MODAL PARAMETERS IDENTIFICATION USING ONLY RESPONSE DATA - STOCHASTIC SUBSPACE IDENTIFICATION AND FREQUENCY DOMAIN DECOMPOSITION -

Thiago Caetano de Freitas, tcfreitas@gmail.com
João Antônio Pereira, japereir@dem.feis.unesp.br
UNESP-FEIS "Faculdade de Engenharia de Ilha Solteira", Av. Brasil, 56, CEP: 15385-000, Ilha Solteira – SP, Brasil

Abstract. *Modal analysis involving output-only measurements is still a challenge that requires the use of special modal identification techniques. The present paper discusses the basic concepts and the implementation of a time and a frequency domain output-only based methods, the Stochastic Subspace Identification (SSI) and Frequency Domain Decomposition (FDD), respectively . The advantage of these output-only based methods is that no artificial excitation needs to be applied to the structure, they can use the own operation condition of the structure as the excitation condition. The SSI method estimates the state sequences directly from the response data and the modal parameters are estimated by using the eigenvalues decomposition of the state matrix. The inplementation steps of the procedure are based in the well-known numerical linear algebra algorithms and the SVD and QR decomposition. The FDD method is based on the decomposition of the power spectral density matrix by using SVD. The decomposition of the spectral density matrix for the frequency line corresponding to a resonance amplitude peak, allows the estimating of that corresponding mode shape of the model. The methodologies are evaluated with experimental data and the results have been shown to be promising to identify the structural modal parameters by taking output-only data.*

Keywords: *Output-Only Modal Analysis, Stochastic Subspace Identification, Frequency Domain Decomposition, SVD, QR decomposition*

1. INTRODUCTION

Nowadays, the requirements of the market have demanded machines and equipment more and more elaborated, with performance and production capacity each time higher and to accessible costs. In this context, the interest for the study and development of structural techniques of identification, more efficient for analysis of performance and structural integrity of systems is quickly becoming increasing.

Experimental Modal Analysis is one of these techniques; they have been used to study the structural dynamic behavior of a model through a matricial formulation that allows the estimating of the vibration parameters of the model. This formulation permits one to write a direct relationship between the excitation and the response of the structure in terms of its modal parameters. In the experimental test, this input-output relationship is obtained by measuring the excitation and the response of the structure for a set of selected measuring points. Once obtained the data, one defines a set of complex functions that establish a direct relationship between the excitation force and the responses of the structure. For a force applied in a point *j* and the corresponding response measured in a point *i* of the structure, one can define the called Frequency Response Function (FRF) of the model or its equivalent in the time domain, the Impulse Response Functions (IRF).

The Modal Analysis based on these function as well as its application have been widely studied and currently exists a vast literature covering the main theoretical aspects, as well as the practical aspects related with the experimental tests, (Ewins, 1984; Maia et al., 1997; Allemang, 1999)

In the convectional modal analysis, it is necessary the measuring of the input excitation and the response of the model in order to obtain the respective FRF(s). Generally these tests are carried out in laboratory with well controlled conditions and the excitation of the structure is made artificially. However, the vibrate-acoustic behavior of a structure in operation conditions (real conditions of load), can present different behavior of a laboratory test. It has the effect of pre-tensions, effect of suspensions, effect of ambient conditions and others. Therefore, the study of the behavior of the structure by using its proper operation conditions as excitation, it could be more representative then a laboratory tests and the identification of the modal parameters of the model it would be more realistic.

This paper discusses the application of two techniques of the modal parameters identification using only the response of the structure - a non-parametric technique based on Frequency Domain Decomposition (FDD), (Brincker, 2000a; Brincker, 2001), and a parametric technique based on the raw data time domain, the Stochastic Subspace Identification (SSI) technique, (Van Overschee e De Moor, 1996; Peeters and Roeck,1999). The implemented algorithms are evaluated with experimental data and the identification of the modal parameters of a frame structure using only the response data are discussed.

2. FREQUENCY DOMAIN DECOMPOSITION (FDD)

The Frequency Domain Decomposition technique as discussed in Brincker (2000a), it is an extension of the classic frequency domain approach. In the classic frequency domain approach, the parameters are estimated directly from the use of Discrete Fourier Transform, that allows an estimating of well separated modes directly from the analysis of the power spectrum density matrix (PSD), at the frequency peaks. This technique has some limitations concerning of its accuracy in the identification process. It gives a good estimating of natural frequencies and modes if the modes are well separated and the estimating of the damping it is difficulty in most cases.

The proposed technique removes some disadvantages associated with the classic approach such as the heavy dependency on the frequency resolution of the estimated power spectral density and difficulties of to work with close modes. Nevertheless, it keeps the important characteristic of being users friendly and it is still providing a physical understanding of the estimated parameters of the model if one observes the spectral density function as in the previous case.

The FDD technique estimates the parameters of the model directly from the measured density output matrix, by using Singular Value Decomposition (SVD). The main concept and formulation of the approach come from the relationship between the input *x(t)* and corresponding output *y(t)* for an invariant linear system, (Papoulis, 1991), Eq.(1) and the assumption that force is a white noise type presenting a flat spectral density function in the range of analysis, that means, $G_{xx}(j\omega)$ is equal a constant *C*.

$$G_{yy}(j\omega) = \overline{H}(j\omega)G_{xx}(j\omega)H^T(j\omega) \tag{1}$$

$G_{xx}(j\omega)$ is the input Power Spectral Density Matrix (PSD) and $G_{yy}(j\omega)$ is the output PSD matrix. For *r* inputs, $G_{xx}(j\omega)$ is of order *r*x*r* and the output matrix $G_{yy}(j\omega)$ it is of order *m*x*m*, being *m* the number of measured responses. *H(jω)* it is the Frequency Response Function matrix (FRF) of order *m*x*r*.

The Eq. (1) can be redefined writing the matrix $\overline{H}(j\omega)$ in the partial fraction form and assuming $G_{xx}(j\omega)$ constant. In this case, one can define, after some mathematical manipulations, see detail in (Borges, 2006), a similar expression for the spectral density function in terms of a sun of modes contribution. For a specific frequency ω_k, one can assume that only a limited number of modes will contribute significantly for the residue, typically one or two modes. This permits one to fix the number of modes, which allows a definition of a set of modes of interest, denoted by Sub (ω). Then, in the case of structures lightly damped, the output spectral density matrix can be written like Eq. (2).

$$G_{yy}(j\omega) = \sum_{k=p}^{q} \frac{d_k \psi_k \psi_k^T}{j\omega - \lambda_k} + \frac{\overline{d}_k \overline{\psi}_k \overline{\psi}_k^T}{j\omega - \overline{\lambda}_k} \tag{2}$$

In the FDD identification, the firs step is to estimate the PSD matrix. Once, the estimated matrix $\hat{G}_{yy}(j\omega)$ is known at discrete frequencies $\omega = \omega_i$, it is then decomposed by using Singular Value Decomposition (SVD), Eq. 3:

$$G_{yy}(j\omega_i) = U_i S_{+i} U_i^H \tag{3}$$

where the matrix $U_i = [u_{i1}, u_{i2}, \ldots\ldots, u_{im}]$ is a unitary matrix holding the singular vectors u_{ij} and S_i is a diagonal matrix holding the scalar singular values s_{ij}. In the region near a peak corresponding to the *kth* mode the dominating amplitude of the spectrum is due to that mode, or maybe a possible close mode. If only the *kth* is dominating, the first singular vector u_{i1} is an estimate of the corresponding mode shape, Eq. 4:

$$\hat{\phi} = u_{i1} \tag{4}$$

and the corresponding singular value is the PSD function of a equivalent single degree of freedom (SDOF) system.

This PSD function is identified around of the peak by comparing the estimated mode shape $\hat{\phi}$ with these singular vectors obtained for the others frequency lines around of the peak. As long as the singular vector present a high correlation with $\hat{\phi}$ measured by using the concept of Modal Assurance Criterion (MAC) , this corresponding singular value belongs to the SDOF density function, that will be used to estimate the parameters of the model.

The natural frequencies and damping ratio are obtained from the piece of the SDOF density function around of the peak of the PSD, (Borges, 2006). In this case, the SDOF function is taken back to the time domain by inverse fast Fourier transform (IFFT), and the frequency and the damping is estimated from the crossing times and the logarithmic decrement of the corresponding SDOF autocorrelation function.

3. STOCHASTIC SUBSPACE IDENTIFICATION (SSI)

The Stochastic Subspace Identification (SSI) technique, presented by Van Overschee and De Moor (1996) is becoming a consolidating method, being one of those methods more indicated for identification of systems submitted to natural excitation condition. This method identifies a stochastic state space model from output-only measurements. The state space model is formulated and solved using discrete state space formulations of the form, Eq. (5) e (6):

$$x_{k+1} = A_d x_k + B_d u_k \tag{5}$$

$$y_k = C_d x_k + D_d u_k \tag{6}$$

In the practice, always exist uncertainties in the model, associated with the noise in the processes and in the measurement. If the stochastic components, noise of the process, *w*k, and the measurement, *v*k, they are included in the Eq. (5) and (6), the discrete system model can be redefined to include the uncertainties too, Eq. (7) and (8):

$$x_{k+1} = A_d x_k + B_d u_k + w_k \tag{7}$$

$$y_k = C_d x_k + D_d u_k + v_k \tag{8}$$

The noise is present due to the disturbances and modeling mistakes, as well as, due to the measuring uncertainties due to the sensor. The correct determination of the characteristics of each individual component of the noise is difficult and, therefore, makes necessary some assumptions. In this case, it is assumed that the components of the noise, although non measured, they have zero mean and covariance matrix given by Eq. (9):

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix}\begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}\delta_{pq} \tag{9}$$

where E is expected value operator and δ_{pq} is the Kronecker delta.

The sequences w_k and v_k are assumed independent each other. In the case of natural excitation (operation conditions) only the responses of the structure are measured. The input u_k is not measured, therefore, it is impossible to distinguish the input u_k of the noise terms w_k, v_k in the Eqs. (8) and (9). If the characteristic of the input is typically a white noise, the input u_k can be modeled by the terms w_k and v_k, resulting in a system purely random. The input now is modeled in terms of the noise w_k and v_k, Eq. (10) and (11).

$$x_{k+1} = A_d x_k + w_k \tag{10}$$

$$y_k = C_d x_k + v_k \tag{11}$$

The deterministic knowledge of the input in this case, is substituted by the assumption that the input is an random process (white noise). So, the identification of the parameters will mean the identification parameters of a stochastic system. The Eqs. (10) and (11) constitute the base line for system identification in the time domain by using natural excitation condition. The SSI method identifies the state matrix based on the measurement data, using some robust numerical techniques, such as Singular Values Decomposition - SVD, QR - factorization and least square.

The fundamental concept in the SSI algorithm is the projection of the future rows space outputs in the past rows space output. The great advantage is that the identification is based on the measured data instead of the data covariance, i.e., the explicit formulation of the covariance is avoided. It clearly is a direct method of identification in the time domain, that works directly with time data without the need to convert them in correlation or spectral matrix data. Figure 1 shows a schematic representation of the stochastic state space model, the Δ-block represents a data delay.

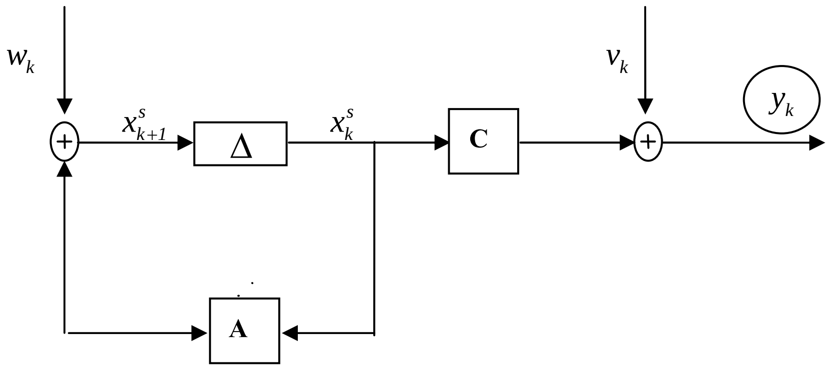

Figure 1. Stochastic state space model representation

The Kalman filter it is used for identifying of the modal parameters. Nunes (2006), discusses the complete formulation and various steps of the approach. The first step of the process is the estimation of the states sequences of the Kalman filter from the measured response data. Basically, they are obtained building blocks matrices of the output data (Hankel matrix) that contains all information of the correlation functions of the measured response data . The matrices are decomposed by using (SVD) and the singular values are used to select the order of the model. In a second stage, the system of matrices *Ad, Cd, Q, S* and *R*, Eq. (9), (10) and (11) they are determined from the states sequences through the solution of a least square problem and finally, the natural frequencies, damping ratio and the modes shapes are directly extracted from the estimated matrix.

4. EXPERIMENTAL ANALYSIS OF A FRAME STRUCTURE

The FDD and SSI techniques were evaluated for identification of modal parameters of a real frame structure, i.e., natural frequencies, modes shapes (residues) and damping ratios of the model. Two experimental tests were conducted, one based in the classical modal analysis and second one output-only modal analysis approach. The conventional modal analysis aims to provide a data base set of reference to compare and evaluate the results obtained in the FDD and SSI procedures. Figure 2a shows the structure and some details of the experimental setup used in the acquisition process. The responses of the structure were measured by accelerometers, placed in the connective of the bars, totalizing 16 points. The test was conducted in a free-free condition. The structure was suspended by means of flexible cable (elastic) aiming to represent appropriately the suspension of the structure.

Figure 2a. Set up of the Experimental Tests

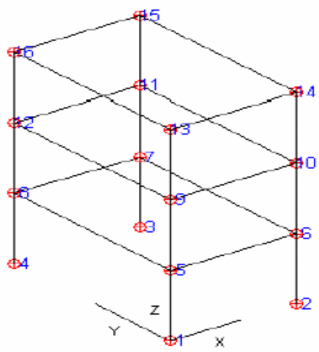

Figure 2b. Points of Measuring

In the second test (output only), the excitation of the structure was not exactly caused by a condition of operation of the structure. To simulate this condition of operation it was introduced an unmeasured excitation force with random characteristics in the structure. And the responses were measures in all nodes in the x-direction, Fig. 2b. This unmeasured excitation was a white noise of spectral density G_{xx}.

The experimental tests, as discussed previously, are composed of a convectional modal test based in the response and the excitation force and a second test based only in the response. In the first test, the input and the responses were both captured. The excitation force it was applied in point 5, x-direction, and measured by a force cell mounted between the shaker and structure and the responses were captured by using a accelerometers in all measured nodes, in x-direction, Fig 2b. The obtained input-output relationship provides the FRF(s) used to estimate the modal parameters of the model. In the second test, the set of data was defined without to measure the excitation force, only the response of the model were measured and the parameters were estimated the by using the FDD and SSI approaches.

The identification of the modal parameters of the structure in the ordinary sense it was carried out by using the method of Ibrahim, (Ibrahim, 1977).These results are used to confront with the results obtained from the FDD and SSI, aiming a better evaluation of the approach as well as to certify the quality of the obtained results and to discus the validation of the implemented methodology. It seems that, in the stage of implementation and validation of the approach, a comparison with a well-defined data base obtained by of consecrated techniques could give a better insite of the reliability of the results.

5. RESULTS OF FREQUENCY DOMAIN DECOMPOSITION (FDD)

The only-output modal analysis using FDD was conducted by using only the output spectral density matrix of the structure. Figure 3 shows a typical measured spectral density function.

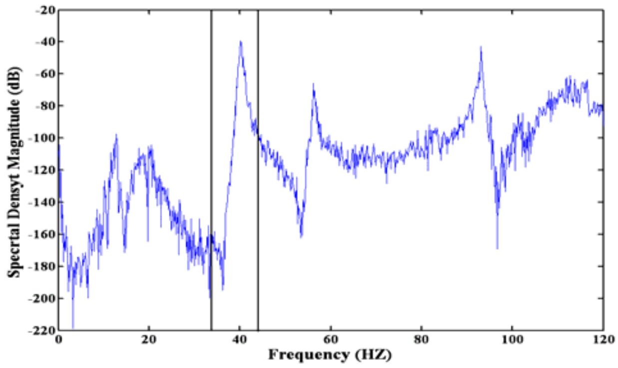

Figure 3. Typical measured output spectral density function

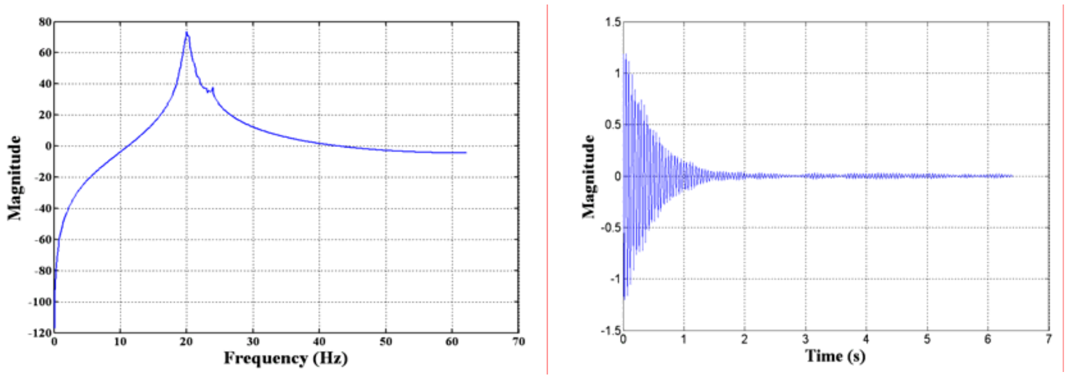

Figure 4. Singular Values of the spectral density matrix and its equivalent time domain function

From the measured output, the spectral density matrix was defined and used to estimate the modal parameters of the model, by using the FDD technique. The decomposition of the output spectral density matrix, it was defined for a set of discreet frequencies lines near to the frequency peaks. Figure 3 shows detail of the peace of the data used to identify the equivalent SDOF corresponding the first mode The extension of the number of frequency line used to define the equivalent spectral density function of the equivalent SDOF, it was defined by using the concepts of MAC-values. This value of MAC is pre-defined by the analyst and, in this analysis, it was used all frequency lines that presented MAC-values higher then 0.5.

Figure 4 show the estimated spectral density function for the first peak of frequency and its corresponding time representation, respectively. This new estimated spectral density function corresponds to a equivalent system of a single degree of freedom that will be used for the estimating of the information of the corresponding mode shape. The estimation of the parameters for the others modes is analogous.

Table 1 shows the natural frequencies, and damping ratio, identified by using the technique FDD and Fig. 5 shows a plot of the firs identified mode shape.

Table 1. Results of FDD

MODES	FDD	
	ω(Hz)	Damping (%)
1	**40,156**	**0,13**
2	**56,094**	**0,18**
3	**92,969**	**0,22**

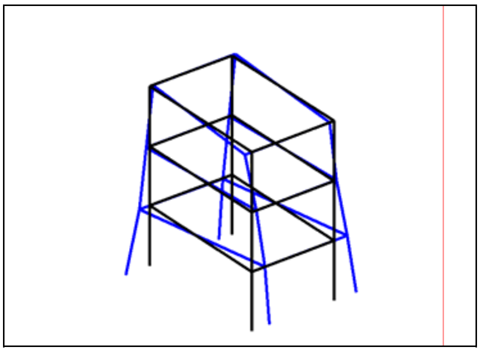

Figure 5. First Mode-DDF

6. RESULTS OF STOCHASTIC SUBSPACE IDENTIFICATION (SSI)

In the only-output modal analysis using SSI, the response signals were sampled at a sampling rate of 4096 data points, measured in all nodes of the frame in a range of frequencies from 0 to 125 Hz. Figure 6 shows a typical data output in the time domain and its corresponding Spectral Density, more specifically the response measured in the point 12, direction x. Just to say, the spectrum function are not used in this case, it is shown only to shown the resonances peaks in the measure range.

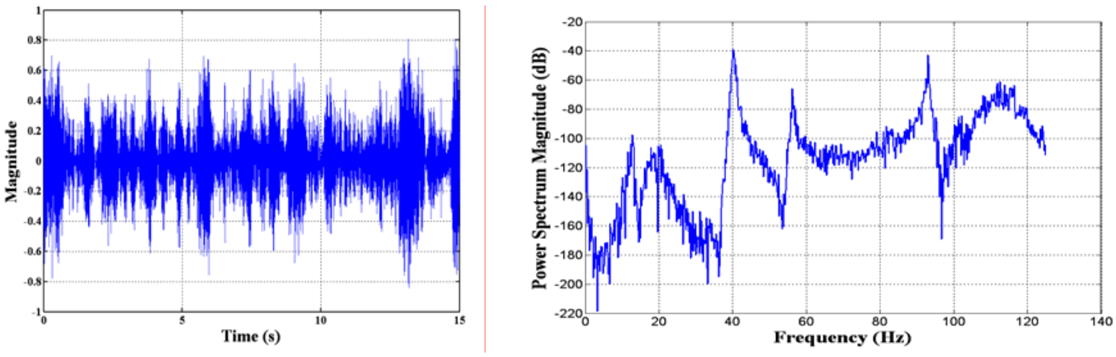

Figure 6. Measured data and Spectral Density of Power of point 12

Figure 7 shows the bar chart used for the identifying of the order of the system. The identified order is n =8. This result is coherent with the previous analysis, since the experimental pre-tests had shown the existence of 4 natural frequencies in the frequency band analyzed.

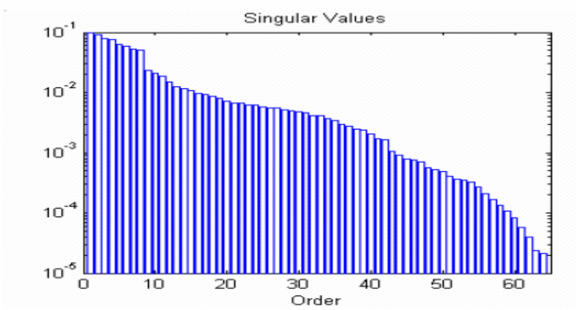

Figure 7. System Order

Once identified the order of the model, then, it is possible to identify the dynamic matrix *A* (nxn), from the measured data.

The modal parameters of the model are gotten through the decomposition of the dynamic matrix in eigenvalue and eigenvector, "(Nunes, 2006)". Table 2 shows the natural frequencies, damping ratio and the modes identified using the technique SSI and Fig. 8 presents the mode shape for the first mode

Table 2. Results of SSI

MODES	**SSI**	
	ω(Hz)	**Damping (%)**
1	**40,98**	**0,34**
2	**56,61**	**0,34**
3	**92,56**	**0,16**
4	**122,81**	**0,38**

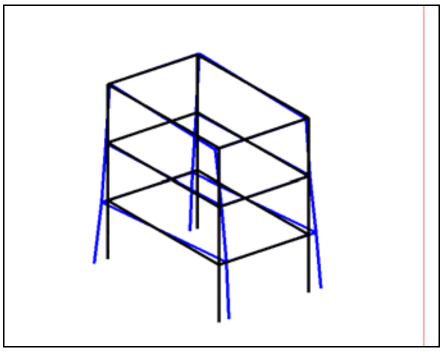

Figure 8. First Mode - SSI

7. COMPARISON BETWEEN FDD AND SSI

The results of the two techniques were validated against each other. In this case, the natural frequencies and the damping ratios can be compared directly and the modes shapes can be compared by MAC values. Additionally a measure of modal significance can also be calculated by using the modal significance concept as discussed in "Brincker (2000b)", Eq. (12):

$$MSC = MAC(i,i)^2 / (MAC_{i\,max}^{FDD} * MAC_{i\,max}^{SSI}) \qquad (12)$$

where $MAC_{i\,max}^{FDD}$ is the maximum off-diagonal element in row no i and $MAC_{i\,max}^{SSI}$ is the maximum off-diagonal element in column no i . Thus the measure compares the modes shapes in one model with the other mode shapes (other modes) in the other model. If the measure is larger than one, it means that the mode shape compares better with the corresponding mode in the other model than with any other mode shape in the other model, "(Brincker, 2000b)".

Table 3 shows the comparison of the two models, the identified the natural frequencies, damping ratio, MAC-values and modal significance for the modes shapes obtained by FDD and SSI techniques. Figure 9 shows MAC- graph.

Table 3. Comparison of the Modes (DDF x SSI)

MODES	**DDF**		**SSI**		**MAC**	**MSC**
	ω(Hz)	**Damping (%)**	**ω(Hz)**	**Damping (%)**		
1	**40,156**	**0,13**	**40,98**	**0,34**	**0,823**	**2.88**
2	**56,094**	**0,18**	**56,61**	**0,16**	**0,807**	**2.00**
3	**92,969**	**0,22**	**92,56**	**0,34**	**0,7993**	**8.70**

The results in table 3 shows a good agreement of the identified frequencies by the two methods, the value of damping estimation seemed to be suitable appropriate, since damping estimating in most cases is quite uncertain. Finally all modes present a satisfactory correlation indices (MAC) and the modal significance for the modes is high, this indicates that identified modes shapes in both cases are really the characteristics modes of system.

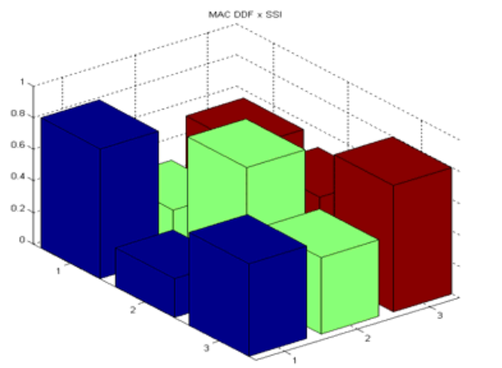

Figure 9. Comparison of the Modes Shapes – MAC
(DDF x SSI)

Additionally, the results obtained from FDD and SSI were confronted with the result obtained with the conventional modal test. This step it is important to guarantee the evaluation of the approach as well as to certify the quality of the obtained results and the validation of the implemented methodology.

Table 4 compares the natural frequencies, damping ratio and the modes using the techniques FDD and Ibrahim.

Table 4. Comparison of the Modes (DDF x Ibrahim)

MODES	DDF		IBRAHIM		MAC
	ω(Hz)	Damping (%)	ω(Hz)	Damping (%)	
1	40,46	0,43	40,46	0,24	0,9758
2	56,25	0,35	56,24	0,40	0,9192
3	92,96	0,23	92,87	0,12	0,9643

Figure 10 presents the MAC-graph shows the modes correlation, taking modes shapes obtained with the Frequency Domain Decomposition technique and the conventional technique (Ibrahim). It is noticed that all modes present a satisfactory correlation indices.

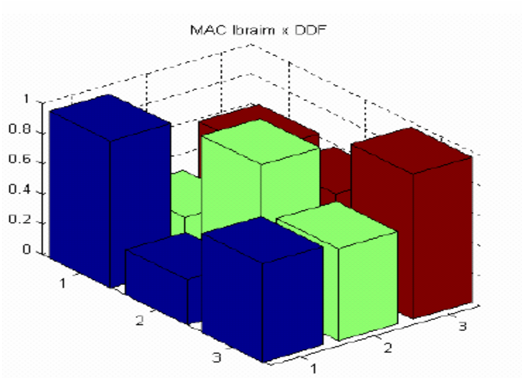

Figure 10. Comparison of the Modes Shapes – MAC
(Ibraim x DDF)

Table 5 shows the natural frequencies, damping ratio and the modes using the techniques Ibrahim and SSI. It is possible to observe a good index of correlation between the modes, evidenced for the values of MAC.

Table 5. Comparison of the Modes (SSI x Ibrahim)

MODES	SSI		IBRAHIM		MAC
	ω(Hz)	Damping (%)	ω(Hz)	Damping (%)	
1	40,98	0,34	40,46	0,24	0,83
2	56,61	0,34	56,24	0,40	0,87
3	92,56	0,16	92,87	0,12	0,86

Figure 11 shows, through the MAC- graph, the index of correlation of the mode shapes. As in the previous comparison, it is noticed that all modes present a satisfactory correlation indices.

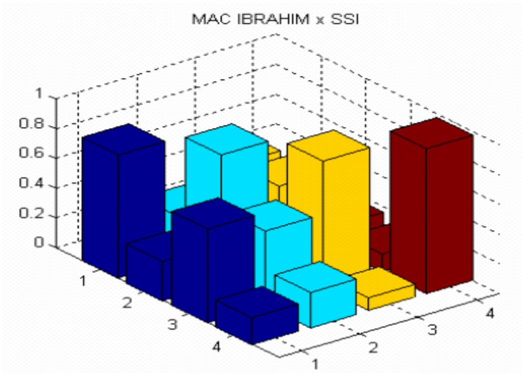

Figure 11. Comparison of the Modes Shapes – MAC (Ibraim x SSI)

8. CONCLUSION

This paper discusses how the modal parameters can be obtained from output-only data modal analysis using a frequency and a time based technique. It discusses the application for a frame structure. The modal parameters of the frame structure were also estimated by a classic modal analysis in order to compare with the results of the output-only based methods. In this case the convectional modal analysis was used as reference to evaluate the quality of the estimated results obtained by the Frequency Domain Decomposition and Stochastic Subspace Identification methods.

The obtained results have been shown promising. The modal parameters of the system were estimated with the same order of precision as compared with the results obtained in the classic modal analysis. This shows that the methodology could be applied in the identification of real structures, using output-only data, it will be next step of the work.

9.REFERENCES

ALLEMANG, R. J. 1999, *Vibrations*: *Experimental Modal Analysis.* Course Notes, Seventh Edition, Structural Dynamics Research Laboratory, University of Cincinnati, OH.

BORGES, A. S. *Análise Modal Baseada Apenas na Resposta – Decomposição no Domínio da Freqüência*. Ilha Solteira, 2006. 104 p. Dissertação (Mestrado em Engenharia Mecânica) – Faculdade de Engenharia de Ilha Solteira, Universidade Estadual Paulista, Ilha Solteira, 2006

BRINCKER, B.; ZHANG, L.; ANDERSEN, P.; 2000, "*Modal Identification from Ambient Responses using Frequency Domain Decomposition*", Proceedings of the XVIIIIMAC, 2000a.

BRINCKER, R., ANDERSEN, P. and MØLLER, N.: "Output-only Modal Testing of a Car Body subject to Engine Excitation", Proc. of the 18th International Modal Analysis Conference, San Antonio, Texas, February 7 – 10, 2000b.

BRINCKER, B. VENTURA, C.; ANDERSEN, P.; 2001, *Damping Estimation by Frequency Decomposition*, IMAC XIX, Kissimmee, 2001.

EWINS, D. J., 1984, "*Modal Testing: Theory and Practice,*" John Wiley & Sons Inc, New York.

IBRAHIM, S. R. A *Method For The Discrete Identification Of Vibration Parameters From The Free Response*. The Shock and Vibration Belletin. Vol 43. Nº 4. 1973, pp 21-37

MAIA, S., et al., 1997, "*Theoretical and Experimental Modal Analysis*", Research Studies Press Lta.

NUNES JUNIOR, O. A. *Identificação dos Parâmetros Modais Utilizando apenas as Respostas da Estrutura – Identificação no Domínio do tempo*. Ilha Solteira, 2006. 111 p. Dissertação (Mestrado em Engenharia Mecânica) – Faculdade de Engenharia de Ilha Solteira, Universidade Estadual Paulista, Ilha Solteira, 2006.

PAPOULIS, A. *Probability, Random Variables, and Stochastic Processes*, Third Edition, McGraw- Hill Inc, Singapore,1991.

PEETERS, B.; ROECK, G. D.; "*Reference-Based Stochastic Subspace Identification for Output-only Modal Analysis*", Mechanical Systems and Signal Processing, Vol.13, pp 855-878, 1999.

VAN OVERSCHEE P., DE MOOR B., "*Subspace identification for linear systems – Theory, Implementation, Applications*", Kluwer academic Publishers, ISBN 0-7923-9717-7, 1996.