

## PIV measurements of air-water vertical two-phase flow across a sudden expansion

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**Abstract.** *The mean velocity and void fraction profiles for the air-water vertical two-phase flow across a sudden expansion are studied. Measurements of the liquid phase were performed with a two-dimensional PIV system as well as with a mini-LDV one dimensional system. Void fraction profiles were obtained with both electro-resistivity sensors and PIV. The expansion was provoked by a change in pipe diameter from 13 mm to 19.5 mm. Profiles were measured in 15 upstream and 17 downstream stations. The experiments aimed at characterizing the flow pattern transitions and the void fraction increase as the flow passes the sudden expansion section. The sudden increase was followed downstream by a gradual relaxation to a fully developed value further downstream.*

**Keywords:** *Two-phase flow, sudden expansion, electro-resistivity, PIV.*

### 1. Introduction

Gas-liquid flows are a tedious occurrence in technology. They commonly appear in process equipment and in many oil and gas applications. In vertical tubes, a variety of flow patterns can be observed. In particular, one of the most persistent of these is a flow structure designated by liquid slugs. When a series of large axisymmetric gas bubbles are inserted into a pipe, they occupy most of the cross section, being separated from each other by continuous regions of liquid that contain small gas bubbles. These regions are termed liquid slugs.

In many early works the emphasis of studies on slug flow was to develop a physically consistent mechanistic model that could be used, e.g., to yield predictions of void fraction, pressure drop, characteristic velocities and frequency. Typical examples are the works of Nicklin et al. (1962), Taitel et al. (1980) and Fernandes et al. (1983). Their most important inheritance has been a system of 17 independent equations – and variables – that can be solved to provide a closed form solution.

Despite the relevant advances achieved in the past, one must realize that prediction and modelling of flow patterns is a problem that is still far from being resolved. The difficulties are well known. The selection of parameters for pattern characterization is not based on solid physical arguments. Hence, this selection is quite arbitrary and thus necessarily limited in scope. Authors at some stage could not even agree whether results should be presented or not in dimensionless parameters. As a result, and for the sake of simplicity, studies have mostly restricted their analyzes to flows geometry configurations that involve vertical or horizontal pipe systems.

In many applications, however, flows are subject to local accidents. Under this condition, the standard results are not expected to hold. Recent studies of the flow over local accidents, i.e., sudden expansions, are the works of Schmidt and Friedel (1997), Bertola (2003) and Ahmed et al.(2003, 2004)). These works, however, have been carried out in horizontal pipes. Petrick and Swanson (1959) were seemingly the first to address the problem in vertical pipes. They conducted an experimental investigation of an air-water system to find data on the effects of a sudden change in cross section – expansion or contraction – on the relative velocities of the two phases. These authors found that the relative velocity and therefore the mean void fraction of the mixture changed following either an expansion or a contraction. The order of magnitude of the change, however, was not great and could be predicted by a semi-theoretical equation. The phase distribution was obtained by the use of a radiation attenuation traversing technique.

Two-phase flow for a vertical bubbly flow in a pipe with a sudden expansion was studied by Rinne and Loth (1996). Using fiber-optic probes, the local void fraction, bubble velocity, bubble frequency, bubble chord length and size, and local inter-facial area concentration were determined in a pipe with a sudden expansion from a 40 mm to a 90 mm diameter. The main concern of the authors was the determination of the local inter-facial area concentration, the ratio of the total surface area of all bubbles in a reference volume to the size of that volume. Different bubble shapes were used to evaluate the surface area. The calculated distribution of bubble sizes depended on the chord length resulting from the product of

measured bubble velocity and bubble residence time at the probe tip. The changes in void fraction in pipes with a sudden contraction were experimentally studied by Fossa and Guglielmini (1998). The experiments, however, were conducted for a horizontal pipe. To find the void fraction, the instantaneous measurements of the electrical impedance of an air-water mixture was made. The instrumentation consisted of basically two ring electrode pairs set on the internal wall of the pipe. Plug and slug flow regimes were investigated. Measurements were taken at five different stations. Hibiki et al. (2004) discussed the structure of downward bubbly flows in vertical pipes. The authors propose an approximate radial phase distribution pattern map based on available data of some flow parameters such as void fraction, inter-facial area concentration, interfacial velocity and bubble Sauter mean diameter. The one-dimensional drift-flux model for a downward two-phase flow and the correlation of the inter-facial area concentration were compared with the downward flow data.

The objective of the present work is study gas-liquid slug flow in a vertical pipe fitted with a sudden expansion. In particular, important flow parameters including the pressure drop, local void fraction and liquid phase velocity will be presented. The measuring techniques will be based on electro-resistivity sensors and on particle image velocimetry (PIV). Results will be compared with the results of other authors. in particular, with the results of Petrick and Swanson (1959).

## 2. Experimental setup and flow instrumentation

The test section consisted of two acrylic pipes with 13 and 19 mm internal diameter and lengths of 0.9 and 0.9 m respectively. As designed, the test section can be operated so as to impose to the flow a sudden expansion or contraction. However, in its present configuration, the loop only operates for upward flows. The air supply is given by a compressor fitted with a pressure valve. The water is supplied by a centrifugal pump connected to a plenum chamber; the purpose here is to avoid intermittent flow. Air and water are introduced into the test section through an injection unit. The liquid and air flow rates were measured with two independent and calibrated rotameters. Figure 1 shows an actual photograph of the rig. The general experimental set up is shown in Fig. 2.



Figure 1. Experimental setup.

## 3. Instrumentation

### 3.1 Electro-resistivity probes

Electro-resistivity sensors built from small needles were simultaneously developed by Neal and Bankoff (1963) and by Nassos (1963). In these studies, almost all efforts were dedicated to the development of the experimental technique rather than to the investigation on the nature of some particular type of flow. Neal and Bankoff used a Nitrogen-Mercury system, whereas Nassos carried out measurements in an air-water system. Chesters et al. (1980) were the first to employ the resistivity technique in gas-liquid non-confined flows with a certain degree of success. The authors used electro-resistivity sensors together with laser-Doppler anemometry to describe the characteristics of the liquid and of the gas phases in a bubble plume.

Tacke et al. (1985), studying gas stirred steel making processes, used the electro-resistivity sensor technique to make

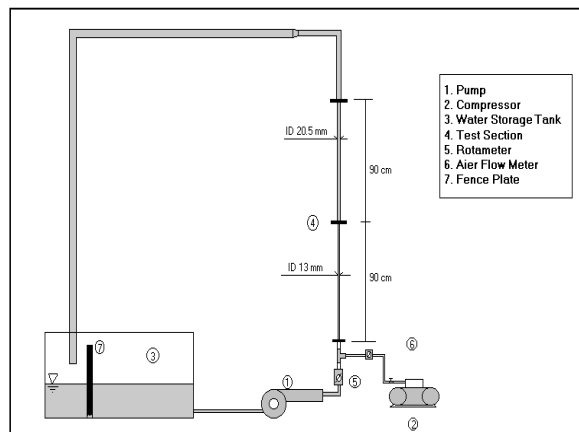


Figure 2. Experimental rig and detail of electroresistivity probe support.

some measurements of the gas phase properties in air-water, Helium-water and Nitrogen-Mercury systems. Castillejos and Brimacombe (1975), also aiming at the application of the bubble plume phenomenon in the steel making industry, developed a comprehensive instrumentation based on the resistivity technique to investigate the problem. In 1988, Teyssedou et al. presented a new AC probe system, together with an analysis of the effect of the geometry of the sensor tip and of other parameters on the performance of the system. More recently, Kocamustafaogullari and Wang (1991), Leung et al. (1992) and Liu and Bankoff (1993) used resistivity sensors to determine local time-averaged mean gas fraction, inter-facial area concentration, bubble rise velocity and bubble pierced length in internal bubbly flows.

The working principle of the experimental technique is based on the difference between the electrical conductivity (resistivity) of the phases. Since the electrical conductivity of water is much higher than that of air, it is assumed, for practical purposes, that only the continuous phase (liquid) is capable of conducting electrical current. Accordingly to Herringe and Davis (1974), resistivity sensors are the most suitable technique for measurements in two-phase mixtures where the continuous phase is conductive. The main adversity of the technique is the existence of an in-stream sensor, which affects the structure of the flow.

In a double channel system (whether AC or DC supply), the difference in electrical resistivity between the phases can be sensed by the electrodes in the two-phase flow so that parameters like the local time-averaged mean gas fraction, the rise velocity and the pierced length of bubbles can be obtained through an analysis of the output signal. The measuring system used in this work has been fully described in Barbosa and Bradbury (1996) and in Barbosa (1997); for any detailed information on the system the reader is, therefore, referred to those works. Next we will just briefly comment on the features of the probes.

Co-axial probes were chosen to be used here. In fact, due to its symmetric geometry, the co-axial probe interference on the flow is recognized as being weaker than that of a parallel probe of nearly equal dimensions. The measuring probes were constructed with the following features: i) 0.2 mm diameter stainless steel internal wire (upstream electrode); ii) 0.4 mm OD 0.2 mm ID hypodermic tubing (downstream electrode); iii) length of electrodes free of insulation equal to 0.1 mm; and iv) distance between electrodes equal to  $1.5 \pm 0.15$  mm. The data acquisition and analysis system consisted of a microcomputer with an interface data acquisition board, an oscilloscope, a signal conditioner module and the electro-resistivity probes. The mean gas fraction at a point in the flow is a time-averaged property given by,

$$f(r, x) = \frac{1}{T} \int_0^T I(r, x, t) dt \quad (1)$$

where T is the total sampling time, I is the digital output signal from the conditioning module and r and x are the coordinates. The output signal, I, consists of a series of pulses which correspond to the transit of bubbles through the probe. Further details concerning the output signal, I, are available in Barbosa and Bradbury (1996) and in Barbosa (1997).

The data were acquired at a sampling rate of approximately 2.5 kHz and about 50 sampling blocks of 10,000 readings (Barbosa and Bradbury (1996), Barbosa (1997)) were shown to be sufficient to describe the flow at each measured point. In fact, in Barbosa (1997), the shape, size and velocity of the rising bubbles was detailed studied; also, the influence of the injection geometry on plume development and the existence of any lateral wandering motion of the plume were investigated.

The probe calibration was carried out in a vertical pipe with 3 cm diameter and 100 cm length. After the pipe had been filled with water taken from the tank and the probe had been placed in position, a large bubble was carefully introduced at its bottom through a syringe. As the bubble rose and hit the probe, the recordings from a Panasonic video camera were

digitalized and analyzed. The pictures which were taken at a frequency of 60 frames per second were then compared with the signal of the conditioning module. The resistance of the flow in the conditioning module was then set so as to furnish the same response in both systems.

An uncertainty analysis of the data was performed according to the procedure described in Kline (1985). Typically the uncertainty associated with the mean gas fraction measurements was:  $f = 0.00035$  precision, 0 bias ( $P = 0.95$ ).

### 3.2 Particle image velocimetry - PIV

In particle image velocimetry (PIV) small tracer particles are added to the flow. Then, provided the displacements of these particles can be tracked in space and time, the general dynamical features of the flow can be determined. In a typical procedure, the particles are successively illuminated in a plane within a short time interval. The light scattered by them is then recorded on separate frames on a CCD sensor. To determine the local displacement vectors, the recordings are split into small sub-areas termed *interrogation windows*. Then by considering the particles to have moved homogeneously between two consecutive exposition times, statistical methods can be used to find the displacements. The great advantages of PIV are its non-intrusive character and the large spatial resolution.

In two-phase flow applications, the information conveyed by each phase needs to be adequately separated. Hassan et al. (1998) have shown how PIV measurements in conjunction with a forward-projection shadow method can be used to find the sizes and velocities of bubbles rising in a vertical pipes. Their experimental arrangement also allowed for an investigation of the dynamic properties of the continuous phase.

Lindken et al. (1999) showed that the application of filtering techniques to separate the signals resulting from both phases permits a high resolution of velocity measurements in regions near to interfaces.

To investigate the velocity field of the continuous phase, the water was seeded with a commonly used particle tracer, rodamin, 10  $\mu\text{m}$  in size. The light source was furnished by two Nd:YAG lasers that produced short duration (10 ns) high energy (200 mJ) pulses of light green (532 nm). The collimated laser beam was transmitted through a cylindrical (15 mm) and a spherical (500 mm) lens to generate a 1 mm thick lightsheet. The reflected light was recorded with a 6300059 Power View Plus 4 Megapixels CCD camera (2048x2048 pixels at 8 Hz). The camera was fitted with a Nikkor 105 mm f/2.8D lens.

During normal operation, bubbles will cross the light sheet provoking large reflections that will hit directly the camera. Moderate disturbances can be corrected by passage of filtering techniques. The digital recordings were evaluated through a minimum quadratic difference method. The signals from the two phases were separated naturally due to the fluorescent character of the tracers. Basically the reflected signals were recorded with the help of an optical high pass filter. A problem with this method is that the bubbles might reflect the signals from the tracers inducing errors in the measurements. The shapes and positions of the bubbles were found from their shadows. A difficult problem of PIV measurements in two-phase flow is the determination of the areas that are filled with bubbles. Here, this problem has been greatly alleviated for the flow pattern is dominated by a large Taylor bubble.

## 4. Measurements

Before quantitative measurements were made, a flow visualization study was performed. For the experiments, the liquid and the gas flow rates were varied between 0.06 to 0.11 ( $\text{Kg/s}$ ) and 0.00016 to 0.00006 ( $\text{Kg/s}$ ) respectively. However, only the conditions shown in Table 1 will be presented here.

Volumetric flux	$W_l$ (liquid) ( $\text{Kg/s}$ )	$W_g$ (gas) ( $\text{Kg/s}$ )
Condition 1	0.066	0.03
Condition 2	0.1	0.06

Table 1. Experimental mass flow rates.

Three measurement station were considered at positions -400 mm, 0 mm and 400 mm, where the origin refers to the location of the expansion (from 13 to 19 mm).

Figure 3 illustrates the flow pattern before the expansion is reached (Condition 2 at -400mm). The flow was dominated by Taylor bubbles with varied length. The spherical noses and flat tails of the elongated bubbles can be seen in Fig. 3 as well as the small bubbles that occur in their wake. The seeding particles are also identified in the figure. The well define shape of the noses can be used to find the rising velocity of the Taylor bubbles. To find the velocity profiles of the rising water, measurements were taken at the wake region. At least 200 instantaneous velocity profiles were used to find the mean profile.

Figure 3 shows some typical vector fields for the mean velocity of the continuous phase before the expansion is reached. The dark areas characterize the location of the bubbles.

The general flow pattern and the liquid velocity field at the expansion are shown in Figs. 4 and 5.

The abrupt change of cross section gives rise to a large recirculation zone downstream of the expansion. When a

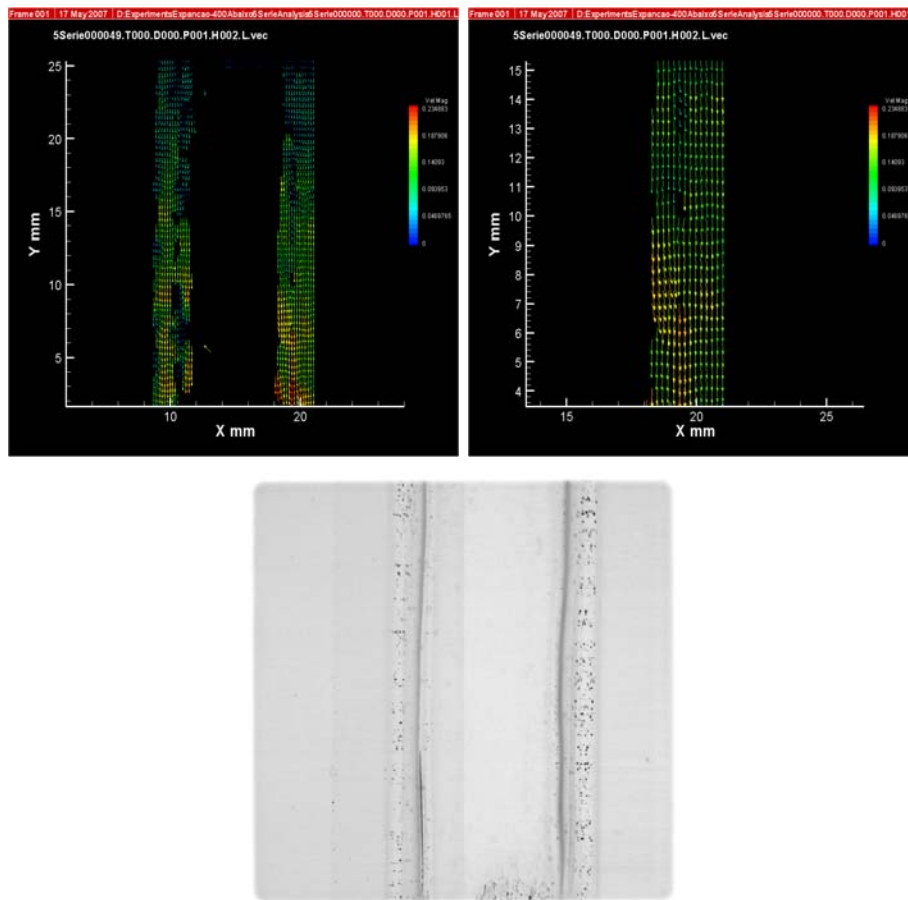


Figure 3. Flow pattern before expansion is reached. A, liquid phase velocity field; B, detail of flow downward in the annulus formed between the bubble and the pipe; C, picture of seeding particles and of the Taylor bubble interface.

Taylor bubble flows through the enlargement, a two-phase shear layer is created between the fast moving bubble and the slow, recirculating liquid. The increase in interfacial friction decelerates the bubble and disrupts the two-phase interface to a point where smaller bubbles are formed as the interface breaks-up (Fig. 4.b). As the Taylor bubble advances into the large diameter tube, part of the liquid ahead of it is pushed upwards by the bubble, but another part is displaced radially and starts flowing downwards as a falling film around it. As the enlargement impedes the downward flowing liquid from entering the smaller diameter section, the recirculation downstream of the expansion is strengthened and may become so intense that the associated instability leads to 'necking' of the Taylor bubble (Fig. 4.c) and, eventually, a large-scale break-up.

Figures 6 and 7 show details of the velocity field around a Taylor bubble, at station 400mm downstream of expansion.

The large region of reverse flow behind the expansion together with the thick film show that the strong instabilities confine the perturbed flow to a region limited by diameter of the smaller section, 13 mm.



Figure 4. Flow pattern at expansion.

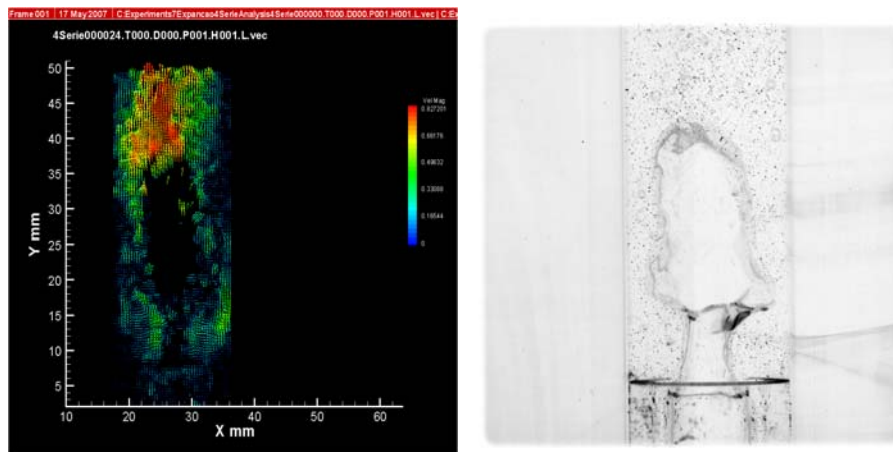


Figure 5. Liquid phase velocity field at expansion. PIV results..

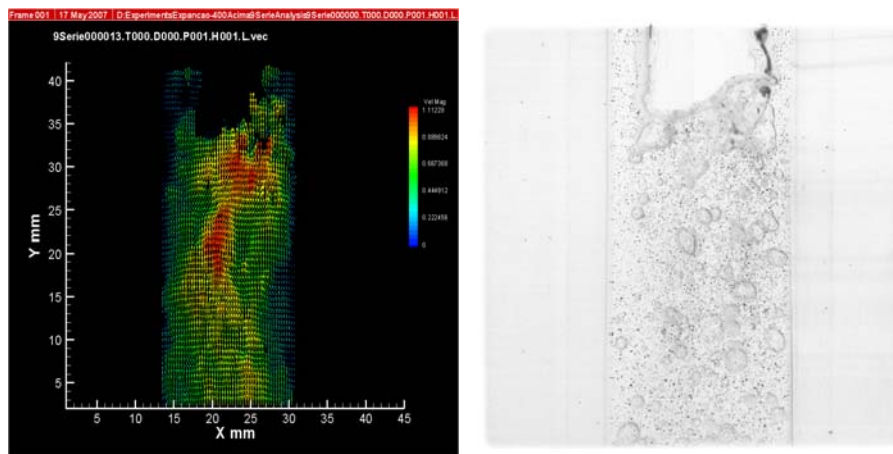


Figure 6. Velocity field in the back region of a Taylor bubble. Station: 400 mm downstream of expansion.

Two-dimensional graphs of the mean velocity profiles of the continuous phase are shown in Fig. 8.

The PIV results concerning the flow velocities are consolidated in Table 2.

The void fraction distribution was measured in 27 different stations, 8 upstream of the sudden expansion, the other 19 stations downstream of the sudden change. The actual position of the measuring stations will become clear in next figures.

Local void fraction distributions for two different measuring stations are presented in Fig.9. Far upstream of the sudden change ( $z = -400$  mm), the void fraction shows a flat shape. As the flow approaches the sudden expansion, the void fraction is observed to decrease. As noted by other authors, this is indicative of an increase in slip due to the reduction in pressure gradient as the flow approaches the sudden expansion. The graphs also show that the increase in void fraction is followed by a gradual decrease toward a constant value. All the downstream void fraction profile exhibit a top-hat profile, with this shape becoming more pronounced for higher values of  $z$ .

The variation of void fraction with section length is shown in Fig.10 and Fig.11. The pressure distribution is shown next in Fig.12.

Petrick and Swanson (1959) have observed that shortly downstream of an expansion, a sharp increase in void fraction may occur. Still, according to these authors this is due to the formation of a jet over the few inches past the transition and creation of air pockets. The jet then dissipates into a very turbulent region, after which a regular pattern is established. The severity of the jet effects were identified with the mixture quality, fluid velocity and area enlargement. For expansions with a small area enlargement, low fluid velocities and mixture quality, severe transition regions were not observed. Thus, given the conditions of the present experiment, Fig. 10 seems to be coherent: no sharp zone of flow transition was observed and a region of established flow ( $\alpha \cong 0.35$ ) pattern was observed.

In fact, all the features described by Petrick and Swanson (1959) have been well recorded by Figs. 4, 6 and 8. Thus, it appears that their simple theory may apply to the presently tested flow.

The changes in pressure along the measuring section were almost due to the hydrostatic effects.

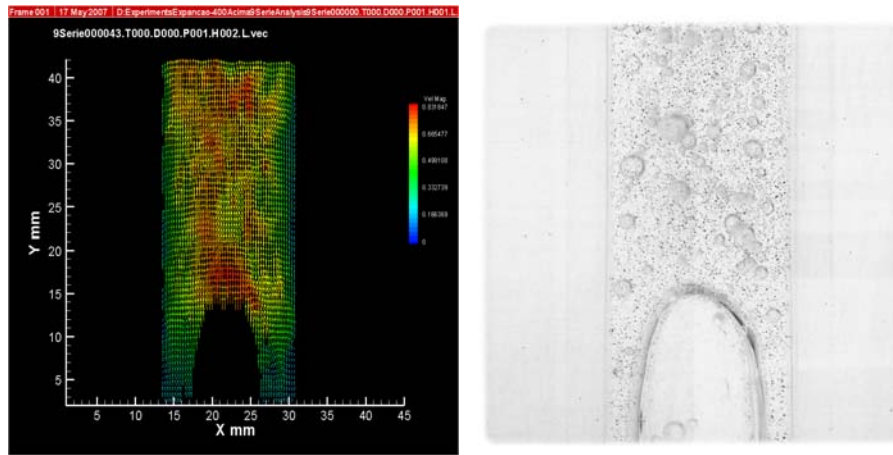


Figure 7. Velocity field in front of a Taylor bubble. Station: 400 mm downstream of expansion.

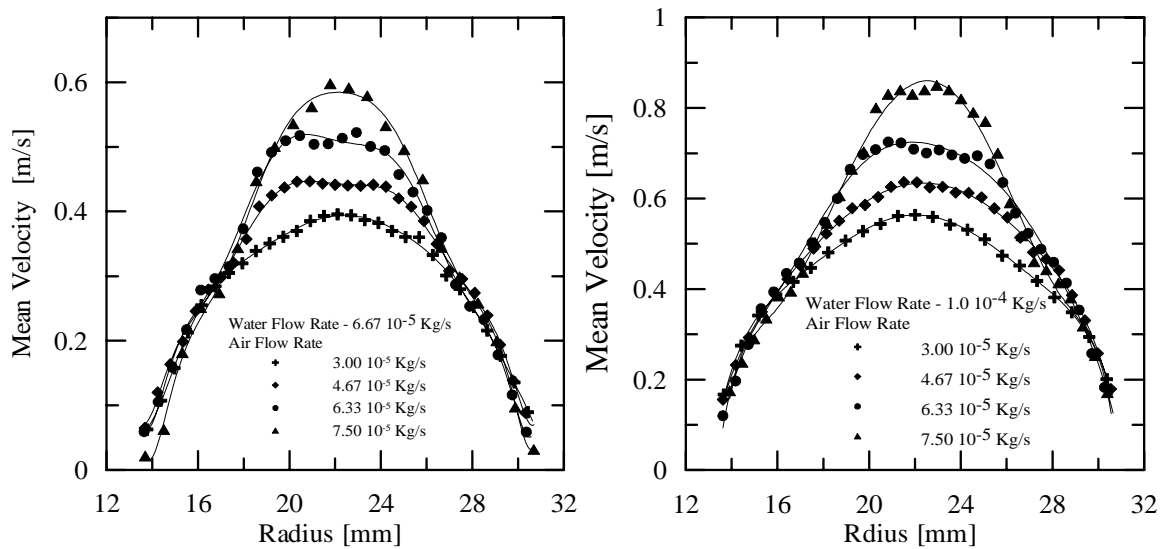


Figure 8. Mean velocity fields of the continuous phase before and after the expansionB.

### 5. The expression of Petrick and Swanson

Petrick and Swanson (1959) showed that the relative velocity and hence the mean void fraction of an air water mixture changed following an expansion. Furthermore, these authors proposed a semi-theoretical expression through which the flow changes could be predicted.

For a two-phase mixture, the liquid phase velocity, (also known as the ‘slip ratio’) can be written in terms of the mass quality and the void fraction as

$$\frac{u_g}{u_f} = \frac{x}{1-x} \frac{1-\alpha}{\alpha} \frac{\rho_f}{\rho_g} \quad (2)$$

For an adiabatic system, where the quality is a constant, the void fraction will only change if the slip ratio changes. Considering that the flow undergoes a sudden change in pipe area, then the following relation holds

$$\frac{\alpha_1}{1-\alpha_1} \frac{1-\alpha_2}{\alpha_2} = \frac{x_1}{1-x_1} \frac{1-x_2}{x_2} \frac{\rho_{f1} \rho_{g2}}{\rho_{g1} \rho_{f2}} \frac{u_{f1} u_{g2}}{u_{g1} u_{f2}} \quad (3)$$

With the postulation of an adiabatic system,  $x_1 = x_2$  and  $\rho_{f1} = \rho_{f2}$ , so that it results

$$\frac{\alpha_1}{1-\alpha_1} \frac{1-\alpha_2}{\alpha_2} = \frac{\rho_{g2}}{\rho_{g1}} \frac{u_{f1}}{u_{g1}} \frac{u_{g2}}{u_{f2}} \quad (4)$$

Petrick and Swanson then took

Table 2. Consolidate PIV results. Measurements taken 400 mm downstream of expansion.

$q_l$ (Kg/s)	$q_g$ (Kg/s)	$u_l$ (ms <sup>-1</sup> )	$u_g$ (ms <sup>-1</sup> )	slip velocity = $u_g/u_l$
0.066	0.0000166	0.2917	0.5	1.714
0.066	0.000033	0.3314	0.63	1.917
0.066	0.00005	0.3806	0.80	2.114
0.066	0.000066	0.4154	1.01	2.443
0.1	0.0000166	0.3994	0,7	1.752
0.1	0.000033	0.4946	0,89	1.809
0.1	0.00005	0.5174	0,95	1.837
0.1	0.000066	0.5664	1.06	1.886

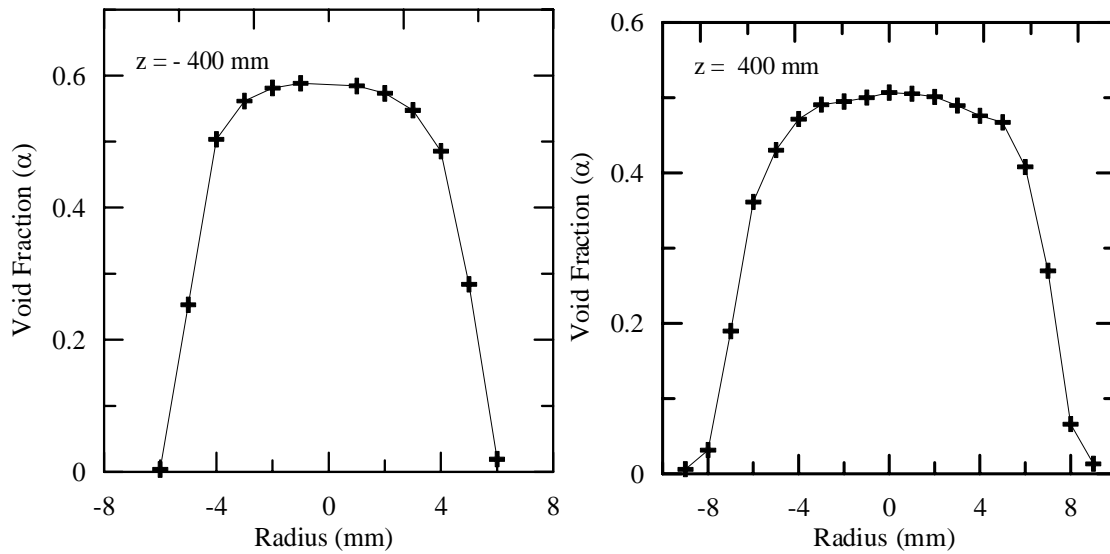


Figure 9. Typical void fraction variation before and after expansion.

$$\frac{u_g}{u_f} = K u_{wo}^N \tag{5}$$

But,

$$W = u_{wo} A \rho, \quad u_{wo} = \frac{W}{A \rho} = \frac{K'''}{A} \tag{6}$$

The result is that the slip ratio can be evaluated from

$$\frac{u_g}{u_f} = \frac{K''}{A^N} \tag{7}$$

Finally, considering that

$$\frac{1}{\rho} = \frac{RT}{PM} \tag{8}$$

the following expression is obtained

$$\alpha_2 = \frac{1}{\left(\frac{P_2}{P_1}\right) \left(\frac{A_1}{A_2}\right)^N \left[\frac{1-\alpha_1}{\alpha_1}\right] + 1} \tag{9}$$

where  $N$  had to be experimentally determined.

For their experiments, Petrick and Swanson found  $N=0.2$ . In the present work we have found  $N=0.3$ , a similar value.



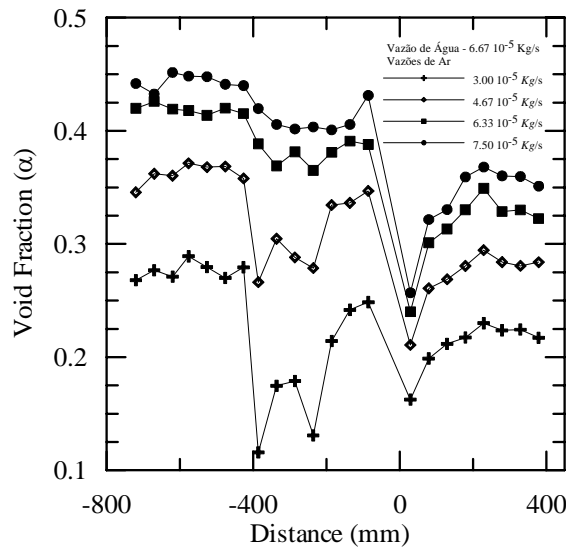


Figure 10. Void fraction variation with section length - Condition1.

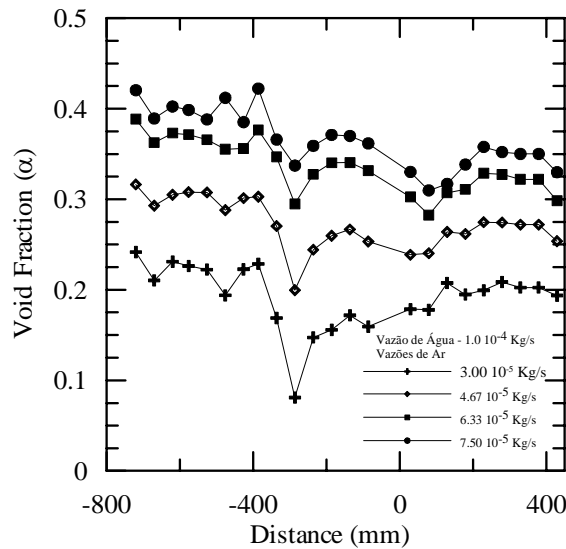


Figure 11. Void fraction variation with section length - Condition2.

## 6. Nicklin correlation

The translation velocity of the Taylor bubbles will now be compared with the theory of Nicklin et al. (1962).

Two different methods were used to find bubble velocities: electro-resistivity probes and PIV. The former technique used two channel probes with sensing elements (tips) 0.5 mm apart. A typical signal is shown in Fig. 14. Bubble velocities were also found by a filtering technique that allowed the front of the bubble to be identified.

The theory of Nicklin et al. (1962) has been developed for bubbles in a process of expansion. Still, according to these authors the velocity of translation can be evaluated through Eq. (10). Their experimental data has further furnished  $C_0 = 1.48$  (Fig. 15). These value is valid for bubbles in a stagnant environment.

Measurements made at station 400 mm are shown in Table 3. The overall agreement is very good.

$$V = C_0 U_m + V_\infty, \tag{10}$$

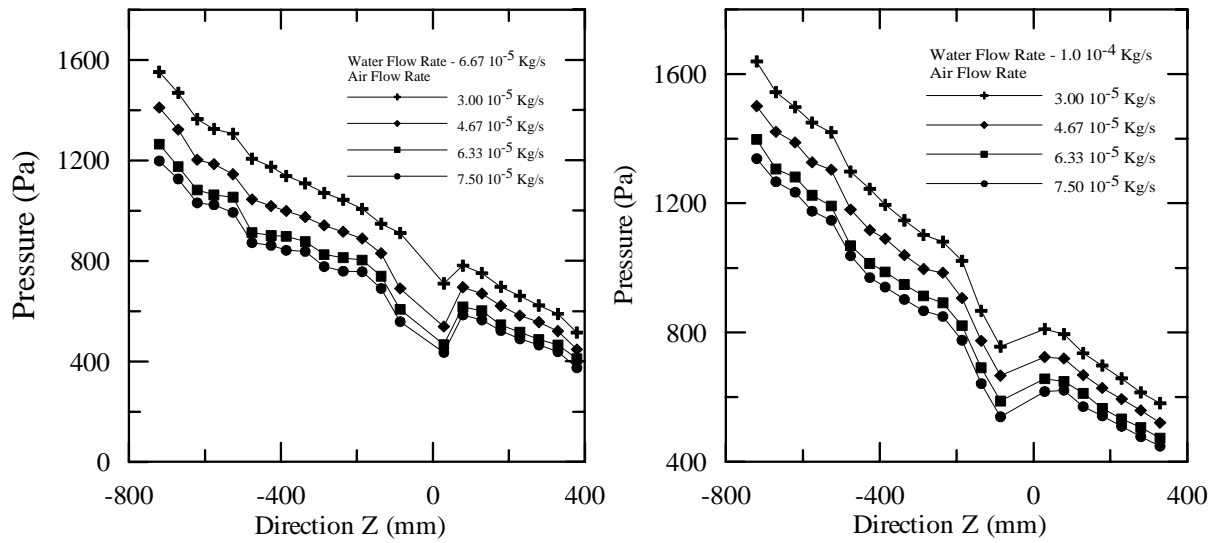


Figure 12. Pressure distribution in both conditions.

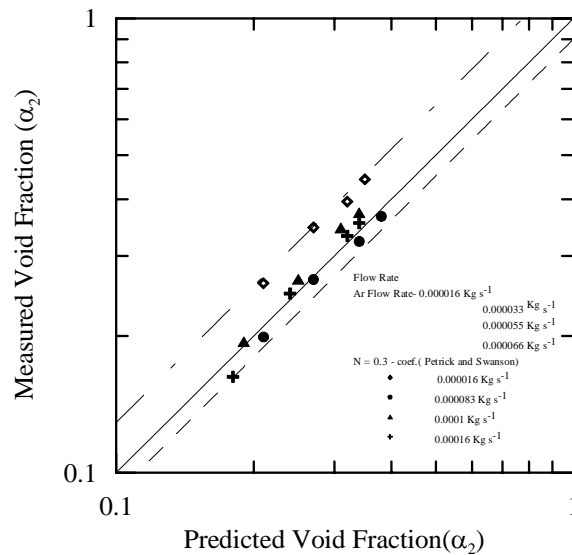


Figure 13. Comparison of the predicted and measured air void fraction.

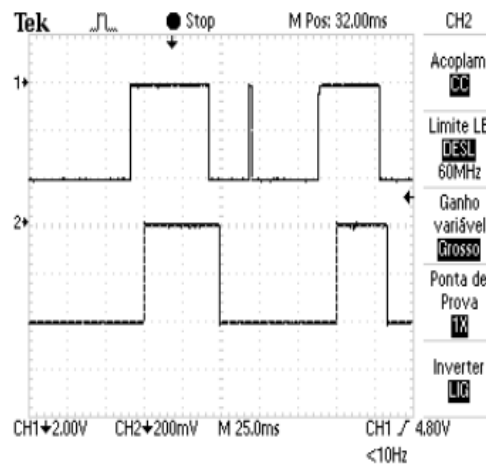


Figure 14. Typical signal of a two-channel electro-resistivity probe.

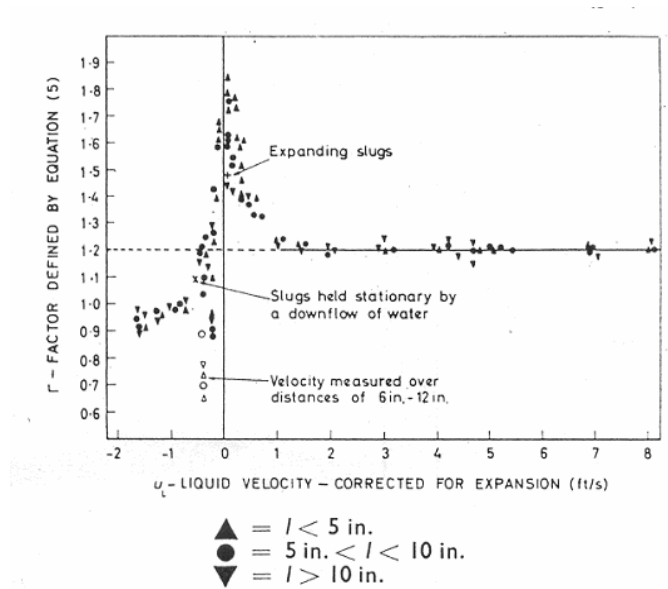


Figure 15. Experimental determination of factor  $C_o$ .

Bubble velocity [m/s]	Translation velocity $U_t$ [m/s]
PIV Measurements	<i>Nicklin et al. (1962)</i>
0.58	0.582821
0.63	0.641577
0.80	0.714393
1.01	0.765897
0.7	0.742217
0.89	0.883113
0.95	0.916857
1.06	0.988889

Table 3. Comparison between theory (Nicklin et al., 1962) and experimental results - PIV

## 7. Final remarks

The velocity profiles, pressure loss and void fraction changes of an air-water flow in a vertical pipe with a sudden area expansion have been investigated experimentally. The expansion is defined when the two-phase flow passes from a 13 mm to a 19.5 mm pipe. PIV was used to find the velocity profiles of both phases. Pressure was measured in 15 upstream and 17 downstream stations through a series of pressure taps. In addition, electro-resistivity probes were used to measure void fraction in 27 different locations. The experiments were aimed at analysing the flow pattern transitions and the void fraction decrease as the flow passes the sudden expansion section. The sudden decrease was followed downstream by a gradual relaxation to a fully developed value further downstream. Upstream values of the void fraction observed were about 0.4; downstream values were about 0.35.

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## 8. References

- Ahmed, W. H., Ching, C. Y., Shoukri, M., 2003, Characteristics of air-oil two-phase flow across a sudden expansion, 4TH ASME-JSME Joint Fluids Engineering Conference, Honolulu, Hawaii, USA, July 6-11.
- Ahmed, W. H., Ching, C. Y., Shoukri, M., 2004, A new model for the pressure recovery of air-oil two-phase flow across sudden expansions, 3rd International Symposium on Two-phase flow Modelling and Experimentation, Pisa, 22-24 September.
- Barbosa, J. R. J. and Bradbury, L. J. S. 1996 "Experimental Investigations in Round Bubble Plumes", *In Proc. 6th Brazilian National Meeting on Thermal Sciences (ENCIT)*, Florianopolis, 1073-1078.
- Barbosa, J. R. J. 1997 "The Electro-resistivity Method Applied to Bubble Plumes", M.Sc. Thesis, Mech. Eng. Dept, Federal University of Rio de Janeiro.
- Bertola, V., 2003, The Estructure of gas-liquid flow in a horizontal pipe with abrupt area contraction, *Experimental Thermal and Fluid Science*, Article in press.
- Castillejos, A. H. and Brimacombe, J. K. 1987 "Measurement of Physical Characteristics of Bubbles in Gas-Liquid Plumes", *Metall. Trans. B*, 18B, 649-671.
- Chesters, A. K., van Dorn, M., and Goossens, L. H. J. 1980 "A General Model for Unconfined Bubble Plumes from Extended Sources", *Int. J. Multiphase Flow*, 6, 499-521.
- Fernandes, R. C., Semiat, R., Duckler, A. E., 1983, Hydrodynamic model for gas-liquid slug flow in vertical tubes, *AIChE J.* 29 981-989.
- Fossa, M., Guglielmini, G., 1988, Dynamic void fraction measurements in horizontal ducts with sudden area contraction, *Int. J. of Heat and Mass Transfer* 41 3807-3815.
- Hassan, Y. A. Schmidt, W. D., Ortiz-Villafuerte, J., 1998, Investigation of three-dimensional two phase flow structure in a bubbly pipe flow, *Meas. Sci. Technol.*, 9 309-326.
- Herringe, R. A. and Davis, M. R. 1974, Detection of Instantaneous Phase Changes in Gas-Liquid Mixtures, *J. Phys. E: Sci. Instrum.*, 7, 807-812.
- Hibiki, T., Ishii, M., Kim, S., Goda, H., Uhle, J., 2004, Structure of vertical downward flow, *Int. J. of Heat and Mass Transfer* 47, 1847-1862.
- Nickin, D. J., Wilkins, J. O., Davidson, J. F., 1962, Two-phase flow in vertical tubes, *Trans. Inst. Chem. Engrs* 40 61-68.
- Kline, S. J., 1985, The Purpose of Uncertainty Analysis, *J. Fluids Engineering*, 107, 153-160.
- Kocamustafaogullari, G. and Wang, Z., 1991, An Experimental Study on Local Interfacial Parameters in Horizontal Bubbly Two-Phase Bubbly Flow, *Int. J. Multiphase Flow*, 17, 553-572.
- Leung, W. H., Revankar, S. T., Ishii, Y., and Ishii, M., 1992, Axial Development of Interfacial Area and Void Concentration Profiles Measured by Double-Sensor Probe Method, *Int. J. Heat Mass Transfer*, 38(3), 445-453.
- Lindken, R. and Merzkirch, W., 2000, Velocity measurements of liquid and gaseous phase for a system of bubbles rising in water, *Exp. Fluids*, S194-S201.

- Liu, T. J. and Bankoff, S. G., 1993, Structure of Air-Water Bubbly Flow in a Vertical Pipe – II. Void fraction, Bubble Velocity and Bubble Size Distribution, *Int. J. Heat Mass Transfer*, 36(4), 1061–1072.
- Nassos, G. P., 1963, Development of an Electrical Resistivity Probe for Void Fraction in Air-Water Flow, , Technical Report 9, Argonne Report ANL – 6738.
- Neal, L. G. and Bankoff, S. G., 1963, “A High Resolution Resistivity Probe for Determination of Local Void Properties in Gas-Liquid Flow”, *AIChE J.*, , 9, 49-54.
- Petrick, M. and Swanson, B. S., 1959, Expansion and Contraction of an Air-Water Mixture in Vertical Flow, *A. I. Ch. E. Journal* Vol. 5, No 4, 440-445.
- Rinne, A. and Loth, R., 1996, Development of Local Two-Phase Flow Parameters for Vertical Bubbly Flow in a Pipe with Sudden expansion, *Experimental Thermal and Fluid Science*; 13:152-166.
- Schmidt, J. and Friedel, L., 1997, Two-Phase pressure drop across sudden contraction in ducts areas, *Int. J. Multiphase Flow* Vol. 23. Nº.2, pp. 283-299.
- Tacke, K. H., Schubert, H. G., Weber, D. J., and Schwerdtfeger, K., 1985, “ Characteristics of Round Vertical Gas Bubble Jets”, *Metall. Trans. B*, 16B, 263-275.
- Taitel, Y., Bornea, D., Duckler, A. E., 1980, Modelling flow pattern transitions for steady upward gas-liquid flow in vertical tubes, *AIChE J.* 26 345-354.
- Teyssedou, A., Tapucu, A., and Lortie, M., 1988, Impedance Probe to Measure Local Void Fraction Profiles, *Rev. Sci. Instrum.*, 59(3), 631–638.