STABLE CAVITATION OF AIR AND VAPOUR MICROBUBBLES IN COMPRESSIBLE WATER

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Abstract. *Stable oscillations of cavities (or bubbles) driven by variable external pressure fields are studied in the present work. In a stable cavitation condition, numerical solutions of a system of equations, based on the Keller-form equation, are presented with the purpose to study the oscillations of the bubble. A more realistic condition is used to model the growth and collapse of each oscillation, that is, a bubble filled with vapour and gas, taking into account the compressibility and the physical properties of the fluids involved.*
Such oscillations are analysed here to test the stable cavitation conditions of pressure and radius, and are also compared to the ones in the references.
Stable non-linear cavitation are observed for the predicted conditions of radius and pressure. The oscillations go on for long periods of time, as expected for stable cavities.

Keywords*: stable cavitation, cavity, bubble*

1. INTRODUCTION

Since Rayleigh (1917), many equations have been proposed to explain the collapse and growth of a spherical bubble in a liquid. They are, in fact, modifications of the classical Rayleigh-Plesset equation (Brennen, 1995). An analysis of these further equations (or modified Rayleigh-Plesset equations) was made by Prosperetti; Lezzi (1986), searching for the best equation or the best set of equations.

In the classical Rayleigh-Plesset equation (Hammitt, 1980), it is described the behaviour of a spherical bubble of radius R (as a function of time t) in an infinite domain of liquid where the pressure far from the bubble (and made as a constant) is P_∞. The surrounding liquid is considered as incompressible, the bubble content is assumed to be homogeneous, and the temperature and the pressure within the bubble are always uniform. A good approximation is to disregard mass and heat transfer across the bubble because the collapse process is very fast. According to Brennen (1995), with such considerations, the bubble will oscillate indefinitely. A simple deduction of the Rayleigh-Plesset equation can be found in Hammitt (1980), and in Brennen (1995).

Fujikawa; Akamatsu (1980) made analytical and numerical analysis of the behaviour of the bubble, taking into account condensation of the water vapour, heat conduction, and temperature discontinuity at the phase interface. The bubble content was considered to obey the perfect gas law. Although this is a very complex model, no significant differences were found when disregarding mass and heat transfer across the bubble wall, as can be seen in Bazanini (2003). Besides, to solve a full set of radial equations for the conservation of mass, momentum and energy in the bubble and in the surrounding liquid would be a huge computation. Recently, a work that has the purpose to investigate efficient methods of incorporating heat and mass transfer effects for spherical bubbles with the aim to reduce computation time was made by Preston et al. (2001). Even though, the authors assumed that the perfect gas law holds for the mixture of air and water vapour, and that the liquid is incompressible. It seems such an unnecessary complication of the problem, since maintains assumptions as incompressible liquid and perfect gas law, what will not be done in the present work, because of the importance of the liquid compressibility and real gas assumption in the process, as can be seen in Brennen (1995), and Bazanini (2001). Let us now focus on the modified Rayleigh-Plesset equations, which take into account the liquid compressibility and the physical properties of the fluids involved in the phenomenum.

Perhaps the most important work about the bubble behaviour since Rayleigh is the one by Gilmore (1952). Working with the liquid enthalpy, it takes into account the compressibility of the surrounding liquid through the use of the sonic velocity in the Rayleigh-Plesset equation. Every important work since then is somehow based on it, like the ones by Trilling (1952), Keller; Kolodner (1956), Prosperetti; Lezzi (1986), and Löfstedt et al. (1993), among others.

The bubble contents (vapour and air) compression and expansion during the oscillations can be treated as isothermal or adiabatic, although adiabatic is a more realistic assumption because of the rapidity of each collapse and growth (Young, 1989).

The problem of bubble dynamics may be summarized by Fig. (1) below, where there is an initial bubble radius R_0, from which the bubble oscillate with an radius $R = R(t)$.

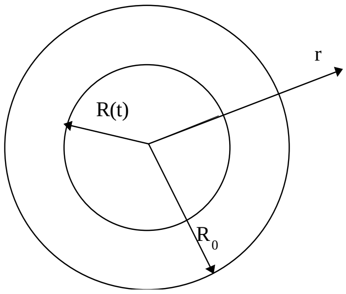

Figure 1. Simplified scheme for bubble oscillations.

An extensive analysis of several types of bubble collapse equations (Herring, 1941; Trilling, 1952) was made by Prosperetti and Lezzi (1986), concluding that an equation in the Keller form (Keller and Kolodner, 1956) is slightly more accurate by describing bubble collapses. Such general form of equation will be once again kept here, but now modified to include terms of frequency and amplitude of imposed external variable pressure signs. Inside the bubble it will be considered the presence of water vapour and air, through the use of their partial pressures. It is necessary to consider the presence of both (disregarded in other models) because bubble is nucleate from microbubbles of air, and growths by being filled by the vapour of the surrounding liquid.

Physical properties of the fluids involved are considered through viscosity, surface tension and density. Because of their importance in the process (Brennen, 1995; Bazanini, 2001), liquid compressibility and real gas assumption are kept. Since the gas (vapour plus air) inside the bubble is compressed as the collapse proceeds, it is necessary an appropriate model for this compression, that is, the real gas assumption. Such model will be used here. A system of equations is presented in an attempt to model the phenomenum in a more complete and realistic way, in the sense to make a contribution to study bubble oscillations controlled by external imposed variable pressure signs.

2. EQUATIONS OF BUBBLE OSCILLATIONS

Some kind of bubble collapse equations (also called Rayleigh-Plesset Equations), have appeared in several works along the last decades. Some remarkable works are the ones by Herring (1941), Trilling (1952), Gilmore (1952), Keller and Kolodner (1956). Prosperetti and Lezzi (1986) made an extensive and accurate analysis of that works, and the conclusion was that all these equations are entirely equivalent and form a family of equations with the same degree of accuracy, being the Keller form equation (Keller and Kolodner, 1956) slightly more accurate than the others. Because of that, an equation of this type is used here, including viscosity, density and surface tension effects.

An important and complex variable is the pressure at the bubble wall P_L. For that pressure at bubble wall shall be used Eq. (1) below (and extracted from Bazanini, 2004), that takes into account the initial gas pressure P_{g0}, the initial vapour pressure P_{v0}, the initial bubble radius R_0, the surface tension S, and the liquid and gas viscosity μ_L and μ_g, respectively. K_v and K_g are the adiabatic constants for vapour and gas, as well. The compressibility of the liquid, what occurs at the final stage of each collapse is also considered here, through the use of the sonic velocity C in Eq. (2) below.

For the gas and the vapour trapped inside the bubble, it will also be considered the effect of the van der Waals hard core a_g and a_v (Barber and Putterman, 1991). In fact, gas and vapour are being considered to obey the van der Waals equation of state for real gases. This is important because of the raising pressures within the bubble during the collapses.

The time dependence is established, since variables are dependent of the bubble radius R, and $R = R(t)$. The prime denotes time derivative.

$$P_L = \frac{P_{g_0}\, R_0^{\,3K_g}}{\left(R^3 - a_g^3\right)^{K_g}} + \frac{P_{v_0}\, R_0^{\,3K_v}}{\left(R^3 - a_v^3\right)^{K_v}} - \frac{2S}{R} - \frac{4\left(\mu_g + \mu_L\right)}{R} R' \qquad (1)$$

Terms of frequency ω and amplitude P_A are added in Eq. (2) below to consider imposed external variable pressure signs. This is done by varying in time the initial external pressure P_∞, through the use of the driving pressure P_A and frequency ω. ρ_L is the liquid density.

$$\left(1 - \frac{R'}{C}\right) RR'' + \frac{3}{2}\left(1 - \frac{R'}{3C}\right) R'^2 = \frac{1}{\rho_L}\left[\left(1 + \frac{R'}{C}\right) P_L + \frac{R}{C}\frac{dP_L}{dt} - \left(1 + \frac{R'}{C}\right)\left(P_\infty - P_A \sin \omega t\right)\right] \qquad (2)$$

Equation (2) above was derived by the first author, and is very close to Keller equation (slightly more accurate than the others, as discussed by Prosperetti and Lezzi, 1986), but it is being included now the compression of the vapour as well as the air trapped inside the bubble in Eq. (1), using the van der Waals equation of state for real gases. To use Eq. (2), it is first necessary to derive Eq. (1) respect to time. Then results for the term dP_L/dt the following Eq. (3):

$$\frac{dP_L}{dt} = -3R^2 R' \left[\frac{K_g P_{g_0} R_0^{3K_g}}{\left(R^3 - a_g^3\right)^{1+K_g}} + \frac{K_v P_{v_0} R_0^{3K_v}}{\left(R^3 - a_v^3\right)^{1+K_v}} \right] + \frac{2SR'}{R^2} - 4(\mu_g + \mu_L)\left(\frac{RR'' - R'^2}{R^2} \right) \qquad (3)$$

The cavitation thresholds usually define if a bubble will oscillate in a stable or in a transient manner. Stable bubbles oscillate, often non-linearly, for many cycles, while the transient bubbles generally exist for less than one cycle, often disintegrating into a mass of smaller bubbles. These thresholds are commonly the threshold radius R_T and the threshold pressure P_T. In a very simple way, it can be said that a bubble will oscillate stably for a radius smaller than the threshold radius, and for external pressures greater then the threshold pressure. These thresholds can be calculated by Eqs. (4) and (5) below, obtained for the condition of minimum pressure outside the bubble (Neppiras, 1980; Young, 1989).

$$R_T = \left[\frac{3R_0^3}{2S} \left(P_\infty + \frac{2S}{R_0} \right) \right]^{1/2} \qquad (4)$$

$$P_T > P_\infty + \frac{4}{3} \left[\frac{2S^3}{3R_0^3 \left(P_\infty + \frac{2S}{R_0} \right)} \right]^{1/2} \qquad (5)$$

3. RESULTS

First of all, the expected conditions for stable cavities, are obtained using Eqs. (4) and (5). Then, after substituting Eqs. (1) and (3) in Eq. (2), the latter is solved using the finite difference method in an explicit time integration scheme to obtain the bubble oscillations in time. The equations need to be solved step by step, becauseIt is being considered a gaseous bubble in water, containing water vapor and air (the later is always present on the formation of the bubble, since the bubble nucleates from micro-bubbles of air, as described by Hammitt, 1980). For the ratios initial radius/hard core, that is R_0/a_g, and R_0/a_v, are used the values 8.54 and 10.79, extracted form Barber et al. (1997). For the sonic velocity C is used the value 1481 m/s (Löfstedt et al., 1993).

It is being used here, whenever possible, the same conditions used by Löfstedt et al. (1993), that is (for Eqs. 4 and 5), S = 0.03 kg/sec^2 ; $P_\infty = 10^5$ Pa; R_0 = 4.5 µm. Equations (4) and (5) then provides a threshold radius of 22.7 µm, and a pressure threshold of 101760 Pa. It is necessary to work with a bubble radius smaller the threshold radius and with a pressure greater than the threshold pressure to obtain the stable cavities. It is being used the values of 4.5 µm for the initial bubble radius and 135000 Pa for the driving pressure for Fig. 2 and of 10.5 µm and 107500 Pa for Fig. 4, respectively.

Equation (2) is now solved to obtain the bubble oscillations in time. These solutions produced the results shown in Fig. (2) below, for the initial conditions described. It is possible to see similarities between these calculations, and those by Löfstedt et al., (1993), reproduced in Fig. 3, although some boundary conditions are not the same, since we had to assume some fluid properties not revealed by that reference. Those differences are basically: our model takes into account the liquid compressibility, what occurs in the final stages of each collapse (Bazanini, 2001); we consider the presence of water vapor (K_v = 1.329) plus air (K_g = 1.4) in the bubble, and it was assumed an initial partial pressure of 2340 Pa for the vapor and of 40 Pa for the air (Knapp; Hollander, 1948).

In Fig. 4 below, the simulations were allowed to proceed for a longer period of time, to see the numerous collapses and growths of the oscillating bubble. The boundary conditions are also taken from Löfstedt et al., (1993).

Figure 5 was built using he same boundary conditions of Fig. 4, but for an even longer period of time, characterizing the stable cavity, or bubble, oscillating for many cycles. It is clearly seen the stable cavity behaviour of the bubble, by its numerous oscillations. So, the test conditions to obtain stable cavities (given by Eqs. 4 and 5) worked quite well in the case studied here.

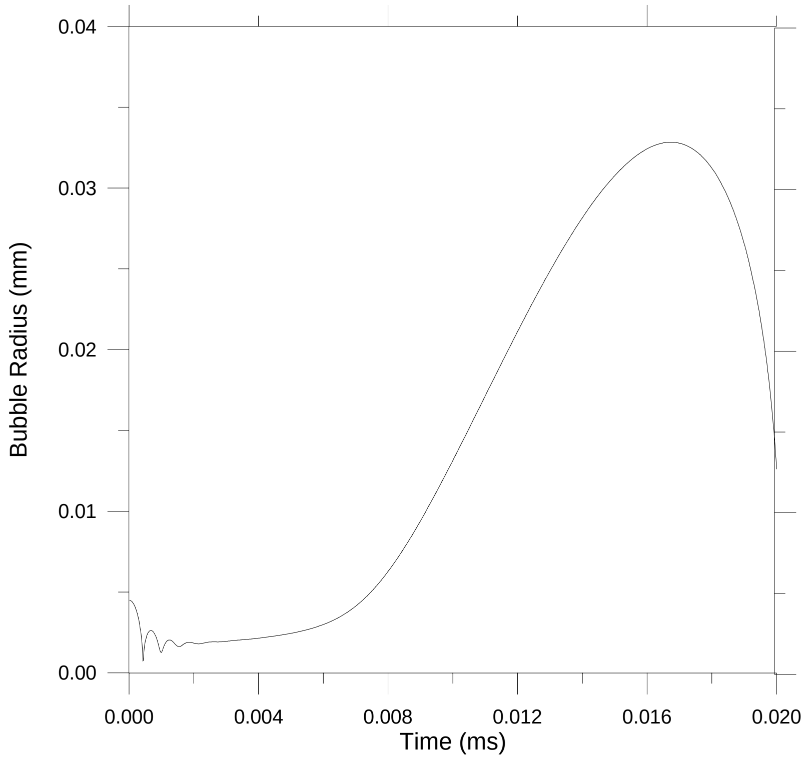

Figure 2. Bubble radius versus time for: $P_\infty = 10^5$ Pa; R_0 = 4.5 μm; P_A = 135000 Pa; ω =26500 Hz.

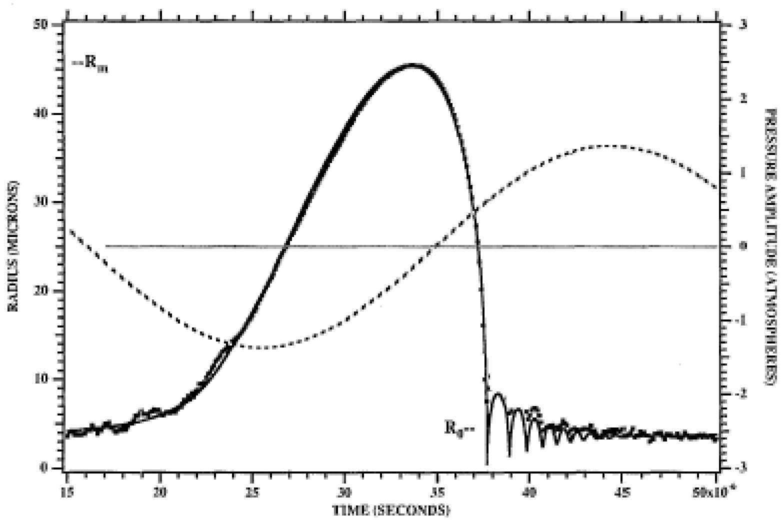

Figure 3. Bubble radius versus time for: $P_\infty = 10^5$ Pa; R_0 = 4.5 μm; P_A = 135000 Pa; ω =26500 Hz.

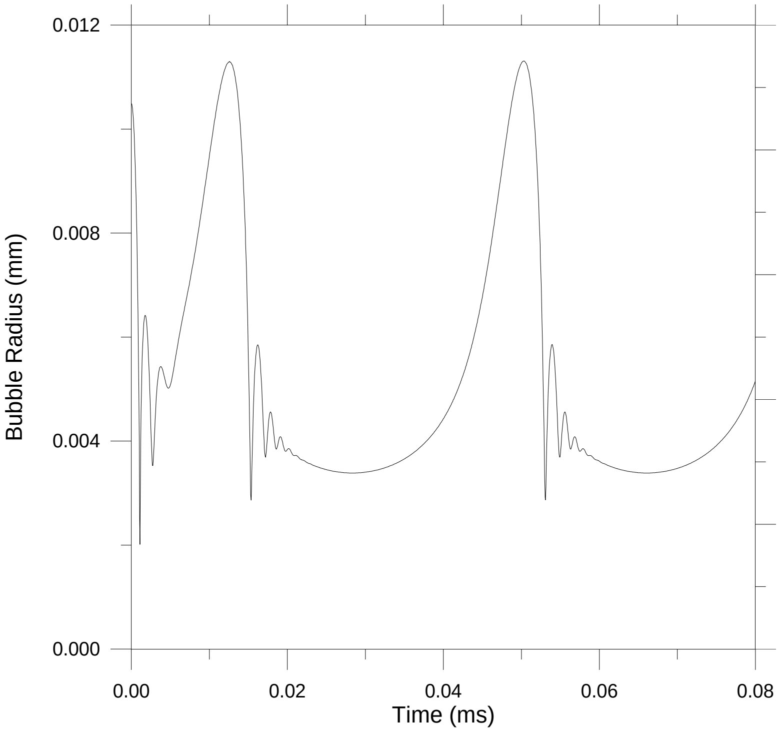

Figure 4. Bubble radius versus time for: $P_\infty = 10^5$ Pa; R_0 = 10.5 µm; P_A = 107500 Pa; ω = 26500 Hz.

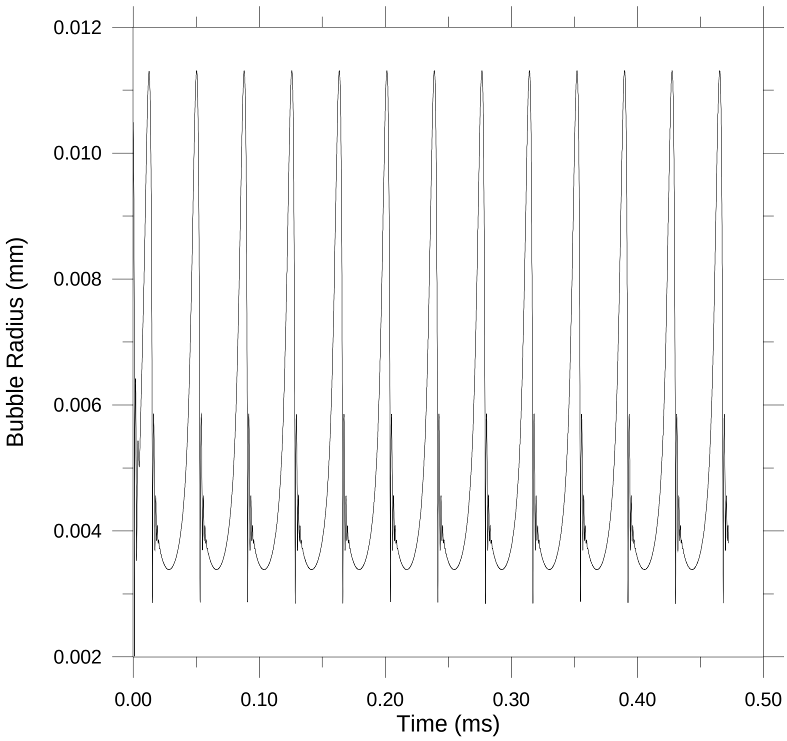

Figure 5. Bubble radius versus time for: $P_\infty = 10^5$ Pa; R_0 = 10.5 µm; P_A = 107500 Pa; ω = 26500 Hz.

4. CONCLUSIONS

Stable non-linear cavitation is succesfully observed for the predicted conditions of radius and pressure from Eqs. (4) and (5). The bubble growths and collapses many times and so the oscillations continue for long periods of time, as expected for stable cavities. So, the test conditions to obtain stable cavitation (given by Eqs. 4 and 5) seem to work quite well.

The result obtained for one oscillation is very close to the reference (Löfstedt et al., 1993), although the reference showed the result for one oscillation only, and it is well known that the phenomenom continues, specially if the external pressure signal is maintened. The differences in the results obtained here to those by Löfstedt et al., (1993), are basically due to the fact that the model used here is expected to be slightly more precise, and that we had to assume some data not revealed by that reference, such as the physical properties used there. But, for one oscillation (that reference presents just one), the model used here worked reasonably well.

The model presented here is somehow more realistic then the existing ones, since it considers real gas assumption, the presence of vapour inside the bubble (it is filled by vapour as it grows), physical properties of the fluids, such as viscosity, density, surface tension, and compressibility of the liquid.

Although the model is suggested to be comparatively better than the other ones, it is still to be validated by future experimental data.

Other different cases of initial and boundary conditions are about to be tested, case by case, in a future or future works, since the model now appears to be in a somehow complete form for one bubble. But it is still also necessary to include the effect of each bubble on the others, since no bubble exists alone. Perhaps it is possible, even due to computational limitations, to simulate the influence of one bubble on the three or four surrounding ones, considering a symmetrical pressure field originated by each collapse. It is the intention of the author to include such reciprocal effect it in a future work.

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6. RESPONSIBILITY NOTICE

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OSCILAÇÕES FORÇADAS DE BOLHAS EM LÍQUIDOS VISCOSOS E COMPRESSÍVEIS

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Resumo*. No presente trabalho são estudadas oscilações estáveis de cavidades ou bolhas, excitadas por um campo externo variável de pressão. Na condição de cavitação estável, são apresentadas soluções numéricas de um sistema de equações, baseado na equação de Keller. É utilizada uma condição mais realista para modelar o crescimento e colapso da bolha, que consiste numa bolha preenchida por gás e vapor, levando em conta a compressibilidade e as propriedades físicas dos fluidos envolvidos no fenômeno.*

Estas oscilações são estudadas para verificar as condições de pressão e raio para cavitação estável, comparando-se com as existentes na literatura.

Foi observada cavitação estável não linear para as condições previstas de raio e pressão. As oscilações prosseguem por longos períodos de tempo, conforme previsto para cavidades estáveis.

Palavras-chaves*. cavitação estável, cavidade, bolha.*