

BLADE OPTIMIZATION OF SMALL HORIZONTAL AXIS WIND TURBINE

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Abstract. *Inexpensive wind power energy can be achieved by a proper design of a wind turbine coupled with an electric generator. Such devices can be made small enough to be used on isolated systems. The objective of this work is to present a procedure to optimize the wind turbine blade design by mathemathical programming. The procedure is based on a sequential minimization of penality functions by golden section search. The wind turbine aerodynamic model is based on the Blade Element Method and the optimization take into account both aerodynamic and structural aspects.*

Keywords: *wind turbine, optimization , hawt*

1. INTRODUCTION

In the present reality where the global warming is being considered serious issue, the wind-generated power presents itself as an alternative for low pollutant and renewable power generation supply. This kind of power supply solution is already in activity in the United States, India and several countries of Europe and with potential to further growth (GWEC,2006 and AWEA,2005). Brazil although is among the ten world's largest producers of wind energy produced only 29MW in 2004 of its 30 GW estimated potential (Ribeiro, 2006).

In Ribeiro (2006) it is proposed a systematized methodology of design for blades of small sized wind turbines. This methodology focused in simplicity for making the project easy to implement and low cost but not necessarily less efficient. With aid of this methodology it was designed the CEAWT 01 horizontal wind turbine witch has a 3.5 meters radius, operating point is at a 10 m/s wind speed produce 10.5 kW and is IEC 61400-2 compliant. The power output of 10kW is a trend for supplying small cities and has an estimated market of 6.6 millions of units by 2020 (AWEA, 2001).

This paper work is an attempt to further increase the power output of CEAWT 01 wind turbine by optimizing the torsion of the blade. This was done on the purpose to investigate how much more efficient would be CEAWT 01 if blade torsion constraints were free. Also this work investigates if an optimum design for a certain design point has actually inferior efficiency in off-design points.

2. AERODYNAMIC MODEL

2.1 Blade Element Method Equations

The aerodynamic model used for evaluating the wind turbine power is based in the Blade Element Method (BEM) in the same way it was used by Ribeiro (2006) in order to size CEAWT 01. Although it could be implemented more complex methods such as a Vortex Lattice Method (VLM) or a Modified Lifting Line Method (MLLM), the BEM was chosen on two basis. The first one is that the BEM makes it easy to perform simpler algorithms of minimum search, such as the golden section, since all the elements are independent of the other elements influences. The second and strongest reason for using BEM is the technical note NREL/TP-500-29494 "Unsteady Aerodynamics Experiment in the NASA-Ames Wind Tunnel: A Comparison of Predictions to Measurements" presented by Simms et al (2001) and cited by Ribeiro (2006). In the technical note eighteen different methods of CFD for estimating the output torque of a wind turbine were tested against wind tunnel data of a 1:1 model. Data presented by Ribeiro (2006) and reproduced in Fig. 1 shows that no method was more precise than any other for this experiment. Since BEM was among the eighteen methods tested, it was used in Ribeiro (2006) as well as in this work as a "time proven" robust method.

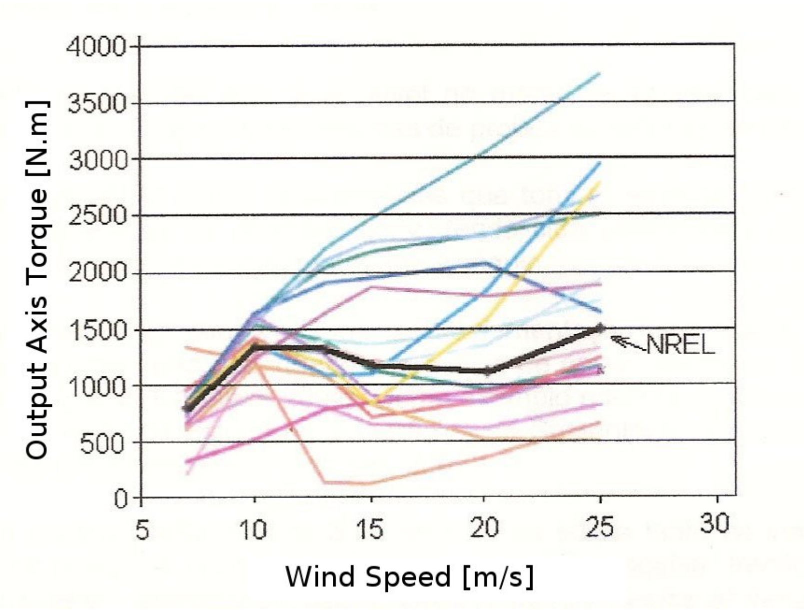

Figure 1. Results for NASA-Ames wind tunnel wind power turbine test - NREL is the wind tunnel data

The Blade Element Method after some manipulations, can be reduced to solve the following equation:

$$\tan(\phi(i)) + \frac{\sigma_r(i) \cdot C_l(i)}{4 \cdot F(i)} \cdot \csc(\phi(i)) + \frac{\sigma_r(i) \cdot C_l(i)}{4 \cdot F(i) \cdot \lambda_r(i)} \cdot \sec(\phi(i)) - \frac{V_{inf}}{\Omega \cdot r(i)} = 0 \tag{1}$$

Where:

$\lambda_r(i)$	Local ratio between tangential velocity and axial velocity.
$\phi(i)$	Angle between the resultant local wind speed vector and the blade plane.
$\sigma_r(i)$	Solidity of the chord in the ith disc.
Ω	Angular velocity of the blade.
$C_l(i)$	Lift coefficient on the ith element.
$F(i)$	Prandtl factor for correction to finite blade number
V_{inf}	Non perturbed speed of the air.
$r(i)$	Distance of the ith element to the turbine's axis

Further:

$$\phi(i) = \beta(i) - \alpha(i) \tag{2}$$

$$F(i) = \left(\frac{2}{\pi}\right) \cos^{-1}\left[e^{-\dfrac{B \cdot (R - r(i))}{2 \cdot r(i) \cdot \sin(\phi(i))}}\right] \tag{3}$$

$$\sigma_r(i) = \frac{B \cdot c(i)}{2 \cdot \pi \cdot r(i)} \tag{4}$$

$$\lambda_r(i) = \lambda \cdot \mu(i) \tag{5}$$

$$\lambda = \frac{\Omega \cdot R}{V_{Inf}} \tag{6}$$

$$\mu(i) = \frac{r(i)}{R} \tag{7}$$

Where:

$\alpha(i)$	Local angle of attack at the ith element of the blade.
$\beta(i)$	Local torsion angle of the blade at the ith element to the disk plane.
λ	Ratio of the blade's tip tangetial velocity to the axial unperturbed velocity
$\mu(i)$	Adimensional distance from the elemnt to the blade's hub
$c(i)$	Local chord of the ith blade element
B	Number of blades of the wind turbine.
R	Radius of the blade of the wind turbine.

2.2 Solution of Aerodynamic Equations

The first thing in nottice is that on Eq. (1) there is no term that relates the ith blade element to some other blade element so the blade elements are independent among themselves. This means that according to the Blade Element Theory it is possible to analyse each element individualy.

Also in Eq. (1) it is possible to check out that there is only one unknown variable which is $\phi(i)$. The inverse equation of Eq. (1) is difficult to be obtained because of the trigonometric operations on ϕ in $F(i)$ and also because of the term $C_l(i)$ witch allways is a non-linear function of β and ϕ.

This problem is generally solved by using goalseek algorithm in wich is normally based in some sort of minimum seek algorithm. Problem arises when the function have local minima. In this case most of the algorithms may converge to any of the minima. This is what happens in this case as can be seen in Fig. 2 where the NACA 23015 presents a local minimum and an absolute minimum (correct answer). This problems are observable in all the blade elements of the CEAWT 01.

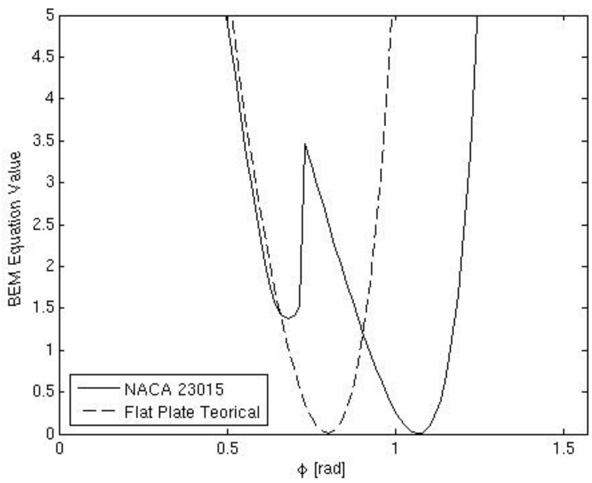

Figure 2. BEM equation value for various ϕ with two diferent airfoils for element $\mu = 0.1$ at $\lambda = 3$ on CEAWT 01

Benchmarking to the flat plate curve (wich does not have stall model) led to the hypothesis that the local maximum observed was indeed caused by the singularity of the C_l curve in the stall. A code was designed using the MATLAB^{TM} to pinpoint the local maxima based on the singularity of the C_l versus α curve by simply calculating ϕ using the stall α in Eq. (2) and then solving Eq. (1). The results are showed in Fig. 3 where is possible to say that the singular local maximum is really the stall of the blade in that section.

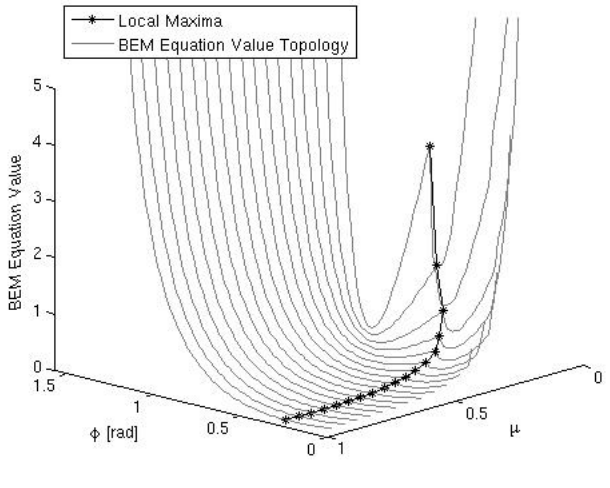

Figure 3. BEM equation value for various ϕ and μ on CEAWT 01 geometry with $\lambda = 3$

Being able to divide the search area in two parts, it was then possible perform a minimum seek based on golden section method algorithm as written in Luenberger (1984), and therefore determining two minima. One of the two is always the global minimum. Another program was implemented using MATLAB™ to perform the task of finding the value of ϕ for a given blade of wind turbine and wind conditions.

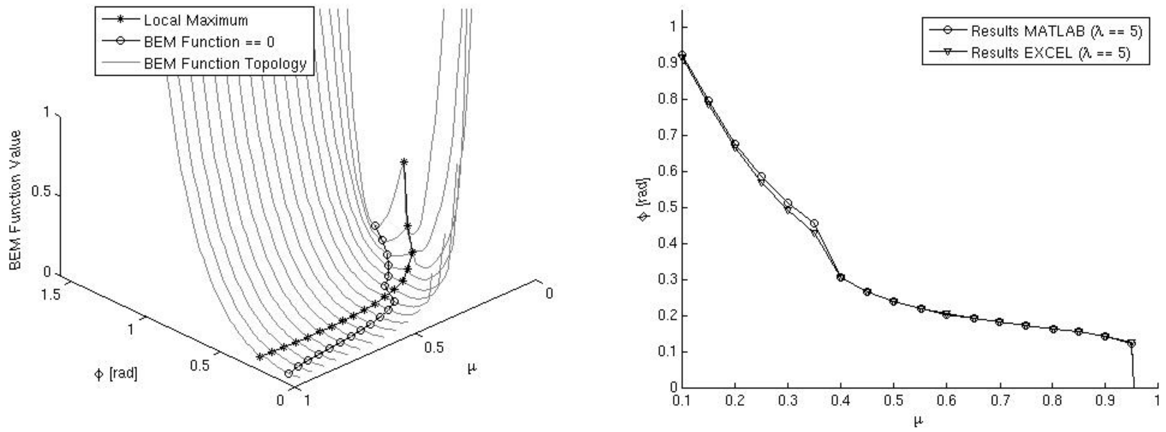

Figure 4. Left: ϕ witch zeroes the BEM equation. Right: Comparison between EXCEL™ software (Reference) and MATLAB™ software

Figure 4 left and right shows the result for the ϕ angles for CEAWT 01 at $\lambda = 5$ and $V_{Inf} = 10\ m/s$. In the left part of Fig. 4 it shows the BEM function values (Eq. 1) obtained on MATLAB™ on the right part it shows comparisons of the program written in MATLAB™ against one made in EXCEL™ used to create the curves in Ribeiro (2006). The benchmark was made to validate the software up to this point in several operation points (just one point is represented). Note should be made that the EXCEL™ counterpart of the present software uses the "goalseek" function witch is proprietary from MICROSOFT™ and therefore the algorithm is not revealed.

3. POWER OUTPUT OPTIMIZATION

3.1 Power Generation Model

Once the aerodynamics model is solved, the power can be obtained according to Ribeiro (2006) from the following formulas:

$$P = \frac{1}{2} \cdot \rho \cdot V_{Inf}^2 \cdot \pi \cdot R^3 \cdot \lambda \cdot \Omega \cdot I_P \tag{8}$$

$$I_P = \int_0^R \mu^2 \cdot \left[8 \cdot a' \cdot (1 - a) \cdot \mu - \frac{W \cdot B \cdot \dfrac{c}{R}}{V_{Inf} \cdot \pi} \cdot C_D \cdot (1 + a') \right] \mathrm{d}\mu \tag{9}$$

Where:

ρ	Local air density.
a	Axial influence coefficient.
a'	Tangential influence coefficient.
c	Local blade element chord.
P	Power output of the turbine.
W	Local total velocity (vector sum of axial and tangential velocity).

Since the problem is beeing solved in computers, discretization of the integral in 11 is needed:

$$I_P \simeq \frac{1}{2} \cdot (f_1 + f_N) + \sum_{i=2}^{N-1} f_i \tag{10}$$

$$f_i = \mu(i)^2 \cdot \left[8 \cdot a'(i) \cdot (1 - a(i)) \cdot \mu(i) - \frac{W(i) \cdot B \cdot \dfrac{c(i)}{R}}{V_{Inf} \cdot \pi} \cdot C_D(i) \cdot (1 + a'(i)) \right] \cdot \frac{R}{N} \tag{11}$$

$$a(i) = \frac{\dfrac{\sigma_r(i) \cdot C_l(i) \cdot \cos(\phi(i))}{4 \cdot F(i) \cdot \sin^2(\phi(i))}}{1 + \dfrac{\sigma_r(i) \cdot C_l(i) \cdot \cos(\phi(i))}{4 \cdot F(i) \cdot \sin^2(\phi(i))}} \tag{12}$$

$$a'(i) = \frac{\dfrac{\sigma_r(i) \cdot C_l(i)}{4 \cdot F(i) \cdot \lambda_r(i) \cdot \sin(\phi(i))}}{1 + \dfrac{\sigma_r(i) \cdot C_l(i) \cdot \cos(\phi(i))}{4 \cdot F(i) \cdot \sin^2(\phi(i))}} \tag{13}$$

$$W(i) = \sqrt{V_{Inf}^2 \cdot (1 - a(i))^2 + \Omega^2 \cdot r(i)^2 \cdot (1 + a'(i))^2} \tag{14}$$

Where:

$a(i)$	Axial influence coefficient for the ith blade element
$a'(i)$	Tangential influence coefficient for the ith blade element
N	Total number of blade elements.
$W(i)$	Component velocity for the ith blade element

It is interesting also to define the power coefficient C_P as an adimensional coefficient for comparing turbines in several diferent atmospheric situations:

$$C_P = \frac{P}{\dfrac{1}{2} \cdot \rho \cdot V_{Inf}^3 \cdot \pi \cdot R^2} = \lambda^2 \cdot I_P \tag{15}$$

3.2 Optimizing The Output Power

Firstly it is required an operating point in wich the performance of the blade will be optimized because as it can be seen in Eqs. (11) to (15) there is fixed values of V_{Inf} and λ. In Ribeiro (2006), the selected design point was $\lambda = 7$ and $V_{Inf} = 10\ m/s$. This point will be also the one selected for this optimization but it could be other operating point because the methodology is not constrained.

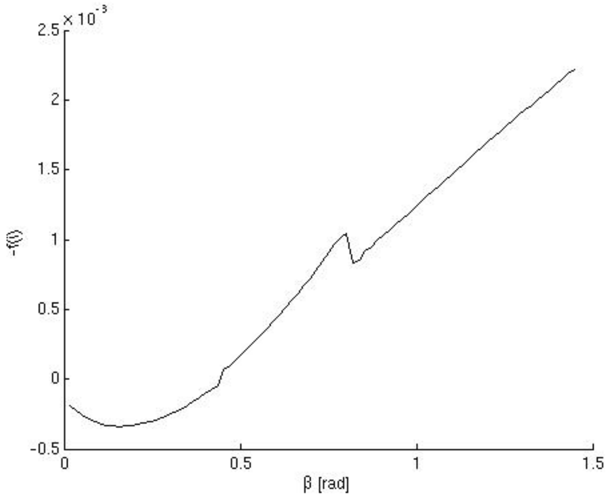

Figure 5. Topology of $-f_i$ function for $\mu = 0.3$, $\lambda = 7$ and $V_{Inf} = 10\ m/s$

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The Eq. (11) is readly solved once the vector of $\phi(i)$ is determined and by doing so, I_P is readly determined by Eq. (10) and C_P is made readly available by Eq. (15) thus giving the output power for a given wind turbine in a given atmosphere. Analysing the Eq. (11) once again it can be noticed that the blade elements are independent of each other. The same technique to solve the problem of finding the $\phi(i)$ can once again be employed.

The target is to maximize the individual values of f_i by varying the torsion β of each blade element. Maximizing f_i maximizes C_P thus rising the power output of the wind turbine.

It is chosen on this work to vary the β angle rather than vary the chord distribution presented by Ribeiro (2006) that take into account structural and constructive aspects. This assures a small variation to the available space for the structural beam. This decision facilitates the adaptation of the non optimized blade structural project to the new optimized project. It also leave some room for possible increase of beam size due to increased aerodynamic load that is spected from the optimization.

In order to recycle the previous algorithm, the cost function was made the following so no reprogramming would be needed:

$$Cost_i = -f_i \tag{16}$$

As it happened in Eq. (1), in the equation Eq. (11) it is seen the presence of local minimum due to C_l singularity effects in the $a(i)$ and $a'(i)$ terms. Altought there is the risk of the code converge to a local minimum, in tests performed with the golden seek using the fixed starting interval of $\beta(i) \in [0,\ 83\pi/180]$ the phenomenon was not observed one single time during the tests. In Fig. 5, it can be seen the topology of the cost function for one of the tested cases. In this figure, there is a local minimum at $\beta \approx 0.8\ rad$. The golden seek method did not fail in this case due to the inclination of the topology. In Fig. 6, it can be seen on left the topology of the function $-f_i$ with its minimization resulted from golden section on each blade element and on the right there is a comparation of the optimized blade to the original CEA WT 01 blade.

3.3 Verifying for Optimization

The blade described to the right of fig 6 was then tested in its design point and the power output compared to CEAWT 01 is presented on Tab. 1. The optimization resulted in a 13.7% increase in the power output generated by the device at the operating point.

To analyse if the performance of the optimized blade is better or not over CEAWT 01 it is required to analyse off-design points. In Fig. 7 it is plotted the C_P vs. λ for both blades for off-design analysis. Also in that figure there is

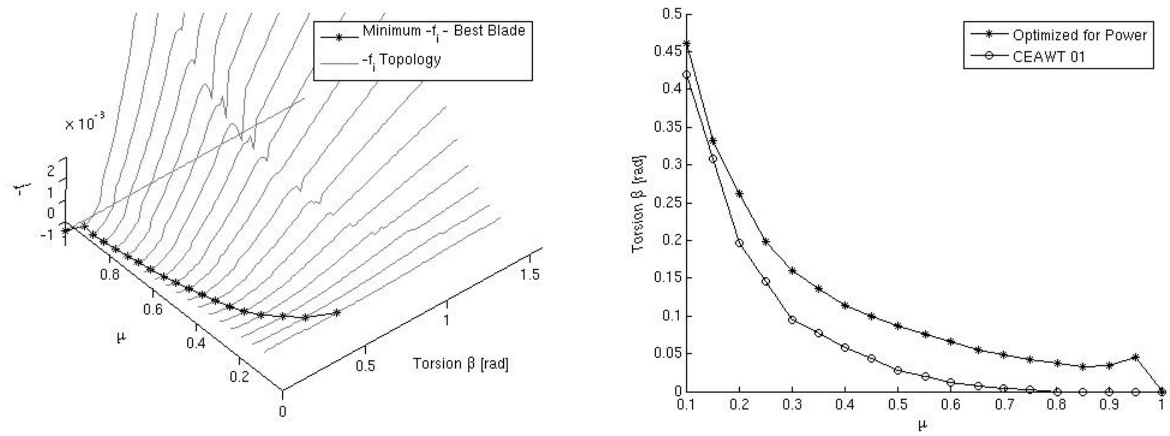

Figure 6. Left: Topology of $-f_i$ for varius μ and β with $\lambda = 7$ and $V_{Inf} = 10\ m/s$. Right: Comparison between optimized and original CEAWT 01

Table 1. Power output comparation at the design point for CEAWT 01 and the Optimized Blade

Blade	V_{Inf} $[m/s]$	λ	Power Output $[kW]$	C_P
CEAWT 01	10	7	10.5	0.437
Optimized Blade	10	7	11.8	0.495

a comparison between EXCEL™ program reference benchmark calculus and this work's MATLAB™ calculus for the CEAWT 01 windturbine.

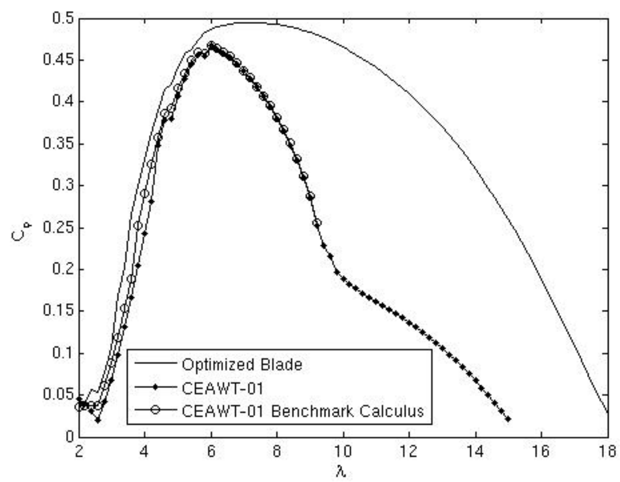

Figure 7. Comparison of performance between CEAWT 01 and Optimized Blade and Comparison between this work calculus and benchmark calculus in EXCEL™

4. CONCLUSION

The optimized design was able to produce more power over a wide off-design points. Up to this point this design can not be declared an full optimized design because it was not analysed structural impacts as well as environment impacts (mainly noise). But the optimized design shows on calculus promissing optmized aerodynamics without changing chord lengths.

Also, it was conducted a extensive benchmark with the existent software in order to validate this results. Figure 7 shows that the present software has a similar performance in results as the previous one.

5. REFERENCES

AWEA - American Wind Energy Association, Small Wind Turbine Comitee, 2001, "U.S. Small Wind Turbine Industry Roadmap".

AWEA - American Wind Energy Association, 2005, "Global Wind Energy Market Report - 2004".

GWEC - Global Wind Energy Council, 2006, "Record year for wind enegy: Global wind power market increased by 40.5% in 2005", Belgium.

Luenberger, D. G, 1984, "Linear and Nonlinear Programming", Addison-Wesley Publishing Co., 491p.

Ribeiro, R. P., 2006, "Metodologia de Projeto de Pás de Turbinas Eólicas de Eixo Horizontal para Baixa Potência", Universidade Federal de Minas Gerais, Belo Horizonte.

Simms. D., Schreck. S., Hand. M., Fingersh. L.J., 2001, "NREL Unsteady Aerodynamics Experiment in the NASA-Ames Wind Tunnel: A Comparison of Predictions to Measurements", NREL/TP-500-29494.

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