MODELING OF TURBULENT COMBUSTION IN INERT POROUS MEDIA

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Abstract. *Combustion in inert porous media has been extensively investigated due to the many engineering applications and demand for developing high efficiency power production devices. The growing use of efficient radiant burners can be encountered in the power and process industries and, as such, proper mathematical models of flow, heat and mass transfer in porous media under combustion can benefit the development of such engineering equipment. This paper proposes a new mathematical model for computing temperature and flow variables inside a porous burner. A new concept called "double-decomposition" is used to represent all transported variables. A set of governing equations is presented and the numerical solution method proposed is discussed. Computations are carried out for a test case considering a simple one-energy equation model and one-step reaction rates. Simulations are presented comparing the inclusion of turbulence and radiation transfer in the model. It is shown that for high Re flows, inclusion of turbulence is as important as modeling radiation for obtaining reliable temperature distribution within the porous material.*

Keywords: *porous combustor, turbulence modeling*

1. INTRODUCTION

The advantages of having a combustion process inside an inert porous matrix are today well recognized [1,2,3,4,5,6,7]. Hsu *et al* (1993) [8] points out some of its benefits including higher burning speed and volumetric energy release rates, higher combustion stability and the ability to burn gases of a low energy content. Driven by this motivation, the effects on porous ceramics inserts have been investigated in Peard *et al* (1993) [9].

Turbulence modeling of combustion within inert porous media has been conducted by Lim & Matthews (1993) [10] on the basis of an extension of the standard k-ε model of Jones & Launder (1972) [11]. Work on direct simulation of turbulence in premixed flames, for the case when the porous dimension is of the order of the flame thickness, has also been reported in Sahraoui & Kaviany (1995) [12].

Motivated by the foregoing, this paper presents computations of combustion flow in an inert porous media and compares the effects of radiation and turbulence in soothing temperature distribution within the domain of calculation.

Two geometries are here investigated. The first one considers an array of porous burners as schematically shown in Figure 1a. The burners are of rectangular shape and displaced in an orderly fashion imbedded in a large porous medium subjected to a cross stream of a mixture of air and fuel (methane). The arrangement is such that one can identify a repetitive cell along the transversal direction. A computational grid for calculating such cell is shown in Figure 1b. One assumes that only within the porous burners combustion takes place (Figure 1b) whereas in the surrounding medium unburned gases pass without ignition. Such hypotheses could be associated with the fact that, depending upon certain medium characteristics (such as porosity), ignition is suppressed and the flame vanishes in the porous matrix. The second geometry is shown in Figure 2 where a two-dimensional porous burner is built with a porosity such that ignition will occur somewhere inside the matrix, provided that conditions are such that the flame is stabilized inside the burner. For solving the flow and heat transfer within such two configurations, the mathematical model below is used.

2. GOVERNING EQUATIONS

The mathematical model here employed is based on the "*double-decomposition*" concept, which has been described in detail in a recently published book, de Lemos (2006) [13]) as well as in book chapters, Lage *et al.* (2002) [14], de Lemos (2004) [15], de Lemos (2005a) [16], de Lemos (2005b) [17]. In that work, transport equations are volume averaged according to the Volume Averaging Theorem (Slattery (1967) [18], Whitaker (1969) [19], Gray & Lee (1977) [20] in addition of using time decomposition of flow variables followed by standard time-averaging procedure for treating turbulence. As the entire equation set is already fully available in open literature, these equations will be reproduced here and details about their derivations can be obtained in the aforementioned references. Basically, in all the above-mentioned work the flow variables are decomposed in a volume mean and a deviation (classical porous media analysis) in addition of being also decomposed in a time-mean and a fluctuation (classical turbulent flow treatment). Because mathematical details and proofs of such concept are available in a number of worldwide available papers in the literature, they are not repeated here. These final equations are:

2.1. Macroscopic continuity equation:

$$\nabla . \overline{\mathbf{u}}_D = 0 \tag{1}$$

where, $\overline{\mathbf{u}}_D$ is the average surface velocity (also known as seepage, superficial, filter or Darcy velocity). Equation (1) represents the macroscopic continuity equation for an incompressible fluid.

2.2. Macroscopic momentum equation:

$$\rho \nabla \cdot \left(\frac{\overline{\mathbf{u}}_D \overline{\mathbf{u}}_D}{\phi} \right) = -\nabla\left(\phi \langle \overline{p} \rangle^i\right) + \mu \nabla^2 \overline{\mathbf{u}}_D + \nabla \cdot \left(-\rho\phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i \right) + \phi\rho\mathbf{g} - \left[\frac{\mu\phi}{K} \overline{\mathbf{u}}_D + \frac{c_F \phi\rho |\overline{\mathbf{u}}_D| \overline{\mathbf{u}}_D}{\sqrt{K}} \right] \tag{2}$$

where the last two terms in equation (2), represent the Darcy and Forchheimer contributions. The symbol K is the porous medium permeability, $c_F = 0.55$ is the form drag coefficient, $\langle p \rangle^i$ is the intrinsic (fluid phase averaged) pressure of the fluid, ρ is the fluid density, μ represents the fluid viscosity and ϕ is the porosity of the porous medium.

Turbulence is handled via a macroscopic $k-\varepsilon$ model given by,

$$\rho \nabla \cdot \left(\overline{\mathbf{u}}_D \langle k \rangle^i \right) = \nabla \cdot \left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_k} \right) \nabla\left(\phi \langle k \rangle^i\right) \right] - \rho \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D + c_k \rho \frac{\phi \langle k \rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}} - \rho\phi \langle \varepsilon \rangle^i \tag{3}$$

$$\rho \nabla \cdot \left(\overline{\mathbf{u}}_D \langle \varepsilon \rangle^i \right) = \nabla \cdot \left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_\varepsilon} \right) \nabla\left(\phi \langle \varepsilon \rangle^i\right) \right] + c_1 \left(-\rho \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D \right) \frac{\langle \varepsilon \rangle^i}{\langle k \rangle^i} + c_2 f_2 c_k \rho \frac{\phi \langle \varepsilon \rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}} - c_2 f_2 \rho\phi \frac{\langle \varepsilon \rangle^{i2}}{\langle k \rangle^i} \tag{4}$$

where

$$-\rho\phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i = \mu_{t_\phi} 2 \langle \overline{\mathbf{D}} \rangle^{\text{v}} - \frac{2}{3} \phi\rho \langle k \rangle^i \mathbf{I} \tag{5}$$

and

$$\mu_{t_\phi} = \rho c_\mu f_\mu \frac{\langle k \rangle^{i2}}{\langle \varepsilon \rangle^i} \tag{6}$$

Details on the derivation of the above equations can be found in Pedras & de Lemos (2001) [21]

2.3. Macroscopic Energy Equation:

As mentioned, for the sake of simplicity, we are assuming a local thermal equilibrium between the fluid and solid phases. This simplified model is known to be inappropriate to handle large temperature differences between the solid matrix and the burning gas, but for investigating the role of the mechanisms of turbulence and radiation, this simple mathematical framework may as well provide insight for more elaborated simulations later. As an example, one energy equation models in combustion in porous media has been applied by Mohamad et al (1994) [22] and de Neef et al (1999) [23].

The governing equation for energy transport is:

$$\left\{ \left(\rho c_p\right)_f \phi + \left(\rho c_p\right)_s (1-\phi) \right\} \frac{\partial \langle \overline{T} \rangle^i}{\partial t} + \left(\rho c_p\right)_f \nabla \cdot \left(\overline{\mathbf{u}}_D \langle \overline{T} \rangle^i \right) = \nabla \cdot \left\{ \mathbf{K}_{eff} \cdot \nabla \langle \overline{T} \rangle^i \right\} + \Delta H \, S_{fu} \tag{7}$$

where, $\langle T \rangle^i$ is the averaged temperature for both the solid and the liquid, $\mathbf{K}_{eff}$, given by:

$$\mathbf{K}_{eff} = \left\{ \overbrace{\phi k_f + (1-\phi)[k_s}^{conduction} + \overbrace{\frac{16\sigma(\langle \overline{T} \rangle^i)^3}{3\beta_r}}^{radiation}] \right\} \mathbf{I} + \underbrace{\mathbf{K}_{tor}}_{tortuosity} + \underbrace{\mathbf{K}_{disp}}_{dispersion} + \underbrace{\mathbf{K}_t + \mathbf{K}_{disp,t}}_{turbulence} \tag{8}$$

is the effective conductivity tensor, ΔH is the heat of combustion [J/kg], β_r is the extinction coefficient [1/m] and S_{fu} is the rate of fuel consumption [kg/m^{2}s] to be commented below. In Equation (8) all mechanisms contributing to heat transfer within the medium, together with turbulence and radiation, are included in order to compare their effect on temperature distribution.

A steady state form of (7) reads,

$$\left(\rho c_p\right)_f \nabla \cdot \left(\overline{\mathbf{u}}_D \langle T \rangle^i\right) = \nabla \cdot \left\{\mathbf{K}_{eff} \cdot \nabla \langle T \rangle^i \right\} + \Delta H\, S_{fu} \tag{9}$$

where all additional mechanisms of transfer, as mentioned, are included in $\mathbf{K}_{eff}$. Also, a transport equation for the fuel mass fraction reads,

$$\nabla \cdot (\overline{\mathbf{u}}_D \langle \overline{m}_{fu} \rangle^i) = \nabla \cdot \mathbf{D}_{eff} \cdot \nabla(\phi \langle \overline{m}_{fu} \rangle^i) - S_{fu} \tag{10}$$

where m_{fu} is the local mass fraction for the fuel. The effective dispersion, $\mathbf{D}_{eff}$, and the dispersion tensor, $\mathbf{D}_{disp}$, are defined as:

$$\mathbf{D}_{eff} = \mathbf{D}_{disp} + \mathbf{D}_{diff} + \mathbf{D}_t + \mathbf{D}_{disp,t} = \mathbf{D}_{disp} + \frac{1}{\rho}\left(\frac{\mu_\phi}{Sc_\ell} + \frac{\mu_{t_\phi}}{Sc_{\ell,t}}\right)\mathbf{I} = \mathbf{D}_{disp} + \frac{1}{\rho}\left(\frac{\mu_{\phi,ef}}{Sc_{\ell,ef}}\right)\mathbf{I} \tag{11}$$

and

$$-\rho \langle {}^i\overline{\mathbf{u}}\ {}^i\overline{m}_{fu} \rangle^i = \rho\, \mathbf{D}_{disp} \nabla . \langle \overline{m}_{fu} \rangle^i \tag{12}$$

The boundary conditions for solving the above equation set in the domain of Figure 1b were given values for all flow properties in $x/L=0$. Further, at the north and south boundaries, symmetry boundary conditions were applied. At the exit, null diffusion flux condition was implemented also for all variables.

3 Simple Combustion Model

In this work, for simplicity, the chemical exothermic reaction is assumed to be instantaneous and to occur in a single step, which is given by the chemical reaction,

$$CH_4 + 2(1+\Psi)(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2\Psi O_2 + 7.52(1+\Psi)N_2 \tag{13}$$

where ψ is the excess air in the reactant stream at the inlet of the porous foam. For the stechiometric ratio, ψ=0.

The rate of fuel consumption over the total volume (gas plus solid) was determined by a one step Arrehnious reaction [24] given by

$$S_{fu} = \rho\phi A \overline{m}_{fu}\, \overline{m}_{ox} \exp[-E / R \langle \overline{T} \rangle] \tag{14}$$

where m_{fu} and m_{ox}, are the local mass fractions for the fuel and oxidant, respectively, A is the pre-exponential factor [1x10^{10} m^3/(kg.s)] and E is the activation energy [1.4 x 10^8 J/kmol], where all values used are the ones commonly used in the literature for combustion of methane,

The reaction rate was assumed to occur only within the burning muffin by having its porosity ϕ set up in such way that the flame was hold inside it, according to several previous studies in the literature which indicate that below a certain value of porosity combustion can not be sustained. So, the flow around it was assumed to be not ignited. This is in fact an over simplification of the problem but it was assumed for the sake of simplicity and after considering the main objective of this work, which was to investigated the sole effect of including turbulence and radiation in the transport equations above. Also, the dispersion mechanism in (8) and (11)_was 'turned off' so that $\mathbf{K}_{disp}$ and $\mathbf{D}_{disp}$ were set to zero.

3 RESULTS AND DISCUSSION

3.1 Arrays of combustors of Figure 1

Effect of Extinction Coefficient β Figure 3 presents two-dimensional plots for temperature distribution calculated with laminar (left) and turbulence models (right). When radiation is not considered, Figure 3*a,b*, one can see that turbulence spreads at a faster rate heat than when using the laminar solution model. If the radiation model is included (*c,h*), the lower the extinction coefficient, the greater the distribution effect of heat at the domain outlet. This behavior can be understood by noting the radiation model implemented in (8) where an increase in $\mathbf{K}_{eff}$ is obtained for lower values of β_r. Therefore, increasing $\mathbf{K}_{eff}$ one increases the effective diffusisive effect leading to flatter temperature profiles. For the sake of comparisons when using both turbulence and radiation mechanisms, the value of β_r.= 1000m^{-1} was used in all computations below.

Effect of Inlet Velocity

Figure 4 shows the effect of inlet velocity U_{in} on the cross-stream temperature profiles at the exit (x/L = 1.0, see Figure 1b). The figure indicates that for laminar flows, a substantial reduction on the *T* profiles is obtained for low inlet velocity cases. For high velocity runs (Figure 4*d*), little differences are observed when the laminar and laminar-plus-radiation models are compared. On the other hand, when the turbulence mechanism is included, temperature profiles are substantially reduced and energy transfer to the cross-stream direction is enhanced. Comparing all plots one can infer that radiation plays an important role for low velocity runs whereas turbulent transfer cannot be neglected when the fluid mass flow rate is of higher value. In fact, if one intends to model high speed flow in highly porous media, such as ceramic foams used in modern combustion systems, one cannot neglect turbulent transport as it affects dramatically energy transfer within the porous medium.

Effect of Excess Air

Figure 5 finally shown the effect of excess air coefficient ψ on temperature profiles at x/L=1. One can see a substantial reduction on temperature values for ψ greater than zero. Also important to observe is that for ψ>0 (Figure 5*b*), radiation seems to play a less important role in reducing temperature levels. For either laminar or turbulent solution, Figure 5*b* indicates that the most considerable *T* reduction is obtained when the turbulence mechanism is included.

3.2 Porous combustor of Figure 2

Figure 6 finally presents mass the effect of flow model on mass concentration distribution for fuel. As the flow enters the chamber, the flame is bended by the expanding flow, contrary to what occurs in a flat one-dimension burner, which is extensively studied in the literature. The use of more elaborate model, including turbulence mixing, gives a thicker region of unburnt fuel (Figure 6b). Also, using a turbulence only, without accounting for the radiant heat transfer, the flame gets open and methane exits the burner (Figure 6e). Further, using the turbulence model shown above predicts high levels of *k* as the accelerating flow crosses the porous bed transforming mean kinetic energy of the fluid stream into turbulence (Figure 6f). When both a turbulence and radiation models are not applied, no ignition was computed inside the burner under the same flow conditions used for calculating results shown in Figure 6. **Figure 7** finally shows the centerline combustor temperature (a) and non-dimensional *k* (b) when using distinct models. One can see that consideration of radiant transfer and turbulence gives results quite spread apart, so that inclusion of all transfer mechanisms must be considered for reliable porous combustor modeling.

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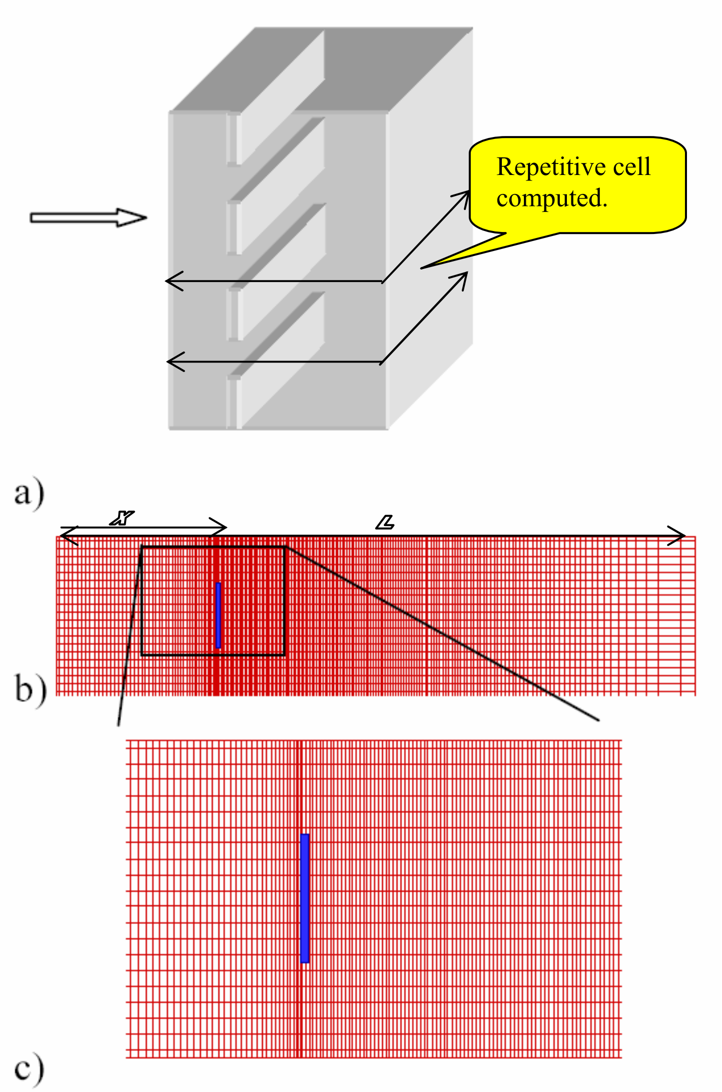

Figure 1 – Geometry under consideration: a) Array of porous burners of a rectangular bar form embedded in a porous medium of different porosity, b) Repetitive cell and computational grid, c) Detail of Porous muffin

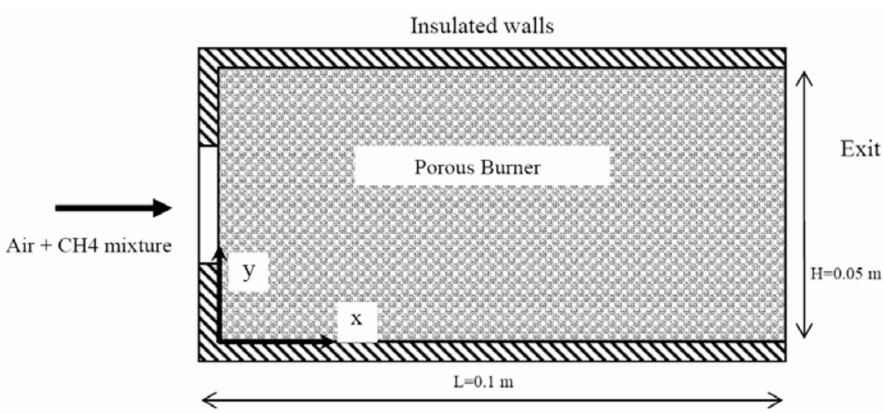

Figure 2 - Two-dimensional porous combustor.

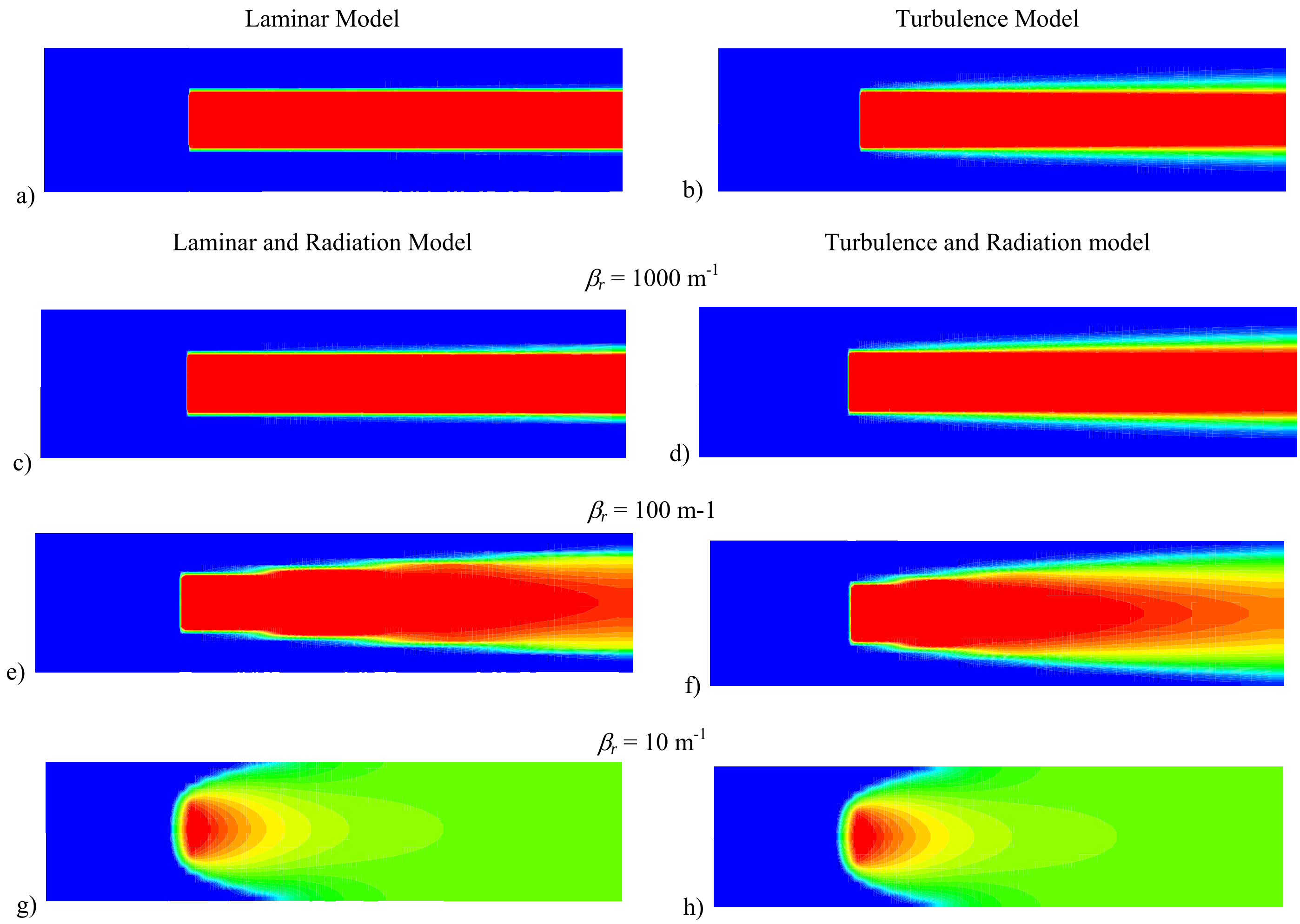

Figure 3 - Effect of extinction coefficinet β_r.

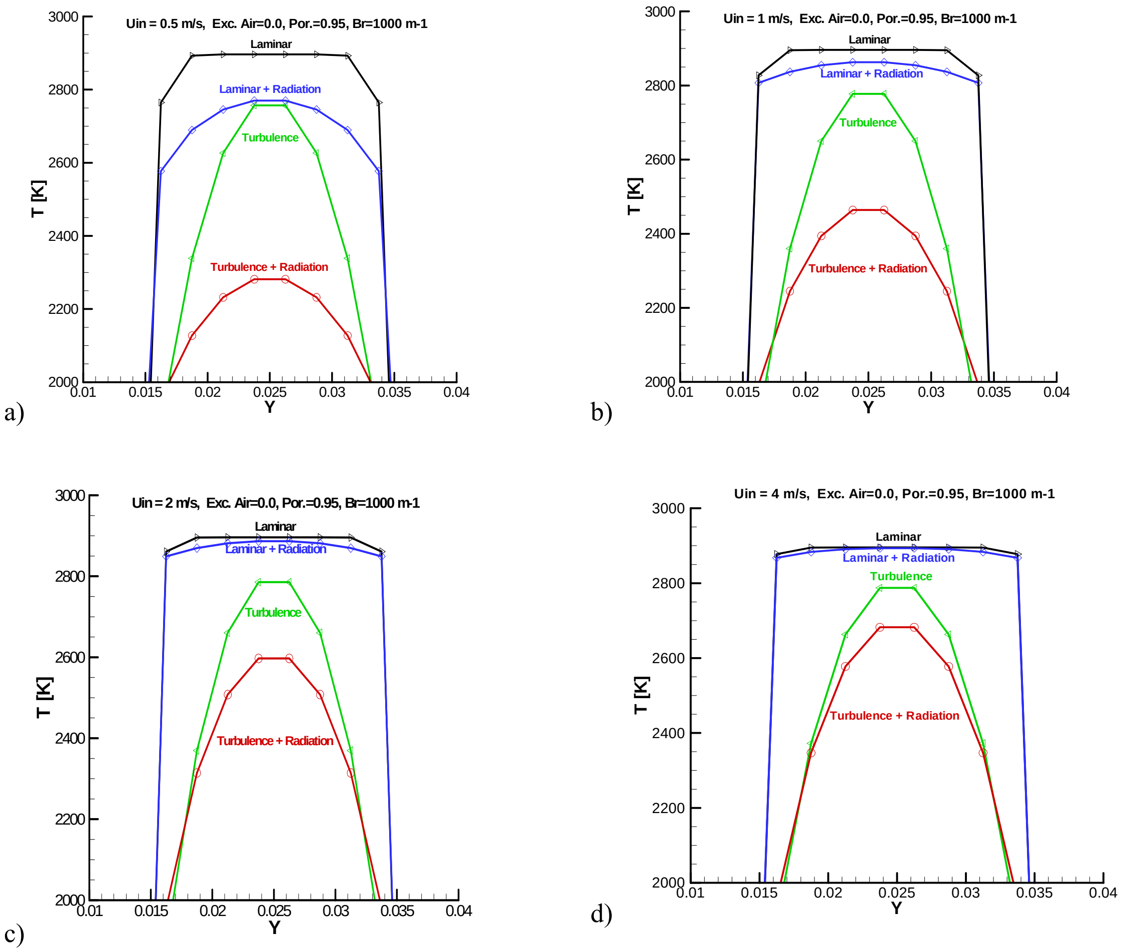

Figure 4 - Effect of inlet velocity, ψ=0.0, ϕ = .95, β_r= 1000m^{-1}, *x/L*=1; *a)* U_{in}=0.5 m/s, *b)* U_{in}=1 m/s, *c)* U_{in}=2 m/s, *d)* U_{in}=4 m/s,

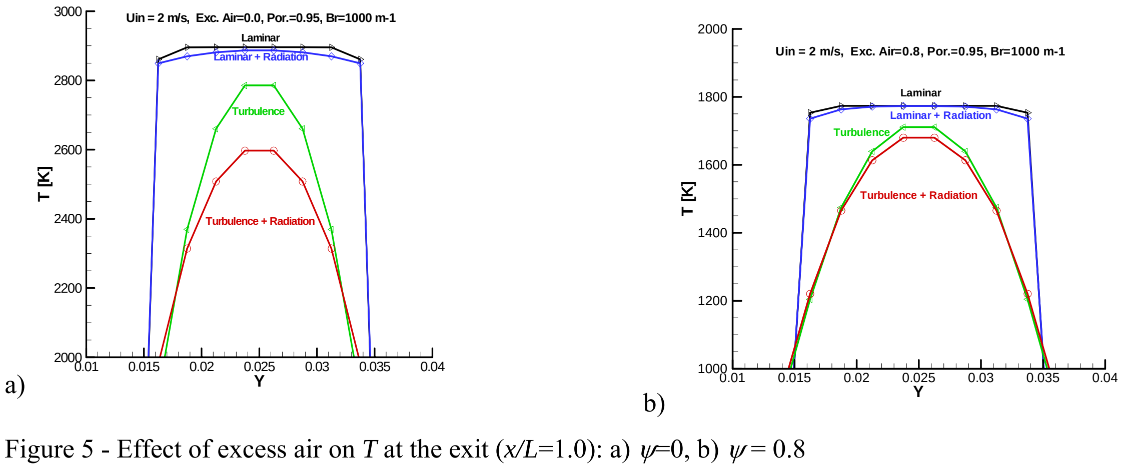

Figure 5 - Effect of excess air on *T* at the exit (*x/L*=1.0): a) *ψ*=0, b) *ψ* = 0.8

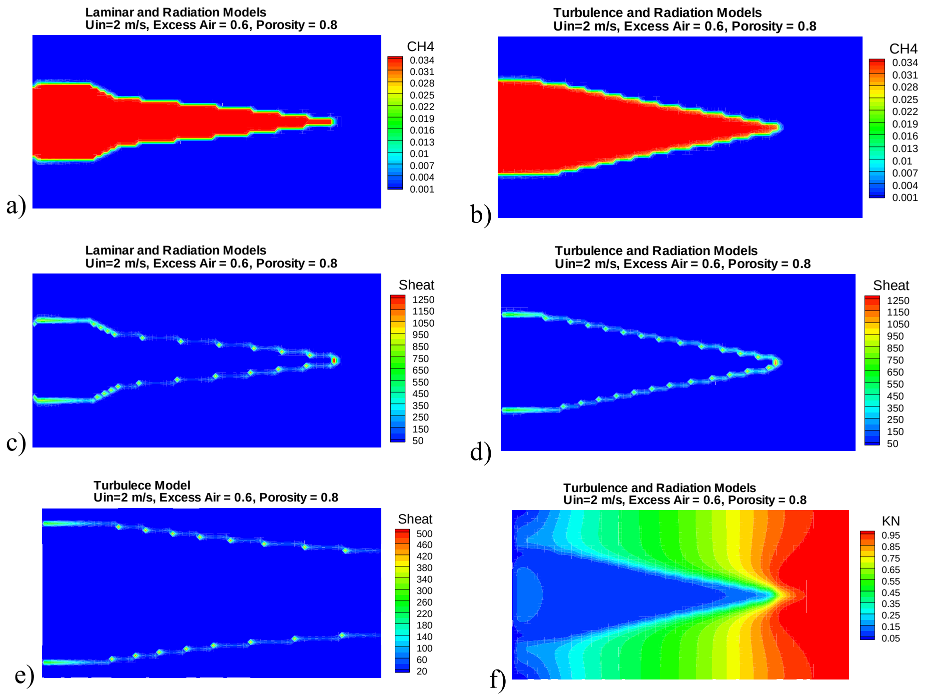

Figure 6 - Effect of model on fuel mass fraction distribution, m_{CH4}: a) Laminar and radiation model; b) Turbulence and radiation models; c) Flame front for laminar and radiation models; d) Flame front for turbulence and radiation models; e) Flame front for turbulence model, f) Normalized turbulent kinetic energy.

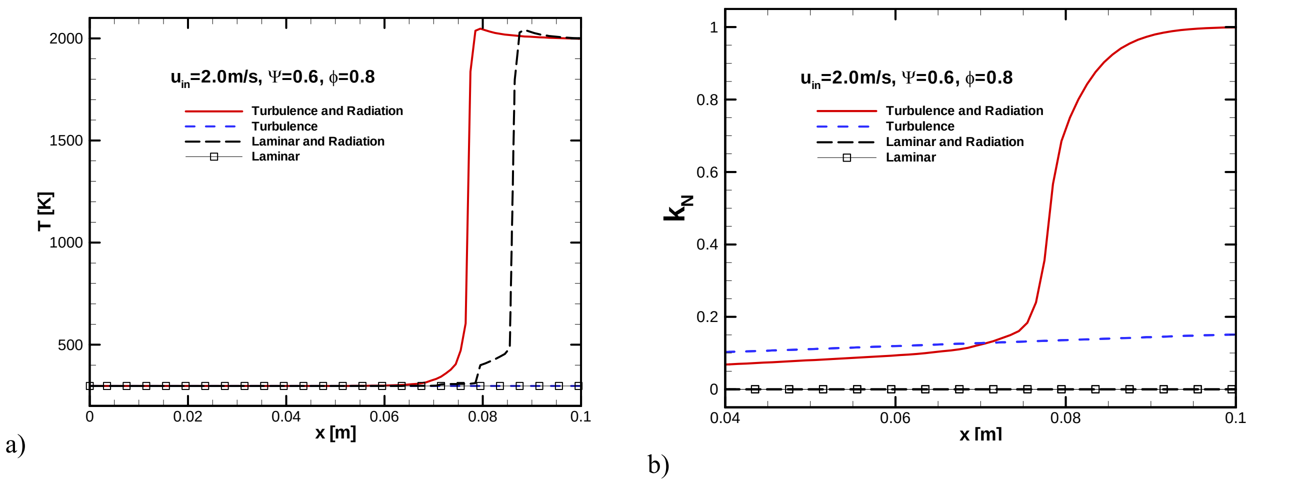

Figure 7 - Effect of distinct mathematical models on temperature (a) and non-dimensional turbulent kinetic energy (b) along combustor center line y/H=0.5 (see Figure 2).