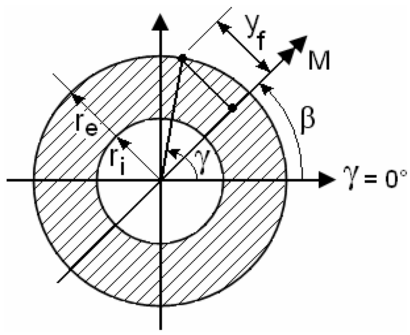
$$\sigma_F = \frac{M\ y_f}{I} \tag{6}$$

where,

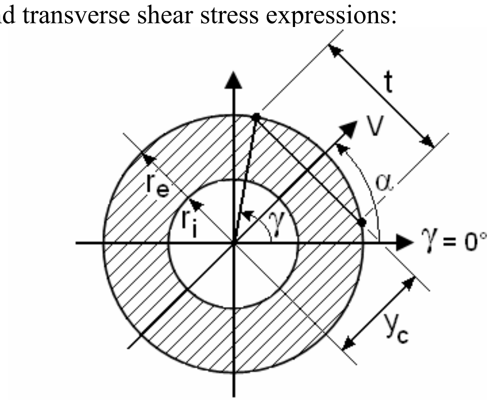
$$A = \pi\left(r_e^{\ 2} - r_i^{\ 2}\right),\ y_f = r_e\ sin(\gamma - \beta) \text{ and } I = \frac{\pi}{4}\left(r_e^{\ 4} - r_i^{\ 4}\right)$$

where, r_e is the outer radius, r_i is the inner radius, A is the area of transversal section, y_f is the perpendicular distance from neutral axis to surface of bone and I is the second moment of area.

Figure 2 shows angles and variables used at bending stress and transverse shear stress expressions:



(a)



(b)

Figure 2: Angles and the variables to use in: (a) bending stress expression and (b) transverse shear stress expression.

The transverse shear stress expression τ_V is:

$$\tau_V = \frac{VQ}{It} \tag{7}$$

The torsional stress expression τ_T is:

$$\tau_T = \frac{T\ r_e}{J} \tag{8}$$

where,

$$t = \left|2\, r_e sin(\gamma - \alpha)\right| \text{ and } J = \frac{\pi}{2}\left(r_e^{\ 4} - r_i^{\ 4}\right)$$

$$\text{for } 0 < y_c \le r_i:\ Q = \frac{2}{3}\left[\left(r_e\ sin(\gamma - \alpha)\right)^3 - \left(r_i^{\ 2} - \left(r_e \cos(\gamma - \alpha)\right)^2\right)^{3/2}\right] \text{and for } r_i < y_c \le r_e:\ Q = \frac{2}{3}\left[\left(r_e\ sin(\gamma - \alpha)\right)^3\right]$$

where, Q is the first moment of area, t is a width at a distance y_c from hollow circle center and J is the polar second moment of area.

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The Mohr circle is used to access principal stresses and maximum shear stress of a determinate point at bone model external surface, as well as its respective principal planes, as shown at Figure 3.

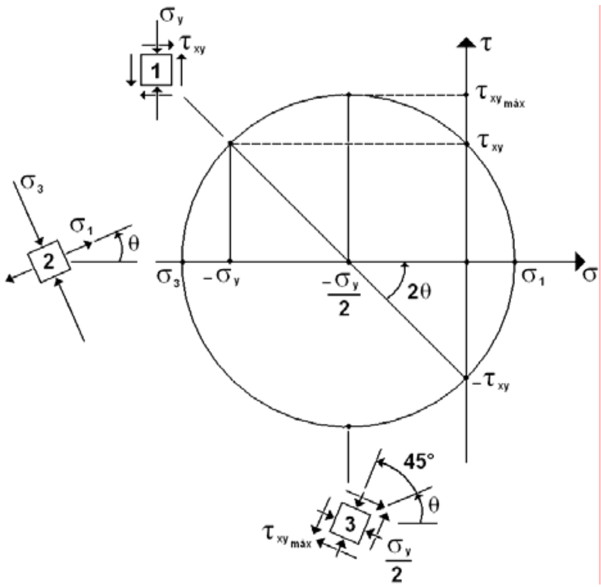


Figure 3: Mohr circle.

The principal stresses σ_1 and σ_3 and the maximum shear stress $\tau_{xy_{max}}$ for this Mohr circle are:

$$\sigma_1, \sigma_3 = \frac{-\sigma_y}{2} \pm \sqrt{\frac{\sigma_y^{\,2}}{4} + \tau_{xy}^{\,2}} \tag{9}$$

$$\tau_{xy_{max}} = \sqrt{\frac{\sigma_y^{\,2}}{4} + \tau_{xy}^{\,2}} \tag{10}$$

Where, $\sigma_y = \sigma_N + \sigma_F$ and $\tau_{xy} = \tau_V + \tau_T$. Numbers 1, 2 and 3 represent the same point at external surface of bone model of fig.1.c, at various inclinations: *1* correspond to θ = 0°, *2* correspond to the angle θ of σ_1 (notice that the normal fracture angle is $\theta_\sigma = \theta + 90^\circ$) and *3* corresponds to the angle were shear stress is maximum $\theta^* = \theta + 45°$ (notice that the shear fracture angles are $\theta_\tau^1 = \theta^*$ and $\theta_\tau^2 = \theta^* + 90^\circ$). Angle θ is calculated as follows:

$$\theta = \frac{1}{2}\arctan\left(\left|\frac{2\tau_{xy}}{\sigma_y}\right|\right) \text{ (Notice that the angle of Mohr circle is } 2\theta \text{ and the angle at bone model surface is } \theta\text{)} \tag{11}$$

3. FAILURE CRITERIA

Failure criteria are used to find out if a material will fail when submitted to a certain combination of normal and shear stresses in comparison with a simple test, as tension test, of same material. The failure can be, for example, by yielding or by rupture. In fact, there is no way of calculate theoretically the relation between stress components with yielding of a three-dimensional state of stress with the yielding of a uniaxial tensile test (Crandall, 1978). It is even more difficult to present failure criterion to such an unusual material like bone. To overcome such difficulties various authors as (Keyak, 2000) tried to use well known criteria created for ductile, brittle and composite materials to bone tissue. For failure criterion based in yielding, for example, there are six quantities that may be used (Boresi, 1985). The maximum principal stress, the maximum shear stress, the maximum strain, the strain energy density, the strain energy density of distortion and the maximum octahedral shear stress.

The maximum principal stress, also called Rankine criterion, has limited applicability in ductile materials, but works properly in brittle materials (only changing S_y by S_u, respectively yielding strength and ultimate strength).

The maximum shear stress, also called Tresca criterion, uses the idea that yielding begins when maximum shear stress τ_{max} reach the value of the yield shear stress $S_{S_y} = 0.5 S_y$, that occurs in the beginning of yielding in a tensile test.

The maximum strain, called Saint-Venant criterion, has limited applicability in ductile materials, but works properly in brittle materials (only changing ε_y by ε_u , respectively yielding strain and ultimate strain).

The strain energy density propose that yielding begun when the strain energy per unit of volume (or strain energy density) absorbed by a point of material is equal to strain energy density of the same material in the beginning of yielding in a tension test.

The strain energy density of distortion, also called von Mises criterion, uses the experimental fact that hydrostatic pressure have little effect in beginning yielding in ductile materials, in other words, variations of a volume of material not begin yielding. Only distortion of a volume of material can begin it, so this criterion propose that yielding begun when the strain energy density of distortion absorbed by a point of material is equal to strain energy density of distortion of the same material in the beginning of yielding in a tension test.

The maximum octahedral shear stress, where the octahedral planes are the eight symmetrical surfaces relative to principal directions, which normal stresses are hydrostatic and the shear stresses are called of octahedral shear stresses, states that beginning of yielding is supposed to occur whenever octahedral shear stress equals the octahedral shear stress for the same material submitted to beginning of yielding in a tension test.

3.1. Ductile criteria

Two of these six criteria of failure based in yielding are usual for determination of beginning of yielding of ductile and isotropic materials. They are Tresca and von Mises criteria. The Tresca criterion, for plane stress, predicts beginning of yielding whenever one or more of following expressions are true:

$$|\sigma_A| = S_y\,,\ |\sigma_B| = S_y \ \text{ or } |\sigma_A - \sigma_B| = S_y \tag{12}$$

Where, S_y is yielding strength, which is supposed be the same in tension and compression for ductile and isotropic materials. σ_A and σ_B are two of three principal stresses σ_1, σ_2 and σ_3. The von Mises criterion can be represented by the following expressions (material begins to yield when $\sigma_{eq} = S_y$):

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)^{1/2} \tag{13}$$

$$\sigma_{eq} = \left(\sigma_A{}^2 - \sigma_A\sigma_B + \sigma_B{}^2\right)^{1/2} \quad \text{(Plane stress)} \tag{14}$$

Notice that for ductile, isotropic materials, like steel, yielding is based only on the magnitude of principal stresses and not in principal stresses orientation, and as hydrostatic state of stress not affect yielding, only the magnitude of differences between principal stresses are important (Crandall,1978). A graphical comparison between Tresca and von Mises criteria, generated by a VB program, using expressions (12) and (14) is shown at Fig. 4:

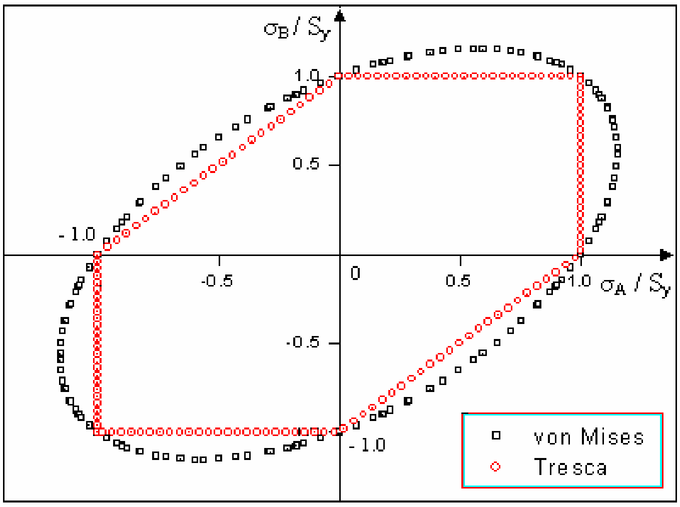


Figure 4 - Graphical comparison between Tresca and von Mises criteria.

Figure 4 shows Tresca and von Mises criteria (notice that the expressions (12) and (14) must be divided by S_y to reproduce fig.4): Every combination of σ_A / S_y and σ_B / S_y that is inside its graphical representation is elastic. At or outside borders it is plastic. Notice that exists combinations that already yielded if consider Tresca criterion but it is still elastic if consider von Mises criterion.

3.2. Brittle criteria

For brittle materials, like cast iron, five classical criteria are shown: Rankine, Saint-Venant, Coulomb-Mohr, Coulomb-Mohr modified and Hoffman. These criteria recognize differences between tensile strength S_{ut} and compressive strength $-S_{uc}$, which occurs to bones.
The Rankine criterion, also know as Maximum Normal Stress criterion, for plane stress, predicts that the fracture will occur whenever the most positive principal tensile stress reach the tensile strength S_{ut} or the most negative compressive principal stress reach the compressive strength $-S_{uc}$ of material:

The material will break if $\sigma_A = S_{ut}$, $\sigma_A = -S_{uc}$, $\sigma_B = S_{ut}$ or $\sigma_B = -S_{uc}$ (15)

The Saint-Venant criterion, also known as Maximum Normal Strain criterion, for plane stress, states that fracture will occur whenever the most positive principal strain reach the tensile ultimate strain ε_{ut} or the most negative compressive principal strain reach the compressive ultimate strain - ε_{uc} of material:

The material will break if $\varepsilon_A = \varepsilon_{ut}$, $\varepsilon_A = -\varepsilon_{uc}$, $\varepsilon_B = \varepsilon_{ut}$ or $\varepsilon_B = -\varepsilon_{uc}$ (16)

The expression (16), with principal strains ε_A and ε_B, must be transformed in an equivalent expression that shows the relations with σ_A and σ_B. To accomplish it elastic stress-strain relations are used (Beer, 1982):

$$\sigma_A = \frac{E(\varepsilon_A + \nu\,\varepsilon_B)}{1-\nu^2} \quad \text{and} \quad \sigma_B = \frac{E(\varepsilon_B + \nu\,\varepsilon_A)}{1-\nu^2}$$

Where ν is the Poisson ratio and the coordinates of the four extremities of figure of plane stress of Saint-Venant criterion are: 1st quadrant $\left(\frac{S_{ut}}{1-\nu}, \frac{S_{ut}}{1-\nu}\right)$, 2nd quadrant $\left(\frac{(-\alpha+\nu)S_{ut}}{1-\nu^2}, \frac{(1-\alpha\nu)S_{ut}}{1-\nu^2}\right)$, 3rd quadrant $\left(\frac{-\alpha S_{ut}}{1-\nu}, \frac{-\alpha S_{ut}}{1-\nu}\right)$ and 4th quadrant $\left(\frac{(1-\alpha\nu)S_{ut}}{1-\nu^2}, \frac{(-\alpha+\nu)S_{ut}}{1-\nu^2}\right)$. Notice that $S_{uc} = -\alpha S_{ut}$. (17)

The Coulomb-Mohr criterion is variation of Mohr criterion constructed with the aim of three simple tests: tension, compression and torsion. Coulomb-Mohr criterion, also know as Internal-Friction criterion, only needs tension and compression tests. In a plane $\sigma \times \tau$ two Mohr circles, correspondent to tension and compression tests are drawn, forming a classical figure, where every combination that is inside the set of the two tangent lines and the two circles are safe. In plane stress, as a function of two principal stresses σ_A and σ_B, the material will break as shown:

1st quadrant $(\sigma_A \geq 0, \sigma_B \geq 0)$: if $\sigma_A = S_{ut}$ or/and if $\sigma_B = S_{ut}$, 2nd quadrant $(\sigma_A \leq 0, \sigma_B \geq 0)$: if $-\frac{\sigma_A}{S_{uc}} + \frac{\sigma_B}{S_{ut}} = 1$, 3rd quadrant $(\sigma_A \leq 0, \sigma_B \leq 0)$: if $\sigma_A = -S_{uc}$ or/and if $\sigma_B = -S_{uc}$ and 4th quadrant $(\sigma_A \geq 0, \sigma_B \leq 0)$: if $\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = 1$.
Notice that σ_A and σ_B cannot be zero at same time. (18)

Experimental observations of failure of brittle materials show that Coulomb-Mohr criterion is non conservative in 2nd and 4th quadrants. An empirical adaptation on Coulomb-Mohr criterion called Coulomb-Mohr modified criterion was proposed to overcome it. In plane stress, as a function of two principal stresses σ_A and σ_B, the material will break as shown:

1st quadrant and 3rd quadrant are the same of expression (18), 2nd quadrant $(\sigma_B \geq 0)$: if $-S_{ut} \leq \sigma_A \leq 0$ then $\sigma_B = S_{ut}$ or if $-S_{uc} \leq \sigma_A < -S_{ut}$ then $-\dfrac{\sigma_A}{S_{uc}} + \dfrac{(S_{uc} - S_{ut})}{S_{ut} S_{uc}} \sigma_B = 1$ and 4th quadrant $(\sigma_A \geq 0)$: if $-S_{ut} \leq \sigma_B \leq 0$ then $\sigma_A = S_{ut}$ or if $-S_{uc} \leq \sigma_B < -S_{ut}$ then $\dfrac{(S_{uc} - S_{ut})}{S_{ut} S_{uc}} \sigma_A - \dfrac{\sigma_B}{S_{uc}} = 1$. Notice that σ_A and σ_B cannot be zero at same time.

(19)

The Hoffman criterion is referred by (Doblaré, 2004) and (Keyak, 2000) as criterion for brittle material used to predict fracture load and fracture pattern of proximal femur. The material breaks as shown:

$$C_1(\sigma_2 - \sigma_3)^2 + C_2(\sigma_3 - \sigma_1)^2 + C_3(\sigma_1 - \sigma_2)^2 + C_4\sigma_1 + C_5\sigma_2 + C_6\sigma_3 = 1 \quad (20)$$

where, C_i are materials parameters defined as: $C_1 = C_2 = C_3 = \dfrac{1}{2S_{ut}S_{uc}}$ and $C_4 = C_5 = C_6 = \dfrac{1}{S_{ut}} - \dfrac{1}{S_{uc}}$.

Figure 5 show graphically a comparison between failure criteria of brittle materials, generated by a VB program, using expressions (15), (17), (18), (19) and (20) (notice that these expressions must be divided by S_{ut} to reproduce fig.5):

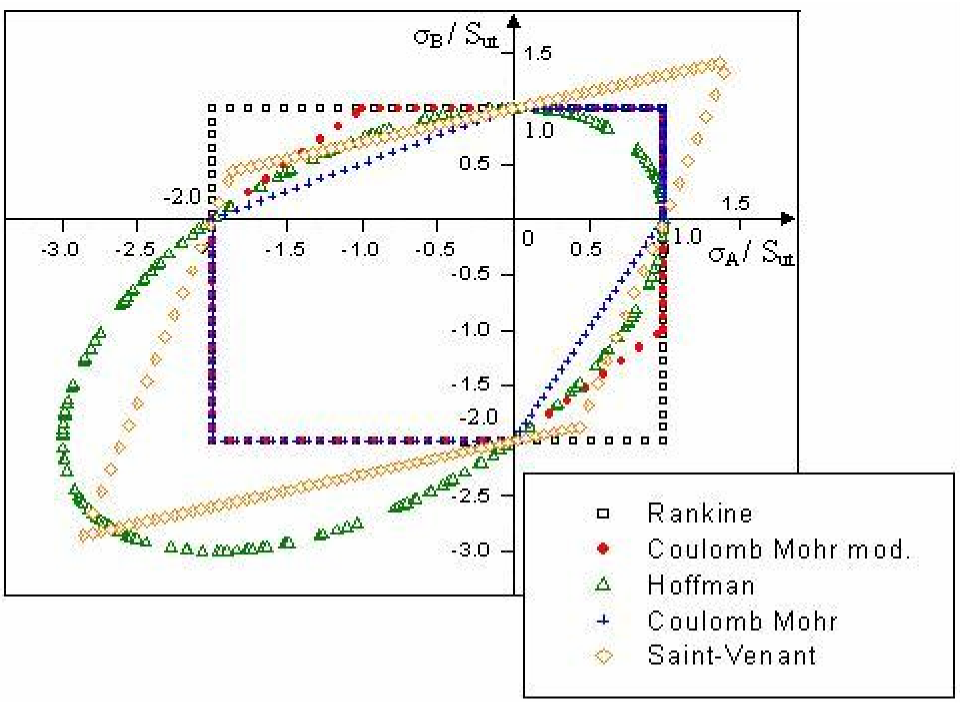


Figure 5 - Graphical comparison between Rankine, Saint-Venant, Coulomb-Mohr, Coulomb-Mohr modified and Hoffman criteria.

Notice that for fig. 5 was used $S_{uc} = -2S_{ut}$. For Saint-Venant criterion was used $\alpha = 2$ and $\nu = 0.3$. Every combination of σ_A and σ_B that is inside every graphical representation is elastic. At borders of graphical representation of each criterion the material breaks. Also notice that for Coulomb-Mohr modified criterion the points $(\sigma_A/S_{ut}, \sigma_B/S_{ut}) = (-1,1)$ and $(\sigma_A/S_{ut}, \sigma_B/S_{ut}) = (1,-1)$ represents pure torsion loading condition.

3.3. Composite materials criteria

Bone tissue has porous medium which failure is affected by all three stress invariant, with different strengths in tension and compression, and sensitive to hydrostatic pressure. Different measures of micro structure can be used to model bone material. Cowin introduced the term *fabric tensor* in bone mechanics. It was defined as "any positive definite, second-rank tensor that gives a local description of the architectural anisotropy, also called *fabric*". Many works has evidence that the fabric approach can be utilized for quantification of anisotropy of bones (Odgaard, 1997).
For non-isotropic materials, like composite materials, three criteria are shown: Tsai-Hill, Tsai-Wu and Cowin.
The Hill criterion is a modification of von Mises criterion to include effects of induced anisotropic behavior in initially isotropic metals during plastic deformation (Gibson, 1994):

$$A(\sigma_2-\sigma_3)^2+B(\sigma_3-\sigma_1)^2+C(\sigma_1-\sigma_2)^2+2D\tau_{23}^2+2E\tau_{31}^2+2F\tau_{12}^2=1 \quad (21)$$

where, the constants A, B, C, D, E and F are experimentally obtained through yield tests in six different directions assuming that yield strength are the same in tension and in compression:

$$A=\frac{1}{2}\left(\frac{1}{Y_2^2}+\frac{1}{Y_3^2}-\frac{1}{Y_1^2}\right),\ B=\frac{1}{2}\left(\frac{1}{Y_3^2}+\frac{1}{Y_1^2}-\frac{1}{Y_2^2}\right),\ C=\frac{1}{2}\left(\frac{1}{Y_1^2}+\frac{1}{Y_2^2}-\frac{1}{Y_3^2}\right), D=\frac{1}{2}\frac{1}{Y_{23}^2},\ E=\frac{1}{2}\frac{1}{Y_{31}^2} \text{ and } F=\frac{1}{2}\frac{1}{Y_{12}^2}$$

where, Y_i are the yield strength along axes 1,2 and 3, and Y_{ij} are the shear yield strength along axes 12, 23 and 31.
The Tsai-Hill criterion is an extension of Hill criterion for orthotropic composite material. The equation of Tsai-Hill criterion is:

$$\frac{\sigma_1^2}{s_L^2}-\frac{\sigma_1\sigma_2}{s_L^2}+\frac{\sigma_2^2}{s_T^2}+\frac{\tau_{12}^2}{s_{LT}^2}=1 \quad (22)$$

where,

$$s_L=Y_1,\ s_T=Y_2=Y_3 \text{ and } s_{LT}=Y_{12}$$

The Tsai-Wu criterion express in terms of the stress tensor and two material dependent tensors. The basic hypothesis is the existence of a failure surface in the stress space of the following form (Doblaré, 2004):

$$f(\sigma_k)=F_i\sigma_i+F_{ij}\sigma_i\sigma_j=1 \text{ for i, j, k = 1,2, …6} \quad (23)$$

where, F_i and F_{ij} are tensors of material and σ_i are the principal stresses.
The main disadvantage of Tsai-Wu criterion, as is usual in composite materials criteria, is the large number of constants that have to be determined experimentally. Also (Pietruszczak, 1999) refer that porous materials, as bones, are sensitive to third stress invariant and this criterion "may quite inadequate to describe the conditions at failure".
Cowin proposed a fracture criterion useful for porous materials and/or composites, based on the properties of the homogenized microstructure. The fracture criterion is a function of the stress state, the porosity and the fabric tensor (Doblaré, 2004):

$$G_{11}\sigma_{11}+G_{22}\sigma_{22}+G_{33}\sigma_{33}+F_{1111}\sigma_{11}^2+F_{2222}\sigma_{22}^2+F_{3333}\sigma_{33}^2+2F_{1122}\sigma_{11}\sigma_{22}+2F_{1133}\sigma_{11}\sigma_{33}+2F_{2233}\sigma_{22}\sigma_{33}=1 \quad (24)$$

where,

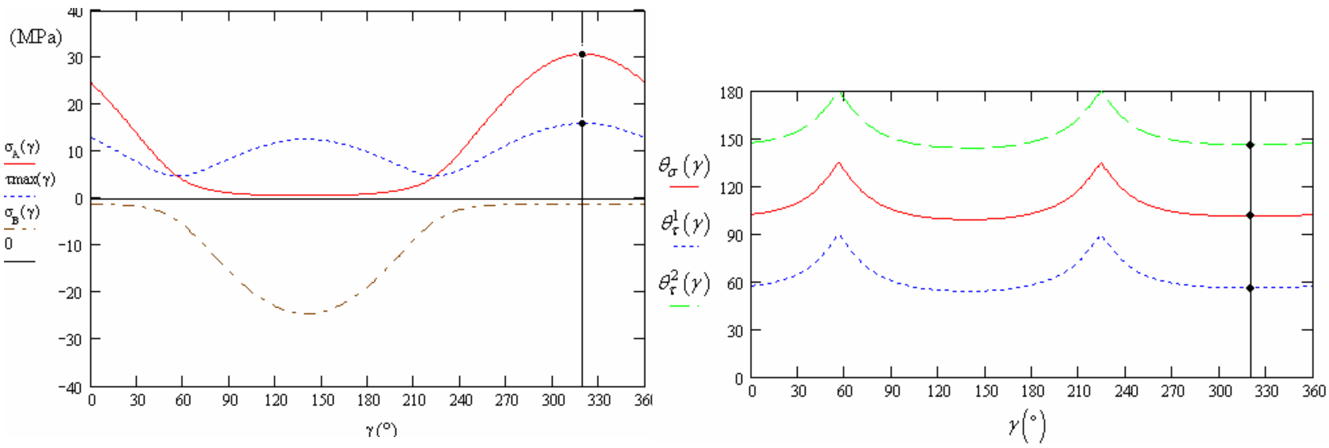
$$F_{iiii}=\frac{1}{\sigma_i^+\sigma_i^-},\ G_{ii}=\frac{1}{\sigma_i^+}-\frac{1}{\sigma_i^-} \text{ and } F_{iijj}=\frac{1}{2}\left(\frac{1}{\sigma_i^+\sigma_i^-}+\frac{1}{\sigma_j^+\sigma_j^-}-\frac{1}{2\sigma_{ij}^2}\right)-g(A)$$

The symbols σ_i^+,σ_i^- and σ_{ij} are the ultimate strength in tension, compression and shear respectively and F_{iiii}, G_{ii} and F_{iijj} are tensors, function of porosity and fabric tensor A.

Although many well established failure criteria were tentative used as failure criteria for bones, until now no model truly describe it. Apparently contrasting conclusions are obtained about which failure criterion describe more closely the failure behavior of bones. For one hand experimental evidences points that the use of failure theories as based on strain energy density of distortion, as von Mises criterion and based in maximum shear stress theories, as Tresca criterion, furnish reasonable performance, even not recognizing the existence of differences between tensile and compressive proprieties of bone material. For another hand other experimental evidences points to failure theory based in the maximum strain theories, as Saint-Venant criterion that recognizing the existence of differences between tensile and compressive proprieties, produce also reasonable results (Keyak, 2000). Finally the composite material criteria recognize the existence of differences between tensile and compressive proprieties and also account for the interaction between stresses, requires a large number of experimental constants that makes almost impracticable its utilization.

4. EXAMPLE

The data of an experimental test with a human femur was selected from specialized literature (Bergmann, 2001) to enter at the simple analytic model with inputs of geometry and loading. The section of human femur bone chosen has external diameter D = 0.032 m and internal diameter d = 0.016 m, the component of forces were F_x = -415 N, F_y = -424 N and F_z = -1654 N. The components of position vector were Δ_x = 0.07 m, $\Delta_y = 1.67 \cdot 10^{-3}$ m and Δ_z = 0.129 m. Using the expressions from (1) to (11), with the aid of a software like MathCad, the principal stresses σ_A , σ_B and maximum shear stress τ_{max} were calculate at bone model external surface, of chosen section, for $0 \le \gamma < 360^\circ$. Also the normal fracture θ_σ and shear fracture θ_τ^1 and θ_τ^2 angles were calculated at bone model external surface, of chosen section, for $0 \le \gamma < 360^\circ$. Figure 6.a shows the principal stresses σ_A and σ_B, and maximum shear stress τ_{max} in function of angle γ. Figure 6.b shows the normal fracture angle θ_σ and shear fracture angles θ_τ^1 and θ_τ^2 (the two shear fracture angles have a 90º difference), all at external surface of bone model, in function of angle γ :



(a) (b)

Figure 6: (a) Principal stresses $\sigma_A(\gamma)$ and $\sigma_B(\gamma)$, and maximum shear stress $\tau_{max}(\gamma)$ and (b) angles θ_σ , θ_τ^1 and θ_τ^2 .

From fig. 6a it is clear that both $\sigma_A = 31$ MPa and $\tau_{max} = 16$ MPa are maximum at $\gamma = 320^\circ$. Figure 6b shows the angles θ_σ , θ_τ^1 and θ_τ^2 in function of γ. Notice that the lines aren't the surface of fracture but the inclination of fracture surface at every γ. In this example the initial fracture angles would be $\theta_\sigma(320^\circ) = 101^\circ$, $\theta_\tau^1(320^\circ) = 56^\circ$ and $\theta_\tau^2(320^\circ) = 146^\circ$, when factor of safety of each criteria reach 1. To estimate the factor of safety n two different failure criteria were used, Tresca and Saint-Venant. The bone properties used were S_{yt} = 115 MPa, S_{ut} = 133 MPa, S_{uc} = 195 MPa and ν = 0.371 (Rapoff, 2007). From fig.6a at chosen point is clear that σ_A is positive and σ_B is negative configuring fourth quadrant point of $\sigma_A \times \sigma_B$ plane. Using expressions (12) and (17) is possible to estimate the factors of safety n_{Tresca} = 3.6 and $n_{\text{Saint-Venant}}$ = 3.7. If loading condition increase to the point of n =1 (at the same load line), then Tresca criterion would predict a initial shear fracture respectively at angles 56º and 146° and Saint-Venant criterion would predict a initial normal fracture angle of 101°.

5. CONCLUSIONS

Various existent failure criteria, for ductile, brittle and composite materials, were analyzed and compared. A couple of existent criteria that presented better correlation with experimental data of referenced literature were selected. A stress analysis of a simple analytic model of human femur was used in conjunction with each select failure criteria, Tresca or Saint-Venant, to estimate comparatively the factor of safety and the initial fracture angle at external surface of bone model. With the development of this research the analytic model will be improved to include the loading effect of tendons positioned sideways, the effect of trabecular bone and transversal sections different from a hollow circle. Also it is in course the utilization of finite element method to implement a more realistic model of human femur.

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