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Numerical Investigation of the Inviscid Linear Stability of an Asymmetric Wake

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Abstract
The presente work investigates the effect of the shear in the vortex shedding of an asymmetric 2D wake profile under temporal development. The aim of such investigation is to predict the relationship between amplification rate of the sinuous mode and the asymmetry parameter of the flow. The effect is relevant for the understening of vortex shedding suppression on bluff bodies. The study was performed in a 2D flow without the cylinder. The velocity profile was obtained by a combination between a gaussian wake profile and a hyperbolic tangent mixing layer, which represents the asymmetry of the velocity profile. This work was done by Direct Numerical Simulation (DNS) of the characteristic formulation of the compressible Navier-Stokes equations in non-conservative form. The code used a compact scheme with high-order of accuracy to compute the spatial derivatives. Filtering scheme was also employed to keep the simulation without aliasing problems. The y-direction adopted free-slip non-reflecting boundary conditions. The results obtained showed that as the asymmetry parameter increases the temporal amplification rate of the sinuous mode decreases.

Keywords: *asymmetric wake, flow instability, temporal development*

1. Introduction

Bluff body wake flows have been a subject of interest to engineers and scientists for many years as they have direct engineering application. The alternate shedding of vortices may cause, among other things, structural vibrations and acoustic noise. One aspect of interest is to establish a relationship between the vortex shedding from bluff bodies and stability theory (Monkewitz, 1988).

The study of bluff body wake flows presents difficulties. Bluff body wakes are complex, as they involve the interaction of various shear layers in the same problem, namely, a boundary layer, a separating mixing layer and a wake, (Williamson, 1996). Several authors investigated the stability of a two-dimensional wake behind a cylinder. The cylinder geometry has less complexity in relation to other bluff body and is representative of the phenomenon. Besides this aspect, cylindrical structures are found in several engineering applications such as risers, transmission cabled and landing gears.

(Williamson, 1996) revised works of several authors that describe the vortex dynamics in the cylinder wake. He discusses the various instabilities and flow regimes. The definition of flow regimes is based on measurements of velocity fluctuation. He found a laminar vortex shedding regime, a transition regime and an "irregular" regime.

One of the first works to investigate the linear instability of the wake is (Betchov and Criminale, 1966). They studied the inviscid spatial instability of the jet and the wake using a hyperbolic-secant-squared to model the velocity profiles. They related the concept of absolute instability of (Briggs, 1964) to the vortex shedding of bodies, via a branch-point singularity for the symmetric modes. Simply stated, the type of instability is determined by the location in the complex angular frequency plane of a certain branch-point singularity in the complex dispersion relation $\omega = \omega(\alpha)$, namely $\frac{\partial\omega}{\partial\alpha} = 0$, where $\omega = \omega_r + i\omega_i$, $\alpha = \alpha_r + i\alpha_i$ and i is an imaginary unit. The occurrence of singularities, was completely unexpected. Details on the location the singularity can be seen found in (Mattingly and Criminale, 1972). They suggested a re-evaluation of the hypotheses of parallel flow and incompressible inviscid flow. The description of the initial development of a vortex street contrasts with the description given by (Sato and Kuriti, 1961) which used a spatial model. (Hultgren and Aggarwal, 1987) also used linear parallel-flow stability theory to investigate the effect of viscosity on the local absolute instability of a family of symmetric gaussian wake profiles. (Monkewitz, 1988) used a symmetric wake with sinh-profile to establish a relationship between vortex shedding from bluff bodies and stability theory. However this analysis is not linked to a critical Reynolds number.

The study of the instability of symmetric wake profile is well justified, since the hypothesis of parallel flow can be described for Orr-Sommerfeld equation, also considered the normal mode assumption (Betchov and Criminale Jr., 1966), (Hultgren et al., 1987) and (Monkewitz, 1988). There are methods of solution of the Orr-Sommerfeld equation which determine the location in the complex angular frequency plane of a certain branch-point singularity in the complex dispersion relation. The asymmetric wake is found in practical applications of engineering. For example, in an aircraft wing, high-lift devices operating in high angle of attack provide asymmetric wake that, due to hydrodynamics instability, can exert an influence in the aerodynamic performance and generated noise.

In the effects of shear in the vortex shedding, two effects can be considered: the presence of a wall and the combination with mxing layer. In the case of a wall, two simultaneous effects exist: the shear and the presence of a wall. In general, the works report a significant influence of the shear in the hydrodynamics instability of the wakes. However, these is no consensus in the literature whether the shear promotes the instability or stability of the wake (Vitola, 2006).

There is also no consensus on the mechanism that provokes the suppression of the vortex shedding. The influence of shear in wake instability has other applications beyond the current aeronautical applications. For example, the shear flow on bodies, in particular, cylinders, occurs in risers (conducting duct of oil), transmission cables, etc.

This work, in development, investigates the effect of the shear in the vortex shedding of cylinder, namely, in far wake considering only asymmetric wake profile.

The current work presents the tests performed to investigate the flow instability, it is possible of satisfy the parallel flow hypotheses by canceling the viscous diffusion of the base flow in the y-direction of a compressible two-dimensional wake at low Mach number and infinite Reynolds number. The present work used the code originally developed by (Germanos and Medeiros, 2005) to investigate the flow instability of a compressible mixing layer.

2. Methodology

The methodology used to investigate the effect of the shear in the vortex shedding of an asymmetric wake was the direct numerical simulation (DNS) of the flow without the cylinder, only considering the base flow given by mean velocity flow profile (2). This methodology differs from the other that investigated wakes.

The mechanism of flow instability depends mainly on the base flow. The body in itself exerts a secondary influence by promoting new instabilities at high Reynolds number or modifying the first instability.

Numerical simulations that considered the presence of the cylinder have some inconveniences. They need a long computational domain and the presence of the body is a huge problem.

In spite of new techniques such as the virtual boundarys condition and the increasing capacity of the computers. The computatinal time are very long. A large time of the simulation used for establishing the base flow. Moreover during this time errors can grow and trigger the flow instability. On the other hand, considering only the velocity profile and it is possible of reproduce the parallel hypotheses by canceling the viscous diffusion at the base flow in the y-direction it possible study the hydrodynamics instability in smaller computational. As our objective is to understand the essence of the phenomenon this approach is interesting.

This work was done by Direct Numerical Simulation (DNS) using the characteristic formulation proposed by (Sesterhenn, , 2001) of the compressible Navier-Stokes equations in the non-conservative form.

The code, originally developed by (Germanos and Medeiros, 2005) to investigate the flow instability of a compressible mixing layer, it use a compact finite difference scheme with 6th-order of accuracy to compute the spatial derivatives and a 4th order Runge-Kutta scheme for the time integration.

A filtering scheme was also employed to keep the simulation without aliasing problems an the y-boundaries adopted free-slip non-reflecting boundary conditions.

3. Statement of the Problem

We consider the two-dimensional flow of a compressible viscous fluid and assume that the unperturbed flow has a velocity $U(y)$ in the x-direction:

$$U(y) = (1-\kappa)U_w(y) + \kappa U_s \tag{1}$$

where $U_w(y)$ is symmetric wake profile, U_s is tan-hyperbolic profile of the mixing layer and κ is parameter that control the amount of asymmetry. Figure 1 shows the variation of the profile with κ.

The symmetric wake profile $U(y)$ is defined as:

$$U_w(y) = 1 - \Lambda + 2\Lambda \left[1 + \sinh^{2N}(y \sinh^{-1}(1))\right]^{-1} \tag{2}$$

where $\Lambda = (U^*_{w_c} - U^*_{w_{max}})/(U^*_{w_c} + U^*_{w_{max}})$, $U^*_{w_{max}}$ is the maximum velocity of the profile (2), $U^*_{w_c} = U^*_w(y=0)$ is the centerline velocity and the $*$ superscript denotes dimensional quantity. U_w represents a parallel mean flow in the streamwise direction, namely x, and y is the cross-stream coordinate where $y = 0$ is the wake centerline. The reference length scale is the local half-width b^* of the wake defined as $U^*_w(b^*) = \bar{U}^*_w$, where $\bar{U}^*_w = (U^*_{w_c} + U^*_{w_{max}})/2$ is the average mean velocity by which the velocities are made non-dimensional.

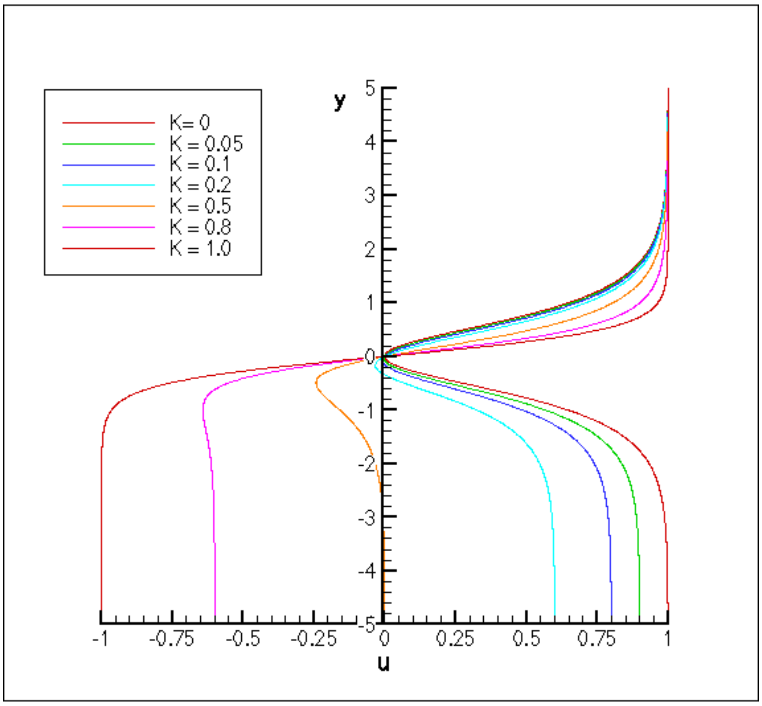

Figure 1. Kappa variation on profile (1), $\kappa = 0, 0.05, 0.1, 0.2, 0.5, 0.8$ and 1.0 to $\Lambda = -1$ and $N = 2$

The parameters are the velocity ratio Λ and N sthe "shape parameter". Figure (2) shows the variation of the profile eith of the velocity ratio and shape parameters, but throughout the paper, we will consider a fixed profile with the parameters $\Lambda = -1$ and $N = 2$.

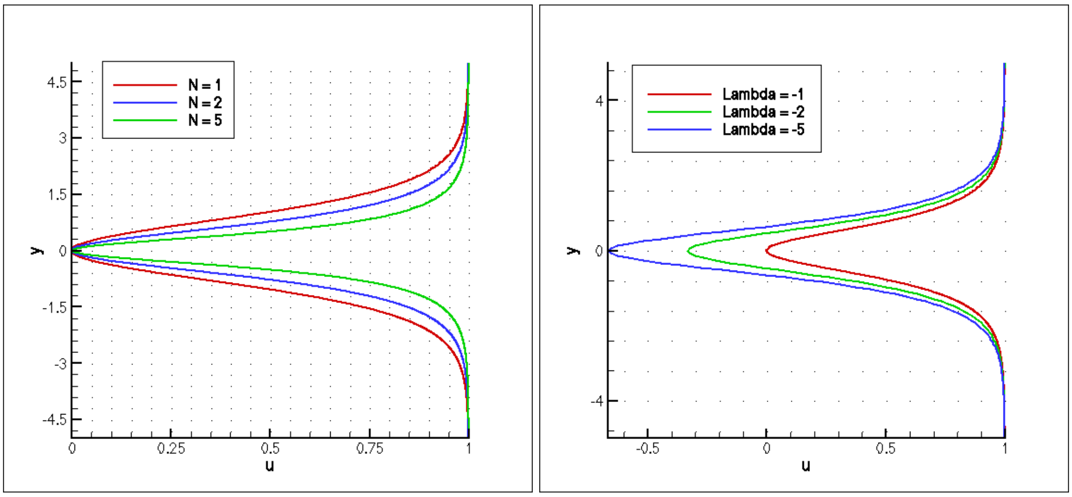

Figure 2. a) Shape parameter variation $N = 1, 2$ and 5 with a fixed velocity ratio parameter $\Lambda = -1$ and b)Velocity ratio parameter variation $\Lambda = -1, -2$ and -5 with a fixed shape parameter $N = 2$

The tan-hyperbolic profile of the mixing layer is given by:

$$U_s(y) = U_{s_{max}} \tanh\left(\frac{2y}{\delta_w}\right), \tag{3}$$

where δ_w is the mixing layer vorticity thickness. The problem parameter are:

$$\delta_w = b^* = 1, \quad c_{ref} = 340.21, \quad Ma = 0.1, \quad 0 < \kappa < 1, \quad Re = \frac{\rho_{max} U_{max} \delta_w}{\mu} \to \infty$$

where c_{ref} is the reference speed of sound, Ma is the Mach number and Re is the Reynolds number.

A domain $0 < x < 2\pi\alpha$ and $-16 < y < 16$ was adopted, where α is the wave number, with a grid of 64 x 128 points along the x and y direction, respectively. Grid stretching in the y-direction was used and compact filter in the x and y directions.

After herein the variables will be shown in the non-dimensional form, where the reference scale used was b^*, the reference temperature was $T_{ref} = \frac{c_{ref}^2}{\gamma R}$, where $\gamma = \frac{c_p}{c_v}$ and the reference pressure was $\rho_{ref} U_{max}^2$. We superpose a small velocity disturbance and the composite fluid motion is described by

$$\begin{aligned} u(x,y,t) &= U(y) + \tilde{u}(x,y,t) \\ v(x,y,t) &= \tilde{v}(x,y,t) \\ p(x,y,t) &= P(x) + \tilde{p}(x,y,t), \end{aligned} \tag{4}$$

where the tilde denotes the disturbance quantities. The linear equations of motion allow solutions of the form

$$\begin{aligned} \tilde{u}(x,y,t) &= \mathbf{u}(\mathbf{y})[e^{i(\alpha x - \omega t)} + cc] \\ \tilde{v}(x,y,t) &= \mathbf{v}(\mathbf{y})[e^{i(\alpha x - \omega t)} + cc] \\ \tilde{p}(x,y,t) &= \mathbf{p}(\mathbf{y})[e^{i(\alpha x - \omega t)} + cc], \end{aligned} \tag{5}$$

where cc is the complex conjugate, α is a real wave number, ω is a complex frequency and $\mathbf{u}$, $\mathbf{v}$, $\mathbf{p}$, complex amplitude function. Thus, the stability analysis provides a dispersion relation that determines the sign of ω_i, namely, imaginary part of ω, for a give wavenumber α. For $\omega_i < 0$ the wave grows exponentially in time and is termed a temporally unstable disturbance.

Linear stability properties of a flow can be studied by investigating its impulse response, that is, finding the time evolution of an impulse disturbance that ideally contains modes of all frequency. However, our interest is to investigate the linear regime, thus, the Linear Stability Theory consists of investigating the amplification rate of the first unstable mode. in the currente study, the initial disturbance was given by

$$\tilde{v}(x,y) = A_0 \cos(\alpha x) e^{-2y\sigma} \tag{6}$$

$$\tilde{u}(x,y) = 2\sigma y A_0/\alpha \sin(\alpha x) e^{-2y\sigma}, \tag{7}$$

for the sinuous instability of the profiles (2), where $A_0 = 10^{-9}$ and $\sigma = 1$.

4. Results

In this section is to show the results.

The temporal amplification rates were obtained from exponential fit of the linear region of the curves showed in figure 3. This figure shows the evaluation of the power spectrum for $\alpha = 0.8$ and $\kappa = 0.05, 0.1, 0.15$ and 0.2, i.e., the temporal development of $\left(A_{Re_1}^2(t) + A_{Im_1}^2(t)\right)^{0.5}$, where $A_{Re_1}(t)$ is the coefficient of the real part and $A_{Im_1}(t)$ is the coefficient of the imaginary part, both from the fundamental mode obtained by Fast Fourier Transform of the v signal along the x-direction at $y = 0$ for a given t time-instant.

Figure 4 shows the temporal amplification rates at infinite Reynolds number and Mach number 0.1 for various wave numbers. For the symmetric cases, the theoretical curve was taken from (Delfs et al., 1997). They solved the Orr-Sommerfeld equation for large Reynolds number.s

The temporal amplification rates for $\alpha = 0.7$ for a symmetric wake profile was interpolated since the rate obtained from the simulation did not represent the rate of growth of the first mode, condition necessary to represent the linear stability theory.

The time development of the vorticity field obtained from this simulation considering a symmetric wake profile is shown in the frame sequence of figure 5. The symmetric case represents the sinuous mode known as mode of Von Kàrman. For this wave number ($\alpha_r = 0.4$), the amplification rate is $\omega_i = 0.1291$ for $\kappa = 0.05$. and ($\alpha_r = 0.4$) the amplification rate is $\omega_i = 0.09111$. Figure 7 shows the temporal development of the vorticity field of the asymmetric wake for $\alpha_r = 0.4$ and $\kappa = 0.2$.

The figures 6 and 7 show the time development of the vorticity field obtained from this simulation considering an asymmetric wake profile,$\kappa = 0.1$ and 0.2, respectively. They show results inducing the linear region to the nonlinear region. The frames corresponds to non-dimensional time 180, 211.5, 225, 236.5, 245.25, 252, 262.35 and 268.8.

Tests of computational mesh convergency had been carried through for meshes 16x32, 32x64, 64x128, 128x256 and 256x512, x and y axis, respectively. The results presented in this section did not depend on the chosen mesh.

5. Summary and Conclusions

The results obtained shows that as the asymmetry parameter increases the temporal amplification rate of the sinuous mode decreases.

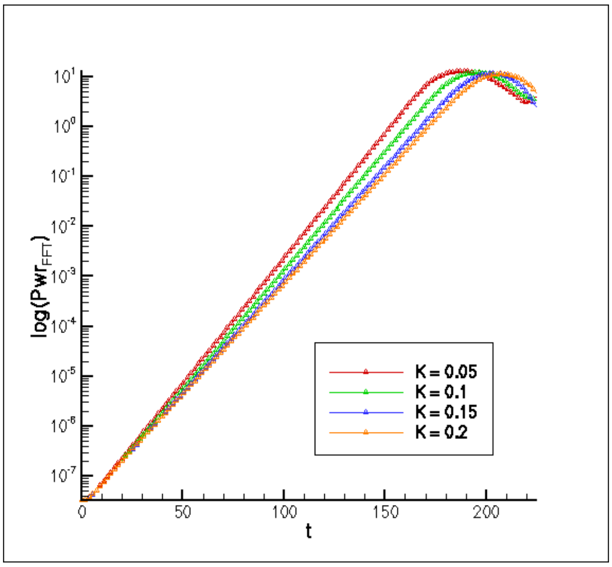

Figure 3. The evaluation of the power spectrum for $\alpha = 0.8$ and $\kappa = 0.05, 0.1, 0.15$ and 0.2 in a log scale

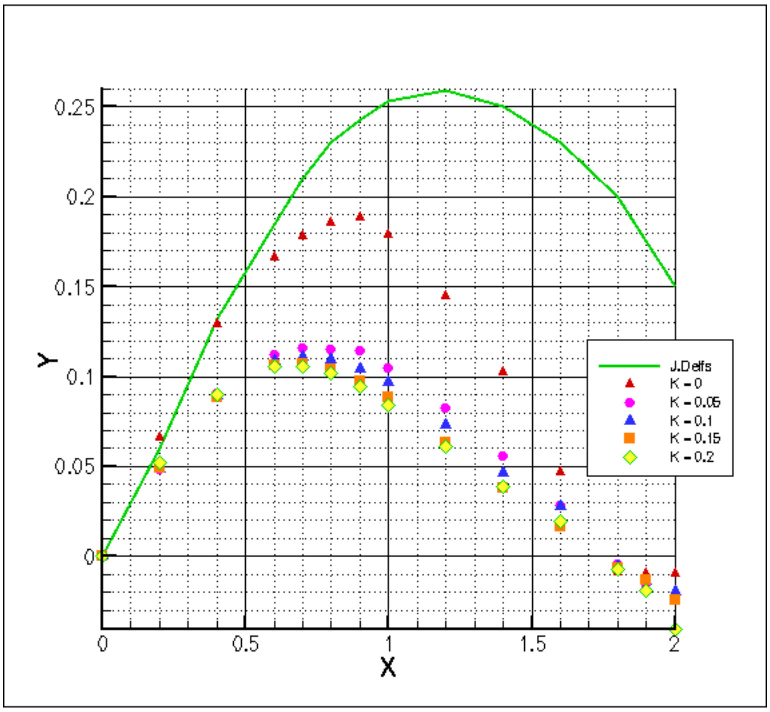

Figure 4. Temporal amplification rates for base flow (1) to $Re = \infty$, $Ma = 0.1$s

The simulations presented for $\kappa = 0, 0.05, 0.1, 0.15$ and 0.2, indicate that asymmetry reduces the temporal amplification rates.

Figure 4 shows that the neutral mode is between $\alpha = 1,7$ and $\alpha = 1.8$ for $\kappa = 0.05, 0.1, 0.15$ and 0.2 dor a larger them $\alpha = 1.8$ the flow remains stable, that is, $\omega_i < 0$.

Our interest was to study the asymmetry effect for $Ma = 0.1$, however, in future works, the effect of Mach can be investigated together to the effect of the asymmetry.

The methodology adopted presented good qualitative results and the code was efficient in computing the physics of the problem described.

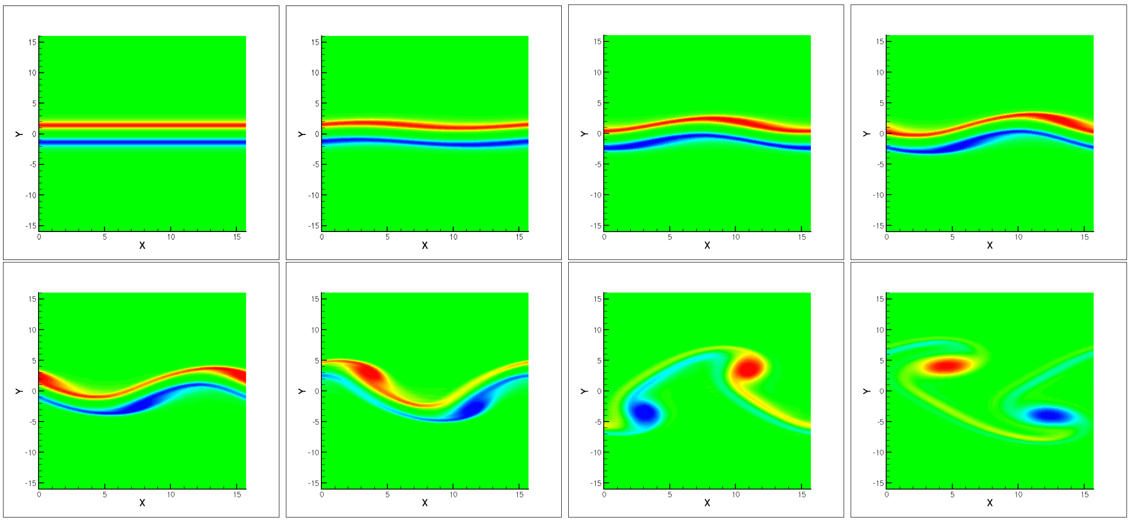

Figure 5. Time development of the vorticity field of the symmetric wake to $Re = \infty$, $\alpha_r = 0.4$ and $Ma = 0.1$

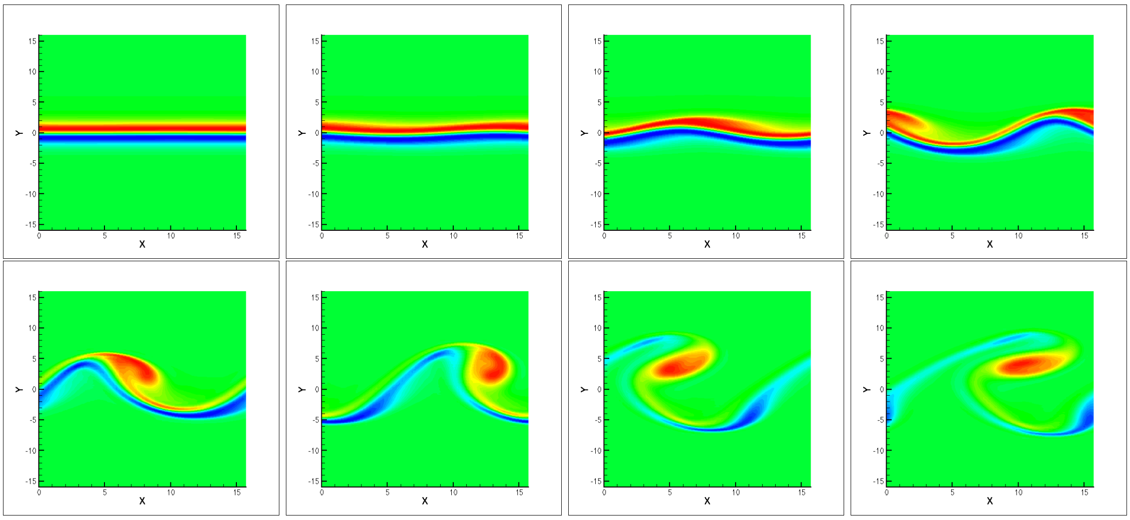

Figure 6. Time development of the vorticity field of the asymmetric wake to $Re = \infty$, $\alpha_r = 0.4$, $Ma = 0.1$ with $\kappa = 0.1$

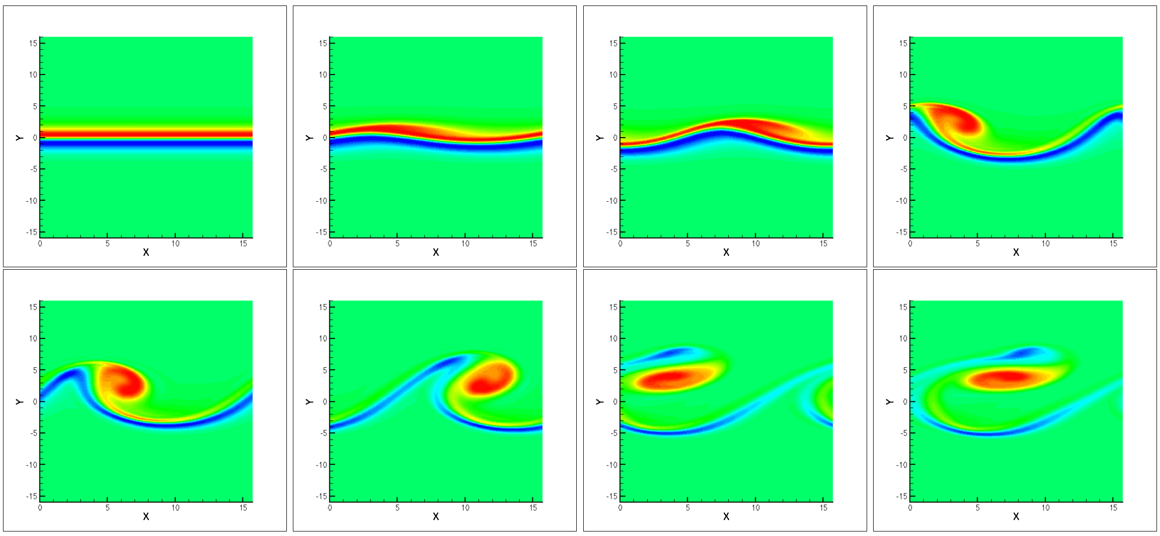

Figure 7. Time development of the vorticity field of the asymmetric wake to $Re = \infty$, $\alpha_r = 0.4$, $Ma = 0.1$ with $\kappa = 0.2$

6. ACKNOWLEDGMENTS

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