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ON ROBUST CONTROL FOR MOBILE ROBOTS

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Abstrac.: *The problem of control and autonomous navigation of mobile robots has been largely studied recently, because of the possibility of the automation of several industrial processes, including those in the conduction of tasks that can hardly be carried out successfully by the human being. This work deals with the specific problem of motion control with Direct Current servo motor. Based on convex optimization, we propose a new digital controller that is practical and able to provide a robust performance and stability, in face of the specified uncertainty bounds and input constraints of the dynamic model of the servo motor under consideration. Based on the proposed controller, we evaluated its performance in real time applications through experiments conducted in a real mobile robot.*

Keywords: *Robust Control, Mobile Robot, Servo Motor.*

1. INTRODUCTION

The mobile robots have a system of movement control that enables them to navigate through its work environment, by interacting with this environment in the conduction of tasks by the use of its own sensing and decision taking resources. The movement control can be defined as a system of technological integration resulting from the control theory, power electronics, and control by microcomputer so as to obtain precision in control of torque, speed, and/or position of the mechanical system responsible for the robot's movement (motors, hydraulic system, pneumatic system, wheels, treadmills, legs) (Sage et. al. (1999), Spong and Vidyasagar (1989)). All of the movements carried out by the robot are obtained through the control of the drive of the traction motors. Servo-motors equipped with encoder of high resolution have made the system of movement control incremental to the best technological approach to obtain precision and control of the drive system. In general, for small robots, as it is here the case, it is possible to use conventional controllers of the PID (Astrom and Hagglund (1995)) type to build servos of speed or position for each wheel. However, it is noticeable that those loops of internal control are fast enough for the system to present the behavior described by the mobile robot's kinematic and dynamic model. In this sense, the characteristics of the control must be robust in face of the disturbances (when the robot is conducting a task, how to carry an object) and variations in the parameters (the mass and the center of gravity of the robot can change) (Laura et. al. (2002), Perez et. al. (2003), Sage et. al. (1999), Spong and Vidyasagar (1989)).

Several compensation techniques in the frequency domain were largely adopted in the synthesis of compensators for the practical project of servo-drivers (Spong and Vidyasagar (1989)). However, such techniques are not effective in the synthesis of controllers of uncertain dynamic systems resulting from the linearization, non-modeled dynamics, noises introduced by the sensors and actuators, and undesirable external disturbances on different parts of the dynamic system. Controllers able to compensate the modeling errors previously mentioned; guarantee an acceptable performance (for instance, in terms of percentage of overshoot, time of accommodation and etc) in spite of the effects of the of the dynamic system's uncertainty: and maintain the stability for all of the models of the system within an a specific range of uncertainty, are called robust controllers. The controller's project that guarantees a robust performance and a robust stability has become an important objective in servomechanism systems (Davison (1981) and Sousa et. al. (1998)). Several methods of controllers based on the robust control theory such as the optimization methodologies H2 and $H\infty$, structured singular value theory (synthesis μ), the LQG/LTR (Linear Quadratic Gaussian/Loop-Transfer Recovery) methodology, and the servomechanism robust theory are techniques that recently have had a lot of attention in the analysis and practical project of controllers for uncertain dynamic systems (Davison (1981), Davison (1981), Garcia et. al. (1998), Costa Filho et. al. (2007)) and more recently in systems of drive of motors (Sousa et. al. (1998), Sage et. al. (1999), Spong and Vidyasagar (1989)). The reasons for that are: i) the growing need for better requisites of performance of the control systems; ii) easy access to modern microprocessors with which sophisticated control strategies can be implemented in real time at a reasonable cost; iii) technological evolution of the power devices; iv) the theory of modern control has been extended and modified so that it can be applied in a practical way to motor control based on microprocessors taking into account the physical restrictions of the drive system.

In this context, a well tuned robust driver of DC motor provides a rather wide scale of speed control with low variation in the torque applied to rotate or to direct the wheels of a mobile robot.

In this work, a new methodology of convex optimization is proposed for practical projects robust digital controllers H_∞ applied to the system of drive of two DC motors independently drived and fixed to a robot's platform. . This methodology contrasts with other approaches, in which the controllers in a closed loop are parameterized in terms of the algebraic equation of Riccati (AER) which characterize most of the current algorithms of robust control (Davison (1981), Garcia et. al. (1998), Costa Filho et. al. (2007)). Since the sixties, equations of that type can be solved efficiently, what ended up discouraging the development of alternative methods. That situation was completely changed with the need for treating problems involving uncertainties, that is, problems of robust control. The research in that direction revealed an enormous potential for application of methods of convex optimization to several problems not yet solved.

To confirm the viability of the proposed methodology in relation to the practical implementation of the robust controllers with good numeric stability and able to generate a family of controllers that guarantee fast dynamic answers, with good properties of performance and stability in a stationary regime, we will show the experimental results of a H∞ synthesis project of speed of the motors of direct current of the robot. These experimental results will be compared to those obtained by the implementation of the PID controllers well tuned (Astrom and Hagglund (1995)).

This work is organized in the following way: In Section 2, we present a brief description of the experimental mobile robot used in this work. In Section 3, we describe the activation system and the dynamic model of the DC motor. The project of the robust control of speed of the DC motor and its details are described in Section 4, 5 and 6. The experiments conducted and the results obtained are discussed and analyzed in Section 7.

2. DESCRIPTION OF THE MOBILE PLATFORM

2.1 Structure

The developed mobile robot is shown in the Fig. 1b. This robot is composed of a series of peripherals, among which ultrasound-based sonars, infrared sensors and with both of them will be used for detection of obstacles and for measurement of distances to feed the navigation system of the robot and an image acquisition system. The ultrasound sensor is fastened to a servo-motor, what allows a scan of approximately 180 degrees and the collection of information about the reflections of the sound waves in the objects, thereby forming "sonic picture" of the environment. A wireless communication system is incorporated into the robot, thereby providing a larger flexibility in the treatment of the information obtained by the sensors with a monitoring purpose. Although the robot has all of those peripherals presented, we will not deal with them in this work, as one must give them special in order to have a proper approach to that problem. Regarding its geometry, it has a cylindrical form with a height of 0.80m and a radius of 0.30m.

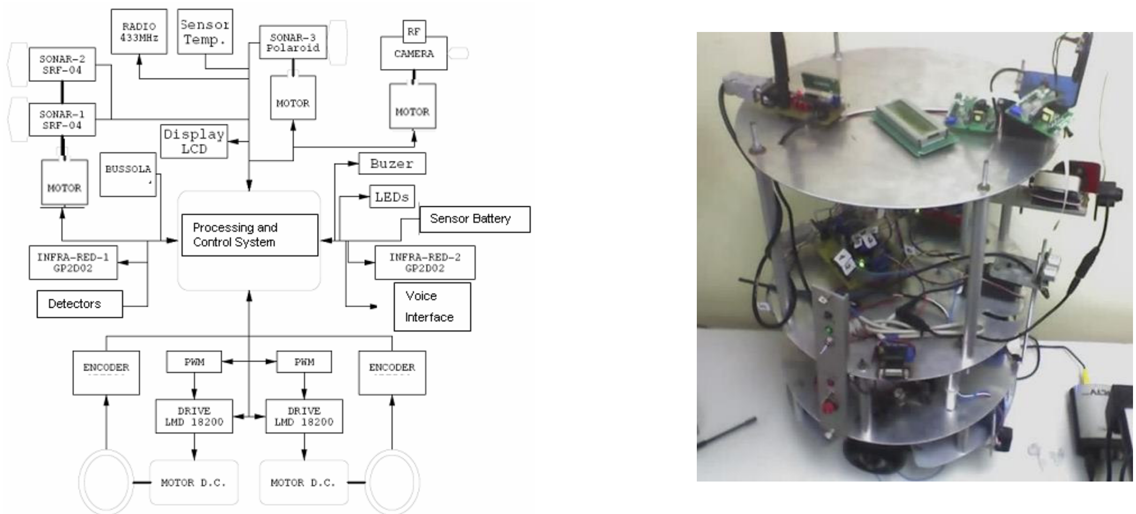

Figure 1.a) Diagram of blocks of the and b) Experimental mobile robot

The traction-differential configuration is the most used among the existing ones for wheel based movement systems (Spong and Vidyasagar (1989)). That fact is due mainly to the construction simplicity for this configuration, and also to the efficiency in the conduction of maneuvers in small areas, with those factors representing a low implementation cost and a good performance of navigation. We can observe that the rotation and translation movements are carried out by two motors DC of high torque directly coupled to the traction wheels of the robot and other two wheels only for support. The mobile robot developed presents the following diagram of blocks of the Fig. 1a.

The hardware for the robot's control (on-board computer) is composed of the following main parts: Two robust controllers, being one for each driving wheel and a PIC 18F452 Microprocessor manufactured by the Microchip company, Among its main characteristics, we can be mention (PIC18F45X Data Sheet): maximum speed operation of 20 MHz, providing cycles of instruction of 200ns; Up to 2,944 bytes of RAM and up to 2,048 bytes of EEPROM memory; Interruptions (up to 14 different sources); Temporizer/counter with resolution of 8/16 bits; Two capture modules, comparison and PWM of 10 bits; Digital analogical converter (A/D) with resolution of 10 bits.

3. DC MOTOR DRIVE

The functional diagram and the scheme of driver of a direct current motor used in this work, is showed in Fig.2, with an arrangement of four switches known as "H" bridge. This bridge allows a bi-directional control of the current that flows in the motor through PWM (also known as Pulse-Width Modulation).

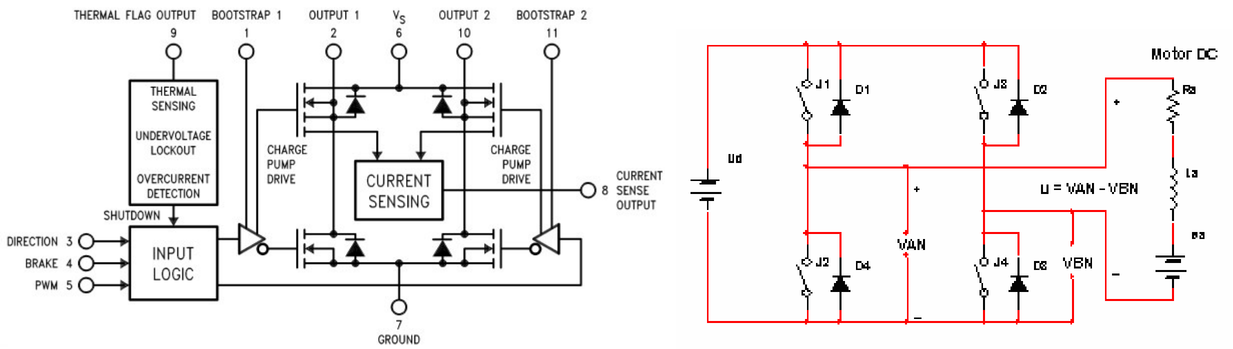

Figure 2. Scheme of activation of a direct current motor

PWM is a technique involving the modulation of the duty ratio of a signal or power supply to transport any information on a communication channel or control the power value delivered to the load.

PWM is a way of encoding digitally analog signal levels. The duty cycle of a square wave is modulated to encode a specic analog signal level, as shown in Fig. 3. The PWM signal is still digital because, at any time instant, it is either on or off. The relation between the on-time and the off-time varies accordingly to the analog level to be represented. The analog level is obtained through a series of on and off pulses. Given a suficient bandwidth, any analog value may be encoded with PWM (Astrom and Hagglund (1995)).

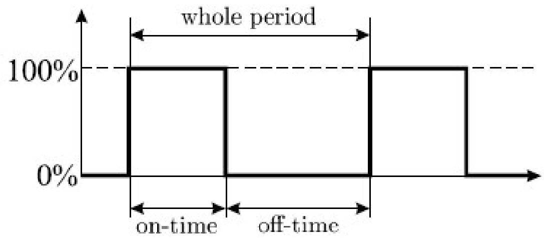

Figure 3. Example of a PWM wave

Using as a reference (Laura et. al. (2002)), the PWM operator can be written in the following way

$$PWM(t_k) = \begin{cases} U_{d,} & t_k < t \le t_k + \bar{\delta} T_s \\ -U_{d,} & t_k + \bar{\delta} T_s < t \le t_k + T_{s,} \end{cases}$$

being $t_k = kT_s$, $k \in \mathrm{N}$, the time increment, and δ a function varies in a closed interval $[0;1] \subset R$ and known as pulse width or work cycle. One considers that the frequency does not vary ($T_s = T_{on} + T_{off}$, with T_s being the switching period, T_{on} the conduction or transport time, and T_{off} the blockade time).

The work cycle can be defined as the relation between the switch conduction interval and the switching period $\bar{\delta} = T_{on}/T_s$ or defined by the relation between the average terminal tension in the load, u, and the tension of the primary source in the following way:

$$\left(2\bar{\delta}-1\right)=\frac{u}{U_d}. \quad (1)$$

The relation between the source of primary tension and the average output tension can be represented in the following way:

$$u(t)=\delta(t)U_{d.} \quad (2)$$

With $\delta=(2\bar{\delta}-1)\in[-1;\ 1]\subset R$.

The direct current electric motors under consideration are being activated with the aid of a power supply whose gain is subjected the saturation, due to the use of power electronic circuits modulated by pulse width. From the Eq. 2, one must notice that the motor control is assumed not by the tension anymore, rather it is assumed by the pulse width $\delta(t)$. The dynamics of that variable is restricted to the interval $[0;1]\subset R$, regardless of the value of U_d. Therefore, in this situation we have that $\delta_{\max}=1$. However, when we start considering the physical imperfections existing in the activation system, we notice that $u(t)$ cannot reach the value of $\pm U_d$ when $\delta(t)=\pm 1$ in Eq. 2 and that U_d can vary with time. In this context, we emphasize two non-structured uncertainties that are related to the dynamics of the pulse width - $\delta(t)$, that we considered in the robust controller's project, namely: a) the switches are transistors that present a tension decrease when used as switches in conduction called saturation tension, V_{sat}, that range from 0.1 to 0.3 volts for common transistors. V_{sat} can be discarded for high values of U_d. However, for low values of de U_d, what happens in embarked systems and having batteries as primary tension source, it cannot be despised; b) in autonomous embarked systems with batteries serving as primary tension source, the value of U_d decreases as the system operation time increases. Therefore, the decrease in the value of U_d can be modeled as a disturbance, $\Delta U_d(t)$. This way, we can substitute U_d in the Eq. 2, maintaining $\delta_{\max}=1$, by $U_d^{'}$, defined as:

$$U_d^{'}(t)=U_d-2V_{sat}-\Delta U_d(t), \quad (3)$$

3.1 Dynamics of the Motor of Direct Current

The dynamic equation of the motor of direct current can be described by the equation:

$$\delta(t)=\frac{LJ_m}{k_tU_d^{'}}\ddot{\omega}_m+\left[\frac{LB_m+RJ_m}{k_tU_d^{'}}\right]\dot{\omega}+$$
$$\left[\frac{RB_m+k_tk_{em}}{k_tU_d^{'}}\right]\omega_m+\frac{L}{\nu Nk_tU_d^{'}}\dot{\tau}(t)+\frac{R}{\nu Nk_tU_d^{'}}\tau_c(t) \quad (4)$$

Where $\delta(t)$ is the pulse width applied to the armature of the motor, ω_m is the angular speed of the motor's axis, L is the inductance of the armature, R is the resistance of the motor of the armature, k_{em} is the constant of the counter-electromotive force, t is the torque constant, N is the factor of speed reduction, $\nu\in[0;\ 1]\subset\Re$ is the efficiency of the mechanical coupling, $J_m = J_a + J_g$ is the momentum of inertia of the actuator that is, the sum of the momentums of inertia of the engine's axis and of the reducer system, B_m is the coefficient of viscous attrition, and $\tau_c(t)$ the torque imposed to the motor by the load coupled to its axis (Laura et. al. (2002)).

4. FORMULATION OF PROBLEM

In this section, we consider a linear time-invariant system described by

$$x(k+1)=Ax(k)+B_1w(k)+B_2u(k) \quad (5)$$
$$z(k)=C_1x(k)+D_1u(k) \quad (6)$$
$$y(k)=C_2x(k)+D_2w(k) \quad (7)$$

where $x(k) \in R^n$ is the state, $w(k) \in R^q$ is the disturbance input, $u(k) \in R^m$ is the control input, $y(k) \in R^l$ is the measured output, $z(k) \in R^l$ is the signal to be controlled. With $C_1^t[C_1 \quad D_1] = [Q \quad 0]$ and $D_1^t[C_1 \quad D_1] = [0 \quad I_{m \times m}]$, where: 0 is a null matrix, $I_{m \ x \ m}$ is an identity matrix and the matrix Q is positive definite. Assume that ($A; B_2$) is controllable and ($C_2; A$) is detectable. The superscript *"t"* denotes the transpose.

The H_∞ problem we address in this paper is that of designing a control law *u(.)* over the horizon *[0,T-1]*, using the available measurements, *y(.)*. The controller is required to reduce the worst case effect of the disturbance signal *w (.)* on the controlled output *z(.)*. More specifically, we consider the following index of performance

$$J(x(k),u(k),w(k),k) = \frac{1}{2} z'(T)z(T) + \frac{1}{2}\sum_{k=0}^{T-1}\{ \ z'(k)z(k) - \gamma^2 w'(k)w(k) \ \} \tag{8}$$

The second term in the right-hand side is the penalty term on *w(.);* γ is a positive constant which represents the magnitude of the penalty.

The admississible control law is assumed to be of the form

$$u(k) = Gx(k) \tag{9}$$

where G is a time-invariant operator.

We denote the optimal solution by $x^*(k)$ and $u^*(k)$, respectively. We call $w^*(k)$ the worst-case disturbance. The minimax control problem is formulated by:

$$\min_{z(T-1)}(\max_{w(T-1)} \cdots \min_{z(k)}(\max_{w(k)} \cdots \min_{z(0)}(\max_{w(0)} J) \cdots) \cdots)$$

Associated to this problem of optimal control, we have the following constraints:

$$\begin{array}{ll} x_{\min} \le x(k) \le x_{\max} & k = 1,...,T \\ u_{\min} \le u(k) \le u_{\max} & k = 0,...,T-1 \end{array} \qquad 10)$$

5. NECESSARY CONDITIONS

We first derive necessary conditions for the existence of the minimax solutions by exploiting the sweep method, which is a straightforward optimization method based on the Lagrange multiplier technique (Davison (1981), Costa Filho et. al. (2007)). Therefore, we can first perform the optimization with respect to *{x1,…,x(T)}, {u(0),…,u(T-1)}* and *{w(0),...,w(T-1)}*. In this sense, we form the Hamiltonian

$$H^k(z(k),w(k),p^*(k)) \equiv \frac{1}{2}[z'(k)z(k) - \gamma^2 w'(k)w(k)] + p^*{}'(k)(Ax(k) + B_1 w(k) + B_2 u(k)) \tag{11}$$

where $p^*(k)$, $k \in K$ is the co-state vector, with $p(-1) = 0$ and $p(k) = 0$ for $k > T$.

The Lagrangean function L is related to $H(k)$ by

$$L(z(k), w(k), p^*(k), k) = \sum_{k \in K}\{H^k(z(k), u(k), p^*(k)) - p^*{}'(k-1)x(k)\} \tag{12}$$

Therefore, we have $T + 1$ optimal control problems:

i) For $k = 0$,

$$\begin{array}{l} \min\limits_{u(0)} \quad H^k(z(0), u(0), p^*(0)) \\ \textit{subject to} \quad x(0) = \xi \\ \qquad u_{\min} \le u(0) \le u_{\max} \end{array} \tag{13}$$

ii) For $k = 1,…, T-1$

$$\begin{array}{l} \min\limits_{u(k),x(k)} \quad H^k(z(k),u(k),p^*(k)) - p^*{}'(k-1)x(k) \\ \textit{subject to} \quad x_{\min} \le x(k) \le x_{\max} \\ \qquad u_{\min} \le u(k) \le u_{\max} \end{array} \tag{14}$$

iii) For $k = T$

$$\min_{x(T)} \quad z'(T)z(T) - p^{*}{}'(T-1)x(T) = \min_{x(T)} \quad x'(T)C_1'C_1x(T) - p^{*}{}'(T-1)x(T)$$
$$s.ubject\ to: \quad x_{min} \le x(T) \le x_{max} \tag{15}$$

Then, the necessary conditions of optimality is given by

$$p^*(k-1) = \frac{\partial H^k}{\partial z(k)} = Qx(k) + A'p^*(k), \quad \therefore \quad x(k) = sat[-Q^{-1}\{p(k-1) - A'p(k)\}], k = 0,\dots,T-1 \tag{16.a}$$

$$0 = \frac{\partial H^k}{\partial u(k)} = u(k) + B_2'p^*(k), \quad \therefore \ u(k) = sat[-R^{-1}B_2'p(k)] \quad , \quad k = 0,\dots,T-1 \tag{16.b}$$

$$0 = \frac{\partial H^k}{\partial w(k)} = -\gamma^2 w(k) + B_1'p^*(k), \quad k = 0,...,T-1 \tag{16.c}$$

$$p^*(T-1) = Qx(T) \tag{16.e}$$

where: $sat(\eta) = \begin{cases} \eta_{max} & if \quad \eta \ge \eta_{max} \\ \eta & if \quad n_{min} < \eta < n_{max} \\ \eta_{min} & if \quad \eta \le x_{min} \end{cases}$

Let *(x*(k),p*(k),u*(k))* be the trajectories of *(x(k),p(k),u(k))* corresponding to the worst case disturbance *w*(k).* In this sense, we can observe that for all fixed *(x*(.),w*(.),u*(.)),* the optimization problem can be solved as a dual maximization problem.

$$\underset{p(k)}{Max}\, L(p(k)) = \underset{p(k)}{Max} \underset{u(k),x(k)}{Min} \sum_{k=0}^{T-1} [\frac{1}{2}(x'(k)Qx(k) + u'(k)u(k)) - \gamma^2 w'(k)w(k) +$$
$$p'(k)(Ax(k) + B_1w(k) + B_2u(k)) - p'(k-1)x(k)] + \frac{1}{2}x'(T)Qx(T) - p'(T-1)x(T) \tag{17}$$

Subject to: (10) e (16).

In this paper, the control problem is reformulated as a convex o optimization problem. From duality theory in convex optimization, dual problems can be derived for these convex problems. These dual problems can be in turn be reinterpreted in control terms, often yielding new results or news proofs for existing results from control theory. In this sense, we propose the reformulation of the dual problem as the following static optimization problem (SOP) of order *n.T*.

SOP: Find the vector $p = [p(0) ... p(T-1)]' \in R^{nT}$ that minimizes

$$\underset{p}{Max}\, L(p) = \frac{1}{2}p'Hp + p'b + c$$
$$subject\,to: p \in R^{nT} \tag{18}$$

where: $H = H' \in R^{nT \times nT}$ is a positive definite matrix, $b \in R^{nT}$ e c is a scalar.

We start the creation of the problem (18), by substituting the constraints in (16) in *L(p(k))* obtaining:

$$L(p(k)) = -\frac{1}{2}p'(T-1)Q^{-1}p(T-1) + \frac{1}{2}\xi' Q\xi - \frac{1}{2}p'(0)\overbrace{(B_2B_2' - \gamma^{-2}B_1B_1')}^{V_1}p(0)$$
$$+ p'(0)(A\xi) + \frac{1}{2}\sum_{k=1}^{T-1}\{ \ p'(k-1)Q^{-1}p(k-1) + p'(k)\overbrace{(AQ^{-1})}^{V_2}p(k-1)$$
$$- p'(k)\overbrace{(AQ^{-1}A' + B_2B_2' - \gamma^{-2}B_1B')}^{V_3} + p'(k-1)Q^{-1}A'p(k) \ \} \tag{19}$$

where: $k \in K$, $V_{kk} = -V_3 - Q^{-1}, V_{kk+1} = V_2', V_{kk-1} = V_2, V_{00} = -V_1 - Q^{-1}$

$$H = \begin{bmatrix} V_{00} & V_{01} & & \\ V_{10} & V_{11} & V_{12} & \\ & : & : & : \\ & & V_{T-1,T-2} & V_{T-1,T-1} \end{bmatrix} \qquad b = [A\xi \ 0 \cdots 0]' \text{ and } c = \frac{1}{2}\xi' Q \xi$$

A serious drawback of direct application of non-linear programming (NLP) algorithms to dynamic problems is due to the increased dimension of the resulting optimization problem and the corresponding high computational effort and condition number (system is ill- or well-conditioned) requirements as it is the case here. In this paper, we adapt an active set NLP algorithm to the particular structure of the studied dynamic problems in the aim reducing substantially the computational effort and the condition number for problem solution. The algorithm is well suited for problems where the number of constraints is much higher than the number of optimization variables as it is the case in the considered dynamic problem.

6. NEW ROBUST CONTROL

In this section, we obtain the values of steady-state for robust control and gain matrix. For better understand of the proposed methodology, we can refer the readers to the paper (Sousa et. al. (1998), Costa Filho et. al. (2007)).

Proposition 1 – To build an equivalent problem in such that the condition number of the matrix *H* with respect to any norm is small. This optimization problem has the form

$$\text{SOP} \quad \begin{aligned} &\underset{p}{Min}\, L(p) = \frac{1}{2} p' \tilde{H} p + p' \tilde{b} + \tilde{c} \\ &subject\, to : \| p \|_2 \le \Delta, \Delta > 0 \end{aligned} \tag{20}$$

Where: $\tilde{H} = -H$; $\tilde{b} = -b$ e $\tilde{c} = c$

This problem is the basis of the model-trust region approach to minimization (Costa Filho et. al. (2007), Sorensen (2002)). Its solution is given in Lemma 1 below.

Lemma 1 - Let $L : R^{nT} \to R$ be twice continuously diffentiable, $\tilde{H} \in R^{nt\,x\,nT}$ be symmetric and positive definite, and let $\|.\|$ designate the l_2 norm. Then problem (20) is solved by

$$p(\alpha) \equiv -(\tilde{H} + \alpha I)^{-1} \tilde{b} \tag{21}$$

For the unique $\alpha \ge 0$ such that $\|p(\alpha)\| = \Delta$, unless $\|p(0)\| \le \Delta$, in which case $p(0) = p^N$ (the quasi-Newton step) is the solution. For any $\alpha \ge 0, p(\alpha)$ defines a descent direction for *L*.

Proof: See (Sorensen (2002)).

The parameter $\alpha \ge 0$ is basic to obtain a family of robust controllers as we will see to follow.

Proposition 2 - Successively to solve this open loop control problem for intervals. Through the experiments, we verified that, for *T* small e *x(0)* close to the solution, the error in the residue of the system: $\tilde{H}p + \tilde{b} = 0$ it is reduced. Then, this system is well conditioned. In general, when the matrix *H* increases of dimension in function of *T*, its condition number also increases. In this sense, a significant reduction of the dimension of the matrix is possible of *N.T* for *N. t*, where $t << T$.

We may define the robust control based on the propositions 1 and 2. We obtain the values of steady-state for robust control and gain matrix. After to apply propositions 1 and 2 to the problem (20), we obtain following the system of equations for $k = 0,1$

$$\left(\begin{bmatrix} B_2 B_2' - \gamma^{-2} B_1 B_1' + Q^{-1} & -Q^{-1} A' \\ -AQ^{-1} & B_2 B_2' - \gamma^{-2} B_1 B_1' + AQ^{-1} A' + Q^{-1} \end{bmatrix} + \alpha I \right) \begin{bmatrix} p(0) \\ p(1) \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix} x(0) \tag{22}$$

Therefore we have that the vector of co-state *p(0)* and *p(1)* is given by:

$$\begin{bmatrix} p(0) \\ p(1) \end{bmatrix} = \begin{bmatrix} \overline{H}_{11} & \overline{H}_{12} \\ \overline{H}_{21} & \overline{H}_{22} \end{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix} x(0) \tag{23}$$

where: 0_{nxn} is a null matrix .

From (16.c) and (23), the worst case disturbance is given by

$$\begin{aligned} w^*(k) &= K_w x(k) \\ w^*(k) &= \gamma^{-2} B_1' \overline{H}_{11} A x(k) \end{aligned} \tag{24}$$

Substituting (23) in the Eq. 16.a, *for k = 1*, we have:

$$\begin{aligned} x(1) &= Q^{-1} p(0) - A' p(1) \\ x(1) &= Q^{-1} (\overline{H}_{11} - A' \overline{H}_{21}) A x(0) \end{aligned}$$

The gain matrix can be defined as

$$A + B_2 G + B_1 K_w = Q^{-1} (\overline{H}_{11} - A' \overline{H}_{21}) A \tag{25}$$

Note that $A + B_2 G + B_1 K_w$ depends on α and therefore, through this parameter, is possible to determine the control input *u(.)* which minimizes the H_∞ norm of the transfer function between *w(.)* and *z(.);* being used the *G* gain matrix, whose conditions of existence are given in (Sousa et. al. (1998), Costa Filho et. al. (2007)).

7. NUMERICAL EXPERIMENTS

In this section, it is shown the experimental results of the project of robust digital control of a motor D.C. applying the optimal control methodology.

Initially, we define the plant increased with the inclusion of an integrator in the entrance of the plant in order to take the errors of steady state the zero. In this case, we define the plant of the dynamic system (5) as:

$$\begin{aligned} \Delta x(k+1) &= A^a \Delta x(k) + B_1^a w(k) + B_2^a \Delta u(k) \\ z^a(k) &= C_1^a \Delta x(k) + D_1^a \Delta u(k) \\ y^a(k) &= C_2^a \Delta x(k) \end{aligned} \tag{26}$$

Where: $\Delta x(k) = [e(k) \quad [x(k) - x(k-1)]']'$ e $\Delta u(k) = u(k) - u(k-1)$ are the new state and control variables respectively, with:

$$A_1^a = \begin{bmatrix} I_1 & -C_2 A_1 \\ 0 & A_1 \end{bmatrix} \quad B_2^a = \begin{bmatrix} -C_2 B_2 \\ B_2 \end{bmatrix} \quad B_1^a = \begin{bmatrix} -E_1 B_1 \\ B_1 \end{bmatrix} \quad C_2^a = \begin{bmatrix} I & 0 \\ 0 & C_2 \end{bmatrix}$$

C_1^a and D_1^a are chosen to satisfy the conditions given in formulation problem.. The state variable *e(k),* defined as error of adjustment of a system of control in relation to a reference *r(k),* will be introduced in the system:

$$e(K) = r(k) - Cx(k) \tag{27}$$

Considering two instants of sampling consecutive and admitting that reference is constant *(r(k+1) = r(k)),* the following system of equations the differences is obtained:

$$e(k+1) = e(k) - C(x(k+1) - x(k)) = e(k) - CA\Delta x(k) - CB\Delta u(k)$$

For an analysis of the dynamic performance of the motor D.C. (speed control), we carry through a test with 300 samples and period of sampling 5 milliseconds. The matrices and parameters obtained and used in the H_∞ control project had been:

$$A^a = \begin{bmatrix} 1 & 0{,}9587 \\ 0 & 0{,}9587 \end{bmatrix}; B_1^2 = \begin{bmatrix} -0{,}1 \\ 0{,}1 \end{bmatrix}; B_2^a = \begin{bmatrix} -0{,}0401 \\ 0{,}0401 \end{bmatrix}; C_1^a = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; D_1^a = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \alpha = 0{,}05\,; \gamma = 2{,}2$$

In Fig. 4a, it is shown the motor speed in terms of the encoder tension for a reference of 6 Volts, using the robust controller and a well syntonized PID controller (Astrom et. al (1995), Sousa et. al. (1998)). The robust controller got an excellent performance, showing a fast reply, without overshoot and null steady error.

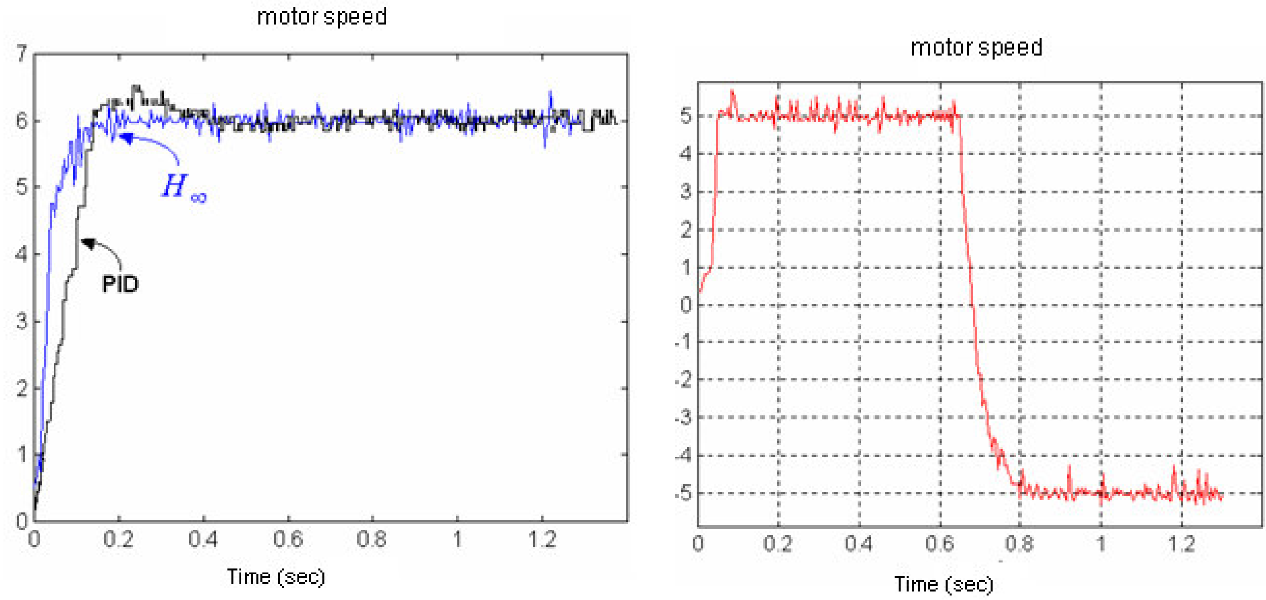

Figure 4.a) Reply to the step of the system and b) Reply the variation of the reference signal

To observe the behavior of the system front the variation of the reference signal, was applied a reference of 5 Volts to the system. When the value of regimen was reached, is applied a value of reference of -5 Volts. Fig. 4b shows to the results for this case. Again the results demonstrate the good performance of the system.

8. CONCLUSION

Some appropriate hypotheses had been carried through on the dynamic of speed of the discrete model of the motor D.C. for the use of the proposed method of robust digital control. The project of digital controllers with static gain of state feedback was developed without the use of modified equations of Riccati. The strategy of H_∞ control revealed robust in relation the saturation of the actuators. The results show the high robustness of the controller in relation to the variations of the parameters, load disturbances and tracking of the reference signal. Although this methodology to have been developed for the attendance a specify problem of control, it can be affirmed that, in general, the basic stages of method can be considered valid for any type of application involving systems contend uncertainties.

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10. REFERENCES

Astrom, K. and Hagglund, T., 1995, “PID Controllers: Theory, Design, and Tuning”, Instrument Society of America, ISBN 1-55617-516-7.

Costa Filho, J. T., Nascimento, V. A., 2007, “A Nice Method For Linear Quadratic Regulator Problem”, 6o Brazilian Conference on Dynamics, Control and Their Applications,vol 1, May.

Davison, E. J. and Ferguson, I. J., 1981, “The Design of Controllers for the Multivariable Robust Servomechnism Problem using Parameter Optimazation Methods”, IEEE Transactions on Automatic Control, vol. 26, no. 1, pp. 93-110, February.

Doyle, J. C., Francis, B. A. e Tannenbaum, A. R., 1992, “Feedback Control Theory”, Macmillan Publishing Company.

Garcia, J. A. A., Niu, X. e Cabrera, L. A., 1998, "Optimization of Stability Robustness Bounds for Linear Discrete-Time Systems", Journal of Optimization Theory and Applications, vol. 99, no. 2, pp. 303-330, November.
Laura, T. L., Cerqueira J. F. and Paim C., 2002, "Modelo Dinâmico da Estrutura de Base de Robôs Móveis com Inclusão de Não-Linearidades – O Desenvolvimento do Modelo", XIV Congresso Brasileiro de Automática, vol. 04, September.
Perez, T., G. C. Goodwin and M. M. Seron, 2003, "Performance degradation in feedback control due to constraints". IEEE Transactions on Automatic Control 48(8), 1381-1385.
PIC18F45X Data Sheet – Microchip Tecnology Inc 2005.
Sage, H. G., M. F. De Mathelin and E. Ostertag, 1999, "Robust control of robot manipulators: A survey". International Journal of Control 72(16), 1498-1522.
Sousa, F. C. and Costa Filho, J. T., 1998, "Projeto de Controlador Robusto via Otimização Multinível", XII Congresso Brasileiro de Automática, vol. 06, pp. 2055-2061, September.
Sorensen, D.C., 2002, "Newtons's Method with a Model Trust Region Modification". SIAM Journal on Numerical Analysis, 19(2): 409-426,1982.
Spong, M. W. and M. Vidyasagar, 1989, "Robot Dynamics and Control". John Wiley & Sons.

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