DIMENSIONAL SYNTHESIS OF PARALLEL PLANAR MECHANISMS USING GENETIC ALGORITHMS, SCREW THEORY AND ASSUR VIRTUAL CHAINS

Marcial Trilha Junior, marcial_junior@yahoo.com.br
Daniel Martins, daniel@emc.ufsc.br
Universidade Federal de Santa Catarina - Departamento de Engenharia Mecânica - Laboratório de Robótica, Caixa Postal 476 - Campus Universitário - Trindade - CEP 88040 900 - Florianópolis-SC - Brazil

Abstract.
The dimensional synthesis of closed kinematic chains is still an open problem in kinematics. In fact, there are not general analytical methods to the dimensional synthesis of mechanisms. The proposed method allows to carry on the dimensional synthesis of parallel planar kinematic chains using a genetic algorithm by optimizing the dimensions of the links of the chain to follow a trajectory with minimum error. This trajectory is pre-specified to the end-effector, a chosen link of the chain, and is constrained to a limited workspace. The procedure is relies on the Davies' method, a method based on the Kirchhoff laws and screw theory. For the imposition of the movement in the end-effector, it is applied Assur Virtual Chains. The movement of the end-effector is described in the operational space; when the mobility of the chain is smaller than the dimension of the operational space, the system is redundant and a pseudoinverse solution is necessary. The objective function developed for the genetic algorithm is based on the joint errors. The method is applied to a slider-crank and the results are compared with a commercial mechanism simulations package (Pro-Engineer).
Keywords: *Synthesis of Mechanisms, Genetic Algorithms, Assur Virtual Chains, Screw Theory, Davies Method.*

1. INTRODUCTION

During the last few years, important advances in the solution of problems related with synthesis of mechanisms appeared in the literature. These advances are mainly connected to enhancements in the computers performance as well as the successive improvements in the optimization techniques. This allowed the application of techniques of mathematical programming to the dimensional synthesis of mechanisms, however the majority of the developed procedures encloses only particular problems and generally it leads the methods of difficult application and not taking in account important parameters in the project as the work in confined space.

The graphics method to kinematic synthesis of mechanisms are applied only in simpler mechanisms due to the increase of the complexity of the graphical constructions when the number of elements and/or the amount of position or control points increases. The analytical methods are limited by the increase in the complexity of its equations and the increase of the number of links. In general, they are constrained to a pre-determined number of precision points where the problem has closed-form explicit mathematical solutions. Moreover, most of the analytical methods show defects such as orber, circuit and branch problems [2] [10] [1] [7].

Therefore, the use of optimization methods for the synthesis of parallel manipulators becomes inevitable when the limits in precision poinst and number of bodies is reached, as discussed above and in the references therein. However, the synthesis of more complex manipulators involves a large number of variable and the solution space contains also a large number of local minima. Luckily, normally only one global minimum exists [11].

Methods based in direct search, such as the gradient-based method and finite element methods, tend to converge to the closer local minimum, thus to reach a global minimum will only be possible if the initial parameters are close to the global minimum. In all other cases the method will converge to a distant local minimum.

Although not totally guaranteed, the method of global search has a better possibility of convergence to a global minimum, and in average require less computational time. Global search methods are structured to search in the space of all possible solutions and to find regions where the values of the function have low cost. However, methods of global search, such as genetic algorithms, have to scrutiny the space of all possible solutions. Since the size of the population is limited, there is always a possibility of not reaching the global minimum.

This work show a general method to the synthesis of parallel planar manipulators using genetic algorithm for dimensional optimization of the links of the manipulator. The method is based on a series of other techniques such as: Davies method to solve the kinematic, Newton-Raphson to integrate, and the inverse Moore-Penrose method to work out when the manipulator has to possess inferior mobility to the number of given actuators and Assur Virtual Chains. Assur Virtual Chains have a twofold application: a) they impose the desired trajectory to the end-effector and b) they guarantee the closure of the chain during the integration process.

With the proposed method, the dimensional synthesis of parallel planar manipulators working in confined spaces is feasible. Furthermore, this method is not limited to the configuration of the manipulator, to the number of control points of the trajectory or to the complexity of the chain. We start the next section describing the method and the techniques which the proposed method relies on.

2. METHOD

2.1 DAVIES METHOD

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. It is based on the so called Kirchhoff-Davies circulation law. Davies solves the differential kinematics of closed kinematic chains from the Kirchhoff circulation law for electrical circuits. The Kirchhoff-Davies circulation law states that "The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" [6].

Using this law, the relationship among the velocities of a closed kinematic chain may be obtained in order to solve its differential kinematics, as is showed in and [9].

The movement of a joint can be univocally represented for a screw coincident with the axis joint, being the referential system inertial. This screw can be represented to normalized screw multiplied for a magnitude , by means of:

$$\$ = \hat{\$}\Psi \tag{1}$$

This normalized screw, is constituted by:

$$\$ = \begin{bmatrix} S \\ S_0 \times S + hS \end{bmatrix} \tag{2}$$

where S is the normalized vector parallel to screw axis, S_0 is the vector that the inertial system of coordinate to vector S and h is the pitch of the screw.

Normally the joints have one degree of freedom and are of type rotative or prismatic. To rotative joint, the pitch h is equal to zero, and the normalized screw is provided to:

$$\$ = \begin{bmatrix} S \\ S_0 \times S \end{bmatrix} \tag{3}$$

to a prismatic joint where $h = \infty$, the normalized screw is provided to:

$$\$ = \begin{bmatrix} 0 \\ S \end{bmatrix} \tag{4}$$

The Kirchhoff law fix that the sum of the differences of potential throughout any electric circuit is null, therefore the initial and the end are the same. Davies Method fix of the analog form that the sum of the relative velocities between adjacent links, throughout of the loop of a closed chain, is kinematically null. With this we can denote that:

$$\begin{aligned} \sum \$ &= 0 \\ \$_A + \$_B + \$_C + \$_D + \ldots + \$_i &= \vec{0} \end{aligned} \tag{5}$$

This equation, substituting to normalized screws provided to in the eq.2, is called of constrain equation, and can be writing as:

$$\hat{\$}_A\Psi_A + \hat{\$}_B\Psi_B + \hat{\$}_C\Psi_C + \hat{\$}_D\Psi_D + \ldots + \hat{\$}_i\Psi_i \tag{6}$$

The constrain equation it can be written in the matricial form:

$$[\$_A + \$_B + \$_C + \$_D + \ldots + \$_i]_{(\lambda l \times F)} \begin{Bmatrix} \lambda_A \\ \lambda_B \\ \lambda_C \\ . \\ . \\ . \\ \lambda_i \end{Bmatrix}_{(F \times 1)} = \vec{0}_{(\lambda_l \times 1)} \tag{7}$$

or in the compact form:

$$\left[\hat{N}\right]_{(\lambda l \times F)} \{\Psi\}_{(F \times 1)} = \vec{0}_{(\lambda l \times 1)} \tag{8}$$

where $[N]$ is the net matrix of the normalized movements. Separating in active joints and passive joints we have:

$$\hat{N}_s\Psi_s = -\hat{N}_p\Psi_p \tag{9}$$

where $\left[\hat{N}_s\right]$ is the net matrix of normalized movements agreeable to passive joints, $\left[\hat{N}_p\right]$ is net matrix of the normalized movement agreeable to active joints, Ψ_s is the vector of the magnitudes of the secondary variables and Ψ_p is the vector of the magnitudes of the primary variables. Isolating the vector of the magnitudes of the secondary variable we have that.

$$\Psi_s = -\hat{N_s}^{-1}\hat{N}_p\Psi_p \tag{10}$$

To obtain the kinematic position we can integrate the eq.10 for example, using the Newton-Raphson method [4]:

$$\begin{aligned} &\frac{q_{s_{(t_k)}} - q_{s_{(t_{k-1})}}}{\Delta_t} = -\hat{N_s}^{-1}\hat{N}_p\frac{q_{p_{(t_k)}} - q_{p_{(t_{k-1})}}}{\Delta_t} \\ &q_{s_{(t_k)}} = q_{s_{(t_{k-1})}} - \hat{N_s}^{-1}\hat{N}_p\left\{q_{p_{(t_k)}} - q_{p_{(t_{k-1})}}\right\} \end{aligned} \tag{11}$$

where $q_{s_{(t_k)}}$ is the vector with secondary variables in the time t_k, $q_{s_{(t_{k-1})}}$ is the vector with secondary variables in the time t_{k-1}, $q_{p_{(t_k)}}$ is the vector with primary variables in the time t_k and $q_{p_{(t_{k-1})}}$ is the vector with primary variables in the time t_{k-1}, being the screws mounted with the configuration in the time t_{k-1}. In such a way have the initial configuration, is provided to $q_{p_{(t_k)}}$, it is gotten $q_{s_{(t_k)}}$.

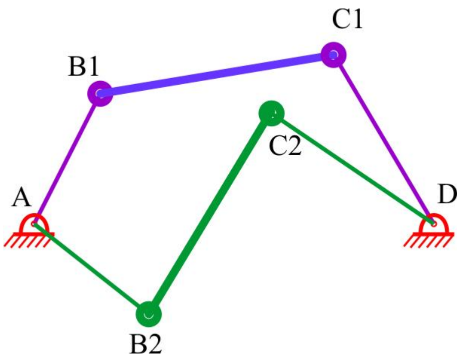

Figure 1. Representation of a geometric inversion.

Therefor, Davies Method it needs a initial configuration, to leave this, to follow the movements imposed. This initial configuration is not necessarily a kinematic coherent position, is possible stipulate a configuration "almost any" and iteratively to reach the first kinematically coherent configuration, case exist. This method consist in give a any initial configuration, $q_{s_{(t_{k-1})}}$ and $q_{p_{(t_{k-1})}}$ and a point to be reached, it mounted the matrix N_s e N_p and it gotten $q_{s_{(t_k)}}$. Making $q_{s_{(t_{k-1})}} = q_{s_{(t_k)}}$ and $q_{p_{(t_{k-1})}} = q_{reached}$ it calculated again $q_{s_{(t_k)}}$ and iteratively it arrived the values of $q_{s_{(t_k)}}$ of the initial configuration possible, case exist. The O flow of operations can be seen in Fig.1. However depending on the variable of the joints given initially, the first kinematically coherent configuration reached can be a configuration geometrically inverted, thus the method is leashed the first initial configuration given. The Fig.2 it show schematically a geometric invention.

2.2 ASSUR VIRTUAL CHAIN

An Assur Virtual Chain is essentially a tool to obtain information above the movement some point of the chain or to impose movement in a kinematic chain. The approach of the utilization of Assur Virtual Chain is normally applied to analyze of serial manipulators and/or to apply of movements to this type of manipulators, however, nothing hinders that this technique is applied the parallel manipulators.

Assur Virtual Chains are kinematics chains composed of virtual bodies and joints, that satisfy the following properties: a) the Assur Virtual Chain is open; b) the normalized screws of the joints are linearly independent; c) they do not modify the mobility of the real kinematics chain in which it will be joined [4] [9].

For example, a slider-crank mechanosm with 3 rotative joints and 1 prismatic (Fig.2-a), has 1 DOF. A Assur Virtual Chain to represent the end-effector movement of manipulator it could be a chain with three joints being two prismatic and one rotative (PPR), of 3 DOF (Fig.2-b). the union this two chains result in a close kinematic chain (or parallel) with 6 bodies and 7 joints (Fig.2-c), with mobility 1, as it can be verified by general of mobility equation showed in eq.12, where M is the mobility of open chain, λ is the dimension of operational space, n the number of bodies and j the number of joints.

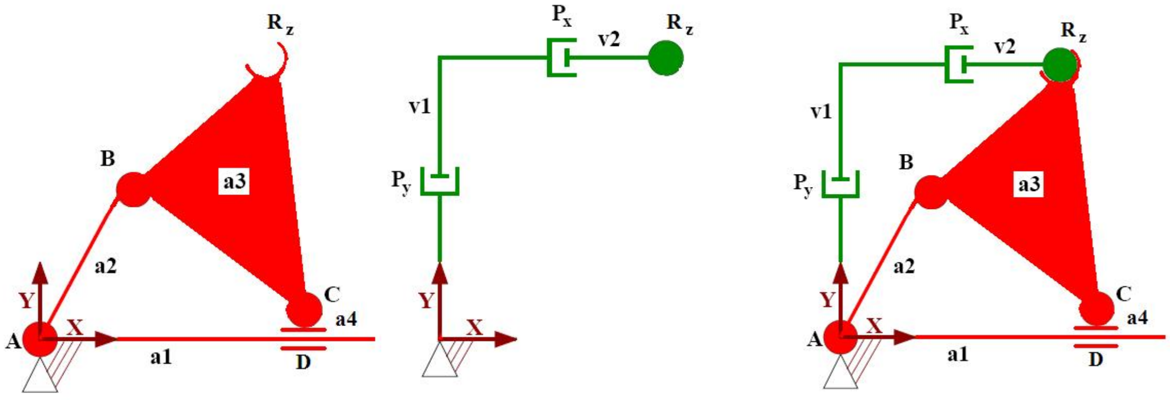

Figure 2. Representation of a parallel manipulator, your Assur Virtual Chain PPR and the interaction between both.

$$
\begin{aligned}
M &= \lambda * (n - j - 1) + j \\
M &= 3 * (6 - 7 - 1) + 7 \\
M &= 1
\end{aligned}
\tag{12}
$$

2.3 IMPOSITION OF THE TRAJECTORY

Depending of the type of problem that if it desired solve, the trajectory can be depends of the time, however in this study the time is not a variable, and our trajectory is only position.

One of the great problems in the optimization of mechanisms is the a comparison of the desired trajectory with the gotten trajectory. This, because the direction of the trajectory it influence directly in the error evaluation, it can cause a erroneous evaluation.

The method more general of comparison is the discretização of the trajectory in control points, however this cause others problems, such as, i) the trajectories, obtained and desired, they most have the same numbers of points; ii) and the comparison between the curves is strong dependent of the direction. A detailed description can be obtained in [15].

With the objective not to fall again into these problems, our method considers the addition of the Assur Virtual Chain (PPR) joining the origin of the coordinated system of the end effector, and being with that the active joints they are the meeting virtual chain. This form imposing to the manipulator the desired trajectory. Normally this process accumulate some error, or either, the difference in module of the point to be reached and the point that the end-effector really reached. This error is here call error of trajectory.

This approach it possess the advantages of in liberating them of the necessity of comparison of curves, being necessary only the comparison between the control points of the trajectory. In second lugar the problem of the necessary initial configuration for the application of Davies method is decided, for the fact of the imposition of the trajectory of active form on the end-effector it hinders that the manipulator initialize its movement in a geometrically inverted configuration. In third place, the method here proposed solves the problem of initial position of the active joints, being these treated now as passive joints.

However when manipulator to be synthesized will have inferior mobility the joints of Assur Virtual Chain, that they had been become active, occurs a problem in inversion of the matrix N_s, therefore this leave of being squared, as it can be verified in eq.10. To make this inversion the Moore-Penrose method it was used.

2.4 CLOSURE OF THE CHAIN

The closing error of the cinematic chain is the modulate of the sum of the Euclidean distance among the two elements of cinematic pair that uncouple. These erors are obtained of Davies Method.

The numerical integration carried through to passed of the velocities to get the position, accumulates some errors, calls integration errors. These errors make with that the chain has a closing error, which is, be open.

As already seen previously, the form with that imposes the movement to the chain also accumulates error. As form to reduce these errors, an Assur Virtual Chain Px Py Rz, active, is added to the circuit forcing the closing of the real chain. In the specific case study here, was possible to reduce the virtual chain of PPR to PP.

This chain called closing chain tends to guarantee the closing of the chain to be fact of their prismatic joints tend to a null length, this way uniting the links that are uncoupled.

2.5 INITIAL CONFIGURATION

With the used artifice to promote the closing of the chain, we are unleashed of an initial configuration for this, therefore the closing "will be guaranteed" by the closing chain and the first kinematically coherent configuration reached never will be a geometric inverted position, because the Assur Virtual Chain will be forcing the structure for the ideal configuration.

2.6 GENETIC ALGORITHM

The GA is a search algorithms based in a natural evolution an in environment adaptation, where the more apt survive and they reproduce, producing bigger number of descendants and passing its children its characteristics. Thus, the population evolves of form to improve its environment adaptation [14]. Computationally the environment adaptation is given from an evaluation function, being that the individual with better evaluation is more apt than another with lesser evaluation.

Holland supplies a form to describe the evolution of the population of a genetic algorithm in function of the time, proving the convergence from Schema Theorem [**?**]. With this, if see the GA as a parallel process that executes a search through the space of possible schemas and at the same time executes a search through the space of individuals. Then the equation of the theorem indicates that schemas of bigger aptitude will grow in influences with passing of the generations, inducing the convergence of the system for a point of optimum global, if admitting an enough great population interacting for a number enough great of generations.

In general a genetic algorithm is divided in representation or codification, creation of the initial population, evaluation of the individuals of the population in terms of a function evaluation, genetic operations including selection of the reproducers, reproduction and mutation.

2.6.1 ENCODING AND INITIAL POPULATION

Each individual of the population is represented by a vector of real numbers that corresponds the dimensions of a link or the variable of the joint. The genes of each individual of the population are composed for the variable to be optimized, that is a length of link or a variable of joint. Nor all the variable of the mechanism are genetic variable, but it can come to be. For example, the points where the mechanisms are fixed in the base, they can be a variable to be optimized or can be a predefine place. Only the genetic variables suffer the process of recombination and mutation.

Having been defined the type of codification and the genetic variable, are created the individuals that composes the initial population, being this formed by **n** individuals where each genetic variable to be optimized is initiated of random form, equally random it is the definition of the variable of joint in the initial configuration.

2.6.2 EVALUATION FUNCTION

The evaluation used here is initially based in the sum of the norm of the closing errors of the kinematic chain, errors of trajectory and of numbers of geometric inversions multiplied by a adaptation coefficient. After certain generation number, the function evaluation starts to be only the sum of the error of trajectory and the relative error the geometric inversions, being the closing error chain is equal the zero.

This technique guarantees an accented improvement of the population in the initial generations and good convergence in excessively, but it can be disastrous if the exchange of the evaluation method will be premature.

When is desired the synthesis of a manipulator to be used in confined environment, other evaluations must be carried through and its values must be added to the sum of the errors in the evaluation function. This was made defining a border, which the manipulator will not have to transpose and when some point of the manipulator transpose this border the norm of this error is added to the evaluation function. The Fig.3 illustrates the attainment of the aptitude of individual.

Other problems that can be associates the mechanisms synthesis, such as, angles of transmission, variations of velocities or acceleration, among others, they can be detected and be added in the evaluation function, thus, influencing the convergence to a configuration that guarantees angles of transmission or variations of velocities and acceleration inside of the specified limits.

Figure 3. Flowchart of Davies Method.

Figure 4. Flowchart of the attainment of the fitness of each individual.

2.6.3 SELECTION AND REPRODUCTION

Similarly the nature, where the individuals most apt if reproduce greater number of times that de others, computational one individual with better evaluation, or aptitude, too tends to reproduce more.

For the process of recombination in the genetic algorithm, is carried through a selection between the groups of individuals pertaining the population. A relation of reproducers is mounted on the basis of the evaluation of each individual, most apt having appeared bigger number of times.

A selection is based only in the aptitude of the individuals tends to diminish the robustness of the algorithm being able to direct the convergence for a not maximum point. The great problem in the direct use of the values of the evaluation of the individuals is the occurrence of crowding, where a superindividual with evaluation very bigger that the others could monopolize the reproductions, eliminating the diversity of the population.

Similarly as it occurs in the animal life, for example, in the flocks of monkeys, the dominant male of the flock, stronger than excessively, produces greater numbers of descendants, taking for itself all the females. However other males in the flock exist that also they will reproduce being able to arrive the female before the dominant male. Thus, exactly that the dominant male to copule with all the females, it will arrive behind in some, thus limiting its reproductions. In the same way we apply this theory to the algorithm, limiting the number of reproductions of individual with bigger aptitude.

The new individual generated will have its genes formed for an average between the values of the genes of the parents, which is selected of the reproducers, and is chosen of random form by the method of the roulette, with the algorithm below.

$$individualselected = m + 1 - \lfloor b \rfloor \qquad (13)$$
$$b = \left[\frac{-1 + \sqrt{1 + 4 * rand * (m^2 + m)}}{2} \right]$$

where: m is the number of individuals of the reproducers population, rand is a random number between 0 and 1 and $\lfloor b \rfloor$ return the major value minor that b. The elitism is used to clone the best individual of the population in order to guarantee that this is showed in the next generation facilitating the convergence.

2.6.4 MUTATION

The mutation is a random process of modification of genes, very useful for the maintenance of the population diversity and the creation of new individuals, enclosing all the ranges of the search.

In this study the mutation occurs randomically with a 6% of possibility. The whole process is carried out after the reproductions, where the individual selected for the mutation suffers two local mutations, where two random genes chosen are substituted by a random number between a maximum and a minimum previously defined, thus generating a new individual. This new individual only will be evaluated in the next generation, without mattering if is a viable or not individual.

3. METHOD APPLICATION

The algorithm considered was applied the dimensional synthesis of the 4 links manipulator, RRRP, with mobility 1, in order to get the combination of the dimensions of its links so that this manipulator follows the stipulated trajectory. The manipulator was synthesized to execute a task in a confined work space, being the definite border of confinement as the union of a horizontal line that passes for the point $y = 12mm$ and of a vertical line that passes for the point $x = 35mm$. This confinement being defined only to the joints B, C e D.

The border of confinement together with the topological schema of the mechanism and the Assur Virtual Chain, which imposes the movement to the mechanism, they are illustrated in Fig.5.

As genetic variables we have the dimensions of bodies a2 (distance AB), a3 (3 variables, distance BC, CE and BE), a4 (distance AD) and the coordinate Ax. The values of Ay and Dy they had been defined as equal to zero.

Expanding and applying the eq.10, to the exemplificated mechanism, together with Assur Virtual Chain for imposition of the movement to one Assur Virtual Chain to promote the closing of loop of the chain, which together the joint A to joint D, it is represented mathematically in agreement eq.15.

$$\Psi_{s_{(1\times5)}} = -\hat{N_s}^{-1}_{(6\times5)} \hat{N_p}_{(6\times4)} \Psi_{p_{(1\times4)}} \qquad (14)$$

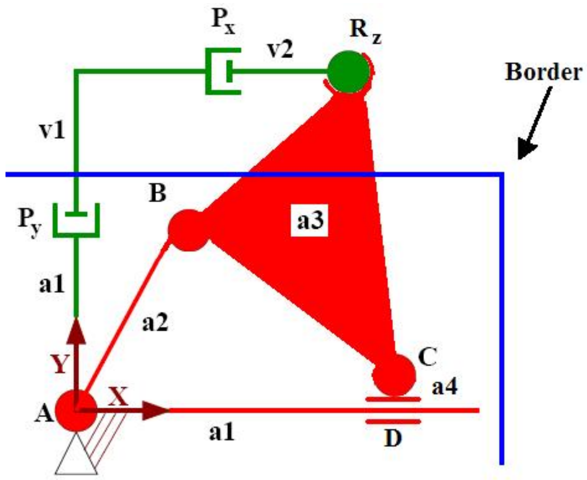

Figure 5. RRRP manipulator, with Assur Virtual Chain and the border confined.

$$\left\{\begin{array}{c}\lambda_A\\ \lambda_B\\ \lambda_C\\ \lambda_D\\ \lambda_{Rz}\end{array}\right\}_{(1\times5)} = -\left[\hat{N}_{s-1}\right]_{(6\times5)} - \left[\hat{N}_{p-1}\right]_{(6\times4)} \left\{\begin{array}{c}\lambda_{P_x}\\ \lambda_{P_y}\\ \lambda_{P_{ex}}\\ \lambda_{P_{ey}}\end{array}\right\}_{(1\times4)}$$

where P_{ex} and P_{ey} are the prismatic joints of the closing virtual chain. To the position kinematic we can integrate the eq.15 for in eq.12, we have that:

$$q_{s_{(t_k)}} = q_{s_{(t_{k-1})}} - \hat{N}_s^{-1}\hat{N}_p \left\{q_{p_{(t_k)}} - q_{p_{(t_{k-1})}}\right\} \left\{\begin{array}{c} q_{A_{(t_k)}}\\ q_{B_{(t_k)}}\\ q_{C_{(t_k)}}\\ q_{D_{(t_k)}}\\ q_{Rz_{(t_k)}}\end{array}\right\} \tag{15}$$

$$q_{s_{(t_k)}} = \left\{\begin{array}{c} q_{A_{(t_{k-1})}}\\ q_{B_{(t_{k-1})}}\\ q_{C_{(t_{k-1})}}\\ q_{D_{(t_{k-1})}}\\ q_{Rz_{(t_{k-1})}}\end{array}\right\} - \left[\hat{N}_s\right]^{-1}\left[\hat{N}_p\right] \left\{\begin{array}{c} q_{px_{(t_k)}} - q_{px_{(t_{k-1})}}\\ q_{py_{(t_k)}} - q_{py_{(t_{k-1})}}\\ q_{p_{ex}(t_k)} - q_{p_{ex}(t_{k-1})}\\ q_{p_{ey}(t_k)} - q_{p_{ey}(t_{k-1})}\end{array}\right\}$$

Being $[B]$ the circuit matrix, $\left[\hat{M}\right]$ the matrix with the normalized screws, applying the eq.2 and 3 and being the joints A, B, C and R_z rotatives and the joints D, P_x, P_y, P_{ex} e P_{ey} prismatic, we have that:

$$\begin{aligned} [B] &= \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}\\ \left[\hat{M}\right] &= \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ A_y & B_y & C_y & 1 & R_{zy} & 1 & 0 & 1 & 0 & \\ -A_x & -B_x & -C_x & 0 & -R_{zx} & 0 & 1 & 0 & 1 & \end{bmatrix}\\ \left[\hat{N}\right] &= \left[\hat{M}\right]_{3\times9} diag\left([B]\right) \end{aligned} \tag{16}$$

Separating the matrix $\left[\hat{N}\right]$ in $\left[\hat{N}_s\right]$ and in $\left[\hat{N}_p\right]$, we have that:

$$\hat{N}_s = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & B_y & 0 & 0 & R_{zy} \\ -A_x & -B_x & 0 & 0 & R_{zx} \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & C_y & 1 & R_{zy} \\ 0 & 0 & -C_x & 1 & R_{zx} \end{bmatrix} \qquad \hat{N}_p = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{17}$$

And finally we have that the system of balance discreted in the time, written in the matricial form as:

$$q_{s_{(t_k)}} = q_{s_{(t_{k-1})}} - \hat{N}_s^{-1}\hat{N}_p \left\{q_{p_{(t_k)}} - q_{p_{(t_{k-1})}}\right\} \tag{18}$$

$$\begin{Bmatrix} q_{A_{(t_k)}} \\ q_{B_{(t_k)}} \\ q_{C_{(t_k)}} \\ q_{D_{(t_k)}} \\ q_{Rz_{(t_k)}} \end{Bmatrix} = \begin{Bmatrix} q_{A_{(t_{k-1})}} \\ q_{B_{(t_{k-1})}} \\ q_{C_{(t_{k-1})}} \\ q_{D_{(t_{k-1})}} \\ q_{Rz_{(t_{k-1})}} \end{Bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & B_y & 0 & 0 & R_{zy} \\ -A_x & -B_x & 0 & 0 & R_{zx} \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & C_y & 1 & R_{zy} \\ 0 & 0 & -C_x & 1 & R_{zx} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} q_{px_{(t_k)}} - q_{px_{(t_{k-1})}} \\ q_{py_{(t_k)}} - q_{py_{(t_{k-1})}} \\ q_{pex_{(t_k)}} - q_{pex_{(t_{k-1})}} \\ q_{pey_{(t_k)}} - q_{pey_{(t_{k-1})}} \end{Bmatrix}$$

The initial population was constituted of 300 individuals with dimensions of its bodies being inside of the dimensional limits of $5mm$ e $30mm$.

The stopped criterion of the algorithm was the maximum number of iterations of 80 generations or diversity of the population lesser that $15mm$. The diversity of the population was defined as being the amount of the differences between the greater and the lesser dimension of the bodies the each one mechanisms of the population.

In the graph showed in the Fig.6, we have the resultant error in each one of the 25 control points given as desired trajectory for the end-effector. The gotten average error was of 0.129 mm, being that the biggest error occurred in point 1 showing value of 0.221 mm and the minor erro occurred in point 5 with value of 0.017 mm.

The manipulator its trajectory and the desired trajectory are showed in the Fig.7.

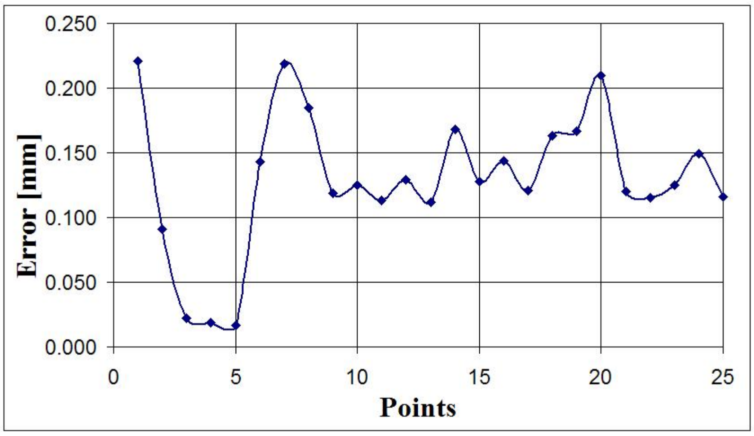

Figure 6. Schematical representation of mechanism RRRP generated.

4. DISCUSSION AND CONCLUSION

In the example showed, the border of the confinement was defined by lines, however this region of confinement can be defined by several others forms, needing only to implement an algorithm to monitor these borders.

The showed method is not bounded to the initial configuration of the manipulator as in many other cases listed in the literature; therefore, it has the great advantage of being able to start the process of dimensional synthesis without being worried by getting a kinematically-coherent initial configuration.

In many cases of mechanisms synthesis, the desirable beyond the imposition of the position impose the orientation to the end effector. With the method showed here, position and orientation of the end-effector are imposed to the mechanism not having mattered which the mobility of this. This strategy, which uses the Inverse of Moore-Penrose method, promotes a small accumulate of error, what many cases it does not intervene in the final result.

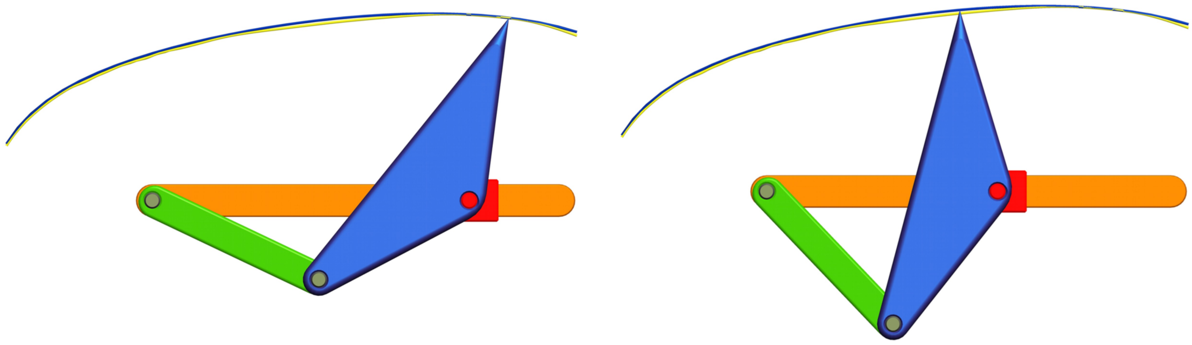

Figure 7. Schematical representation of mechanism RRRP generated.

In great number of applications of dimensional synthesis of mechanisms which use genetic algorithm as optimization tool, this genetic algorithm is programmed and configured for each specific case not being a flexible tool. In the study show here, the feeding of the genetic algorithm is carried through totally in function of error evaluations, either of position of the end effector, closing the chain and confinement, in such a way, the topology of the mechanism to be synthesized it does not influence in nothing the performance of the genetic algorithm, being totally generic.

This method, although being a proved viable method, not yet it is concluded, therefore as in many other methods he is being applied to a useful, but simple mechanism and in a planar space. The next steps to this work will be, the expansion of the space of work of planar for special, the implementation of detention of geometric inversions which will entered together as feeding of the genetic algorithm in error form and using de same genetic algorithm, allowing to the application of the method the mechanisms with bigger mobility and greater number of circuits. Future steps will be the incorporation of the kinostatic to the method as well as the synthesis of dwell mechanisms.

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