

A LIMIT ANALYSIS THEORY FOR POROUS MATERIAL

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Abstract. *This work proposes a limit analysis theory for porous materials. This study is analyzed from the basic equations of kinematics, equilibrium and constituve, and in the formulations achieved the total plastic strain rate is assumed to be given by the sum of the plastic strain rates considering the yielding and the compaction. The compaction is considered because the onset of the yielding in porous materials depends on the hydrostatic stress besides the deviatoric stress component. Hence in the development of the calculations, it is necessary to use the yield and compaction criterions as functions of the relative density. The von Mises's yield given for fully dense materials is changed, as proposed by Doraivelu. The collapse factor is obtained using the statical, kinematical and mixed formulations. The statical formulation, that is obtained from the modified form of Drucker's postulate and the kinematical compatibility, points the set of stresses out which the plastic admissibility, compaction and equilibrium are taken into account. A velocities field is given by the kinematical formulation, that is obtained from the subdifferential of the dissipation function and the kinematical compatibility, so that the value one is attributed to the reference external power. The mixed formulation is a combination of the plastic dissipation which is used in the kinematical formulation.*

Keywords: *Porous material, limit analysis, collapse factor.*

1. INTRODUCTION

Powder compaction constitutes an important step in the manufacture of products which use the powder as raw material, for example, advanced ceramics, pharmaceutics industry, automotive and aerospace applications. To obtain the components of these areas, with desire forms and final adequate properties, it is of extreme importance to fulfill all the technical conditions of the project and of the production.

Nowadays manufacturing of components and parts by forging of sintered powder materials is a subject considered of interest to industries. The success of the manufacturing processes depends on factors such as control of deformation to ensure uniform densification (Park, 1995).

Metal-working operations such as powder-metallurgy (P/M) extrusion and forging of sintered materials are currently employed for achieving required shape, full densification and mechanical properties (Doraivelu *et al*, 1984).

Theoretical approaches in conventional deformation processes have been made so as to obtain the working load required to cause plastic deformation and the density of the products, when sintered porous metals are employed as starting materials in working process such as forging and extrusion (Shima and Oyane, 1976).

In conventional plasticity concept, various theories and methods for analyzing problems in ordinary metal working processes have been developed, although they are not capable of being applied to the deformation of porous materials. Constancy of volume in conventional plasticity theory is assumed for the material undergoing deformation, and this assumption applies to pore-free metals. In the deformation of porous metals the volume does not remain constant (Shima and Oyane, 1976).

The studies to analyze the yielding of porous materials are more complicated than that of the fully dense materials because the yielding is caused not only as function of the deviatoric stress but also as function of the hydrostatic stress, therefore the formulation for the yield criterion considering such materials should be a function of both the second invariant of the deviatoric stress tensor and the first invariant of the stress tensor (Doraivelu *et al*, 1984).

In this work, a limit analysis theory for porous material is developed on the basis of kinematic, equilibrium and constitutive equations. All the concepts concerned with the traditional plasticity theory, pore-free material, are considered to this study. The statical, kinematical and mixed formulations are used to obtain the stresses and velocities fields.

In order to achieve the purpose of this study, the total plastic strain rate is assumed to be given by the sum of the strain rates considering the yielding and the compaction of the porous material. The compaction must be taken into account because of the volume does not remain constant when the yielding is reached in porous material. The yielding depends on both the deviatoric stress component and the hydrostatic stress.

The statical formulation is reached from the global form of Drucker's postulate combined with the equilibrium. The kinematical formulation is given from the global form of the subgradient of the dissipation function considering the admissible plastic strain rate field. The mixed formulation is a combination of the plastic dissipation which is used in the kinematical formula.

2. BASIC EQUATIONS

In order to model the problem, relations based in the kinematic, equilibrium and mechanical behavior of the material are shown in the following sections.

2.1. Kinematics

The small deformations are considered and all the formulations that are shown in this work are related to this assumption.

The contour Γ of a body that occupies a region limited β of the R^n is divided in two supplementary parts Γ_τ and Γ_v.

Forces on the surface Γ_τ and null velocities in the part Γ_v are assigned.

The admissibility of the motion is given so that if $v \in V$ then $v = 0$ if on the contour Γ_v.

The following compatibility equation relates the rate of deformation $\dot{\varepsilon}$ to the velocity field v.

$$\dot{\varepsilon} = Dv \tag{1}$$

where $v \in V$ and $\dot{\varepsilon} \in W$.

In the equation (1), D is considered the tangent operator of deformation and the set of rate of deformation fields $\dot{\varepsilon}$ is represented by W.

The rate of the total deformation is represented by the sum of the rates of the elastic and plastic deformations respectively:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \tag{2}$$

2.2. Equilibrium

The space of the external loads is V' or the set of all linear functions $< F, v >$ that is defined by their dual V and the space of the stresses is represented by W' dual of W.

The external power given by a reference load is defined by the linear function $< F, v >$:

$$< F, v > = \int_\beta b.v \, d\beta + \int_{\Gamma_\tau} \tau.v \, d\Gamma_\tau \tag{3}$$

In the equation (3), b and τ are body and surface forces respectively.

If the Principle of the Virtual Power is verified, an external power $\alpha < F, v >$ will be equilibrated with an internal power given by a field of stresses $T \in W'$, as shown below:

$$< T, Dv > = \alpha < F, v > \forall\ v \in V \tag{4}$$

The product $< T, Dv >$ is defined by the equation:

$$< T, Dv > = \int_\beta T.Dv \, d\beta \tag{5}$$

The proportionality factor α is used as the relationship between the real external power and the reference power.

2.3. Constitutive Equations

The set of stresses, where the plastic admissibility is taken into account, is defined by the following condition:

$$P(\rho) = \{T \in W' \mid f(T, \rho) \leq 0\} \tag{6}$$

The yield function $f(T,\rho)$ defines, in the space of the stresses, a convex region limited by $f(T,\rho)=0$ which is called elastic region where $f(T,\rho)\leq 0$

Yield patterns to porous materials, such as Corapcioglu, Shima and Doraivelu, have been developed. For instance, The Doraivelu's model is shown below (Doraivelu *et al*, 1984):

$$f(T,\rho) = (2+\rho^2)J_2 + \left(\frac{1-\rho^2}{3}\right)I_1^2 - (2\rho^2 - 1)T_D^2 \leq 0 \tag{7}$$

In the equation (7), ρ is the relative density – the relationship between the specific mass of the porous material and the dense material, J_2 is the second invariant of deviatoric stress component, I_1 is the first invariant of stress tensor, T_D is the yield stress of fully dense material.

The complementary equation is defined by the dot product of the yield parameter $\dot{\lambda}$ and its yield fuction $f(T,\rho)$, as the following equation:

$$f.\dot{\lambda} = 0 \mid f \leq 0, \dot{\lambda} \geq 0 \tag{8}$$

The set of the stresses $Q(\rho)$, where the compaction function is taken into account, is defined as a function of the sum of the elements of the tensor's main diagonal and the relative density ρ :

$$Q(\rho) = \{T \in W' \mid g(trT, \rho) \leq 0\} \tag{9}$$

The complementary equation is defined by the dot product between the compaction parameter $\dot{\mu}$ and the compaction function $g(trT,\rho)$, as the following equation:

$$g.\dot{\mu} = 0 \mid g \leq 0, \dot{\mu} \geq 0 \tag{10}$$

When porous material is considered, the complementary equation (10) must be observed. If the porous material is not being compacted, the compaction function will be represented by negative values $g(trT,\rho)<0$ and the compaction parameter will be given by a null value $\dot{\mu}=0$. When the plastic case is considered, $g(trT,\rho)=0$ and $\dot{\mu}>0$, the material will be compacted and the relative density's value will be increased.

The plastic strain rate is assumed to be given by the sum of the strain rates considering the yield $\dot{\varepsilon}_f^p$ and the compaction $\dot{\varepsilon}_g^p$.

$$\dot{\varepsilon}^p = \dot{\varepsilon}_f^p + \dot{\varepsilon}_g^p \tag{11}$$

The internal variable $\dot{\varepsilon}^p$ can be defined by the flow law, that is, the mathematical representation of the normality rule.

The plastic strain rate, which is related to the yield function f by the normality law, is represented by the following equation:

$$\dot{\varepsilon}_f^p = \dot{\lambda}\, \nabla_T f(T,\rho) \tag{12}$$

The plastic strain rate, which is related to the compaction function g by the normality law, is represented by the following equation:

$$\dot{\varepsilon}_g^p = \dot{\mu}\, \nabla_T g(trT, \rho) \tag{13}$$

The equation (13) can be rewritten to simplify the next formulations:

$$\dot{\varepsilon}_g^p = \dot{\mu}\, \nabla_{trT} (g(trT, \rho)) I_{3x3} \tag{14}$$

The trace of the deformation tensor represented by the expression (14) is:

$$\dot{\varepsilon}_{gv}^p = tr(\dot{\varepsilon}_g^p) = 3\, \dot{\mu}\, \nabla_{trT} g(trT, \rho) \tag{15}$$

As a result:

$$\dot{\varepsilon}_g^p = \frac{1}{3} (\dot{\varepsilon}_{gv}^p)\, I_{3x3} \tag{16}$$

To model the problem the Drucker's postulate (Lubliner, 1990) is considered.

$$(T - T^*).\dot{\varepsilon}^p \geq 0\ \forall\ T^* \in P \cap Q \tag{17}$$

P is the set of the stresses considering the porous materials, these stresses are function of the relative density in a space of stresses where there is the admissibility plastic.

Q is the set of the stresses where the compaction of the material occurs, these stresses function of the relative density.

Using the equality (11):

$$(T - T^*).(\dot{\varepsilon}_f^p + \dot{\varepsilon}_g^p) \geq 0\ \ \forall\ \ T^* \in P \cap Q \tag{18}$$

The condition (18) can be rewritten:

$$(T - T^*).\dot{\varepsilon}_f^p + (T - T^*).\dot{\varepsilon}_g^p \geq 0\ \ \forall\ \ T^* \in P \cap Q \tag{19}$$

The following condition is based on equation (16):

$$(T - T^*).\dot{\varepsilon}_f^p + (T - T^*).\frac{1}{3}(\dot{\varepsilon}_{gv}^p) I_{3x3} \geq 0\ \ \forall\ \ T^* \in P \cap Q \tag{20}$$

By performing the dot product:

$$(T - T^*).\dot{\varepsilon}_f^p + \frac{1}{3}\dot{\varepsilon}_{gv}^p\, tr(T - T^*) \geq 0\ \ \forall\ \ T^* \in P \cap Q \tag{21}$$

The condition (21) can be rearranged:

$$T.\dot{\varepsilon}_f^p + \frac{1}{3}\dot{\varepsilon}_{gv}^p\, trT \geq T^*.\dot{\varepsilon}_f^p + \dot{\varepsilon}_{gv}^p \frac{1}{3} trT^*\ \ \forall\ \ T^* \in P \cap Q \tag{22}$$

The function of the plastic dissipation is given by:

$$\aleph(\dot{\varepsilon}^p) = T.\dot{\varepsilon}^p \tag{23}$$

According to the equality (11), the dissipation is represented as:

$$\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) = T.\dot{\varepsilon}_f^p + T.\dot{\varepsilon}_g^p \tag{24}$$

Using the equality (16):

$$\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) = T.\dot{\varepsilon}_f^p + T.\frac{1}{3}(\dot{\varepsilon}_{gv}^p) I_{3x3} \tag{25}$$

or by performing the dot product, the dissipation is given by the following expression:

$$\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) = T.\dot{\varepsilon}_f^p + \dot{\varepsilon}_{gv}^p \frac{1}{3} trT \tag{26}$$

where $\frac{1}{3}trT$ is the hydrostatic pressure associated with the stresses tensor T .

According to the expressions (22) and (26), the next condition is obtained:

$$\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) \geq T^*.\dot{\varepsilon}_f^p + \dot{\varepsilon}_{gv}^p \frac{1}{3} trT^* \quad \forall \ T^* \in P \cap Q \tag{27}$$

The variation T^* is restricted to the set $P \cap Q$ and it can be concluded that the dissipation is equal the supreme value of the plastic admissible stresses, which gives the highest value of the dot product, as the equality:

$$\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_{gv}^p) = T.\dot{\varepsilon}_f^p + \dot{\varepsilon}_{gv}^p \frac{1}{3} trT = \sup_{T*} \{T^*.\dot{\varepsilon}_f^p + \dot{\varepsilon}_{gv}^p \frac{1}{3} trT^*\} \ \Big| \ T^* \in P \cap Q \tag{28}$$

The subgradient of the dissipation function $\partial\aleph(\dot{\varepsilon}^p)$ is defined by the following condition:

$$\partial\aleph(\dot{\varepsilon}^p) \equiv \{T \ \Big| \ \aleph(\dot{\varepsilon}^{p*}) - \aleph(\dot{\varepsilon}^p) \geq (\dot{\varepsilon}^{p*} - \dot{\varepsilon}^p).T \ \ \forall \ \ \dot{\varepsilon}^{p*} \in W\} \tag{29}$$

According to equality (11), the expression (29) is modified:

$$\partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) \equiv \{T \ \Big| \ \aleph(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_g^{p*}) - \aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) \geq (\dot{\varepsilon}_f^{p*} - \dot{\varepsilon}_f^p).T + (\dot{\varepsilon}_g^{p*} - \dot{\varepsilon}_g^p).T \ \ \forall \ \ (\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_g^{p*}) \in W\} \tag{30}$$

Taking into account the dissipation (24):

$$\partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_g^p) \equiv \{T \ \Big| \ \aleph(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_g^{p*}) - T.\dot{\varepsilon}_f^p - T.\dot{\varepsilon}_g^p \geq (\dot{\varepsilon}_f^{p*} - \dot{\varepsilon}_f^p).T + (\dot{\varepsilon}_g^{p*} - \dot{\varepsilon}_g^p).T \ \ \forall \ \ (\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_g^{p*}) \in W\} \tag{31}$$

Using the equality (16), the expression above is modified:

$$\partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_{gv}^p) \equiv \{T \ \Bigg| \ \aleph(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) - T.\dot{\varepsilon}_f^p - \frac{1}{3}\dot{\varepsilon}_{gv}^p trT \geq (\dot{\varepsilon}_f^{p*} - \dot{\varepsilon}_f^p).T + \frac{1}{3}(\dot{\varepsilon}_{gv}^{p*} - \dot{\varepsilon}_{gv}^p) trT \ \ \forall \ \ (\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \in W\} \tag{32}$$

In such case $\partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_{gv}^p)$ is represented by the expression:

$$\partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_{gv}^p) \equiv \{T \;\Big|\; \aleph(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \geq \dot{\varepsilon}_f^{p*}.T + \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} trT \;\; \forall \;\; (\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \in W\} \qquad (33)$$

Then the compact form of the constitutive equation can be given as:

$$T \;\in\; \partial\aleph(\dot{\varepsilon}_f^p, \dot{\varepsilon}_{gv}^p) \qquad (34)$$

3. COLLAPSE FACTOR BY THE STATICAL FORMULATION

The Drucker’s postulate in global form is pointed out:

$$<(T^* - T), \dot{\varepsilon}^p> \;\leq\; 0 \;\; \forall \;\; T^* \in P \cap Q \qquad (35)$$

According to the compatibility equation (1), the Drucker’s postulate is shown:

$$<(T^* - T), Dv> \;\leq\; 0 \;\; \forall \;\; T^* \in P \cap Q \qquad (36)$$

The following expression is the condition (36) solved:

$$<(T^*, Dv> \;\leq\; <T, Dv> \;\; \forall \;\; T^* \in P \cap Q \qquad (37)$$

According to the equilibrium (4), the above condition is represented:

$$<T^*, Dv> \;\leq\; \alpha <F, v> \;\; \forall \;\; T^* \in P \cap Q \qquad (38)$$

If the variation of T^* is restricted to equilibrated stresses:

$$S_{\alpha*} = \{T^* \in W' \;\Big|\; <T^*, Dv> \;=\; \alpha^* <F, v> \;\; \forall \;\; v \in V\} \qquad (39)$$

In such case the representation of the expression (38) is modified:

$$\alpha^* <F, v> \;\leq\; \alpha <F, v> \;\; \forall \;\; T^* \in P \cap Q \cap S_{\alpha*} \qquad (40)$$

The optimum problem to statical formula is rewritten:

$$\alpha = \sup_{\alpha*, T*, \rho*} \alpha^* \;\Big|\; T^* \in P \cap Q \cap S_{\alpha*} \qquad (41)$$

The field of stresses and the collapse factor is calculated when the limit analysis problem is solved with the statical formulation.

The statical formulation matches the first collapse theorem:

Any external loading is supported by a structure if the produced field of stresses satisfies plastic admissibility and obeys the equation of equilibrium (Sneddon and Hill, 1964).

4. COLLAPSE FACTOR BY THE KINEMATICAL FORMULATION

Taking into account the kinematical condition (1) and if the variations in the plastic strain rate $\dot{\varepsilon}^{p*}$ are restricted to kinematically admissible plastic strain rate field, the following equality is obtained:

$$\dot{\varepsilon}^{p*} = Dv^* \qquad (42)$$

where $v^* \in V$ and $\varepsilon^{p*} \in W$.

By the equation (11), the plastic strain rate is compounded by two components of plastic deformation, considering the yield $\dot{\varepsilon}_f^{p*}$ and the compaction $\dot{\varepsilon}_g^{p*}$:

$$\dot{\varepsilon}^{p*} = \dot{\varepsilon}_f^{p*} + \dot{\varepsilon}_g^{p*} \tag{43}$$

where $\dot{\varepsilon}_f^{p*} \in W$ and $\dot{\varepsilon}_g^{p*} \in W$.

Using the equality (16), $\dot{\varepsilon}_g^{p*}$ is shown as a function of the trace of deformation tensor to simplify the next formulations:

$$\dot{\varepsilon}_g^{p*} = \frac{1}{3}\dot{\varepsilon}_{gv}^{p*}\ I_{3x3} \tag{44}$$

where $(\dot{\varepsilon}_{gv}^{p*}\ I_{3x3}) \in W$.

Taking into account the equalities (43) and (44), the kinematical compatibility (42) can be rewritten:

$$\dot{\varepsilon}_f^{p*} + \frac{1}{3}\dot{\varepsilon}_{gv}^{p*}\ I_{3x3} = Dv^* \tag{45}$$

where $(\dot{\varepsilon}_{gv}^{p*} I_{3x3}), \dot{\varepsilon}_f^{p*} \in W$ and $v^* \in V$.

Using the equality (45) in the equilibrium (4) the expression below is obtained:

$$< \dot{\varepsilon}_f^{p*}, T > + < \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} trT > \ = \ \alpha < F, v^* > \quad \Big| \quad \dot{\varepsilon}_f^{p*} + \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} I_{3x3} = Dv^* \tag{46}$$

The global form of the condition (33) is rewritten in order to obtain the kinematical formulation:

$$\chi(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \ \geq \ < \dot{\varepsilon}_f^{p*}, T > + < \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} trT > \ \forall \ (\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*} I_{3x3}) \in W \tag{47}$$

Replacing the equilibrium (46) in the condition (47):

$$\chi(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \ \geq \ \alpha < F, v^* > \quad \Big| \quad \dot{\varepsilon}_f^{p*} + \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} I_{3x3} = Dv^* \tag{48}$$

If the value one is attributed to the reference external power, that is, field of velocities is chosen so that the value one is attributed to the reference external power:

$$< F, v^* > = 1 \tag{49}$$

Hence the optimum problem to the kinematical case is obtained:

$$\alpha = \inf_{v^*, \dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}} \chi(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) \quad \left| \begin{array}{l} < F, v^* > = 1 \\ \dot{\varepsilon}_f^{p*} + \frac{1}{3}\dot{\varepsilon}_{gv}^{p*} I_{3x3} = Dv^* \end{array} \right. \tag{50}$$

The kinematical formulation matches the second collapse theorem:

The inequality sign in (48) is for all kinematically admissible strain rate fields, that is, all admissible collapse mechanisms. The stronger inequality – where the equality sign is suppressed – is a condition imposed for the capability of the structure of supporting the external loads (Sneddon and Hill, 1964).

5. COLLAPSE FACTOR BY THE MIXED FORMULATION

The mixed formulation is a combination of the plastic dissipation and the kinematical formulation.

Considering the global form of the plastic dissipation (28):

$$\chi(\dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}) = \sup_{T^*} < T^*, \dot{\varepsilon}_f^{p*} + \dot{\varepsilon}_{gv}^{p*} \frac{1}{3} trT^* > \Big| \; T^* \in P \cap Q \qquad (51)$$

Replacing the plastic dissipation (51), in its global form, in the kinematical formulation (50), the mixed condition is reached:

$$\alpha = \inf_{v^*, \dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}} \; \sup_{T^*} < T^*, \dot{\varepsilon}_f^{p*} + \dot{\varepsilon}_{gv}^{p*} \frac{1}{3} trT^* > \left| \begin{array}{l} < F, v^* > = 1 \\ T^* \in P \cap Q \\ \dot{\varepsilon}_f^{p*} + \frac{1}{3} \dot{\varepsilon}_{gv}^{p*} I_{3x3} = Dv^* \end{array} \right. \qquad (52)$$

A more compact form of the mixed formulation can be obtained:

$$\alpha = \inf_{v^*, \dot{\varepsilon}_f^{p*}, \dot{\varepsilon}_{gv}^{p*}} \; \sup_{T^*} < T^*, Dv^* > \left| \begin{array}{l} < F, v^* > = 1 \\ T^* \in P \cap Q \end{array} \right. \qquad (53)$$

The fields of both velocities and stresses are obtained by the result of the condition (53), which is a stationary point, that is, a saddle point.

6. CONCLUSIONS

The use of the limit analysis theory can improve several factors in the project development, in order to obtain lighter products, with greater resistance and more economically, as well as to avoid the plastic collapse.

A variational formulation for the limit analysis of porous materials was proposed. The strains due to both yield and compaction stresses were considered in the formulations, apart from the equilibrium, constitutive equation and the compatibility equation.

Drucker's postulate in global form is taken into account to obtain the statical formulation, which coincides with the first collapse theorem and shows the field of stresses that satisfies the compaction of the material and the plastic admissibility, besides obeying the equilibrium condition. The kinematical formulation is obtained taking into account the condition of the subgradient of the dissipation function, the kinematical compatibility and the reference external power. From this formulation all admissible collapse mechanisms are given, and the capacity of the body of supporting the external loads. By the combination of the plastic dissipation and the kinematical formulation, the mixed formulation is obtained, which points out a stationary point, that is, the fields of velocities and stresses.

7. REFERENCES

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