DOUBLE DIFFUSIVE NATURAL CONVECTION IN A TRAPEZOIDAL ENCLOSURE FILLED WITH POROUS MEDIA

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Abstract. *This work presents derivations of double diffisuve transport for turbulent buoyant flows in trapezoidal permeable structures. Equations are developed following two distinct procedures. The first method considers time averaging of the local instantaneous mass transport equation before the volume average operator is applied. The second methodology employs both averaging operators but in a reverse order. This work is intended to demonstrate that both approaches lead to equivalent equations when one takes into account both time fluctuations and spatial deviations of velocity and mass concentration. A modeled form for the final transport equation is presented where turbulent double diffusive transported is based on a macroscopic version of the k-ε model.*

Keywords*: Equation Numerical Method, Double Diffusion, Porous Media, Turbulent Flow*

1. INTRODUCTION

During last years many experimental and numerical studies have been carried out concerning convective phenomena. Most of these studies deal with fluid motion due only to temperature gradients. Nevertheless, fluid motion may be induced by density variations due to gradients of other scalar quantities. On of these quantities can be pollutant concentration within the fluid. Such a phenomenon, combining temperature and concentration buoyancy forces, is called double-diffusion.

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc (Mamou et al., 1995), Mohamad & Bennacer, (2002), (Goyeau et al., 1996), (Nithiarasu et al., 1997), (Mamou et al., 1998), (Bennacet et al., 2001), (Bennacet et al., 2003). In some specific applications, the fluid mixture may become turbulent and difficulties arise in the proper mathematical modeling of the transport processes under both temperature and concentration gradients.

Modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat or mass transfer Bear (1972), Bear & Bachmat, (1967), Whitaker (1966), Whitaker (1967). If time fluctuations of the flow properties are considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: a) application of time-average operator followed by volume-averaging Masuoka & Takatsu, (1996), (Kuwahara et al., 1996), Kuwahara & Nakayama, (1998), Nakayama & Kuwahara, (1999), or b) use of volume-averaging before time-averaging is applied Lee & Howell, (1987), Wang & Takle, (1995), Antohe & Lage, (1997), (Getachewa et al., 2000). This work intends to present a set of macroscopic mass transport equations derived under the recently established double decomposition concept Pedras & de Lemos, (2000), Pedras & de Lemos, (2001a), Pedras & de Lemos, (2001b), Pedras & de Lemos, (2001c), through which the connection between the two paths a) and b) above is unveiled. That methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature Rocamora & de Lemos, (2000). Buoyant flows de Lemos & Braga, (2003) and mass transfer de Lemos & Mesquita, (2003) have also been investigated. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published de Lemos & Pedras, (2001) and . Here, double-diffusive turbulent and laminar natural convection flow in porous media is extended for the case there was study of a binary mixture saturating a trapezoidal enclosure filled with porous material.

2. LOCAL INSTANTANEOUS TRANSPORT EQUATION

The steady-state microscopic instantaneous transport equations for an incompressible binary fluid mixture with constant properties are given by:

$$\nabla \cdot \boldsymbol{u} \tag{1}$$

$$\rho \nabla \cdot (\boldsymbol{uu}) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g} \tag{2}$$

$$\rho c_p \nabla \cdot (\boldsymbol{u}T) = \nabla \cdot (\lambda \nabla T) \tag{3}$$

$$\rho \nabla (\boldsymbol{u} m_\ell + \boldsymbol{J}_\ell) = \rho R_\ell \tag{4}$$

where $\boldsymbol{u}$ is the mass-averaged velocity of the mixture, $\boldsymbol{u} = \sum_\ell m_\ell \boldsymbol{u}_\ell$, $\boldsymbol{u}_\ell$ is the velocity of species ℓ, m_ℓ is the mass fraction of component m_ℓ, defined as $m_\ell = \rho_\ell / \rho$, ρ_ℓ is the mass density of species ℓ (mass of ℓ over total mixture volume), ρ is the bulk density of the mixture ($\rho = \sum_\ell \rho_\ell$), p is the pressure, μ is the fluid mixture viscosity, $\boldsymbol{g}$ is the gravity acceleration vector, c_p is the specific heat, T is the temperature and λ is the fluid thermal conductivity. The generation rate of species ℓ per unit of mixture mass is given in (4) by R_ℓ.

An alternative way of writing the mass transport equation is using the volumetric molar concentration C_ℓ (mol of ℓ over total mixture volume), the molar weight M_ℓ (g/mol of ℓ) and the molar generation/destruction rate R_ℓ^* (mol of ℓ /total mixture volume), giving:

$$M_\ell \nabla \cdot (\boldsymbol{u} C_\ell + \boldsymbol{J}_\ell) = M_\ell R_\ell^* \tag{5}$$

Further, the mass diffusion flux $\boldsymbol{J}_\ell$ (mass of ℓ per unit area per unit time) in (4) or (5) is due to the velocity slip of species ℓ,

$$\boldsymbol{J} = \rho_\ell (\boldsymbol{u}_\ell - \boldsymbol{u}) = -\rho_\ell D_\ell \nabla m_\ell = -M_\ell D_\ell \nabla C_\ell \tag{6}$$

where D_ℓ is the diffusion coefficient of species ℓ into the mixture. The second equality in equation (6) is known as Fick's Law, which is a constitutive equation strictly valid for binary mixtures under the absence of any additional driving mechanisms for mass transfer. Therefore, no Soret or Dufour effects are here considered.

Rearranging (5) for an inert species, dividing it by M_ℓ,

$$\nabla \cdot (\boldsymbol{u} C_\ell) = \nabla \cdot (D \nabla C_\ell) \tag{7}$$

If one considers that the density in the last term of (2) varies with temperature and concentration, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density ρ_T,

$$\rho_T \cong \rho [1 - \beta (T - T_{ref}) - \sum_1^\ell \beta_{C_\ell} (C_\ell - C_{\ell_{ref}})] \tag{8}$$

where the subscript ref indicates a reference value and β and $\beta_{C\ell}$ are the thermal and salute expansion for chemical species ℓ coefficients, respectively, defined by,

$$\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}\bigg|_{p,C}, \quad \beta_{C_\ell} = -\frac{1}{\rho_\ell} \frac{\partial \rho_\ell}{\partial C_\ell}\bigg|_{p,T} , \tag{9}$$

Equation (8) is an approximation of (9) and shows how density varies with temperature and concentration in the body force term of the momentum equation.

Further, substituting (8) into (2), one has,

$$\rho \nabla \cdot (\boldsymbol{uu}) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g} [1 - \beta (T - T_{ref}) - \sum_1^\ell \beta_{C_\ell} (C_\ell - C_{\ell ref})] \tag{10}$$

Thus, the momentum equation becomes,

$$\rho\nabla\cdot(\boldsymbol{uu}) = -(\nabla p)^{*} + \mu\nabla^{2}\boldsymbol{u} - \rho\,\boldsymbol{g}[(\beta(T-T_{ref}) + \sum_{1}^{\ell}\beta_{C_\ell}(C_\ell - C_{\ell ref})] \tag{11}$$

where $(\nabla p)^{*} = \nabla p - \rho\,\boldsymbol{g}$ is a modified pressure gradient.

As mentioned, there are, in principle, two ways that one can follow in order to treat turbulent flow in porous media. The first method applies a time average operator to the governing equation (4) before the volume average procedure is conducted. In the second approach, the order of application of the two average operators is reversed. Both techniques aim at derivation of a suitable macroscopic turbulent mass transport equation.

Volume averaging in a porous medium, described in detail in references Slattery (1967), Whitaker (1969), Gray & Lee, (1977), makes use of the concept of a Representative Elementary Volume (REV), over which local equations are integrated. After integration, detailed information within the volume is lost and, instead, overall properties referring to a REV (see Fig. are considered. In a similar manner, statistical analysis of turbulent flow leads to time mean properties. Transport equations for statistical values are considered in lieu of instantaneous information on the flow.

Before undertaking the task of developing macroscopic equations, it is convenient to recall the definition of time average and volume average.

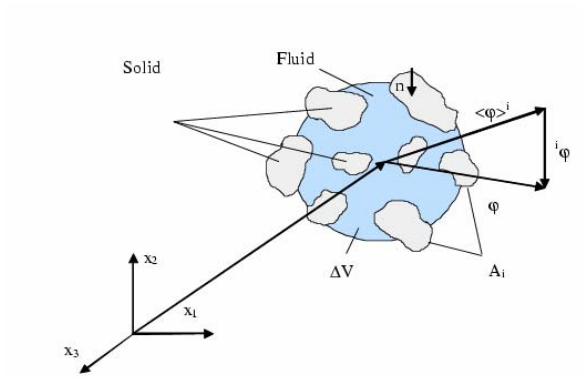

Figure 1 - Representative Elementary Volume, ΔV .

3. TIME AVERAGED TRANSPORT EQUATIONS

In order to apply the time average operator to Eqs. (1), (3), (7) and (11), one considers

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}',\; T = \overline{T} + T',\; C_\ell = \overline{C}_\ell + C_\ell',\; p = \overline{p} + p' \tag{12}$$

Substituting (12) into the governing equations and considering constant property flow,

$$\nabla\cdot\overline{\boldsymbol{u}} = 0 \tag{13}$$

$$\rho\nabla.\left(\overline{\boldsymbol{u}\boldsymbol{u}}\right) = -\left(\nabla\overline{p}\right)^{*} + \mu\nabla^{2}\overline{\boldsymbol{u}} + \nabla.\left(-\rho\overline{\boldsymbol{u}'\boldsymbol{u}'}\right) - \rho g\left[\beta\left(\overline{T} - T_{ref}\right) + \sum_{1}^{\ell}\beta_{C\ell}\left(\overline{C}_\ell - C_{\ell ref}\right)\right] \tag{14}$$

$$\left(\rho c_p\right)\nabla.\left(\overline{\boldsymbol{u}T}\right) = \nabla.\left(k\nabla\overline{T}\right) + \nabla.\left(-\rho c_p\overline{\boldsymbol{u}'T'}\right) \tag{15}$$

$$\nabla.\left(\boldsymbol{u}C_\ell\right) = -\nabla.\left(D\nabla C_\ell\right) + \nabla.\left(-\overline{\boldsymbol{u}'C_\ell'}\right) \tag{16}$$

For clear fluid, the use of the eddy-diffusivity concept for expressing the stress-strain rate relationship for the Reynolds stress appearing in (14) gives

$$-\rho\overline{\boldsymbol{u}'\boldsymbol{u}'} = \mu_t 2\overline{\mathbf{D}} - \frac{2}{3}\rho k \mathbf{I} \tag{17}$$

where $\overline{\boldsymbol{D}} = \left[\nabla\overline{\boldsymbol{u}} + \left(\nabla\overline{\boldsymbol{u}}\right)^T\right] / 2$ is the mean deformation tensor, $k = \overline{\boldsymbol{u}'\boldsymbol{u}'} / 2$ is the turbulent kinetic energy per unit mass, μ_t is the turbulent viscosity and $\boldsymbol{I}$ is the unity tensor. Similarly, for the turbulent heat flux on the R.H.S. of (15) and (16) the eddy diffusivity concept reads

$$-\rho c_p \overline{\boldsymbol{u}'T'} = c_p \frac{\mu_t}{Pr_t}\nabla\overline{T} \; ; \; -\rho c_p \overline{\boldsymbol{u}'C_\ell'} = \frac{\mu_t}{Sc_t}\nabla\overline{C_\ell} \tag{18}$$

where P_{r_ℓ} and S_{C_ℓ} are known as the turbulent Prandtl and Schmidt numbers, respectively.

Further, a transport equation for the turbulent kinetic energy is obtained by multiplying first, by $\boldsymbol{u}'$, the difference between the instantaneous and the time-averaged momentum equations. Thus, applying further the time average operator to resulting product, one has

$$\rho\nabla\cdot\left(\boldsymbol{u}'\right) = -\rho\nabla\cdot\left[\boldsymbol{u}'\left(\frac{p'}{\rho} + q\right)\right] + \mu\nabla^2 k + P + G_T + G_{C_\ell} - \rho\varepsilon \tag{19}$$

where $P = -\rho\overline{\boldsymbol{u}'\boldsymbol{u}'} : \nabla\overline{\boldsymbol{u}}$ is the generation rate of k due to gradients of the mean velocity and

$$G_T = -\rho\beta g \overline{\boldsymbol{u}'T'} \tag{20}$$

$$G_{C_\ell} = -\rho\beta_{C_\ell} g \overline{\boldsymbol{u}'C_\ell'} \tag{21}$$

are the thermal and concentration generation rates of k due temperature and concentration fluctuations, respectively. Also, $q = \boldsymbol{u}'.\boldsymbol{u}'/2$

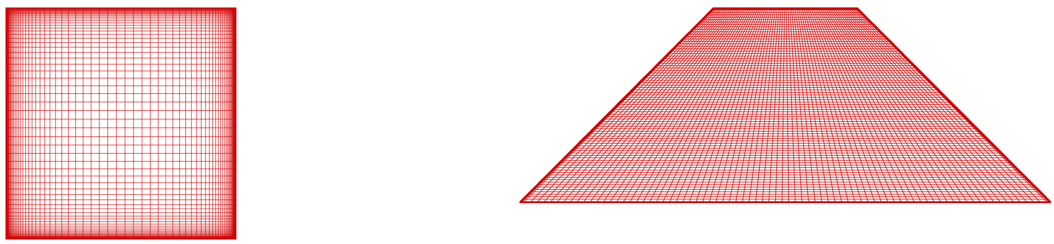

Figure 2 – Square and trapezoidal grid for laminar and turbulent flow simulation.

4. MACROSCOPIC EQUATIONS FOR BUOYANCY FREE FLOWS

For non-buoyant flows, macroscopic equations considering turbulence have been already derived in detail for momentum Pedras & de Lemos, (2001a), heat de Lemos & Braga, (2003) and mass de Lemos & Mesquita, (2003) transfer and for this reason their derivation need not to be repeated here. They read:

Momentum transport

$$\rho\nabla\cdot\left(\frac{\overline{\boldsymbol{u}}_D\overline{\boldsymbol{u}}_D}{\phi}\right) = -\nabla(\phi\langle\overline{p}\rangle^i) + \mu\nabla^2\overline{\boldsymbol{u}}_D - \left[\frac{\mu\phi}{K}\overline{\boldsymbol{u}}_D + \frac{c_F\phi\rho|\overline{\boldsymbol{u}}_D|\,\overline{\boldsymbol{u}}_D}{\sqrt{K}}\right] \tag{22}$$

$$-\rho\phi\left\langle \overline{\boldsymbol{u}'\boldsymbol{u}'} \right\rangle^{i} = \mu_{t_\phi}\, 2\left\langle \overline{\boldsymbol{D}} \right\rangle^{v} - \frac{2}{3}\phi\rho\left\langle k \right\rangle^{i} \boldsymbol{I} \tag{23}$$

$$\left\langle \overline{\boldsymbol{D}} \right\rangle^{v} = \frac{1}{2}\left[\nabla\left(\phi\left\langle \overline{\boldsymbol{u}} \right\rangle^{i} \right) + \left[\nabla\left(\phi\left\langle \overline{\boldsymbol{u}} \right\rangle^{i} \right) \right]^{T} \right] \tag{24}$$

$$\left\langle k \right\rangle^{i} = \left\langle \overline{\boldsymbol{u}' \cdot \boldsymbol{u}'} \right\rangle^{i} / 2$$

$$\mu_{t_\phi} = \rho c_{\mu} \frac{\left\langle k \right\rangle^{i\,2}}{\left\langle \varepsilon \right\rangle^{i}} \tag{25}$$

Heat transport

$$(\rho c_p)_f \nabla \cdot (\boldsymbol{u}_D \langle \overline{T} \rangle^i) = \nabla \cdot \{ \boldsymbol{K}_{eff} \cdot \nabla \langle \overline{T} \rangle^i \} \tag{26}$$

$$\boldsymbol{K}_{eff} = [\phi \lambda_f + (1-\phi)\lambda_s]\, \boldsymbol{I} + \boldsymbol{K}_{tor} + \boldsymbol{K}_t + \boldsymbol{K}_{disp} + \boldsymbol{K}_{disp,t} \tag{27}$$

The subscripts f and s refer to fluid and solid phases, respectively, and coefficients K's come from the modeling of the following mechanisms:

Tortuosity: $\left[\frac{1}{\Delta V} \int_{A_i} \boldsymbol{n}\left(\lambda_f \overline{T}_f - \lambda_s \overline{T}_s \right) dS \right] = \boldsymbol{K}_{tor} \cdot \nabla \langle \overline{T} \rangle^i$ (28)

Thermal dispersion: $-(\rho c_p)_f\, \phi \langle {}^i\overline{\boldsymbol{u}}\, {}^i\overline{T}_f \rangle^i = \boldsymbol{K}_{disp} \cdot \nabla \langle \overline{T} \rangle^i$ (29)

Turbulent heat flux: $-\left(\rho c_p \right)_f \phi \left\langle \boldsymbol{u}' \right\rangle^{i} \left\langle T'_f \right\rangle^{i} = \boldsymbol{K}_t \cdot \nabla \left\langle \overline{T} \right\rangle^{i}$ (30)

Turbulent thermal dispersion: $-(\rho c_p)_f\, \phi \overline{\left\langle {}^i\boldsymbol{u}'\, {}^iT'_f \right\rangle^{i}} = \boldsymbol{K}_{disp,t} \cdot \nabla \langle \overline{T} \rangle^i$ (31)

Mechanisms (30) and (31) were modeled together in de Lemos & Braga, (2003) by assuming,

$$-\left(\rho c_p \right)_f \left\langle \overline{\boldsymbol{u}' T'_f} \right\rangle^{i} = c_{p_f} \frac{\mu_{t_\phi}}{Pr_{t_\phi}} \nabla \left\langle \overline{T}_f \right\rangle^{i} \tag{32}$$

or

$$\boldsymbol{K}_t + \boldsymbol{K}_{disp,t} = \phi c_{p_f} \frac{\mu_{t_\phi}}{Pr_{t_\phi}} \boldsymbol{I} \tag{33}$$

Mass transport

$$\nabla \cdot (\overline{\boldsymbol{u}}_D \langle \overline{C_\ell} \rangle^i) = \nabla \cdot \boldsymbol{D}_{eff} \cdot \nabla (\phi \langle \overline{C_\ell} \rangle^i) \tag{34}$$

$$\boldsymbol{D}_{eff} = \boldsymbol{D}_{disp} + \boldsymbol{D}_{diff} + \boldsymbol{D}_t + \boldsymbol{D}_{disp,t} \tag{35}$$

$$\boldsymbol{D}_{diff} = \langle D \rangle^i \boldsymbol{I} = \frac{1}{\rho_\ell}\frac{\mu_\phi}{Sc_\ell}\boldsymbol{I} \tag{36}$$

$$\boldsymbol{D}_t + \boldsymbol{D}_{disp,t} = \frac{1}{\rho_\ell}\frac{\mu_{t_\phi}}{Sc_{\ell_t}}\boldsymbol{I} \tag{37}$$

Coefficients $\boldsymbol{D}_{disp}$, $\boldsymbol{D}_t$ and $\boldsymbol{D}_{disp,t}$ in (34) appear due to the nonlinearity of the convection term. They come from the modeling of the following mechanisms:

Mass dispersion: $-\langle {}^i\overline{\boldsymbol{u}}\, {}^i\overline{C} \rangle^i = \boldsymbol{D}_{disp} \cdot \nabla \langle \overline{C} \rangle^i$ (38)

Turbulent mass flux: $-\overline{\langle \boldsymbol{u}' \rangle^i \langle C' \rangle^i} = -\overline{\langle \boldsymbol{u} \rangle^{i'} \langle C \rangle^{i'}} = \boldsymbol{D}_t . \nabla \langle \overline{C} \rangle^i$ (39)

Turbulent mass dispersion: $-\left\langle \overline{{}^i\boldsymbol{u}'\, {}^iC_\ell{}'} \right\rangle^i = \boldsymbol{D}_{disp,t} . \nabla \left\langle \overline{C_\ell} \right\rangle^i$ (40)

Here also mechanisms (39) and (40) are added up as de Lemos & Mesquita, (2003)

$$-\left\langle \overline{\boldsymbol{u}' C_\ell{}'} \right\rangle^i = \frac{1}{\rho}\frac{\mu_{t_\phi}}{Sc_{\ell_{t_\phi}}} \nabla \left\langle \overline{C} \right\rangle^i = \langle D_t \rangle^i \nabla \left\langle \overline{C_\ell} \right\rangle^i = \left(\boldsymbol{D}_t + \boldsymbol{D}_{disp,t} \right) . \nabla \left\langle \overline{\boldsymbol{C}_\ell} \right\rangle^i \tag{41}$$

Focusing now attention to buoyancy effects only, aplication of the volume average procedure to the last term of equation (14) leads to

$$\left\langle \rho \boldsymbol{g} \left[\beta \left(\overline{T} - T_{ref} \right) + \sum_1^\ell \beta_{c_\ell} \left(\overline{C}_\ell - C_{ref_\ell} \right) \right] \right\rangle^V = \frac{\Delta V_f}{\Delta V} \frac{1}{\Delta V_f} \int_{\Delta V_f} \rho \boldsymbol{g} \left[\beta \left(\overline{T} - T_{ref} \right) + \sum_1^\ell \beta_{c_\ell} \left(\overline{C}_\ell - C_{ref_\ell} \right) \right] dV \tag{42}$$

Expanding the left hand side of (42) in light of the double decomposition concept and considering that $\left\langle {}^i\varphi \right\rangle^i = 0$ and defining β_ϕ and β_{C_ϕ} are the macroscopic thermal and salute of chemical species ℓ expansion coefficients , respectivelly [for more details see de Lemos & Tofaneli (2004)] . Assuming that gravity is constant over the REV, expressions for them are given as

$$\beta_\phi = \frac{\left\langle \rho \beta \left(\overline{T} - T_{ref} \right) \right\rangle^V}{\rho \phi \left(\left\langle \overline{T} \right\rangle^i - T_{ref} \right)} \tag{43}$$

$$\beta_{C\phi} = \frac{\left\langle \rho \beta_C \left(\overline{C_\ell} - C_{ref} \right) \right\rangle^V}{\rho \phi \left(\left\langle \overline{C_\ell} \right\rangle^i - C_{ref} \right)} \tag{44}$$

Including (43) and (44) into (22), the macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as

$$\rho \nabla \cdot \left(\frac{\overline{\boldsymbol{u}}_D \overline{\boldsymbol{u}}_D}{\phi} \right) = -\nabla (\phi \langle \overline{p} \rangle^i) + \mu \nabla^2 \overline{\boldsymbol{u}}_D - \rho \boldsymbol{g} \left[\beta_\phi \left(\overline{T} - T_{ref} \right) + \sum_1^\ell \beta_{c_\ell} \left(\overline{C}_\ell - C_{ref_\ell} \right) \right] - \left[\frac{\mu \phi}{K} \overline{\boldsymbol{u}}_D + \frac{c_F \phi \rho |\overline{\boldsymbol{u}}_D| \overline{\boldsymbol{u}}_D}{\sqrt{K}} \right] \tag{45}$$

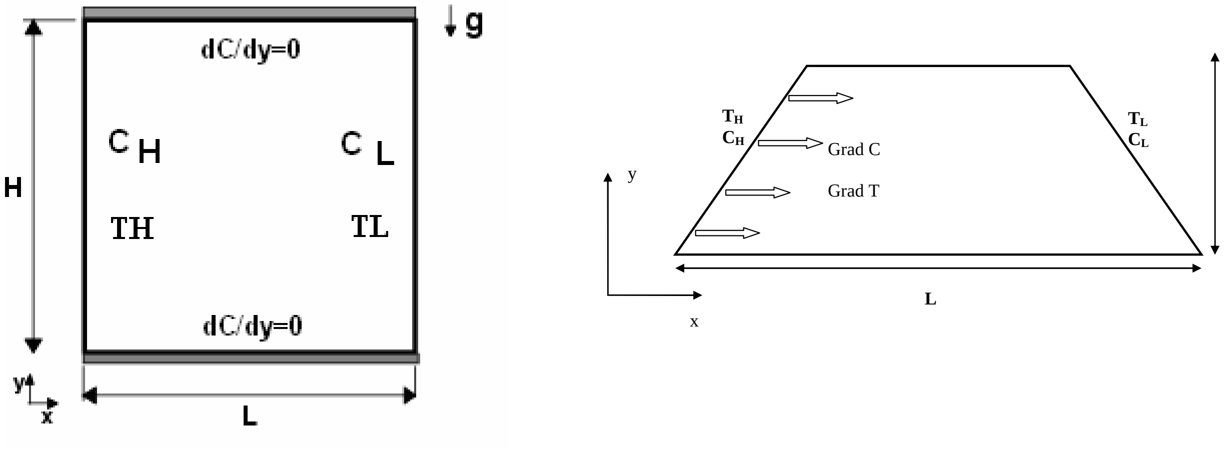

Figure 3 – Geometry and boundary conditions: a) Square cavity and b) Trapezoidal enclosure.

5. RESULTS AND DISCUSSION

All results obtained during the development of this work are presented and discussed upon. Here, results are divided in two main sessions, involving each a certain domain configurations (square and trapezoidal enclosure). First, in clear medium session, the cavities are assumed to be unobstructed so that no extra drag, either of viscous or form nature, are included in the momentum equations. In this session, both laminar flow and turbulent flow regimes are analyzed. Further, in the porous medium session, the cavities are completely filled with porous material and runs are made also for laminar and turbulent flow.

The first problem considered is showed schematically in Fig. 3a and refers to the two-dimensional flow in a clear (or a cavity filled with porous material) rectangular cavity of height and width L , The Schmidt number is assumed to be a unity. The cavity is assumed to be of infinite depth the z-axis and a uniform mass concentration gradient is putting on the left side to opposing side. The second geometry was considered has the form of a trapezoid (Fig.3b). Zero heat and mass values are assumed at the top and bottom walls of the enclosure. In the side walls a uniform heat and mass gradient is putting on left to right side. Numerical computations were performed for square cavity and trapezoidal used a stretched grid with 80 x 80 (CV), see Fig 2 a and b.

Aim order to validate the present work, Figure 4 shows the comparison between the present result and the work of the Goyeau et al. It may be seen on the streamlines that increasing N significantly modifies the flow structure: at low N, the whole enclosure is affected by the flow, and a boundary layer regime progressively appears with larger N. The Figure shows the agreement of the present work and Goyeau et al. it was relatively good.

Figure 5 shows the flow, temperature and concentration fields for different values of N ($Ra_T = 1.E+06, Sc = 1.0$). As just comments for the case where considered the square cavity, it may be seen on the streamlines that increasing N significantly modifies the flow structure: at low N, the whole enclosure is affected by the flow, and a boundary layer regime progressively appears with larger N. This modification of the flow structure has a direct visible consequence on the concentration field, which progressively builds up a vertical stratification. Although this transition towards a solutally dominated regime corresponds to a heat transfer minimum, the consequence on the temperature field is barely visible: the minimum is seen however to coincide with the end of temperature stratification (to more details see Goyeau et al.).

Finally, Figure 6 presents the streamlines, isotherms and isoconcentrations lines mapping for the case where had a trapezoidal enclosure totally filled with porous material for $Ra_T\ 1.E+05$. The streamlines in Fig. 6 indicates the existence of a single vortex with center in the middle of the cavity. Corresponding isotherms and isoconcentration lines are almost parallel to the heated and concentrated walls, indicating that most of the heat and mass transfer is transferred by conduction. The vortex is generated due the horizontal temperature and mass gradient across the section. This gradient, is negative everywhere, inducing a clockwise oriented vorticity.

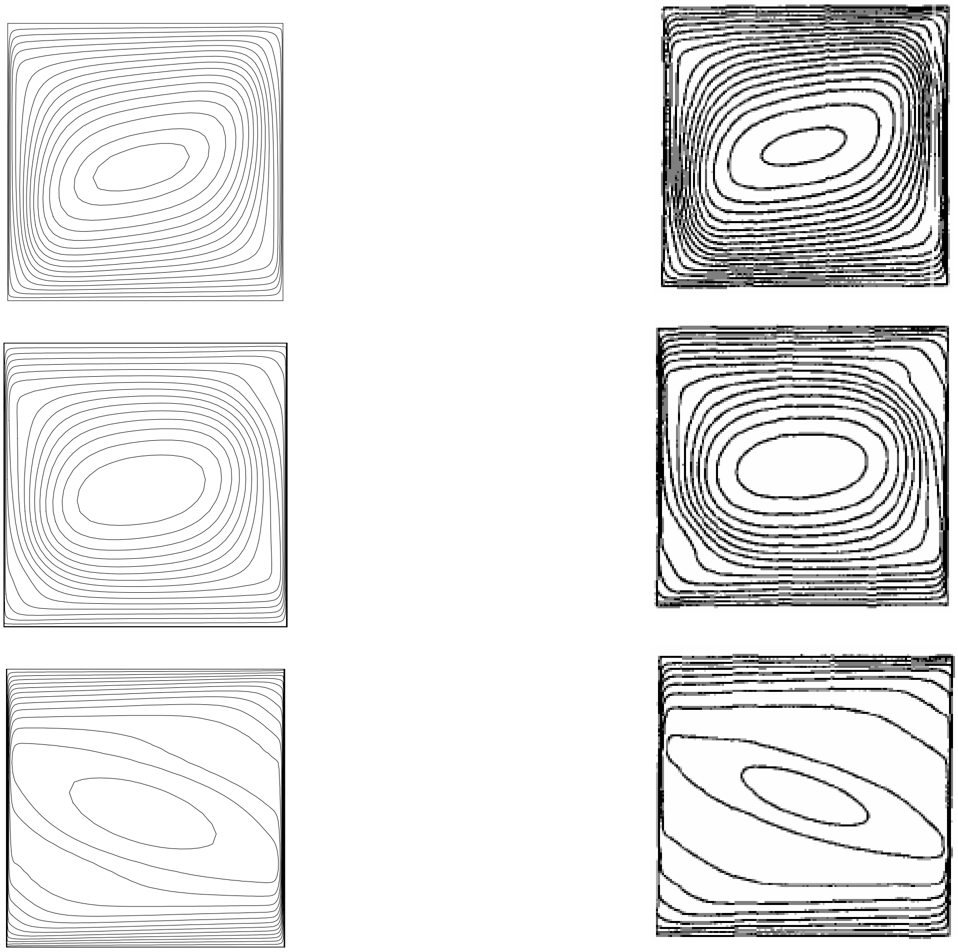

Figure 4 – Streamlines visualization comparison between present work and Goyeau et al., from top to bottom, N = 2, 3 and 5.

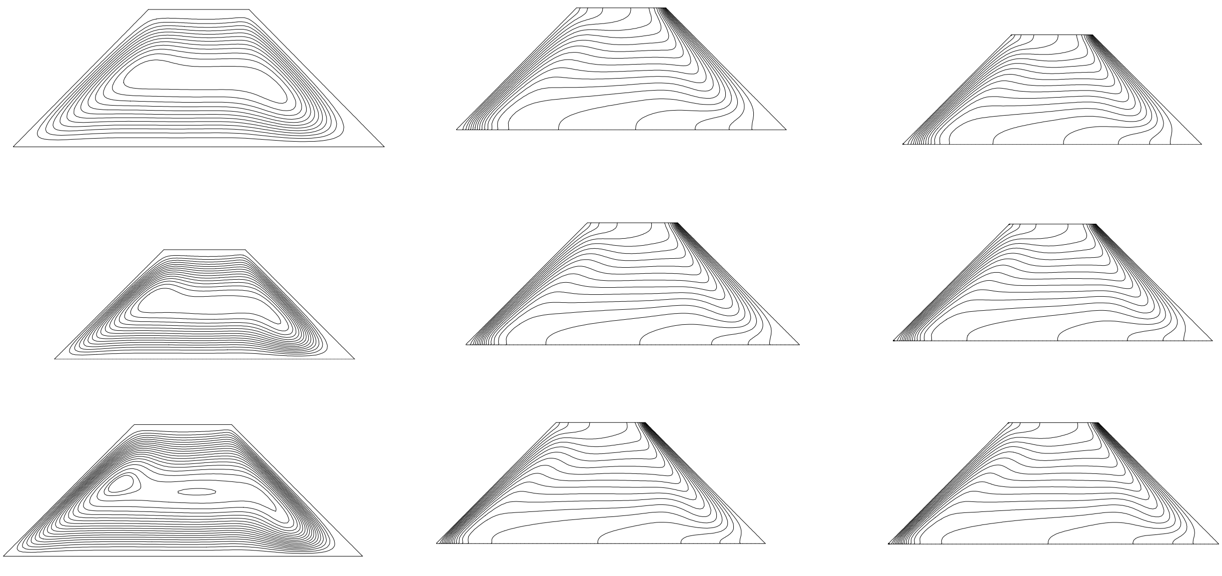

Figure 5 Streamlines, isotherms and isoconcentration lines ($Ra_T = 1.E+06$) from top to bottom , N =2, 3 and 10 .

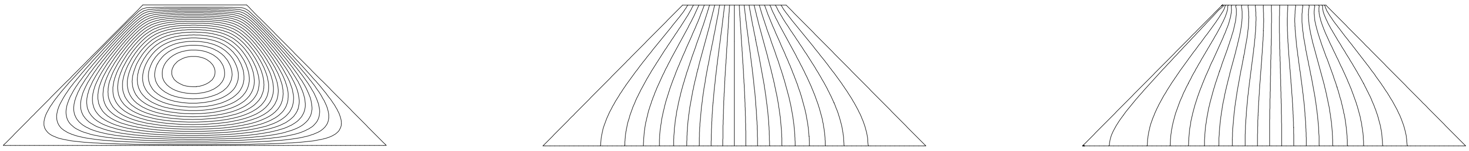

Figure 6 – Streamlines, isotherms and isoconcentration lines for trapezoidal enclosure totally filled with porous material ($dp = 1mm, \phi = 0.8$)

3. ACKNOWLEDGEMENTS

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4. REFERENCES

Antohe, B. V. & Lage, J. L., 1997. A general two-equation macroscopic turbulence model for incompressible flow in porous media, International Journal Heat Mass Transfer, vol.40, n.13, pp. 3013 - 3024.
Bear, J., 1972. Dynamics of Fluids in Porous Media, Dover, New York.
Bear, J. & Bachmat, Y., 1967. A generalized theory on hydrodynamic dispersion in porous media, I.A.S.H. Symp. Artificial Recharge and Management of Aquifers, Haifa, Israel, P.N. 72, pp. 7-16, I.A.S.H..
Bennacer, R., Tobbal, A., Beji, H. & Vasseur, P., 2001. Double diffusive convection in a vertical enclosure filled with anisotropic porous media, International Journal of Thermal Sciences, vol. 40, n. 1, pp.30-41.
Bennacer, R., Beji, H. & Mohamad, A. A., 2003. Double diffusive convection in a vertical enclosure inserted with two saturated porous layers confining a fluid layer, International Journal of Thermal Sciences, vol. 42, n. 2, pp.141-151.
de Lemos, M.J.S. & Braga, E.J., 2003. Modeling of turbulent natural convection in porous media, International Communications Heat Mass Transfer, vol. 30 n. 5, pp. 615 - 624.
de Lemos, M.J.S. & Mesquita, M.S., 2003. Turbulent mass transport in saturated rigid porous media, International Communications Heat Mass Transfer, vol. 30, pp. 105 - 113.
de Lemos, M.J.S. & Pedras, M.H.J., 2001. Recent mathematical models for turbulent flow in saturated rigid porous media, Journal of Fluids Engineering, vol.123, n. 4, pp. 935 - 940.
de Lemos, M.J.S. & Tofaneli, L.A., 2004. Modeling of double diffusive turbulent natural convection in porous media, International Journal Heat Mass Transfer, vol. 47, pp. 4233-4241.
de Lemos, M.J.S. & Rocamora Jr., F.D., 2002. Turbulent transport modeling for heated flow in rigid porous media, Proceedings of the Twelfth International Heat Transfer Conference, pp. 791-795.
Getachewa, D., Minkowycz, W.J. & Lage, J.L, 2000. A modified form of the kappa-epsilon model for turbulent flows of an incompressible fluid in porous media, International Journal Heat Mass Transfer, vol.43, n. 16, pp. 2909 - 2915.
Goyeau, B., Songbe, J.P. & Gobin, D., 1996. Numerical study of double-diffusive natural convection in a porous cavity using the Darcy-Brinkman formulation, International Journal of Heat and Mass Transfer, vol.39, n. 7, pp.1363-1378.
Gray, W.G. & Lee, P.C.Y., 1977. On the theorems for local volume averaging of multiphase system, Int. J. Multiphase Flow, vol.3, pp.333 -340.
Hsu, C.T. & Cheng, P., 1990. Thermal dispersion in a porous medium, International Journal Heat Mass Transfer, vol.33, pp.1587-1597.
Kuwahara, F., Nakayama, A. & Koyama, H., 1996. A numerical study of thermal dispersion in porous media, Journal of Heat Transfer, vol.118, pp.756.
Kuwahara, F. & Nakayama, A., 1998. Numerical modeling of non-Darcy convective flow in a porous medium, Heat Transfer , Proc. 11th Int. Heat Transf. Conf., Kyongyu, Korea, vol. 4, pp. 411-416, Taylor & Francis Washington, D.C..
Lee, K. & Howell, J.R., 1987. Forced convective and radiative transfer within a highly porous layer exposed to a turbulent external flow field, Proceedings of the 1987 ASME-JSME Thermal Engineering Joint Conf., Honolulu, Hawaii, vol. 2, 377-386, ASME, New York, N.Y.
Mamou, M., Vasseur, P. & Bilgen, E., 1995. Multiple solutions for double-diffusive convection in a vertical porous enclosure, International Journal of Heat and Mass Transfer, vol. 38, n. 10, pp. 1787-1798.
Mamou, M., Hasnaoui, M., Amahmid, A. & Vasseur, P., 1998. Stability analysis of double diffusive convection in a vertical brinkman porous enclosure, International Communications in Heat and Mass Transfer, vol. 25, n. 4, pp. 491-500.

Masuoka, T. & Takatsu, Y., 1996. Turbulence model for flow through porous media, International Journal Heat Mass Transfer, vol.39, n. 13, pp. 2803 - 2809.
Mohamad, A. A. & Bennacer, R., 2002. Double diffusion natural convection in an enclosure filled with saturated porous medium subjected to cross gradients; stably stratified fluid, International Journal of Heat and Mass Transfer, vol.45, n. 18, pp. 3725-3740.
Nakayama & Kuwahara, F., 1999. A macroscopic turbulence model for flow in a porous medium, Journal of Fluids Engineering, vol.121, pp. 427 - 433.
Nithiarasu, P., Sundararajan, T., & Seetharamu, K.N., 1997. Double-diffusive natural convection in a fluid saturated porous cavity with a freely convecting wall, International Communications in Heat and Mass Transfer, vol. 24, n. 8, pp.1121-1130.
Pedras, M.H.J & de Lemos, M.J.S., 2000. On the definition of turbulent kinetic energy for flow in porous media, Internernational Communications Heat and Mass Transfer, vol. 27 n. 2, pp. 211 - 220.
Pedras, M.H.J. & de Lemos, M.J.S., 2001. Macroscopic turbulence modeling for incompressible flow through undeformable porous media, International Journal Heat and Mass Transfer, vol. 44, n. 6, pp.1081 - 1093.
Pedras, M.H.J. & de Lemos, M.J.S., 2001. Simulation of turbulent flow in porous media using a spatially periodic array and a lowre two-equation closure, Numerical Heat Transfer - Part A Applications, vol. 39, n. 1, pp.35.
Pedras, M.H.J. & de Lemos, M.J.S., 2001. On the mathematical description and simulation of turbulent flow in a porous medium formed by an array of elliptic rods, Journal of Fluids Engineering, vol. 23, n. 4, pp. 941 - 947.
Rocamora Jr., F.D. & de Lemos, M.J.S., 2000. Analysis of convective heat transfer for turbulent flow in saturated porous media, International Communications Heat and Mass Transfer, vol. 27, n. 6, pp.825 - 834.
Slattery, J.C., 1967. Flow of viscoelastic fluids through porous media, A.I.Ch.E. J., 13, pp. 1066 - 1071.
Trevisan, O. & Bejan, A., 1985. Natural convection with combined heat and mass transfer buoyancy effects in a porous medium, International Journal Heat and Mass Transfer, vol.28, pp.1597-1611.
Wang, H. & Takle, E.S., 1995. Boundary-layer flow and turbulence near porous obstacles .1. derivation of a general equation set for a porous-medium, Boundary Layer Meteorology, vol.74, pp.73 - 78.
Whitaker, S., 1967. Diffusion and dispersion in porous media, J. Amer. Inst. Chem. Eng, vol.3, n. 13, pp. 420.
Whitaker, S., 1969. Advances in theory of fluid motion in porous media, Indust. Eng. Chem., vol.61, pp. 14 - 28.
Whitaker, S., 1966. Equations of motion in porous media, Chem. Eng. Sci., 21, pp.91.

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