ANALYSIS OF TWO-ENERGY EQUATION MODEL FOR LAMINAR FLOW IN POROUS MEDIA

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Abstract. *Two-energy equation model for conduction and convection in porous media is analyzed. In addition, effects of thermal dispersion and local thermal non-equilibrium are evaluated. Macroscopic time-average equations for continuity, momentum and energy are presented based on the recently established double decomposition concept. The numerical methodology employed is based on the control-volume approach with a boundary-fitted non-orthogonal coordinate system. This work intends to calculate the interfacial convective heat transfer coefficient considering a fluid flowing in a porous medium. Furthermore, fully developed forced convection in a porous channel bounded by parallel plates is considered based on a two-energy equation model. Solutions for temperature profile and Nusselt number are obtained and presented for laminar flows.*

Keywords*: Porous media, heat transfer coefficient, thermal non-equilibrium.*

1. Introduction

The assumption of local thermal equilibrium requires several constraints which have been investigated by some authors: Schumann (1929), Quintard and Whitaker (1993), Quintard and Whitaker (1995), Vafai and Sozen (1990) and Whitaker (1991). For example, this condition of local equilibrium is no longer valid when the particles or pores are not small enough, when the thermal properties differ widely, or when convective transport is not important. Furthermore, when there is a significant heat generation in any of the phases, the system will become rapidly far from the local thermal equilibrium, Kaviany (1995). For such extreme conditions, the one-energy equation or one-temperature model is inadequate to describe correctly both the transients associated with the quench front penetrating the hot dry porous layer and regions where dry out occurs. When the assumption of local thermal equilibrium fails to be valid, one possible solution to model such cases is to develop separate transport equations for each phase, Quintard, M., (1998). This leads to macroscopic model which are referred to as thermal non-equilibrium model. Such models tend to become more and more popular in heat and mass transfer problems. For heat transport through a porous medium, a thermal non-equilibrium model has been proposed under the form of a two-energy equation model. However an analysis of heat transfer in a porous medium based on the two-equation model is more concerned because it requires information on the interstitial heat transfer coefficient between the fluid phase and the solid phase as well as the interfacial surface area.

Due to this difficulty, most investigators have been evaluated how to obtain the interfacial heat transfer coefficient. Wakao et al. (1979) obtained a heuristic correlation for closely packed bed and compared with experimental data. A numerical correlation for the interfacial convective heat transfer coefficient was proposed by Kuwahara et al. (2001) for laminar flow and based on porosity dependency.

A comprehensive analysis of these non-equilibrium effects in turbulent heat transport has not yet been achieved. This emphasizes the need for the development of thermal non-equilibrium theories to describe complex heat transfer and turbulent flow in porous media.

When treating turbulent flow in porous media, however, difficulties arise due to the fact that the flow fluctuates with time and a volumetric average is applied Gray and Lee (1977). For handling such situations, a new concept called double decomposition has been proposed for developing macroscopic models for turbulent transport in porous media, Pedras and de Lemos (2000), Pedras and de Lemos (2001a), Pedras and de Lemos (2001c), Pedras and de Lemos (2001b), Pedras and de Lemos (2003). This methodology has been extended to non-buoyant heat transfer Rocamora and de Lemos (2000), buoyant flows Braga and de Lemos (2004), Braga and de Lemos (2005)a, Braga and de Lemos (2005)b, Braga and de Lemos (2006)a, Braga and de Lemos (2006)b, mass transfer de Lemos and Mesquita (2003) and double diffusion de Lemos and Tofaneli (2004). In addition, a general classification of models has been published de Lemos and Pedras (2001). Recently, the problem of treating interfaces between a porous medium and a clear region, considering a diffusion-jump condition for laminar Silva and de Lemos (2003)a and turbulence fields de Lemos (2005), de Lemos and Silva (2006), Silva and de Lemos (2003)b. Following this same concept, de Lemos and Rocamora (2002) have developed a macroscopic turbulent energy equation for a homogeneous, rigid and saturated porous medium, considering local thermal equilibrium between the fluid and solid matrix. Furthermore, Saito and de Lemos (2006) proposed a new correlation for obtaining the interfacial heat transfer coefficient for turbulent flow in a packed bed, which bed is modeled as an infinite staggered array of square rods.

The main purpose of this work is the comparison of longitudinal Nusselt number results herein with those from Alazmi and Vafai (2000). Here, fully developed forced convection in a porous channel, bounded by parallel plates and based on a two-energy equation model, is analyzed. Moreover, the contribution herein consists in obtaining the effects of thermal dispersion and local thermal non-equilibrium on Nu.

2. Macroscopic Transport Equations

Macroscopic transport equations for turbulent flow in a porous medium are obtained through the simultaneous application of time and volume average operators over a generic fluid property φ (see Pedras and de Lemos (2000) Pedras and de Lemos (2001a) Pedras and de Lemos (2001c) Pedras and de Lemos (2001b) Pedras and de Lemos (2003)). Therefore, when the average operators are simultaneously applied or volume integration is performed over a Representative Elementary Volume (REV) (Gray and Lee (1977) and Slattery (1967)), resulting in,

Continuity: $\nabla \cdot \overline{\mathbf{u}}_D = 0$. (1)

where, $\overline{\mathbf{u}}_D = \phi \langle \overline{\mathbf{u}} \rangle^i$ and $\langle \overline{\mathbf{u}} \rangle^i$ identifies the intrinsic (liquid) average of the time-averaged velocity vector $\overline{\mathbf{u}}$.

Momentum:

$$\rho \left[\frac{\partial \overline{\mathbf{u}}_D}{\partial t} + \nabla \cdot \left(\frac{\overline{\mathbf{u}}_D \overline{\mathbf{u}}_D}{\phi} \right) \right] = -\nabla (\phi \langle \overline{p} \rangle^i) + \mu \nabla^2 \overline{\mathbf{u}}_D - \nabla \cdot (\rho \phi \langle \overline{\mathbf{u}' \mathbf{u}'} \rangle^i) - \left[\frac{\mu \phi}{K} \overline{\mathbf{u}}_D + \frac{c_F \phi \rho |\overline{\mathbf{u}}_D| \, \overline{\mathbf{u}}_D}{\sqrt{K}} \right], \tag{2}$$

where the last two terms in Eq. (2), represent the Darcy and Forchheimer contributions by Forchheimer (1901). The symbol K is the porous medium permeability, c_F is the form drag or Forchheimer coefficient, $\langle \overline{p} \rangle^i$ is the intrinsic average pressure of the fluid, and ϕ is the porosity of the porous medium.

The macroscopic Reynolds stress $-\rho \phi \langle \overline{\mathbf{u}' \mathbf{u}'} \rangle^i$ appearing in Eq. (2) is given as,

$$-\rho \phi \langle \overline{\mathbf{u}' \mathbf{u}'} \rangle^i = \mu_{t_\phi} 2 \langle \overline{\mathbf{D}} \rangle^v - \frac{2}{3} \phi \rho \langle k \rangle^i \mathbf{I}, \tag{3}$$

where,

$$\langle \overline{\mathbf{D}} \rangle^v = \frac{1}{2} \left[\nabla (\phi \langle \overline{\mathbf{u}} \rangle^i) + [\nabla (\phi \langle \overline{\mathbf{u}} \rangle^i)]^T \right], \tag{4}$$

is the macroscopic deformation tensor, $\langle k \rangle^i = \langle \overline{\mathbf{u}' \cdot \mathbf{u}'} \rangle^i / 2$ is the intrinsic turbulent kinetic energy, and μ_{t_ϕ}, is the turbulent viscosity, which is modeled in de Lemos and Pedras (2001) similarly to the case of clear flow, in the form,

$$\mu_{t_\phi} = \rho c_\mu \frac{\langle k \rangle^{i\,2}}{\langle \varepsilon \rangle^i}, \tag{5}$$

The intrinsic turbulent kinetic energy per unit mass and its dissipation rate are governed by the following equations,

$$\rho \left[\frac{\partial}{\partial t} \left(\phi \langle k \rangle^i \right) + \nabla \cdot \left(\overline{\mathbf{u}}_D \langle k \rangle^i \right) \right] = \nabla \cdot \left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_k} \right) \nabla \left(\phi \langle k \rangle^i \right) \right] - \rho \langle \overline{\mathbf{u}' \mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D + c_k \rho \frac{\phi \langle k \rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}} - \rho \phi \langle \varepsilon \rangle^i. \tag{6}$$

$$\rho \left[\frac{\partial}{\partial t} \left(\phi \langle \varepsilon \rangle^i \right) + \nabla \cdot \left(\overline{\mathbf{u}}_D \langle \varepsilon \rangle^i \right) \right] = \nabla \cdot \left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_\varepsilon} \right) \nabla \left(\phi \langle \varepsilon \rangle^i \right) \right] + c_1 \left(- \rho \langle \overline{\mathbf{u}' \mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D \right) \frac{\langle \varepsilon \rangle^i}{\langle k \rangle^i} + c_2 c_k \rho \frac{\phi \langle \varepsilon \rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}} - c_2 \rho \phi \frac{\langle \varepsilon \rangle^{i\,2}}{\langle k \rangle^i}. \tag{7}$$

where, c_k, c_1, c_2 and c_μ are nondimensional constants.

Similarly, macroscopic energy equations are obtained for both fluid and solid phases by applying time and volume average operators (see Rocamora and de Lemos (2000), de Lemos and Rocamora (2002)) . As in the flow case, volume integration is performed over a Representative Elementary Volume (REV) resulting in,

$$\left(\rho c_p \right)_f \left[\frac{\partial \phi \langle \overline{T_f} \rangle^i}{\partial t} + \nabla \cdot \left\{ \phi \left(\langle \overline{\mathbf{u}} \rangle^i \langle \overline{T_f} \rangle^i + \langle {}^i \overline{\mathbf{u}}^i \overline{T_f} \rangle^i + \langle \overline{\mathbf{u}' T_f'} \rangle^i \right) \right\} \right] = \nabla \cdot \left[k_f \nabla \left(\phi \langle \overline{T_f} \rangle^i \right) \right] + \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \, k_f \overline{T_f} \, dA \right] + \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_f \nabla \overline{T_f} \, dA, \tag{8}$$

$$\left(\rho c_p \right)_s \left\{ \frac{\partial (1-\phi) \langle \overline{T_s} \rangle^i}{\partial t} \right\} = \nabla \cdot \left\{ k_s \nabla \left[(1-\phi) \langle \overline{T_s} \rangle^i \right] \right\} - \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \, k_s \overline{T_s} \, dA \right] - \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_s \nabla \overline{T_s} \, dA, \tag{9}$$

where $\langle\overline{T_s}\rangle^i$ and $\langle\overline{T_f}\rangle^i$ denote the intrinsically averaged temperature of solid and fluid phases, respectively, A_i is the interfacial area within the REV and $\mathbf{n}_i$ is the unit vector normal to the fluid-solid interface, pointing from the fluid towards the solid phase. Eqs. (8) and (9) are the macroscopic energy equations for the fluid and the porous matrix (solid), respectively.

Further, using the *double decomposition* concept, Rocamora and de Lemos (2000) have shown that the fourth term on the left hand side of Eq. (8) can be expressed as:

$$\langle\overline{\mathbf{u}'T_f'}\rangle^i = \langle\overline{(\langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}')(\langle T_f'\rangle^i + {}^iT')}\rangle^i = \overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i} + \langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i . \tag{10}$$

Therefore, in view of Eq. (10), Eq. (8) can be rewritten as:

$$\begin{aligned}&\left(\rho c_p\right)_f\left[\frac{\partial\phi\langle\overline{T_f}\rangle^i}{\partial t} + \nabla\cdot\left\{\phi\left(\langle\overline{\mathbf{u}}\rangle^i\langle\overline{T_f}\rangle^i + \langle{}^i\overline{\mathbf{u}}\,{}^i\overline{T_f}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i} + \langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i\right)\right\}\right] = \\ &\nabla\cdot\left[k_f\nabla\left(\phi\langle\overline{T_f}\rangle^i\right)\right] + \nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\,k_f\,\overline{T_f}\,dA\right] + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_f\nabla\overline{T_f}\,dA\end{aligned} . \tag{11}$$

Two-energy equation model for conduction and convection in porous media considering a heat transfer coefficient between the fluid and solid phases are given by, respectively,

$$\begin{aligned}&\left(\rho c_p\right)_f\left[\frac{\partial\phi\langle\overline{T_f}\rangle^i}{\partial t} + \nabla\cdot\left\{\phi\left(\langle\overline{\mathbf{u}}\rangle^i\langle\overline{T_f}\rangle^i + \langle{}^i\overline{\mathbf{u}}\,{}^i\overline{T_f}\rangle^i + \overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i} + \langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i\right)\right\}\right] = \\ &\nabla\cdot\left[k_f\nabla\left(\phi\langle\overline{T_f}\rangle^i\right) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\,k_f\,\overline{T_f}\,dA\right] + h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right)\end{aligned} , \tag{12}$$

$$\left(\rho c_p\right)_s\left\{\frac{\partial\left(1-\phi\right)\langle\overline{T_s}\rangle^i}{\partial t}\right\} = \nabla\cdot\left\{k_s\nabla\left[\left(1-\phi\right)\langle\overline{T_s}\rangle^i\right] - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\,k_s\,\overline{T_s}\,dA\right\} - h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right). \tag{13}$$

where, h_i and a_i are the interfacial convective heat transfer coefficient and surface area per unit volume, respectively. Underscored terms in Eq. (11) the following physical significance can be attributed as: turbulent heat flux due to the fluctuating components of macroscopic velocity and temperature; turbulent thermal dispersion in a porous medium due to both time fluctuations and spatial deviations of both microscopic velocity and temperature; and thermal dispersion associated with deviations of microscopic time average velocity and temperature. Note that this term is also present when analyzing laminar convective heat transfer in porous media.

2.1 Macroscopic Two-Energy Equation Modeling

In order to apply Eq. (12) and Eq. (13) to obtain the temperature fields for turbulent flow in porous media, unknown terms in Eq. (12) have to be modeled in some way as a function of the intrinsically averaged temperatures of solid and fluid phases, $\langle\overline{T_s}\rangle^i$ and $\langle\overline{T_f}\rangle^i$. To accomplish this, a gradient-type diffusion model is used for all unknown terms, i.e., thermal dispersion due to spatial deviations, turbulent heat flux due to temporal fluctuations and turbulent thermal dispersion due to both temporal fluctuations and spatial deviations. Also needed is a model for local conduction.

Using these gradient type diffusion models, we can write:

Thermal dispersion: $$-\left(\rho c_p\right)_f\left(\phi\,\langle{}^i\overline{\mathbf{u}}\,{}^i\overline{T_f}\rangle^i\right) = \mathbf{K}_{disp}\cdot\nabla\langle\overline{T_f}\rangle^i . \tag{14}$$

Turbulent heat flux: $$-\left(\rho c_p\right)_f\left(\phi\,\overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i}\right) = \mathbf{K}_t\cdot\nabla\langle\overline{T_f}\rangle^i . \tag{15}$$

Turbulent thermal dispersion: $$-\left(\rho c_p\right)_f\left(\phi\,\langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i\right) = \mathbf{K}_{disp,t}\cdot\nabla\langle\overline{T_f}\rangle^i . \tag{16}$$

Local conduction: $$\begin{cases}\dfrac{1}{\Delta V}\displaystyle\int_{A_i}\mathbf{n}_i\,k_f\,\overline{T_f}\,dA = \mathbf{K}_{f,s}\cdot\nabla\langle\overline{T_s}\rangle^i \\ \dfrac{1}{\Delta V}\displaystyle\int_{A_i}\mathbf{n}_i\,k_s\,\overline{T_s}\,dA = \mathbf{K}_{s,f}\cdot\nabla\langle\overline{T_f}\rangle^i\end{cases} . \tag{17}$$

For the above expressions, Eq. (12) and Eq. (13) can be written as:

$$\left\{\left(\rho c_p\right)_f \phi\right\}\frac{\partial\langle\overline{T}\rangle^i}{\partial t}+\left(\rho c_p\right)_f \nabla\cdot\left(\mathbf{u}_D\langle\overline{T_f}\rangle^i\right)=\nabla\cdot\left\{\mathbf{K}_{eff,f}\cdot\nabla\langle\overline{T}_f\rangle^i\right\}+h_i a_i\left(\langle\overline{T_s}\rangle^i-\langle\overline{T_f}\rangle^i\right), \tag{18}$$

$$\left\{(1-\phi)\left(\rho c_p\right)_s\right\}\frac{\partial\langle\overline{T}\rangle^i}{\partial t}=\nabla\cdot\left\{\mathbf{K}_{eff,s}\cdot\nabla\langle\overline{T_s}\rangle^i\right\}+h_i a_i\left(\langle\overline{T_f}\rangle^i-\langle\overline{T_s}\rangle^i\right), \tag{19}$$

where $\mathbf{K}_{eff,f}$ and $\mathbf{K}_{eff,s}$ are the effective conductivity tensors for the fluid and solid phases, respectively, given by:

$$\mathbf{K}_{eff,f}=[\phi k_f]\,\mathbf{I}+\mathbf{K}_{f,s}+\mathbf{K}_{disp}+\mathbf{K}_{disp,t}+\mathbf{K}_t, \tag{20}$$

$$\mathbf{K}_{eff,s}=[(1-\phi)k_s]\,\mathbf{I}+\mathbf{K}_{s,f}, \tag{21}$$

and $\mathbf{I}$ is the unit tensor. Details of interfacial convective heat transfer coefficient are presented in the next section.

Further, in order to be able to apply Eq. (18), it is necessary to determine the components of the conductivity tensor in Eq. (20), i.e., $\mathbf{K}_{f,s}$, $\mathbf{K}_{disp}$, $\mathbf{K}_t$, and $\mathbf{K}_{disp,t}$. Following Kuwahara and Nakayama (1996) and Quintard et al (1997), this can be accomplished for the thermal dispersion and local conduction tensors, $\mathbf{K}_{disp}$ and $\mathbf{K}_{f,s}$, by making use of a unit cell subjected to periodic boundary conditions for the flow together with an imposed linear temperature gradient on the porous medium. The dispersion and conduction tensors are then obtained directly from the distributed results within the unit cell by making use of Eqs. (14) and (17). In addition, the following correlations by Nakayama and Kuwahara (1999) for the thermal dispersion tensor, which are valid for $Pe_D \geq 10$, can be used:

$$\frac{\left(K_{disp}\right)_{xx}}{k_f}=2.1\frac{Pe_D}{(1-\phi)^{0.1}}, \text{ for longitudinal dispersion} \tag{22}$$

$$\frac{\left(K_{disp}\right)_{yy}}{k_f}=0.052(1-\phi)^{0.5}Pe_D, \text{ for transverse dispersion} \tag{23}$$

where $\left(K_{disp}\right)_{xx}$ and $\left(K_{disp}\right)_{yy}$ are the transverse and longitudinal components of $\mathbf{K}_{disp}$, respectively.

The turbulent heat flux and turbulent thermal dispersion components of $\mathbf{K}_{eff,f}$, namely $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, respectively, can not be determined from a distributed calculation. Instead, they are modeled through the classical eddy diffusivity concept, similar to Nakayama and Kuwahara (1999). It should be noticed that these two terms arise only if the flow is turbulent within the void space, whereas the thermal dispersion term, $\mathbf{K}_{disp}$, exists for both laminar and turbulent flow regimes. Starting out from the time-averaged local energy equation coupled with the standard modeling of the turbulent heat flux through the eddy diffusivity concept, ν_t, one can write:

$$-(\rho c_p)_f\,\overline{\mathbf{u}'T_f'}=(\rho c_p)_f\,\frac{\nu_t}{\sigma_t}\nabla\overline{T}_f \tag{24}$$

where σ_t is the turbulent Prandtl number, which is taken here as a constant.

Applying the volume average to the resulting equation, one obtains the macroscopic version of the turbulent heat flux, given by:

$$-(\rho c_p)_f\,\phi\langle\overline{\mathbf{u}'T_f'}\rangle^i=(\rho c_p)_f\,\frac{\nu_{t_\phi}}{\sigma_t}\nabla\langle\overline{T}_f\rangle^i \tag{25}$$

where we have adopted the symbol ν_{t_ϕ} to express the macroscopic eddy diffusivity. Now, adding up equations Eqns. (15) and (16) in light of Eqn. (10) one has

$$\begin{aligned}-(\rho c_p)_f\left(\phi\left[\langle\overline{\mathbf{u}'}\rangle^i\langle T_f'\rangle^i+\langle\overline{{}^i\mathbf{u}'\,{}^iT_f'}\rangle^i\right]\right)=\\ -(\rho c_p)_f\,\langle\overline{\mathbf{u}'T_f'}\rangle^i=(\mathbf{K}_t+\mathbf{K}_{disp,t})\cdot\nabla\langle\overline{T}_f\rangle^i\end{aligned}. \tag{26}$$

According to (25) and (26), the overall turbulent heat transport is the sum of the turbulent heat flux and the turbulent thermal dispersion mechanisms, as proposed by Rocamora and de Lemos (2000). As suggested by Eqn. (25), both mechanisms are modeled together, giving for $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$ the expression:

$$\mathbf{K}_t + \mathbf{K}_{disp,t} = \phi(\rho c_p)_f \frac{\nu_{t_\phi}}{\sigma_t}\mathbf{I} \tag{27}$$

3. Interfacial Heat Transfer Coefficient

In Eqs. (12) and (13) the heat transferred between the two phases was modeled by means of a film coefficient h_i such that:

$$h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right) = \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_f\nabla\overline{T_f}\,dA = \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_s\nabla\overline{T_s}\,dA\,. \tag{28}$$

where $a_i = A_i/\Delta V$ is the interfacial area per unit volume. In porous media, the high values of a_i make them attractive for transferring thermal energy via conduction through the solid followed by convection to a fluid stream.

For obtaining macroscopic transport properties, highly permeable media can be modeled as an infinite array of rods, which, in turn, can be analogous to flow across a bundle of tubes. Accordingly, two tube arrangements are generally found in the literature, i.e., the tube rows in a bundle are either aligned or inline, with rod centers forming a *square* or a rectangle, or else, they are staggered, where a *triangular* shape is obtained when connecting the tube centerlines.

For the staggered configuration of tube banks, Zhukauskas (1972) has proposed a correlation of the form:

$$\frac{h_i D}{k_f} = 0.022 Re_D^{0.84} Pr^{0.36}\text{, for } 2x10^5 < Re_D < 2x10^6\,. \tag{29}$$

where the values 0.022 and 0.84 are for tubes in cross flow.

Wakao et al. (1979) obtained a heuristic correlation for closely packed bed of particle diameter D and compared their results with experimental data. This correlation for the interfacial heat transfer coefficient is given by,

$$\frac{h_i D}{k_f} = 2 + 1.1 Re_D^{0.6} Pr^{1/3}\,. \tag{30}$$

For numerically determining h_i, Kuwahara et al. (2001) modeled a porous medium by considering an infinite number of solid square rods of size D, arranged in a regular triangular pattern. They numerically solved the governing equations in the void region, exploiting to advantage the fact that for an infinite and geometrically ordered medium a repetitive cell can be identified. Periodic boundary conditions were then applied for obtaining the temperature distribution under fully developed flow conditions. A numerical correlation for the interfacial convective heat transfer coefficient was proposed by Kuwahara et al. (2001) for laminar flow as:

$$\frac{h_i D}{k_f} = \left(1 + \frac{4(1-\phi)}{\phi}\right) + \frac{1}{2}(1-\phi)^{1/2}\,\mathrm{Re}_D\,\mathrm{Pr}^{1/3}\text{, valid for } 0.2 < \phi < 0.9\,, \tag{31}$$

Eq. (31) is based on porosity dependency and is valid for packed beds of particle diameter D. Saito and de Lemos (2005) obtained the interfacial heat transfer coefficient for laminar flows through an infinite square rod; this same physical model will be used here for obtaining the interfacial heat transfer coefficient h_i for turbulent flows. Following this same methodology, which the porous medium is considered as an infinite number of solid square rods, Saito and de Lemos (2006) proposed a new correlation for obtaining the interfacial heat transfer coefficient for turbulent flow as,

$$\frac{h_i D}{k_f} = 0.08\left(\frac{Re_D}{\phi}\right)^{0.8} Pr^{1/3}\text{; for } 1.0x10^4 < \frac{Re_D}{\phi} < 2.0x10^7\text{, valid for } 0.2 < \phi < 0.9\,, \tag{32}$$

4. Numerical Method and Boundary Conditions

The problem under investigation is a flow through a channel completely filled with a porous medium, as shown in Fig. 1. Boundary conditions and periodic constraints for laminar flows in porous media are similar to the clear channel flow.

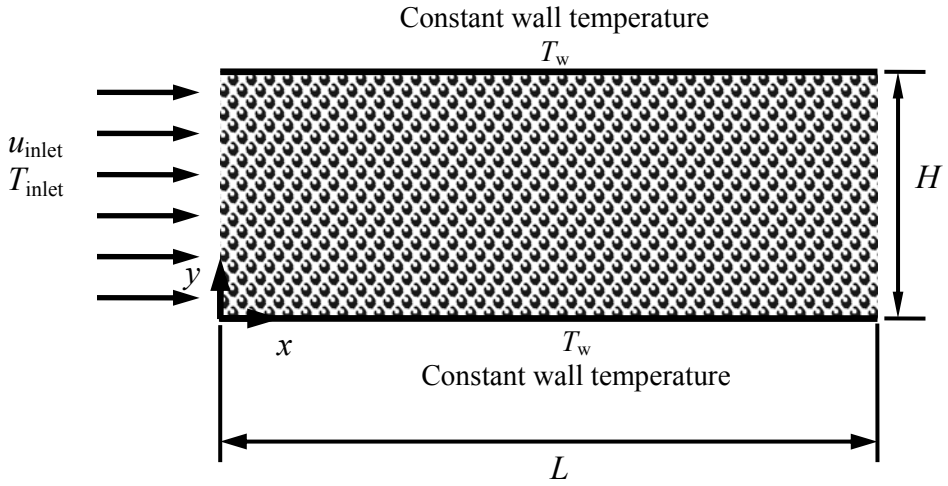


Figure 1. Geometry under investigation and coordinate system.

The numerical method utilized to discretize the flow and energy equations in the unit cell is the Finite Control Volume approach. The SIMPLE method of Patankar (1980). Convergence was monitored in terms of the normalized residue for each variable. The maximum residue allowed for convergence check was set to 10^{-9}, being the variables normalized by appropriate reference values.

Thus, boundary conditions are given by:

On the solid walls:

$$\langle \overline{\mathbf{u}} \rangle^i = 0 \tag{33}$$

$$\langle \overline{T_s} \rangle^i = \langle \overline{T_f} \rangle^i = T_w \ , \tag{34}$$

On the entrance:

$$\langle \overline{T_s} \rangle^i = \langle \overline{T_f} \rangle^i = T_{inlet} \ , \tag{35}$$

5. Results and Discussion

To verify the reliability of the numerical code, numerical results for a fixed porosity is compared with those obtained from Alazmi and Vafai (2000), as shown in Figures 1-4 and it is observed that a reasonable agreement is found. Effects of porosity, Reynolds number, particle diameter and solid-to-fluid thermal conductivity are shown in Figures 1-4.

The longitudinal Nusselt number is calculated for both the fluid and solid phases and is defined as,

Fluid phase Nusselt number,

$$Nu_f = -\frac{2H}{T_w - T_{mf}} \left(\frac{\partial \langle T_f \rangle^i}{\partial y} \right), \tag{36}$$

Solid phase Nusselt number,

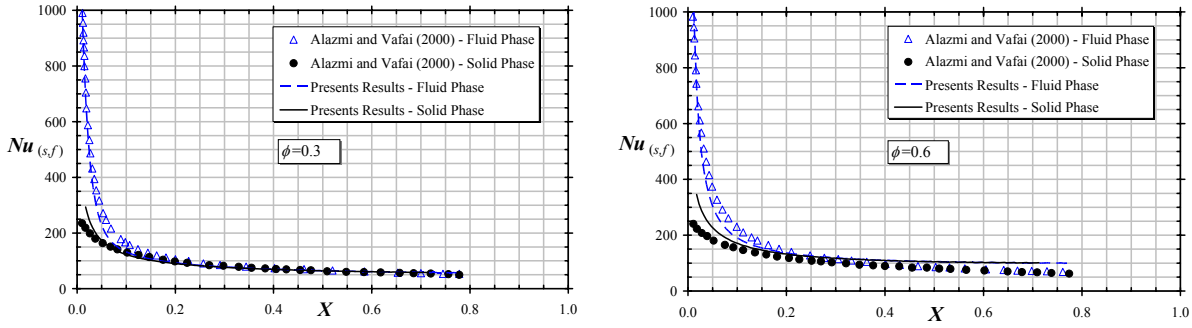
$$Nu_s = -\frac{2H}{T_w - T_{ms}} \left(\frac{\partial \langle T_s \rangle^i}{\partial y} \right), \tag{37}$$

where T_{mf} and T_{ms} are the average temperature of the fluid and the solid phase, respectively, and are defined as follows:

$$T_{mf} = \frac{\int u T_f dy}{u_B H}, \tag{38}$$

$$T_{ms} = \frac{\int T_s dy}{H}, \tag{39}$$

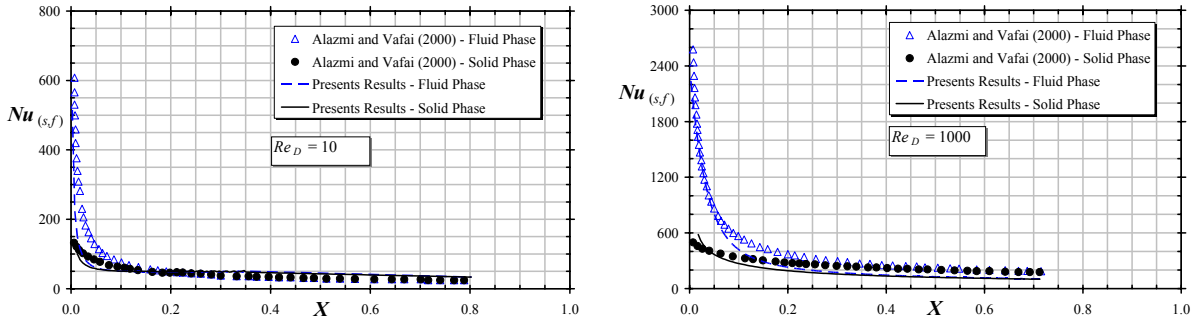
Figure 1 shows the effect of porosity on the Nusselt number distributions along X, where $X = x/L$. It is observed from Fig. 1 that, the higher the porosity, the smaller the differences between present results and those given by Alazmi and Vafai (2000). At low porosities both results are closer to each other. In fact, it is expected that the porosity influences on the temperature distribution since a porosity term appears in a_i.



a) b)

Figure 1. Effect of porosity variations on Nusselt number. $Da = 10^{-4}$; $Re_D = 100$; $D = 0.008$; $k_s / k_f = 25$; a) $\phi = 0.3$, b) $\phi = 0.6$.

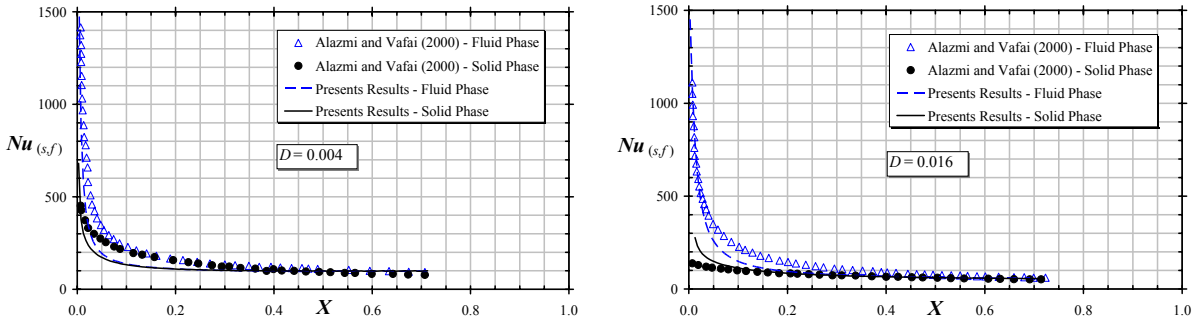
The effect of the Reynolds number is shown in Fig. 2. The Reynolds number is found to have a substantial effect on the variances among the results. For lower Reynolds numbers, the Nusselt number distributions become closer to each other (see Fig. 2). As expected, for lower Reynolds number, the fully developed equilibrium temperature is achieved faster than for higher Reynolds number.



a) b)

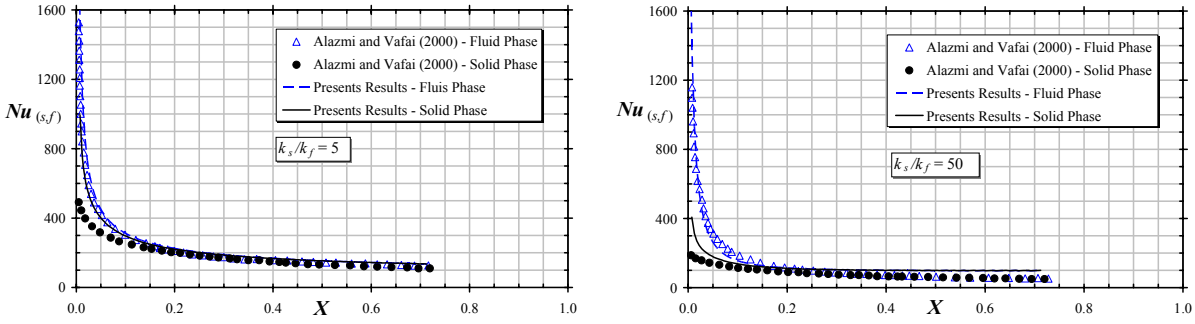
Figure 2. Effect of Reynolds number variations on Nusselt number. $Da = 10^{-4}$; $\phi = 0.6$; $D = 0.008$; $k_s / k_f = 25$; a) $Re_D = 10$, b) $Re_D = 1000$.

The particle diameter appears in the expressions for h_i within all of the correlations. Therefore, the effect of the particle diameter is expected to be critical.3 Smaller particle diameters encourage the local thermal equilibrium, minimizing the variances among the fluid and solid phase. On the other hand, larger particle diameters enhance the local thermal non-equilibrium, increasing the variances among the fluid and solid phase, as seen in Fig. 3. It should be noted that the thermal conductivities of the solid and fluid have a significant effect on the variances among the phases. As seen in Fig. 4, a lower conductivity ratio enhances the local thermal equilibrium and reduces the variances between solid and fluid phase.



a) b)

Figure 3. Effect of D variations on Nusselt number. $Da = 10^{-4}$; $Re_D = 100$; $\phi = 0.6$; $k_s / k_f = 25$; a) $D = 0.004$, b) $D = 0.016$.



a) b)

Figure 4. Effect of solid-to-fluid thermal conductivity ratio variations on Nusselt number. $Da = 10^{-4}$; $Re_D = 100$; $\phi = 0.6$; $D = 0.008$; a) $k_s / k_f = 5$, b) $k_s / k_f = 50$.

Considering all of the above, the one-energy equation model and two-energy equation model can give the same results under some particular circumstances. However, the two-energy equation model is more complete than the one-energy equation model and can deal properly with a higher number of cases considering heat transfer in porous media.

The effect of thermal dispersion on the temperature profile in a channel completely filled with porous material for $X = 0.1$, $Da = 10^{-4}$; $Re_D = 100$; $\phi = 0.6$; $D = 0.008$; $k_s / k_f = 25$ is shown in Fig. 5. Where $Y = y/H$ and $\theta_{(s,f)} = \dfrac{T_w - \langle \overline{T_{s,f}} \rangle^i}{T_w - T_{inlet_{(s,f)}}}$.

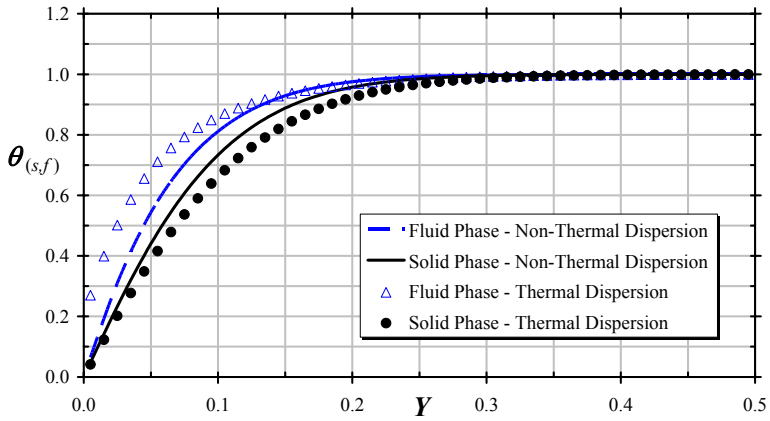


Figure 5. Effect of thermal dispersion on temperature profile.

It is clearly seen from Fig. 5 that the differences between the solid and fluid phase temperature profiles are enhanced when the thermal dispersion is incorporated. An explanation for such behavior is that the thermal dispersion is an additional mechanism of heat transfer and, for that reason, it is expected that the overall results concerning temperature profiles are also enhanced. Furthermore, thermal dispersion is associated with deviations of microscopic time mean velocity and temperature.

6. Concluding remarks

Fully developed forced convection in a porous channel bounded by parallel plates based on a two-energy equation model is analyzed. Details are presented for determining the temperature profile and Nusselt number for laminar flows in a porous medium. Good agreement was obtained when comparing the results herein with those obtained from Alazmi and Vafai (2000). Moreover, comparisons between temperature profiles are shown when the thermal dispersion is included. Further work will be carried out in order to simulate fully turbulent flow and heat transfer in a porous medium with the macroscopic two-energy equation model.

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