LOCAL AND AVERAGE HEAT TRANSFER COEFFICICENTS FOR ROTARY HEAT EXCHANGERS

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Abstract. Heat recovery systems have traditionally being among the most cost effective energy saving techniques, both in power production (pre-heat of combustion air)and acclimatization applications (pre-cool of fresh air stream). Although the design figures for pre-heating and pre-cooling differs significantly, the mathematical modeling of rotary regenerators requires a number of simplifying assumptions regarding the heat transfer and fluid flow across the matrix. In particular, all the models developed so far rely on neglecting the thermal resistance imposed by the air layer when compared to the thermal resistance offered by the storage material, i.e., Bi>>Nu. Accordingly, the temperature across the channel is assumed to be uniform (lumped capacitance), exhibiting a distribution only in the flow direction. The present work aims at the evaluation of the distribution of the heat transfer coefficient (Nu) along the direction of the flow, allowing the air-side thermal resistance to be accounted for. The mathematical model is developed and numerically solved, using a fully-implicit finite-volume technique. The results show that the heat transfer coefficient distribution is remarkably non-uniform regarding both position and time, in opposition to the current design process, which often considers this parameter to be constant throughout the heat transfer process
Keywords*: Heat transfer, Regenerators, Gas Turbines.*

1. INTRODUCTION

The analysis of the periodic behavior of rotary regenerators, before the advent of modern numerical techniques, was restrained to simplified analytical solutions and graphical techniques, as summarized by Coppage and London (1953). Consider Figure 1, which depicts a channel and a storage element. The following simplifying assumptions are common to most of the models developed ever since:

1) The thermal storage within the hot and cold streams is negligible.
2) Constant and uniform thermo physical properties for both fluids and storage material.
3) Fully developed (hydrodynamic) flow in the channel.
4) No carry over and mixing between the two streams.
5) Uniform fluid temperatures in any cross section of the channel.
6) Thermal conductance within the storage material is negligible in the flow direction and infinite in the normal direction to it.

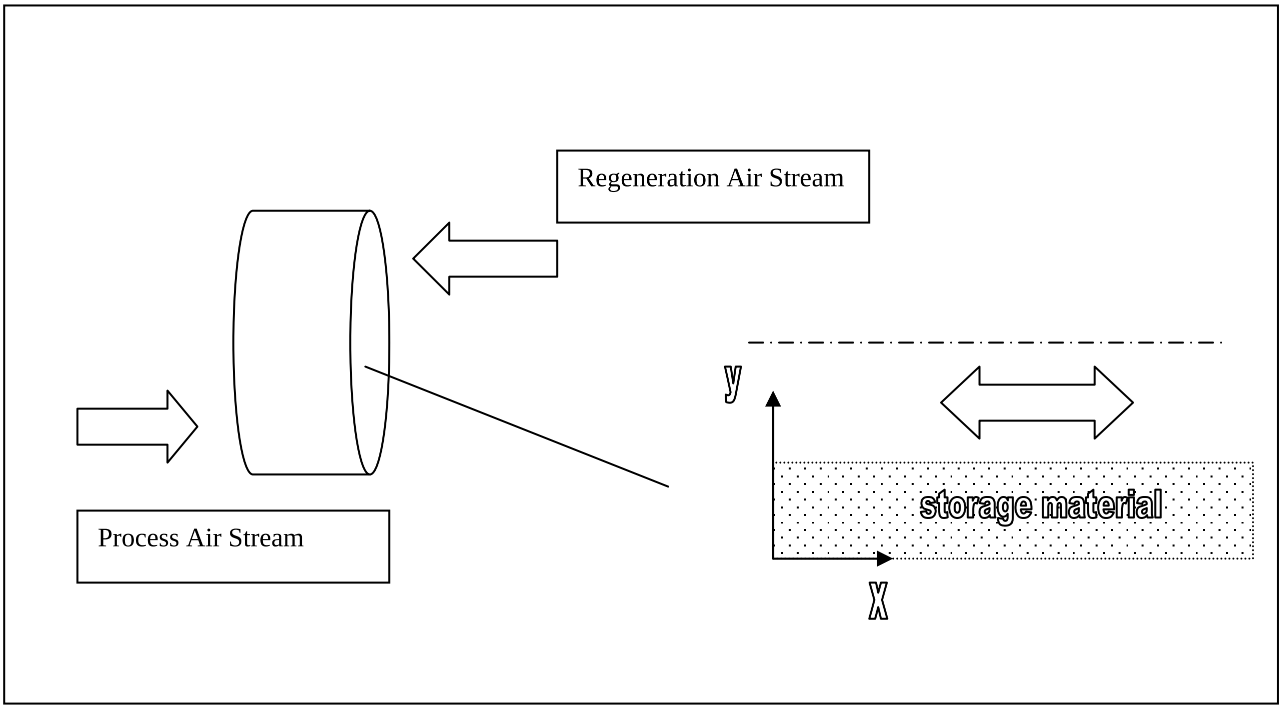

Figure 1: Regenerator Schematic

The low heat capacity of the fluid as compared to the storage material usually justifies assumption (1). Assumption (2) relies on small temperature variation across the regenerator, which is often a better approximation for HVAC rather than gas turbine applications. Also, regenerators are almost exclusively devoted to heat recovery on air or combustion product streams. Accordingly, the low kinetic diffusivity usually claims less than 1% of the regenerator length to develop a steady velocity profile, making assumption (3) a reasonable one. As for assumption (4), the carry over is proportional to the angular speed, whereas leakage is significant for great pressure differences between the two streams. Shah and Skiepko (2005) provide a comprehensive analysis of these effects. Assumption (5) is common to all analysis, as they rely on imposed values for the heat transfer coefficient between the fluid and the storage material, and will be addressed in the present work. Longitudinal (flow direction) conduction has been accounted for by Bahnke and Howard (1964), though it is usually negligible for most regenerator designs. Assumption (6) has been addressed by most of the works in recent years: Szego and Schmidt (1976) considered the resistance to conduction in both directions employing a finite-difference technique. Shen and Worek (1992) employed the same technique to an effectiveness analysis of the rotary regenerator. More recently, Niu and Zhang (2002) accomplished a numerical study in ducts confined by arc and sinusoidal curves, which is a very accurate representation of channel geometry within actual rotary regenerators. All the aforementioned works rely on an imposed non-dimensional (Nu or NTU) heat transfer coefficient in the storage/removal processes. Even though its always possible to represent the phenomena using an average, uniformly-distributed parameter, the knowledge of local heat transfer parameters provides a further insight on the regenerator modeling. Due to the transient nature of the temperature field on the storage element, the flow on the channel is necessarily non-developed from the thermal point of view. Accordingly, the present work aims at obtaining local values of the Nusselt number (Nu) along the channel as it performs a cycle.

2. MATHEMATICAL MODEL

Consider the single channel shown on Figure1, in the counter flow operation. Process air and regeneration flow takes place alternatively in the flow channel. Accordingly,

$$0 \le t \le P_h \ , \ T_{in} = T_{hin} \ \ at \ \ x = 0$$
$$P_h \le t \le P_t \ , \ T_{in} = T_{cin} \ \ at \ \ x = L$$

The flow inside the channel is typically laminar, with Reynolds Number Re as high as 50. Since the hydrodynamic entry length is given by

$$\frac{x_e}{D} \cong 0.05\,\mathrm{Re} \tag{1}$$

we conclude that the entry length is typically as high as 2.5D. Since the regenerator length is typically a hundred times larger than the channel diameter, the flow can be taken as developed, given by

$$u^* = \frac{3}{2}\left[1 - \left(1 - \frac{y}{\delta_c}\right)^2\right] \tag{2}$$

where $u^* = \dfrac{u(y)}{U}$

Neglecting the heat diffusion in the flow direction, the temperature field is described by

$$\left(\rho C_p\right)_f u \frac{\partial T_f}{\partial x} = k_f \frac{\partial^2 T_f}{\partial y^2} \qquad \text{or else}$$

$$\frac{\partial T_f}{\partial x} = \frac{\alpha_f}{u}\frac{\partial^2 T_f}{\partial y^2} \tag{3}$$

defining the following non-dimension variables,

$$y^* = \frac{y}{\delta_{ch}}$$

$$x^* = \frac{x}{\delta_{ch}}$$

$$Pe = \frac{\delta_{ch} U}{\alpha_f}$$

$$\Theta = \frac{(T - T_{ci})}{(T_{hi} - T_{ci})}$$

Applying these definitions to Eq. (3),

$$u^* \frac{\partial \Theta_{fh}}{\partial x^*} = \frac{1}{Pe_h} \frac{\partial^2 \Theta_{fh}}{\partial y^{*2}} \tag{4}$$

subject to the following boundary conditions

$$\Theta_f = 1.0 \quad at \quad x^* = 0 \tag{5}$$

$$\frac{\partial \Theta_f}{\partial y^*} = 0 \quad at \quad y^* = 1.0 \tag{6}$$

$$\frac{\partial \Theta_f}{\partial y^*} = \frac{\partial \Theta_{wh}}{\partial y^*} \quad at \quad y^* = 0 \tag{7}$$

in an analogous way, for the cold period,

$$u^* \frac{\partial \Theta_{fc}}{\partial x^*} = \frac{1}{Pe_c} \frac{\partial^2 \Theta_{fc}}{\partial y^{*2}} \tag{8}$$

subject to the following boundary conditions

$$\Theta_{fc} = 1.0 \quad at \quad x^* = 0 \tag{9}$$

$$\frac{\partial \Theta_{fc}}{\partial y^*} = 0 \quad at \quad y^* = 1.0 \tag{10}$$

$$\frac{\partial \Theta_{fc}}{\partial y^*} = \frac{\partial \Theta_{wc}}{\partial y^*} \quad at \quad y^* = 0 \tag{11}$$

Still referring to Figure 1, the temperature field in the storage element is described by

$$(\rho C_p)_w \frac{\partial T_{wh}}{\partial t_h} = k \frac{\partial^2 T_{wh}}{\partial y^2} \tag{12}$$

defining non-dimensional time as

$$t_h^* = \frac{1}{P_h}\left(t_h - \frac{x}{L} t_{dh} \right) \tag{13}$$

and non-dimensional period as

$$Fo_h = \frac{\alpha_w P_h}{\delta_s^2} \tag{14}$$

Eq.(8) then becomes

$$\frac{\partial \Theta_{wh}}{\partial t_h^*} = Fo_h \frac{\partial^2 \Theta_{wh}}{\partial y^{*2}} \tag{15}$$

as for the cold period,

$$\frac{\partial \Theta_{wc}}{\partial t_c^*} = Fo_c \frac{\partial^2 \Theta_{wc}}{\partial y^{*2}} \tag{16}$$

subject to the following boundary conditions

$$\frac{\partial \Theta_{wh}}{\partial y^*} = 0 \quad at \quad y^* = \frac{\delta_w}{\delta_c} \tag{17}$$

$$\frac{\partial \Theta_{wh}}{\partial y^*} = \frac{\partial \Theta_f}{\partial y^*} \quad at \quad y^* = 0 \tag{18}$$

and for the cold period

$$\frac{\partial \Theta_{wc}}{\partial y^*} = 0 \quad at \quad y^* = \frac{\delta_w}{\delta_c} \tag{19}$$

$$\frac{\partial \Theta_{wc}}{\partial y^*} = \frac{\partial \Theta_{fc}}{\partial y^*} \quad at \quad y^* = 0 \tag{20}$$

The periodicity condition is given by

$$\Theta_{wh}(x^*, y^*, 0) = \Theta_{wc}(x^*, y^*, 2\pi) \tag{21}$$

The domain of equations (4) to (8), (15) and (16) is discretized into finite-volumes, using the fully-implicit and the upwind schemes, to respectively describe the transient and convective terms. The periodic nature of the problem implies an iterative solution. An initial temperature field within the solid is guessed, and sets off the evaluation of the temperature field within the channel and in subsequent time steps. By the end of the cycle, the temperature within the solid is compared to the guessed field, so as to meet the periodicity condition (Eq.21). If the required convergence criteria is not attained, the procedure is repeated, the new guessed field being equal to the previously obtained field.

$$Conv.Crit. = \frac{\Theta_{wh}(x^*, y^*, 2\pi) - \Theta(guess)_{wh}(x^*, y^*, 0)}{\Theta_{wh}(x^*, y^*, 2\pi)} \tag{22}$$

The Nusselt number is defined as the local non-dimensional heat flow at the interface, normalized by the flow bulk temperature

$$Nu_a = \frac{h\,\delta_c}{k_f} = \frac{\left.\partial\Theta_f \middle/ \partial y^*\right|_0}{\Theta_B} \tag{23}$$

where

$$\Theta_B = \int_0^1 u^*\,\Theta_f\,dy^* \tag{24}$$

Also, the non-dimensional heat flux at the interface at any position x* must equal the non-dimensional enthalpy net flow, as energy is to be conserved:

$$Nu_b = \frac{\partial}{\partial x^*}\int_0^1 u^*\,\Theta_f\,dy^* \tag{25}$$

The local heat balance error is defined as the normalized difference between Nu_a and Nu_b. Table 1 shows the sensibility of the HBE to the grid size and time step

3. RESULTS

Table 1: Local Heat Balance Error Sensitivity, Fo=1.0, x* = 0.5, t* = 0.5

i # pts. x dir	j # pts. y dir, solid	k # pts. y dir, channel	δt	Nu	HBE %
100	50	50	10^{-3}	1.83	0.061
100	50	50	10^{-4}	1.74	0.061
100	50	50	10^{-5}	1.75	0.060
100	70	70	10^{-3}	1.87	0.031
100	70	70	10^{-4}	1.75	0.030
100	70	70	10^{-5}	1.75	0.030

Although the modeling using non-dimensional parameters greatly simplify the analysis, actual regenerator dimensional figures should not be disregarded when performing the numerical simulations. Table 2 shows some typical design and operation parameters, as well as the resulting non-dimensional parameters

Table 2: Typical Design and Operation Figures

Dimensional		Non-Dimensional	
Solid Thickness:	0.09 to 0.18 mm	Solid Thickness:	1
Channel Thickness:	0.6 to 1.2 mm	Channel Thickness:	3 to 6
Channel Length:	100 to 200 mm	Channel Length:	1000 to 2000
Flow Velocity (channel)	3.0 to 10 m/s	Pe	100 to 500
Angular Speed	10 to 30 Rpm	Fo	10 to 1000
Thermal Diffusivity (solid)	0.01 to 10 m^2/s		

Figures 2 to 4 show the evolution of the Nu distribution during the hot period. It can be seen that the *Nu* distribution is significantly non-uniform along the channel, especially during the first quarter of the cycle, where the temperature differences are higher. Comparing Figures 2, 3 and 4 it can be learned that shorter non-dimensional periods (*Fo*) lead to higher average heat transfer rates. This effect is consonant with the design practice of rotary regenerators, whose effectiveness increases with the rotation rate (Kays and London, 1984). It is also expected that if the cold period is much longer than the hot period, the temperature distribution in the solid during the hot period will be close to zero. Thus, the model is expected to reproduce the *Nu* value of a thermally developed flow (*Nu* = 7.54), as a significant unbalance between the periods is imposed. This asymptotic behavior is indeed verified in Figures 5 and 6.

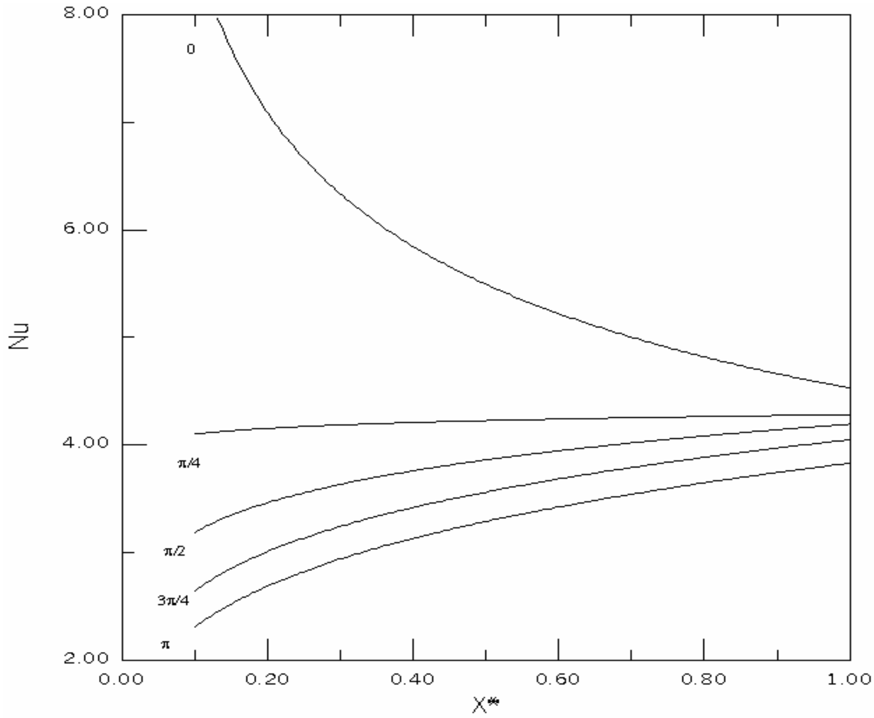

Figure 2: $Fo_h = Fo_c = 1.0$

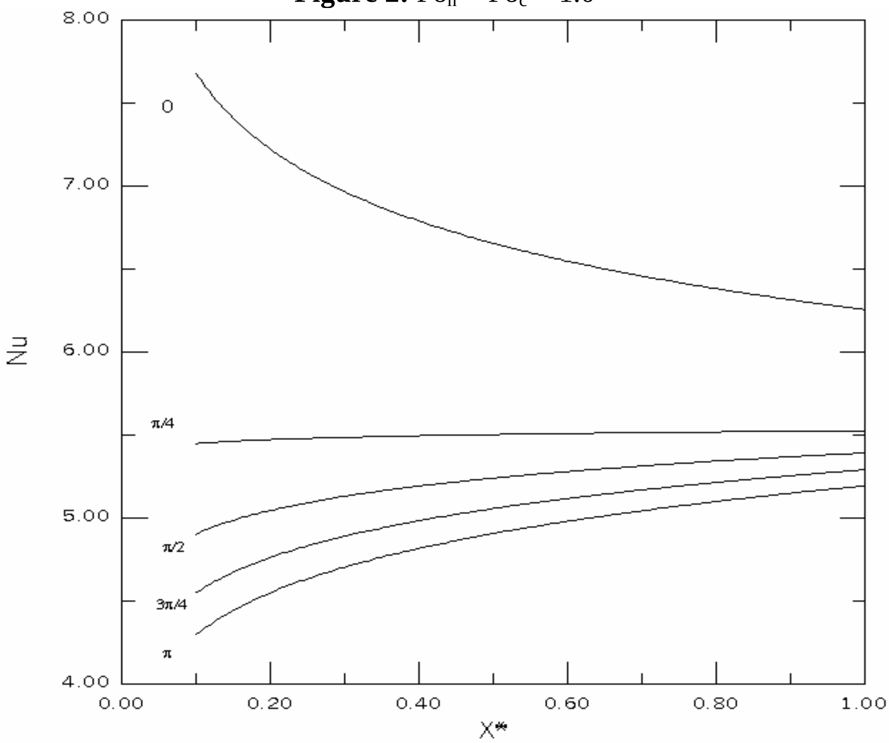

Figure 3: $Fo_h = Fo_c = 0.1$

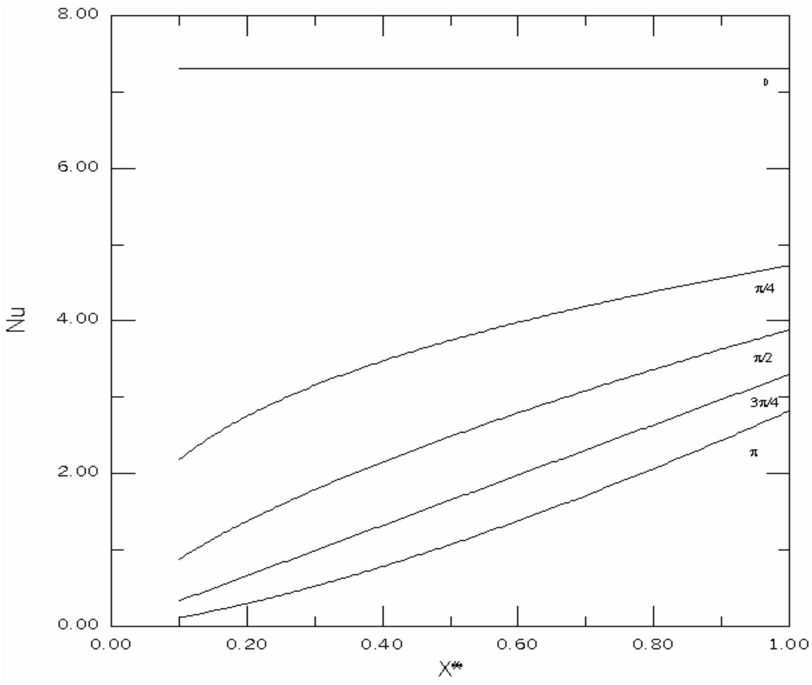

Figure 4: $Fo_h = Fo_c = 10.0$

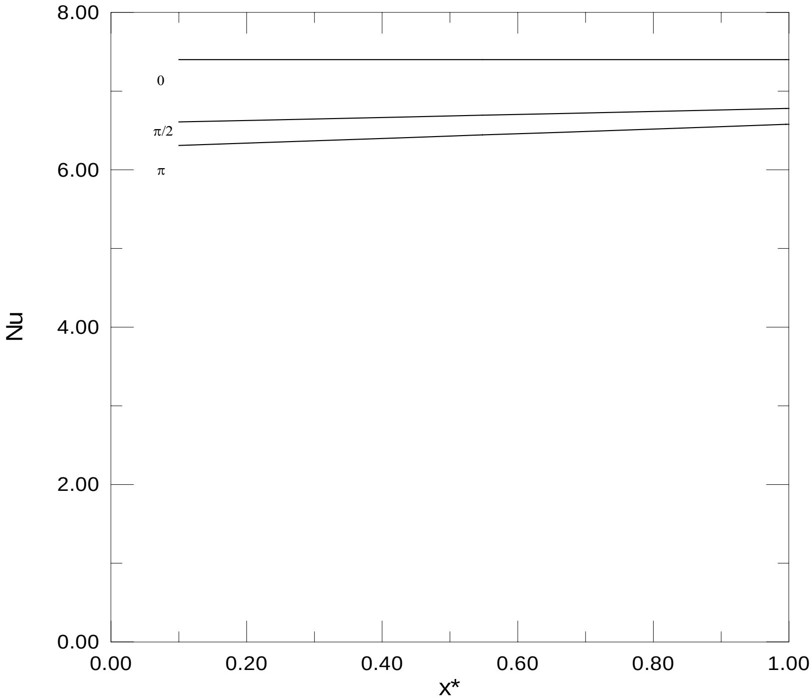

Figure 5: $Fo_h = 10^{-2}$, $Fo_c = 1.0$

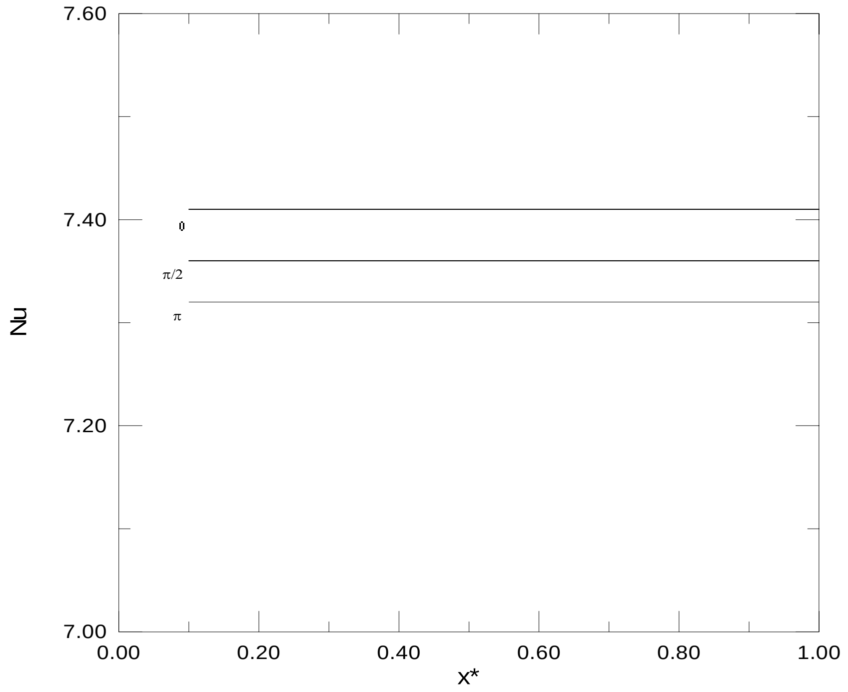

Figure 6: $Fo_h = 10^{-4}$, $Fo_c = 1.0$

4.CONCLUSION

A local model for the heat transfer phenomena in a rotary regenerator was developed, the solution of which, allows a closer view on the periodic thermal recovery process. Whereas most of design practices rely on an average value for the convective coefficient as an imposed value, it was shown that it can be significantly different from the values of a prescribed temperature or heat flux condition within a rectangular channel.

5. NOMENCLATURE

C_p	Specific heat (KJ/Kg K)
Fo	Fourier number
HBE	Heat balance error
HVAC	Heating, ventilation and air conditioning
k	Thermal conductivity (w/m k)
L	Length, (m)
Nu	Nusselt number
P	Period of time, (s)
Pe	Peclet number
T	Temperature (K)
t	Time, (s)
U	Free stream velocity, (m/s)
u	Velocity, (m/s)
u^*	Non-dimensional velocity
x	Longitudinal position, (m)
x^*	Non-dimensional longitudinal position
y	Position, transversal direction (m)
y^*	Non-dimensional position, transversal direction

Greek Letters

α Thermal diffusivity (m^2/s)
δ Thickness (m)
Θ Non-dimensional temperature
ρ Density, (kg/m^3)

Subscripts
h hot period
c cold period
ch channel
w wall
f fluid
t total (period)

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