MULTICRITERIA TOPOLOGY OPTIMIZATION OF HEAT AND MASS TRANSFER PROBLEMS USING BOUNDARY ELEMENTS

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Abstract. *The objective of this work is to present the implementation of a topological-shape sensitivity formulation in a BEM analysis for simultaneous heat and mass transfer optimization problems. A topological derivative estimate is used and evaluated at internal points, and the ones showing the lowest values are used to remove material by opening a circular cavity. As the iterative process evolves, the original domain has holes progressively punched out until a given stop criteria is achieved. Since the sensitivities for each of the differential equations are different, a penalization-type approach has been used to weight the sensitivities associated to each problem. This allows the imposition of distinct penalization factors for each problem, according to specified priorities. The results obtained showed good agreement with the few similar solutions available.*

Keywords: *shape optimization, potential problems, boundary elements, topological derivative, mass transfer*

1. INTRODUCTION

The main objective of this work is focused in presenting a numerical methodology to combine heat and mass transfer designs in a resulting optimal solid shape for both problems. A strategy of compromise (Rozvany et al. 1995) that attributes weights to the respective equations will be employed in order to establish a relationship between both problems. A well known topological derivative (DT) approach (Céa et al. 2000; Sokolowski and Zochowski, 2001) was used to evaluate the sensibilities, as an alternative to the traditional homogenization methods (Bendsøe and Kikuchi, 1988). The boundary element method (BEM) was chosen to provide the numerical solution. Since the BEM does not require domain meshes, a significant reduction in the computational cost during the iterative optimization process can be achieved, in comparison to other traditional numerical methods. The application of a DT + BEM scheme has been introduced in (Marczak, 2007) for the optimization of linear heat transfer problems. In (Anflor and Marczak, 2006) the authors have extended the formulation to non-isotropic materials, using a domain mapping technique to transform the original problem in an isotropic one. In the present work, the natural follow-on development is addressed, namely, the solution of multi-objective optimization problems using the proposed methodology. Firstly, the DT formulation for the Poisson equation is presented. Next, the optimization procedure employed herein for multi-criteria problems detailed. In order to access the formulation, a case of simultaneous two-dimensional heat and mass transfer is optimized for mass reduction and discussed.

2. TOPOLOGICAL DERIVATIVE

A topological derivative for Poisson Equation is applied in this work. A simple example of applicability consists in a case where a small hole of radius (ε) is open inside the domain. The concept of topological derivative consists in to determine the sensitivity of a given function cost (ψ) when this small hole is increased or decreased. The local value of DT at a point ($\hat{x}$) inside the domain for this case is evaluated by:

$$\overset{*}{D}_T(\hat{x}) = \lim_{\varepsilon \to 0} \frac{\psi(\Omega_\varepsilon) - \psi(\Omega)}{f(\varepsilon)} \quad , \tag{1}$$

where ψ(Ω) and ψ(ε) are the cost function evaluated for the original and the perturbed domain, respectively, and f is a problem dependent regularizing function. By equation (1) it is not possible to establish an isomorphism between domains with different topologies. This equation was modified (Feijóo et al. 2002) introducing a mathematical idea that the creation of hole can be accomplished by single perturbing an existing one whose radius tends to zero. This allow the restatement of the problem in such a way that it is possible to establish a mapping between each other (Feijóo et al. 2002).

$$\overset{*}{D}_T(\hat{x}) = \lim_{\varepsilon \to 0} \frac{\psi(\Omega_{\varepsilon+\delta\varepsilon}) - \psi(\Omega_\varepsilon)}{f(\Omega_{\varepsilon+\delta\varepsilon}) - f(\Omega\varepsilon)} \quad , \tag{2}$$

where δε is a small perturbation on the holes's radius. In the case of linear heat transfer, the direct problem is stated as:

$$\text{Solve } \{u_\varepsilon \mid -k\Delta u_\varepsilon = b\} \quad \text{on} \quad \Omega_\varepsilon \tag{3}$$

subjected to

$$\begin{cases} u_\varepsilon = \bar{u} & \text{on} \quad \Gamma_D \\ k\dfrac{\partial u}{\partial n} = \bar{q} & \text{on} \quad \Gamma_N \\ k\dfrac{\partial u_\varepsilon}{\partial n} = h_c\left(u_\varepsilon - u_\infty\right) & \text{on} \quad \Gamma_R \quad , \end{cases} \tag{4}$$

where

$$h(\alpha,\beta,\gamma) = \underbrace{\alpha\left(u_\varepsilon - \bar{u}^\varepsilon\right)}_{Dirichlet} + \underbrace{\beta\left(k\frac{\partial u_\varepsilon}{\partial n} + \bar{q}^\varepsilon\right)}_{Neumann} + \underbrace{\gamma\left(k\frac{\partial u_\varepsilon}{\partial n} + h_c^\varepsilon\left(u_\varepsilon - u_\infty^\varepsilon\right)\right)}_{Robin} = 0 \quad , \tag{5}$$

is a function which takes into account the type of boundary condition on the holes to be created ($u_\varepsilon , \dfrac{\partial u_\varepsilon}{\partial n} = q_\varepsilon$ are the temperature and flux on the hole boundary, while u_∞^ε and h_c^ε are the hole's internal convection parameters, respectively). After an intensive analytical work, (Feijóo et al, 2002) it was developed explicit expressions for DT for problems governed by Eq.(3). Table 1 presents the final expressions for topological derivative, considering the three classical cases of boundary conditions on the holes.

Table 1. Topological derivative for the various boundary conditions prescribed on the holes using the total potential energy as a cost function (Novotny et al. 2003).

Boundary condition on the hole	**Topological derivative**	**Evaluated at**
Neumann homogeneous boundary condition (α = 0, β =1 , γ = 0)	$D_T(\hat{x}) = k\nabla u \nabla u - bu$	$\hat{x} \in \Omega \cup \Gamma$
Neumann non-homogeneous boundary condition (α = 0, β =1 , γ = 0)	$D_T(\hat{x}) = -q_\varepsilon u$	$\hat{x} \in \Omega \cup \Gamma$
Robin boundary condition (α = 0, β = 0, γ = 1)	$D_T(\hat{x}) = h_c^\varepsilon\left(u_\varepsilon - u_{\varepsilon\infty}\right)$	$\hat{x} \in \Omega \cup \Gamma$
Dirichlet boundary condition (α = 1, β = 0, γ = 0)	$D_T(\hat{x}) = -\frac{1}{2}k\left(u - \bar{u}_\varepsilon\right)$	$\hat{x} \in \Omega$
Dirichlet boundary condition (α = 1, β = 0, γ = 0)	$D_T(\hat{x}) = k\nabla u \nabla u - b\bar{u}_\varepsilon$	$\hat{x} \in \Gamma$

3. MULTI-CRITERIA OPTIMIZATION PROCEDURE

The optimization of problems under more than one cost function is becoming quite common in engineering practice. For instance, in the electronic industry the miniaturization of components are leading to excessively slender designs, which demands new and efficient cooling devices, usually base on porous media. The successful design of such components imply in the optimization of both, heat and mass transfer. This technology presents two basic advantages (Jeng et al. 2006): (a) the porous heat sink provides more than 10 times the contact area of a smooth surface; and (b) the irregular structures of the porous heat sinks, at sufficient high velocities, causes irregular fluid flow, increasing the thermal dispersion conductivity. This is a typical case where the heat conduction would lead to an optimum design, while the convection would lead to another one. Therefore, it is necessary to combine both optimization problems in a single one. Clearly, the adoption of one (single criteria) design could result in a less efficient performance of the product from the other criteria(s) point(s) of view.

In engineering design practice, it is usual for a thermal solid to be required to satisfy one of the following optimality criteria:

A – As uniform mass transfer rate as possible;
B – As uniform heat transfer rate as possible;
C – Maximum possible efficiency in both criteria A e B, simultaneously.

From the optimization point of view, the first two criteria means the extremization of a single objective function. The satisfaction of criterion C needs the satisfaction of multiple design criteria, which is the goal of this work. After separate BEM heat and mass analysis, the heat flux and mass densities are determined at internal points. These values are used to evaluate the topological derivative, as described in (Anflor and Marczak, 2006; Marczak, 2007). In order to estimate the relative material usage efficiencies at an internal point, two dimensionless factors are introduced as:

$$\alpha_M^i = \frac{D_T^M\big|_i}{D_T^M\big|_{\max}} \qquad 0 \le \alpha_M^i \le 1 \quad , \tag{6}$$

where α_M^i is the mass flux efficiency factor, $D_T^M\big|_i$ is the mass flux topological derivative at internal point *i*, and $D_T^M\big|_{\max}$ is maximum value of $D_T^M\big|_i$. A heat flux efficiency factor α_H^i is derived accordingly. During the optimization process the basic goal is to remove material where if is less efficient. However, it may happen that for multi-criteria optimization (C), internal points with low heat fluxes does not necessarily will have low mass fluxes. In such cases it is necessary to apply a strategy to generate a compromise between both phenomena. Rosvany et al. (1995) proposed a strategy of compromise in terms of weighted sums of α_H^i and α_M^i:

$$\alpha^i = w_M \alpha_M^i + w_H \alpha_H^i \quad , \tag{7}$$

where w_H and w_M are the weighting factors for the heat and the mass problems respectively. It is important to note that

$$w_M + w_H = 1. \tag{8}$$

The weighting factors provide a meaningful way of assigning different levels of importance to each problem. Therefore, when $w_M = 1$ and $w_H = 0$ the criteria A is obtained as a special case, while $w_M = 0$ and $w_H = 1$ reduces the problem to a simple mass transfer optimization (criteria B).

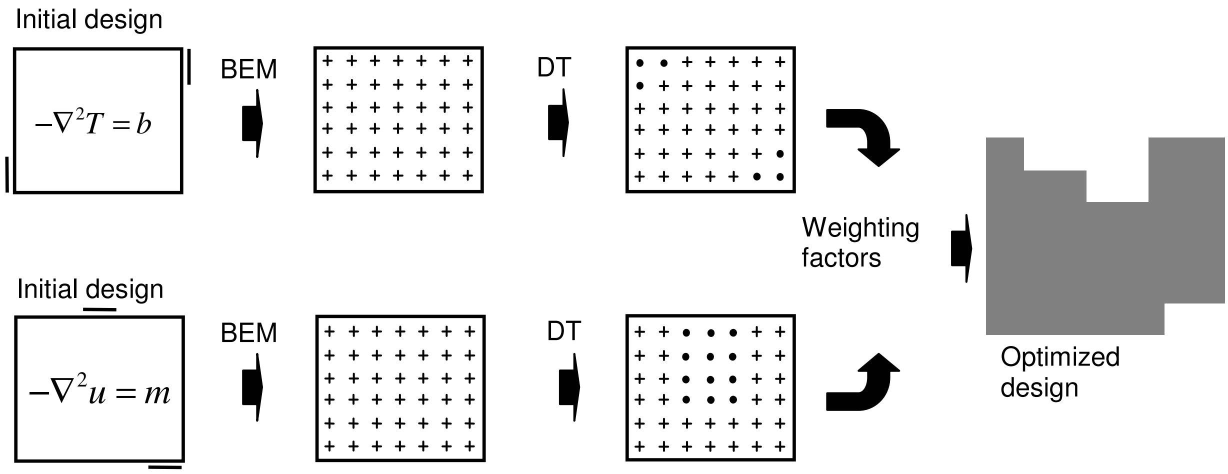


Figure 1. Schematic view of multi-criteria optimization.

4. NUMERICAL EXAMPLE

A porous square domain of dimension 20×20 is submitted to simultaneous heat and mass transfer. The geometry is discretised with 40 linear discontinuous boundary elements integrated with 4 Gauss points. The boundary conditions are depicted in figure 2. The heat transfer problem has a high potential of 25°C at the right upper corner and a low potential of 23°C at the mid lower side. The mass transfer problem has a high potential of 2 kg mol/m^3 on the upper left corner and a low potential of 1 kg mol/m^3 on the mid lower side. The remaining boundary is insulated, as well as all the holes open during the process. The conductivity and diffusivity coefficient are set as 1W/m°C and 1 m^2, respectively. A regularly spaced grid of internal points was automatically generated, taking into account the radius of the holes created during each iteration. The radius was taken as a fraction of a reference dimension of the domain ($r = \alpha l_{\text{ref}}$). Usually $l_{\text{ref}} = \min(\text{height,width})$ was adopted. The objective in all cases is to minimize the material volume. The current area of the domain (A_f) was checked at the end of each iteration until a reference value is achieved ($A_f = \beta A_0$, where A_0 represents the initial area). This numerical example will be studied with $\alpha_M = 0.4$ and $\alpha_H = 0.6$, to illustrate a case where a higher priority is imposed to one of the problems.

Three internal control points were chosen in order to check temperature, mass concentration, heat and mass flux as the process evolves.

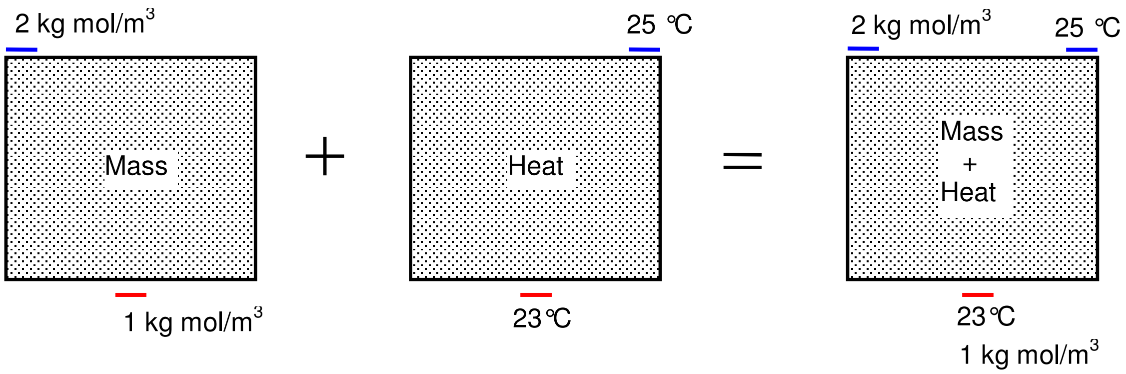
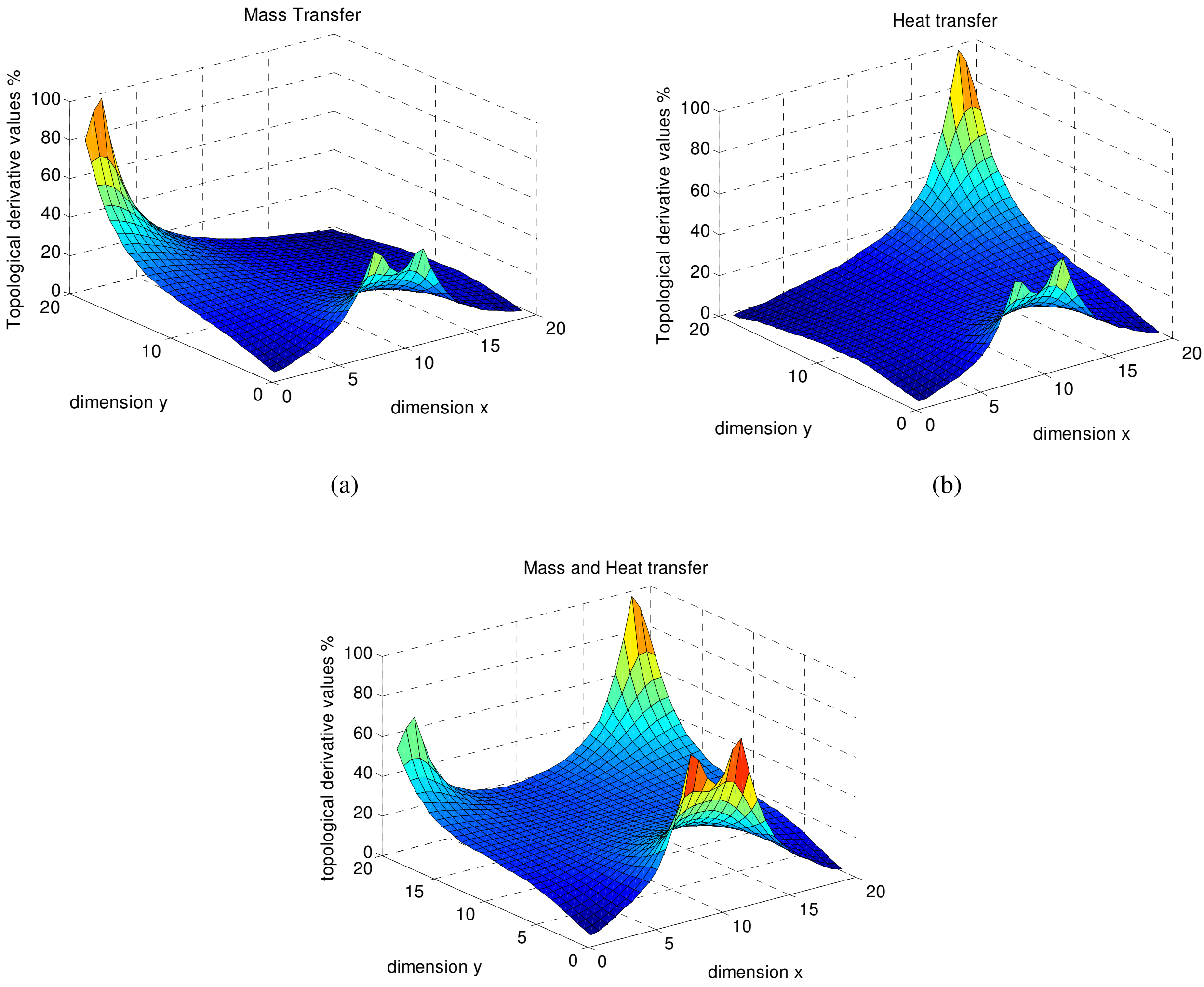


Figure 2. Initial boundary conditions.

Figure 3 illustrates the behavior of the topological derivative values calculated for mass (fig.3a) and heat transfer (fig.3b) inside the domain before the optimization process be initialized. Figure 3c represents the topological derivatives values for both problems obtained by equation (7).

Figure 4 depicts the topology evolution during the iterative process. It is clear the material removal where it is less necessary, according to the weighting factors used. The process iteration was halted when a remaining area of 50 % was achieved. As explained before, three internal points of control were chosen (p_1(10,6), p_2(4.4,14) and p_3 (15.6,14)), to account the internal physical parameters. A mean of the temperature and mass of the boundary elements were considered too.



(a) (b)

(c)

Figure 3. Initial D_T maps for $w_M = 0.4$ and $w_H = 0.6$.

Figures 5 and 6 show the history of temperature and heat flux at the three internal points. Analyzing the heat flux graph, it is possible to verify that the gradient at the points p_1 and p_2 are increasing as the process evolves, while at point p_3 there is a visible decrease of the gradient value. In Fig. 5, the point p_3 shows an increase of temperature due to the decrease of the gradient at that location. It is evident that the region which presents less efficiency is being removed. Consequently, the flux is maximized along the path connecting the points p_1 and p_2. Figures 7 and 8 depict the history of mass concentration and mass flux at the three internal points. The mass flux at the points p_1 and p_2 drops after iteration 56, while it simultaneously increases at the point p_3. This occurs because after iteration 56 the D_T values at the internal points are gradually homogenized, making difficult to select locations with conspicuous lower values. The final geometry resulted in asymmetric Y-shaped design, with material concentrated at the right hand side, due to the weighting factors imposed. However, the mass flux is maximized from the point p_1 to p_3.

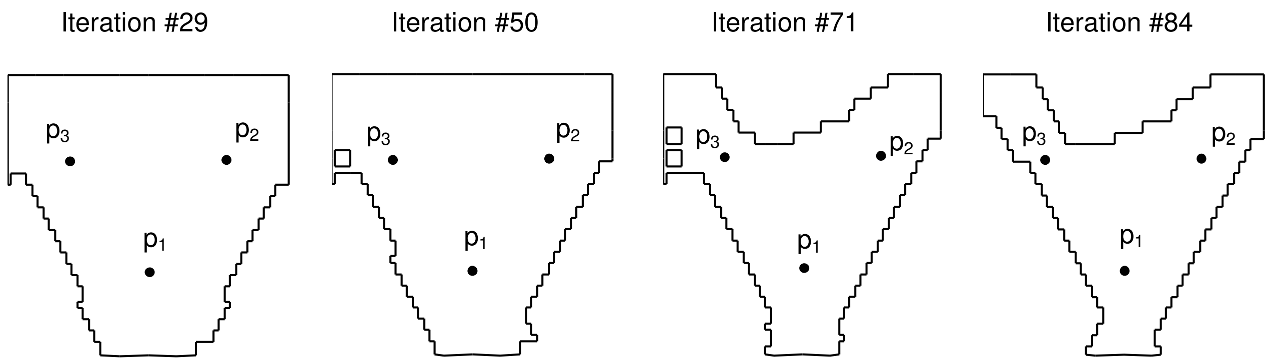


Figure 4. Topological evolution.

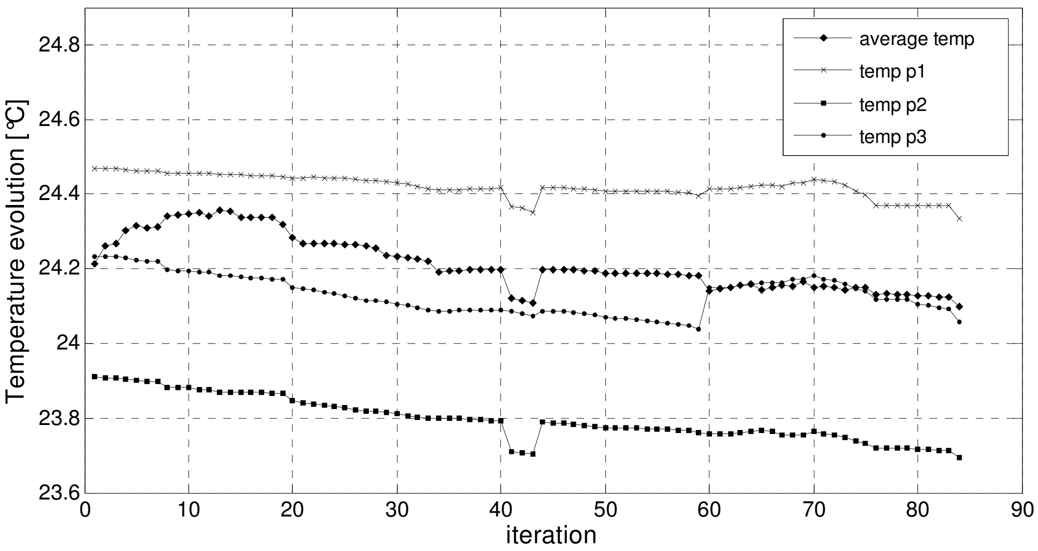


Figure 5. Temperature evolution per iteration for p_1, p_2 and p_3.

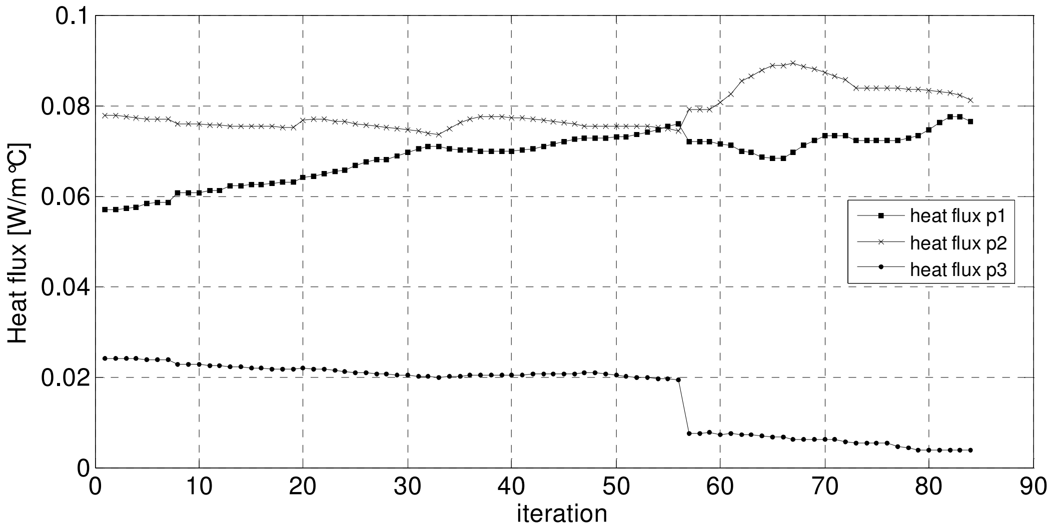


Figure 6. Heat flux evolution per iteration for p_1, p_2 and p_3.

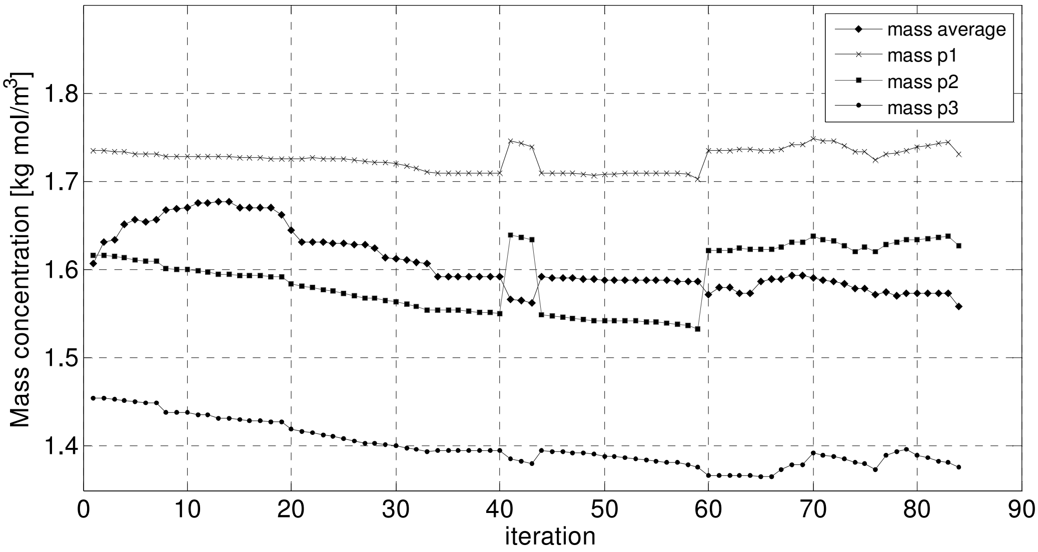


Figure 7. Mass evolution per iteration for p_1, p_2 and p_3.

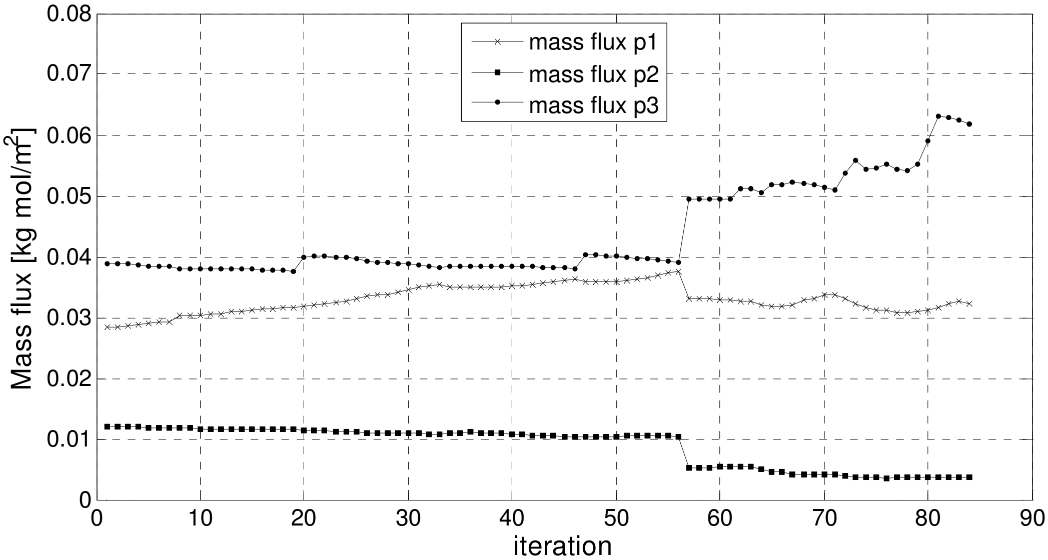


Figure 8. Mass flux evolution per iteration for p_1, p_2 and p_3.

5. ACKNOWLEDGEMENTS

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6. CONCLUSIONS

The goal of this work was to extend the application of topological-shape sensitivity analysis to optimization problems governed simultaneously by two different equations, i.e., heat and mass transfer. In order to achieve this, a compromise optimization scheme was used, relying on topological derivative results for both problems. The BEM was used to provide the numerical solution. It is important to point out that DT has the potential total energy as an implicit cost function. Therefore, the regions which store energy less efficiency are be removed. The final topology obtained for the numerical case presented resulted in an asymmetric Y-shape, showing the weighting factors influence for each problem (mass or heat transfer). It was shown that the present methodology can deliver optimal designs of solids in problems submitted to multi-criteria. It is also interesting to note the importance in specifying the priorities for both problems in order to reach physically meaningful solutions.

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