

## DOUBLE-DIFFUSIVE LAMINAR NATURAL CONVECTION IN TWO - DIMENSIONAL POROUS CAVITIES

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**Abstract.** *This paper presents an analysis and simulation of macroscopic heat and mass transport for laminar flow in a porous cavity. Two driving mechanisms are considered to contribute to the overall momentum transport, namely temperature driven and concentration driven mass fluxes. Aiding and opposing double-diffusive natural convection mechanisms are investigated. By “aiding” and “opposing” flows one means the cases where both temperature and concentration gradients are either in the same direction or of different sign, respectively. Variation on the overall Sherwood number due to changes on Raleigh number, Lewis number and  $N$ , where  $N$  is the ratio of solute and thermal Grashof numbers, are presented. Results indicate that for adding cases mass transfer is enhanced whereas for opposing temperature and concentration gradients the transfer of mass across the cavity is damped.*

**Keywords:** *Porous Media, Natural Convection, Double-Diffusive*

### 1. INTRODUCTION

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc (Bennacer et al, 2001), (Goyeau et al, 1996), (Mamou et al, 1995), (Mamou et al, 1998) and (Mohamad and Bennacer, 2002). In some specific applications, the fluid mixture may become turbulent and difficulties arise in the proper mathematical modeling of the transport processes under both temperature and concentration gradients.

Modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat (Hsu and Cheng, 1990) or mass transfer (Bear, 1972), (Bear and Bachmat, 1967), (Whitaker, 1966), (Whitaker, 1967). If time fluctuations of the flow properties are considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: a) application of time-average operator followed by volume-averaging (Masuoka and Takatsu, 1996), (Kuwahara and Nakayama, 1998), (Kuwahara et al, 1996), or b) use of volume-averaging before time-averaging is applied (Lee and Howell, 1987). This work intends to present a set of macroscopic mass transport equations derived under the recently established double decomposition concept (Pedras and de Lemos, 2000), (Pedras and de Lemos, 2001), (Pedras and de Lemos, 2001b), (Pedras and de Lemos, 2001c), through which the connection between the two paths a) and b) above is unveiled. That methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature (Rocamora and de Lemos, 2000). Buoyant flows (de Lemos and Braga, 2003) and mass transfer (de Lemos and Mesquita, 2003) have also been investigated. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published (de Lemos and Pedras, 2001). Here, double-diffusive laminar natural convection flow in porous media is considered.

### 2. LOCAL INSTANTANEOUS TRANSPORT EQUATION

The steady-state microscopic instantaneous transport equations for an incompressible binary fluid mixture with constant properties are given by:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \quad (2)$$

$$(\rho c_p) \nabla \cdot (\mathbf{u}T) = \nabla \cdot (\lambda \nabla T) \quad (3)$$

$$\rho \nabla \cdot (\mathbf{u} m_\ell + \mathbf{J}_\ell) = \rho R_\ell \quad (4)$$

where  $\mathbf{u}$  is the mass-averaged velocity of the mixture,  $\mathbf{u} = \sum_\ell m_\ell \mathbf{u}_\ell$ ,  $\mathbf{u}_\ell$  is the velocity of species  $\ell$ ,  $m_\ell$  is the mass fraction of component  $\ell$ , defined as  $m_\ell = \rho_\ell / \rho$ ,  $\rho_\ell$  is the mass density of species  $\ell$  (mass of  $\ell$  over total mixture volume),  $\rho$  is the bulk density of the mixture ( $\rho = \sum_\ell \rho_\ell$ ),  $p$  is the pressure,  $\mu$  is the fluid mixture viscosity,  $\mathbf{g}$  is the gravity acceleration vector,  $c_p$  is the specific heat,  $T$  is the temperature and  $\lambda$  is the fluid thermal conductivity. The generation rate of species  $\ell$  per unit of mixture mass is given in Eq. (4) by  $R_\ell$ .

An alternative way of writing the mass transport equation is using the volumetric molar concentration  $C_\ell$  (mol of  $\ell$  over total mixture volume), the molar weight  $M_\ell$  (g/mol of  $\ell$ ) and the molar generation/destruction rate  $R_\ell^*$  (mol of  $\ell$  /total mixture volume), giving:

$$M_\ell \nabla \cdot (\mathbf{u} C_\ell + \mathbf{J}_\ell) = M_\ell R_\ell^* \quad (5)$$

Further, the mass diffusion flux  $\mathbf{J}_\ell$  (mass of  $\ell$  per unit area per unit time) in Eq. (4) or Eq. (5) is due to the velocity slip of species  $\ell$ ,

$$\mathbf{J} = \rho_\ell (\mathbf{u}_\ell - \mathbf{u}) = -\rho_\ell D_\ell \nabla m_\ell = -M_\ell D_\ell \nabla C_\ell \quad (6)$$

where  $D_\ell$  is the diffusion coefficient of species  $\ell$  into the mixture. The second equality in Eq. (6) is known as Fick's Law, which is a constitutive equation strictly valid for binary mixtures under the absence of any additional driving mechanisms for mass transfer (Hsu and Cheng, 1990). Therefore, no Soret or Dufour effects are here considered.

Rearranging Eq. (5) for an inert species, dividing it by  $M_\ell$  and dropping the index  $\ell$  for a simple binary mixture, one has,

$$\nabla \cdot (\mathbf{u} C) = \nabla \cdot (D \nabla C) \quad (7)$$

If one considers that the density in the last term of Eq. (2) varies with temperature and concentration, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density  $\rho_T$ ,

$$\rho_T \cong \rho [1 - \beta(T - T_{ref}) - \beta_c(C - C_{ref})] \quad (8)$$

where the subscript ref indicates a reference value and  $\beta$  and  $\beta_c$  are the thermal and salute expansion coefficients, respectively, defined by,

$$\beta = -\frac{1}{\rho} \left. \frac{\partial \rho}{\partial T} \right|_{p,C}, \quad \beta_c = -\frac{1}{\rho} \left. \frac{\partial \rho}{\partial C} \right|_{p,T} \quad (9)$$

Equation (8) is an approximation of Eq. (9) and shows how density varies with temperature and concentration in the body force term of the momentum equation.

Further, substituting Eq. (8) into Eq. (9), one has,

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} [1 - \beta(T - T_{ref}) - \beta_c(C - C_{ref})] \quad (10)$$

Thus, the momentum equation becomes,

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -(\nabla p)^* + \mu \nabla^2 \mathbf{u} - \rho \mathbf{g} [(\beta(T - T_{ref}) + \beta_c(C - C_{ref}))] \quad (11)$$

where  $(\nabla p)^* = \nabla p - \rho \mathbf{g}$  is a modified pressure gradient.

As mentioned, there are, in principle, two ways that one can follow in order to treat turbulent flow in porous media. The first method applies a time average operator to the governing Eq. (4) before the volume average procedure is conducted. In the second approach, the order of application of the two average operators is reversed. Both techniques aim at derivation of a suitable macroscopic turbulent mass transport equation.

Volume averaging in a porous medium, described in detail in references (Slattery, 1967), (Whitaker, 1969), (Gray and Lee, 1997), makes use of the concept of a Representative Elementary Volume (REV), over which local equations are integrated. After integration, detailed information within the volume is lost and, instead, overall properties referring to a REV are considered. In a similar manner, statistical analysis of turbulent flow leads to time mean properties. Transport equations for statistical values are considered in lieu of instantaneous information on the flow.

Before undertaking the task of developing macroscopic equations, it is convenient to recall the definition of time average and volume average.

## VOLUME AVERAGE OPERATOR

The volume average of  $\phi$  taken over a Representative Elementary Volume in a porous medium can be written as:

$$\langle \phi \rangle^v = \frac{1}{\Delta V} \int_{\Delta V} \phi \, dV \quad (12)$$

The value  $\langle \phi \rangle^v$  is defined for any point  $x$  surrounded by a Representative Elementary Volume, of size  $\Delta V$ . This average is related to the intrinsic average for the fluid phase as:

$$\langle \phi_f \rangle^v = \phi \langle \phi_f \rangle^i \quad (13)$$

where  $\phi = \Delta V_f / \Delta V$  is the medium porosity and  $\Delta V_f$  is the volume occupied by the fluid in a REV. Furthermore, one can write:

$$\phi = \langle \phi \rangle^i + {}^i\phi \quad (14)$$

with  $\langle {}^i\phi \rangle^i = 0$ . In Eq. (14),  ${}^i\phi$  is the spatial deviation of  $\phi$  with respect to the intrinsic average  $\langle \phi \rangle^i$ .

Further, the local volume average theorem can be expressed as (Slattery, 1967), (Whitaker, 1969), (Gray and Lee, 1997):

$$\begin{aligned} \langle \nabla \phi \rangle^v &= \nabla (\phi \langle \phi \rangle^i) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \phi \, dS \\ \langle \nabla \cdot \boldsymbol{\phi} \rangle^v &= \nabla \cdot (\phi \langle \boldsymbol{\phi} \rangle^i) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot \boldsymbol{\phi} \, dS \\ \left\langle \frac{\partial \phi}{\partial t} \right\rangle^v &= \frac{\partial}{\partial t} (\phi \langle \phi \rangle^i) - \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot (\mathbf{u}_i \phi) \, dS \end{aligned} \quad (15)$$

where  $\mathbf{n}$  is the unit vector normal to the fluid-solid interface and  $A_i$  is the fluid-solid interface area within the REV. It is important to emphasize that  $A_i$  should not be confused with the surface area surrounding volume  $\Delta V$ .

## MACROSCOPIC EQUATIONS FOR BUOYANCY FREE FLOWS

For non-buoyant flows, macroscopic equations considering turbulence have been already derived in detail for momentum (Pedras and de Lemos, 2001), heat (de Lemos and Braga, 2003) and mass (de Lemos and Mesquita, 2003) transfer and for this reason their derivation need not to be repeated here. They read:

### Momentum transport

$$\rho \nabla \cdot \left( \frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) = -\nabla(\phi \langle p \rangle^i) + \mu \nabla^2 \mathbf{u}_D - \left[ \frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_f \phi \rho \langle \mathbf{u}_D \mathbf{u}_D \rangle}{\sqrt{K}} \right] \quad (16)$$

### Heat transport

$$(\rho c_p)_f \nabla \cdot (\mathbf{u}_D \langle T \rangle^i) = \nabla \cdot \{ \mathbf{K}_{eff} \cdot \nabla \langle T \rangle^i \} \quad (17)$$

$$\mathbf{K}_{eff} = [\phi \lambda_f + (1 - \phi) \lambda_s] \mathbf{I} + \mathbf{K}_{tor} + \mathbf{K}_{disp} \quad (18)$$

The subscripts f and s refer to fluid and solid phases, respectively, and coefficients K's come from the modeling of the following mechanisms:

- Tortuosity:  $\left[ \frac{1}{\Delta V} \int_{A_i} \mathbf{n} (\lambda_f T_f - \lambda_s T_s) dS \right] = \mathbf{K}_{tor} \cdot \nabla \langle T \rangle^i \quad (19)$

- Thermal dispersion:  $-(\rho c_p)_f \phi \langle \mathbf{u}^i T_f \rangle^i = \mathbf{K}_{disp} \cdot \nabla \langle T \rangle^i \quad (20)$

### Mass transport

$$\nabla \cdot (\mathbf{u}_D \langle C \rangle^i) = \nabla \cdot \mathbf{D}_{eff} \cdot \nabla (\phi \langle C \rangle^i) \quad (21)$$

$$\mathbf{D}_{eff} = \mathbf{D}_{disp} + \mathbf{D}_{diff} \quad (22)$$

$$\mathbf{D}_{diff} = \langle D \rangle^i \mathbf{I} = \frac{1}{\rho} \frac{\mu_\phi}{Sc} \mathbf{I} \quad (23)$$

The coefficients  $\mathbf{D}_{disp}$  in Eq. (21) appear due to the nonlinearity of the convection term. They come from the modeling of the following mechanisms:

- **Mass dispersion:**  $-\langle \mathbf{u}^i C \rangle^i = \mathbf{D}_{disp} \cdot \nabla \langle C \rangle^i \quad (24)$

## MACROSCOPIC DOUBLE-DIFFUSION EFFECTS

Focusing now attention to buoyancy effects only, application of the volume average procedure to the last term of Eq. (11) leads to,

$$\langle \rho \mathbf{g} [ \beta(T - T_{ref}) + \beta_C (C - C_{ref}) ] \rangle^v = \frac{\Delta V_f}{\Delta V} \frac{1}{\Delta V_f} \int_{\Delta V_f} \rho \mathbf{g} [ \beta(T - T_{ref}) + \beta_C (C - C_{ref}) ] dV \quad (25)$$

Expanding the left hand side of Eq. (25) in light of Eq. (14), the buoyancy term becomes,

$$\begin{aligned} & \langle \rho \mathbf{g} [ \beta (T - T_{ref}) + \beta_c (C - C_{ref}) ] \rangle^v \\ & = \rho \mathbf{g} \phi [ \beta \langle (T)^i - T_{ref} \rangle + \beta_{c\phi} \langle (C)^i - C_{ref} \rangle ] + \underbrace{\rho \mathbf{g} \beta \phi \langle (T)^i \rangle}_{=0} + \underbrace{\rho \mathbf{g} \beta_c \phi \langle (C)^i \rangle}_{=0} \end{aligned} \quad (26)$$

where the third and fourth terms on the r.h.s. are null since  $\langle (T)^i \rangle = 0$ . Here, coefficients  $\beta_\phi$  and  $\beta_{c\phi}$  are the macroscopic thermal and solute expansion coefficients, respectively. Assuming that gravity is constant over the REV, expressions for them based on Eq. (26) are given as,

$$\beta_\phi = \frac{\langle \rho \beta (T - T_{ref}) \rangle^v}{\rho \phi \langle (T)^i - T_{ref} \rangle}; \quad \beta_{c\phi} = \frac{\langle \rho \beta_c (C - C_{ref}) \rangle^v}{\rho \phi \langle (C)^i - C_{ref} \rangle} \quad (27)$$

Including Eq. (26) into Eq. (16), the macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as,

$$\begin{aligned} \rho \nabla \cdot \left( \frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) &= -\nabla(\phi \langle p \rangle^i) + \mu \nabla^2 \mathbf{u}_D \\ &+ \rho \mathbf{g} \phi [ \beta_\phi \langle (T)^i - T_{ref} \rangle + \beta_{c\phi} \langle (C)^i - C_{ref} \rangle ] \\ &- \left[ \frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_r \phi \rho \langle \mathbf{u}_D \rangle^i}{\sqrt{K}} \right] \end{aligned} \quad (28)$$

Coefficients  $\beta_\phi$  and  $\beta_{c\phi}$  are used to compose the Grashof numbers associated with the thermal and solute drives, in the form,

$$Gr_\phi = \frac{g \beta_\phi \Delta T H^3}{\nu^2}, \quad Gr_{c\phi} = \frac{g \beta_{c\phi} \Delta C H^3}{\nu^2} \quad (29)$$

where  $\Delta T = T_1 - T_2$  and  $\Delta C = C_1 - C_2$  are the maximum temperature and concentration variation across the cavity, respectively. One should note that for opposing thermal and concentrations drives, such maximum differences are of opposing signs.

The ratio of Grashof numbers define the buoyancy ratio  $N$  in the form

$$N = \frac{Gr_{c\phi}}{Gr_\phi} = \frac{\beta_{c\phi} \Delta C}{\beta_\phi \Delta T} \quad (30)$$

giving for Eq.(28),

$$\begin{aligned} \rho \nabla \cdot \left( \frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) &= -\nabla(\phi \langle p \rangle^i) + \mu \nabla^2 \mathbf{u}_D \\ &+ \rho \mathbf{g} \phi \beta_\phi \left\{ \langle (T)^i - T_{ref} \rangle + N \frac{\Delta C}{\Delta T} \langle (C)^i - C_{ref} \rangle \right\} \\ &- \left[ \frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_r \phi \rho \langle \mathbf{u}_D \rangle^i}{\sqrt{K}} \right] \end{aligned} \quad (31)$$

## AVERAGE NUSSLELT AND SHERWOOD NUMBER

The local Nusselt number on the hot wall for the square cavity at  $x = 0$  is defined as,

$$Nu = hL/k_{eff} \therefore Nu = \left( \frac{\partial \langle T \rangle^i}{\partial x} \right)_{x=0} \frac{L}{T_H - T_C} \quad (32)$$

where  $T_H$  and  $T_C$  refers to the temperature limits imposed at the cavity lateral walls, also named here as  $T_1$  and  $T_2$ , respectively. The average Nusselt number is then given by,

$$\overline{Nu} = \frac{1}{H} \int_0^H Nu dy \quad (33)$$

Likewise, the local Sherwood number on the wall where the highest concentration prevails, or say, at  $x = 0$  for aiding drives and  $x=L$  for opposing cases, can be defined as,

$$Sh = h_c L/D \therefore Sh = \left( \frac{\partial \langle C \rangle^i}{\partial x} \right)_{x_{wall}} \frac{L}{C_1 - C_2} \quad (34)$$

where the subscripts refer to the maximum and minimum concentration values, respectively, and  $h_c$  is a film coefficient for mass transfer. The average Sherwood number is then given by,

$$\overline{Sh} = \frac{1}{H} \int_0^H Sh dy \quad (35)$$

### 2.1. Results and Discussion

The presents work refers to the study of natural convective flows in a porous cavity of height  $H$ , width  $L$  and aspect ratio  $A = H/L = 1$  (see Fig.1), saturated by a binary fluid (such as aqueous solutions, as in numerous experimental studies related to solidification processes). The binary fluid is assumed to be Newtonian and to satisfy the Boussinesq approximation; the flow is incompressible, laminar, 2D and in the steady state. Horizontal temperature and concentration differences are specified between the vertical walls.

For aiding cases, one has  $T_1$  and  $C_1$  on the left wall and  $T_2$ ,  $C_2$  at the right surface. For opposing runs, the values of  $T_1$  and  $C_2$  are assume to prevail on the left wall and  $T_2$  and  $C_1 > C_2$  on the right. For all cases, null mass and heat fluxes are imposed at the horizontal walls.

As seen above, in this work equations were derived for laminar double-diffusive natural convection in porous media. Derivations were carried out under the light of the double decomposition concept (Pedras and de Lemos, 2000), (Pedras and de Lemos, 2001). Extra terms appearing in the equations needed to be modeled in terms of  $\mathbf{u}_D$ ,  $\langle T \rangle$  and  $\langle C \rangle$ .

Table 1 shows average Nusselt e Sherwood numbers compared with those by (Trevisan and Bejan, 1985) and (Goyeau et al, 1996). The table indicates a good agreement with similar results presented in the open literature.

Comparison between aiding and opposing drive cases is shown next, where figures show comparisons in the flow structure for the conditions  $\Delta T = -1, \Delta C = -1$ ,  $\Delta T = -1, N = 0$  and  $\Delta T = -1, \Delta C = +1$ . Thermal and concentration patterns are presented for  $N = 0.1$ ,  $N = 1$  and  $N = 10$ . Strictly speaking, values of  $N$ , as defined by Eq. (30), can be of positive or negative sign depending on the signs of  $\beta_{C_\phi}$  and  $\Delta C$ , in addition to the sign of the product  $\beta_\phi \Delta T$ . Here, only chemical species with  $\beta_{C_\phi} > 0$  are considered, but the overall concentration difference across the cavity can be

either negative (adding case  $\Delta T = \Delta C = -1$ ,) or positive (opposing case,  $\Delta T = -\Delta C = -1$ ). Therefore, opposing cases as here defined would lead to  $N < 0$  and, for consistency, all figures below are show in terms of the absolute value  $|N|$ .

Further, in the figures here presented, positive stream function values are considered in the counter-clockwise direction.

Streamlines are presented in Fig. 4, where the promotion of convective currents are detected when increasing the value of  $N$  for adding cases. The opposing trend, namely the reduction of the recirculation intensity as  $|N|$  increases for opposing cases ( $|N|=0.1$ , Fig. 4b) is clearly seen in the figure. For  $|N|=10$  in Fig. 4c, the drive on the counter-clockwise direction due to weight of the mixture overcomes the clockwise motion caused by higher temperatures closer to the left of the cavity. In this situation, the sign of fluid rotation inside the cavity is changed, causing substantial impact on the corresponding  $T$  and  $C$  fields. Figure 2 indicates such changes for the temperature field where, for  $|N|=10$  in Fig. 2c, the bottom of the cavity is maintained at higher temperatures, in spite of having an overall temperature gradient promoting the stratification of the flow. Finally, Fig. 3 shows corresponding patterns for the concentration field. Stratification of the concentration field, driven by the strong drive of the mass buoyancy term in the momentum equation, prevails for either higher values of  $|N|$ .

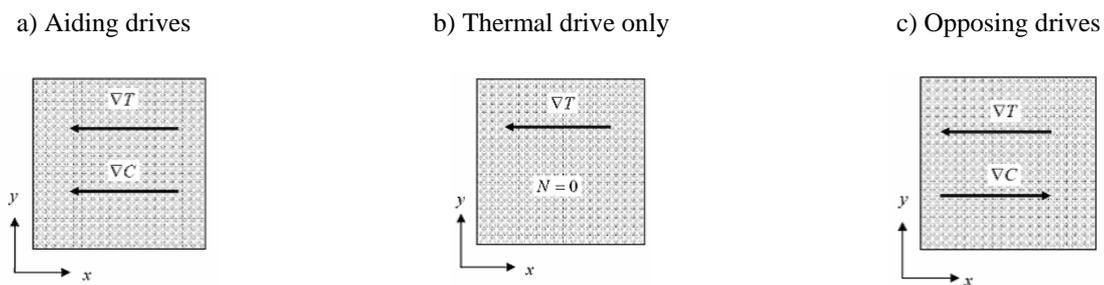


Figure 1- Geometry and imposed conditions: a) Aiding drives:  $\Delta T = -1, \Delta C = -1$ ; b) Thermal drive only:  $\Delta T = -1, N = 0$ ; c) Opposing drives:  $\Delta T = -1, \Delta C = +1$ .

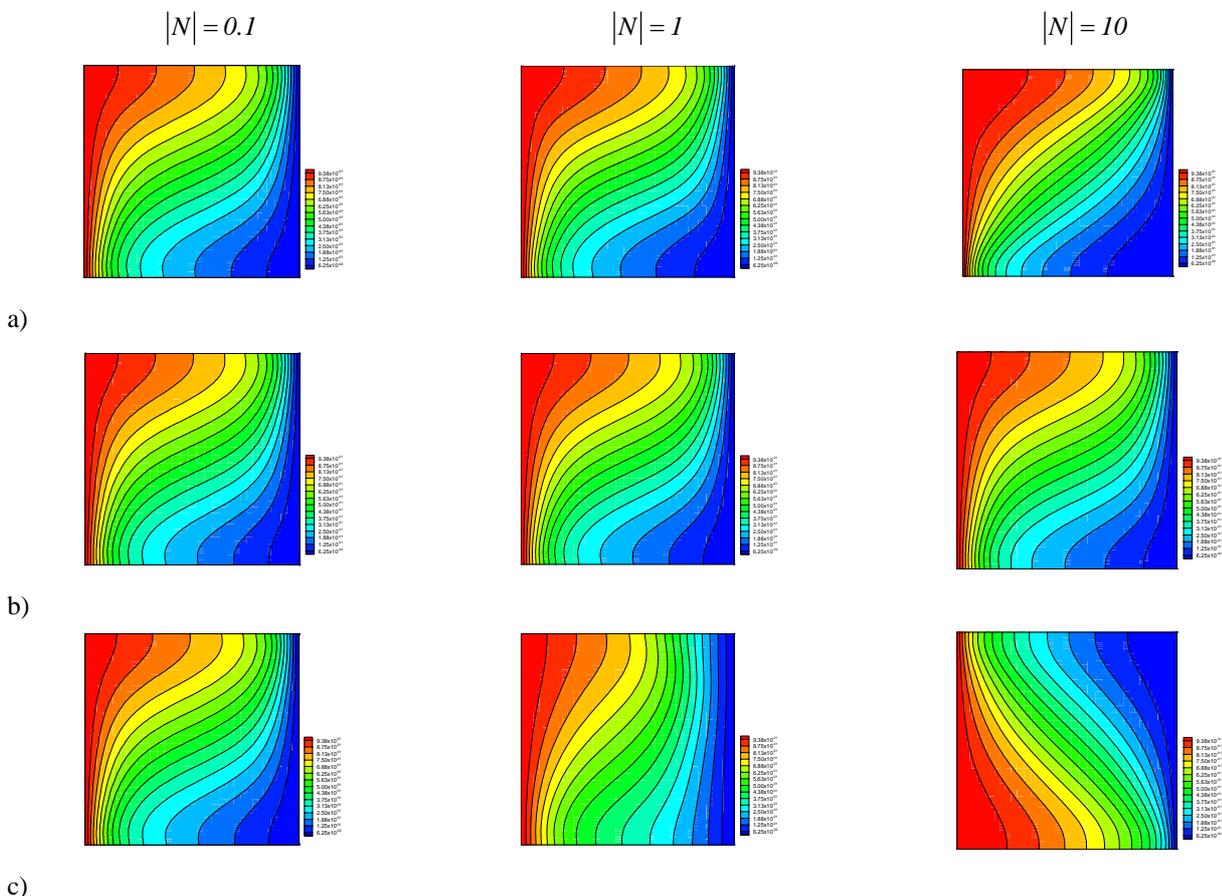


Figure 2 - Effect of  $N$  on isotherms for  $Ra^* = 100$ ;  $Le = 100$ ;  $A = 1$ ,  $Da = 10^{-3}$ : a) Aiding drives,

$\Delta T = -1, \Delta C = -1, N > 0$ ; b)  $\Delta T = -1, N = 0$ ; c) Opposing drives,  $\Delta T = -1, \Delta C = +1, N < 0$ .

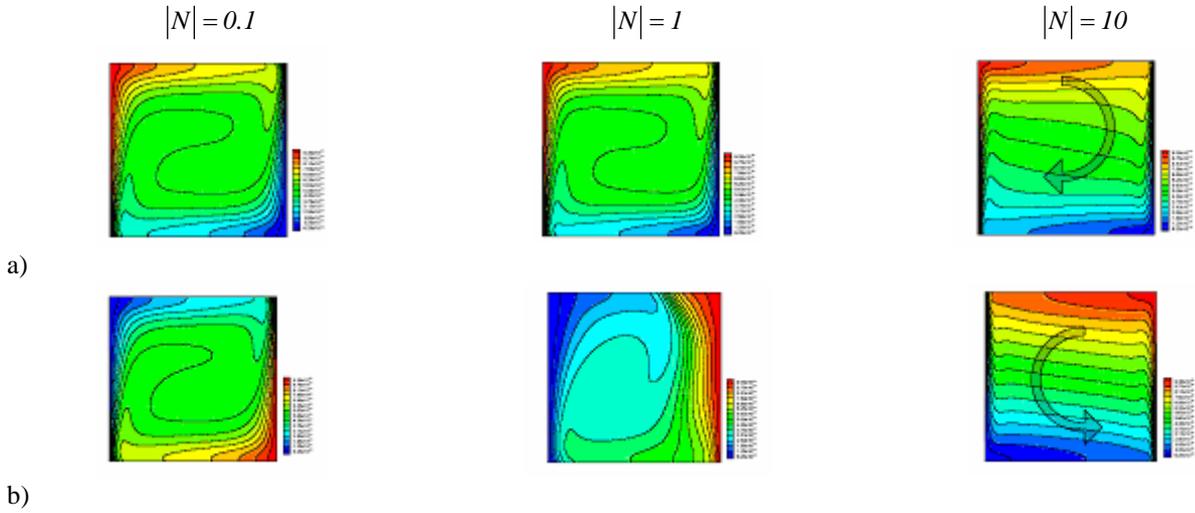


Figure 3 - Effect of  $N$  on isoconcentration lines  $Ra^* = 100$ ;  $Le = 100$ ;  $A = 1$ ,  $Da = 10^{-3}$ : a) Aiding drives,  $\Delta T = -1, \Delta C = -1, N > 0$ ; b) Opposing drives,  $\Delta T = -1, \Delta C = +1, N < 0$ .

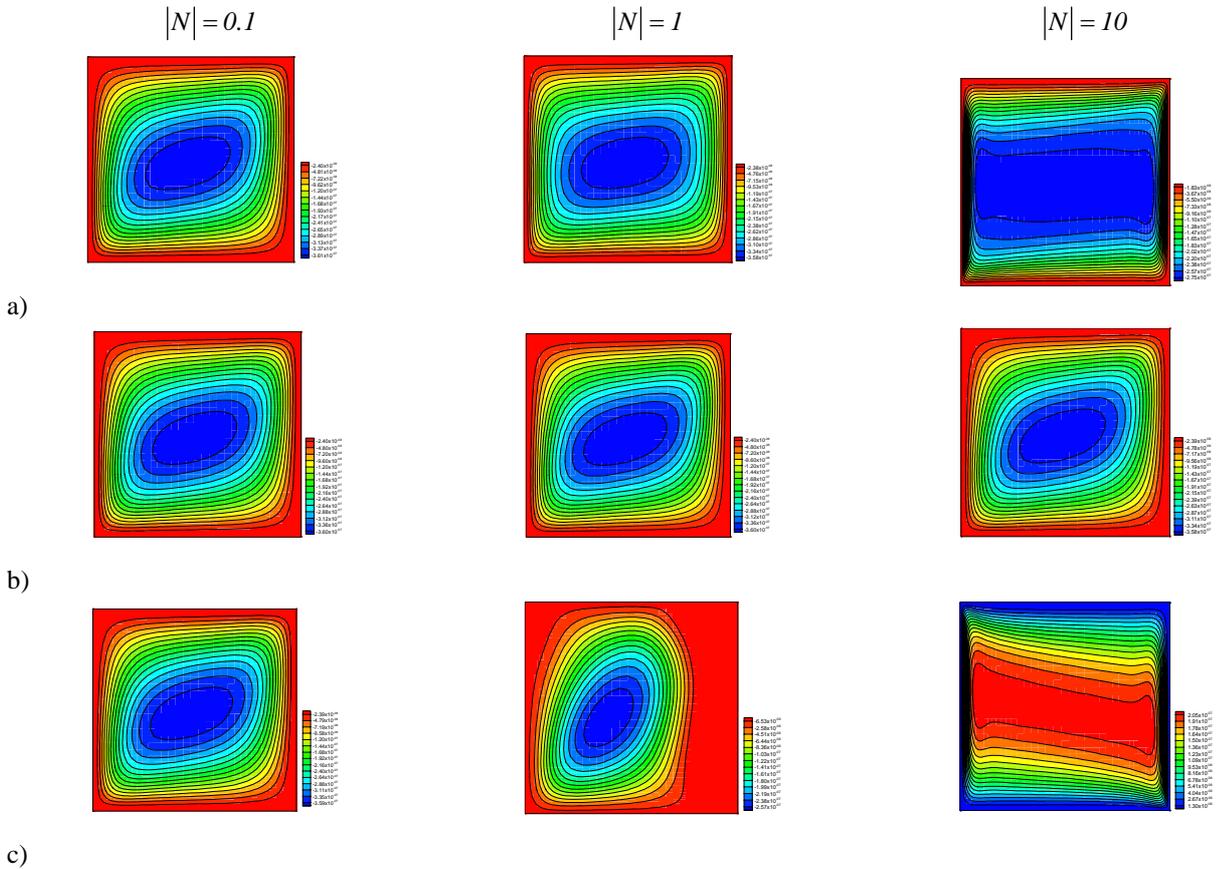


Figure 4 – Effect of  $N$  on streamlines for  $Ra^* = 100$ ;  $Le = 100$ ;  $A = 1$ ,  $Da = 10^{-3}$ : a) Aiding drives,  $\Delta T = -1, \Delta C = -1, N > 0$ ; b)  $\Delta T = -1, N = 0$ ; c) Opposing drives,  $\Delta T = -1, \Delta C = +1, N < 0$ .

Table 1: Average Nusselt and Sherwood numbers (  $N=0$  only thermal drive,  $Le=10$ ,  $A=1$ ).

$Ra^*$	Imposed Directions		100	200	400	1.000	2.000
$Nu$	$\Delta T = -1, \Delta C = -1$	Present Results	3.11	4.90	7.65	13.22	19.54
		(Goyeau et al, 1996)	3.11	4.96	7.77	13.47	19.90
		(Trevisan and Bejan, 1985)	3.27	5.61	9.69	-	-
Sh		Present Results	14.76	22.02	32.55	53.37	76.58
		(Goyeau et al, 1996)	13.25	19.86	28.41	48.32	69.29
		(Trevisan and Bejan, 1985)	15.61	23.23	30.76	-	-
$Nu$	$\Delta T = -1, \Delta C = 0$	Present Results	3.11	4.82	7.65	13.25	19.51
$Nu$	$\Delta T = -1, \Delta C = +1$	Present Results	3.05	4.84	7.59	13.20	19.48
Sh			14.73	22.04	32.58	53.48	76.37

## 2.2. Conclusions

This paper presented numerical simulations for adding and opposing flows in a cavity filled with a fluid saturated porous material. A mathematical model based on volumetric average of transport equations is applied. Numerical simulations made use of the control volume method and algebraic equations were relaxed following the SIMPLE method. For adding flows, both temperature and concentration distributions tend towards stratification as both drives push the fluid along the same angular direction. For opposing flows, mass driven recirculation currents overcome thermal drive and the flow in the entire cavity is pushed into a recirculating motion contrary to that of the adding flow case. Results herein are interesting and might contribute to the improved design and more accurate analysis of important engineering flows.

## 3. ACKNOWLEDGEMENTS

The authors are thankful to CNPq, Brazil, for their financial support during the course of this research.

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