CONTROL OF HIGH SPEED WIND TUNNEL MACH NUMBER

Silva, Maurício Guimarães da
Comando Geral de Tecnologia Aeroespacial (CTA)/
Instituto de Aeronáutica e Espaço (IAE)/ Divisão de Sistemas de Defesa(ASD), Brazil
DENEL Dynamics, Aerospace System, South Africa
maugsilva@iae.cta.br

Falcão, João Batista Pessoa Filho
Comando Geral de Tecnologia Aeroespacial (CTA)/
Instituto de Aeronáutica e Espaço (IAE) / Divisão de Ensaios Aeronáuticos(ALA), Brazil
jb.falcao@ig.com.br

Abstract. *A non-linear mathematical model was developed to analyze the open-loop system thermodynamic characteristics and a linearized mathematical model was used for the controller design. The model for SWT was based on mathematical model due to Fung (1987). Each module of SWT is formulated as an isentropic subsystem. The main difference between this work and the one due to Fung (1987) is that, in present work, the stagnation temperature is the same in all subsystem. Moreover, it was included the test section and supersonic diffuser subsystems in the mathematical model. From this approach, it was developed a relation for the variation of Mach number at test section. It was also possible to define two options of control law for the Mach number at the test section based on stagnation pressure at test section and on the settling chamber. A SIMULINK® block diagram code was used to solve a mathematical model consisted of a set of ordinary differential and algebraic equations. Performance of the supersonic wind tunnel using a PI (proportional-plus-integral) controller was found to be satisfactory, as confirmed by the results.*

Keywords*. Blow-down wind tunnel, pressure control, Mach number control, sensitivity analysis*

1. Introduction

There are many parameters that characterize a blow-down Supersonic Wind Tunnel (SWT) such as the test section dimensions, operating characteristics (Reynolds number x Mach number), general capabilities of the facility (Mach number range, maximum stagnation pressure) and so on. Many types of tests simulated in a high-speed wind tunnel are sensitive in various degrees to the errors in Mach number. One standard task certainly is the measurement of aerodynamic forces and moments. Note that in this type of wind tunnel there is a formation of shock waves inside the test section due to the presence of the model. These waves can reflect on the walls, and may cause a detrimental effect on the measurements of forces and pressures on the tested model. Since the angle of reflection is related to the Mach number (Pope and Goin, 1965), the choice of the size of model is function of the Mach number in test section. Another restriction is the duration of the tests. At a given Mach number it is sometimes required to maximize the test duration by running the tunnel at the lowest possible stagnation pressure – but still maintaining supersonic flow condition. However, it is important to consider the undesirable variation of Reynolds number in the test section during a tunnel run. Therefore, the best choice for the stagnation pressure and temperature at a given Mach number cannot be the best choice for the Reynolds number. Due to the inter relation among these parameters it is very difficult to reproduce experimentally in wind tunnel the conditions required to the project of some aeronautical components. So, it is important to estimate, theoretically, the best test configuration (stagnation pressure, geometrical configuration of nozzles and diffuser) before each experimental run.

In this context, a non-linear mathematical model was developed to analyze the open-loop system thermodynamic characteristics and a linearized mathematical model was used for the controller design. The model for SWT was based on mathematical model due Fung (1987). Each module of SWT is formulated as an isentropic subsystem. The principal difference between this work and that due Fung (1987) is that, in present work, the stagnation temperature is the same in all subsystem. Moreover, it was included the test section and supersonic diffuser subsystems in the mathematical model. From this approach, it was developed a relation for the variation of Mach number at test section. It was also possible to define two options of control law for the Mach number at the test section based on stagnation pressure at test section and setting chamber. A SIMULINK® block diagram code was used to solve a mathematical model consisted of a set of ordinary differential and algebraic equations. Performance of the supersonic wind tunnel using a PI (proportional-plus-integral) controller was found to be satisfactory, as confirmed by the results.

2. Mathematical Model

The dynamic analysis of the control system for SWT is divided into five modules: storage tank, settling chamber nozzle, test section and diffuser, Fig. 1. Control volumes mathematically represent these modules. It is assumed that pressure, temperature and density distribution are uniform over the whole control volume during the test. It should be noted that it is assumed that all the thermodynamic processes are isentropic during the test time (no shock waves, friction and heat transfer are neglected). The change of potential energy of the gas is small and can be neglected.

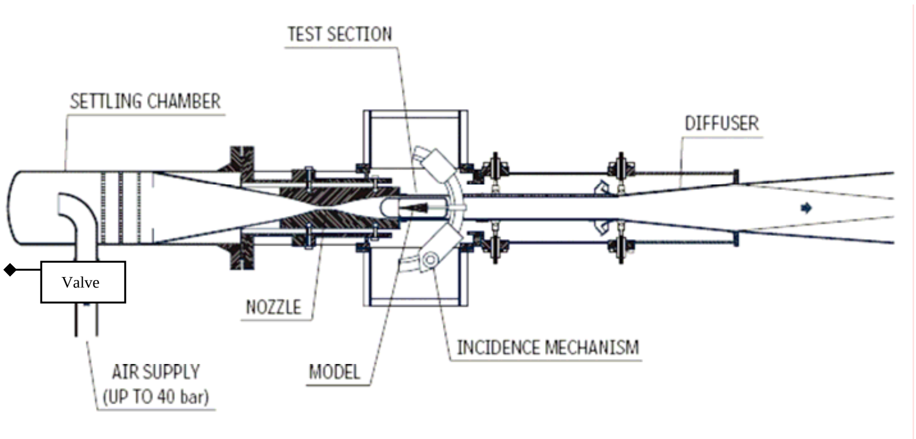

Figure 1. Schematic of a blow-down supersonic wind tunnel

2.1. Storage tank

During a test, it is assumed that the mass influx from the compressor is negligible. Hence, the rate of decrease of mass in air tank is equal to the rate of mass efflux through the valve:

$$\frac{d\rho_T}{dt} = -\frac{1}{V_T}\dot{m}_v , \tag{1}$$

where ρ_T is the storage tank air density, $\dot{m}_v$ is the mass efflux through the valve and V_T is the storage tank volume. The subscript "*T*" refers to storage tank. By assuming the energy loss through the valve is negligibly small, the internal energy change in the storage tank is equal to the enthalpy plus the kinetic energy through the valve. Therefore:

$$\frac{dU_T}{dt} = -\dot{m}_v h_v - \frac{1}{2}\dot{m}_v v_v^2 , \tag{2}$$

where U_T is the storage tank air internal energy, h_v is the specific enthalpy of the air through the valve and v_v is the velocity of the air through the valve. In terms of the stagnation pressure, Eq. (2) can be written (Fung, 1987):

$$\frac{dP_T}{dt} = -\left(\frac{\gamma R T_T}{V_T}\right)\dot{m}_v . \tag{3}$$

The quotient $\gamma = c_p / c_v$ is the specific heats ratio and *R* is the gas constant.

The valve characteristics are described in Fisher Controls Company (1984), from the manufacturer. The mass flow at different valve positions is given by:

$$\dot{m}_v = \frac{2.295810^{-8}}{\sqrt{T_T}} C_g P_T \sin\left(2.71\sqrt{\frac{\Delta P}{P_T}}\right). \tag{4}$$

where C_g is the "gas sizing coefficient". Table 1 shows some characteristic values in the valve operating range. The variables T_T and P_T are the thermodynamics properties (temperature and pressure) of the air into storage tank. ΔP is the pressure difference across the valve. It is assumed that $\Delta P = P_T - P_0$, where P_0 is the stagnation pressure at the settling chamber.

Table 1 – The gas-sizing coefficient of the valve for several valve opening position (θ in degree)

θ	0	10	20	30	40	50	60	70	80	90
C_g	0	194	1680	3767	6230	9288	12835	16351	18942	23120

2.2. Settling Chamber

The second control volume is the settling chamber as shown in Fig. 1. Air flows into the settling chamber from the control valve and goes through the convergent-divergent nozzle to the test section. The energy entering the settling chamber volume with mass flow $\dot{m}_v$ minus the energy exiting through the nozzle with mass flow $\dot{m}_t$ is equal to the internal energy rate in the settling chamber. Therefore, the relation of energy conservation for the settling chamber is:

$$\frac{dU_0}{dt} = \dot{m}_v h_v + \frac{1}{2}\dot{m}_v v_v^2 - \dot{m}_t h_t - \frac{1}{2}\dot{m}_t v_t^2 \ . \tag{5}$$

Subscript "0" refers to the settling chamber and subscript "t" refers to the throat nozzle. Rewriting the Eq.(5) in terms of stagnation pressure, results (Fung, 1987):

$$\frac{dP_0}{dt} = \left(\frac{c_p}{c_v}\frac{R}{V_0}\right)\left(\dot{m}_v T_T - \dot{m}_t T_0\right). \tag{6}$$

The flow is without heat transfer. In this context, it is possible to rewrite Eq.(6):

$$\frac{dP_0}{dt} = \left(\frac{\gamma R T_T}{V_0}\right)\left(\dot{m}_v - \dot{m}_t\right), \tag{7}$$

since $T_0 = T_T$.

2.3. Nozzle

The nozzle of the supersonic wind tunnel is axisymmetric, variable-geometry with converging-diverging geometry. It is assumed that the flow from the settling chamber to the test section runs an isentropic process. Considering the air as a perfect gas and the stagnation state as the reference state, it can be written $\dot{m}_t$ as function of stagnation pressure and the nozzle throat area A_t. The maximum flow through the nozzle will be:

$$\dot{m}_t = P_0^{design} A_t \left(\frac{\gamma}{RT_0}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{or} \tag{8}$$

$$\dot{m}_t = P_0^{design} A_t C_D, \tag{9}$$

where P_0^{design} is stagnation pressure defined in the design of the nozzle and C_D is the discharge coefficient of the nozzle, given as:

$$C_D = \left(\frac{\gamma}{RT_T}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} . \tag{10}$$

The critical area A_t is function of the Mach number (M) desired in the test section and of its transversal section A, namely (Kuethe, 1998):

$$\frac{A_t}{A} = M\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M^2}{\left(\frac{\gamma+1}{2}\right)}\right]^{\frac{-(\gamma+1)}{[2(\gamma-1)]}} . \tag{11}$$

2.4. Test Section

The mathematical formulation for test section module is similar to the settling chamber. Air flows into the test section from the settling chamber and goes through the diffuser module to the atmosphere. Analogously to the Eq.(5) it is possible to write the energy equation applied for test section in terms of pressure:

$$\frac{dP_{TS}}{dt} = \left(\frac{c_p}{c_v}\frac{R}{V_{TS}}\right)\left(\dot{m}_t T_0 - \dot{m}_{dif} T_{TS}\right), \tag{12}$$

or

$$\frac{dP_{TS}}{dt} = \left(\frac{\gamma R T_T}{V_{TS}}\right)\left(\dot{m}_t - \dot{m}_{dif}\right), \tag{13}$$

since $T_{TS} = T_0 = T_T$.

Theoretically, the Mach number of the test section can be determined, for example, from the use of isentropic relations. Consider the definition:

$$P_{TS} = \alpha P_{TS}^{design}, \tag{14}$$

where P_{TS} is the stagnation pressure in test section, P_{TS}^{design} is the stagnation pressure defined in the test section design and α is the "fill" parameter. Since all analysis will be carried out when both throats (nozzle and diffuser) at critical conditions and the drop of stagnation temperature obtained in the process of blow-off the tank do not present high slope or discontinuities, it is possible to use the "quasi-steady" approach to estimate the Mach number at the test section. From the equation of continuity:

$$\frac{A_{TS}}{A_t} = \frac{\rho_t}{\rho_{TS}}\frac{u_t}{u_{TS}} . \tag{15}$$

Note that:

$$\frac{\rho_t}{\rho_{TS}} = \frac{p_t}{P_0^{design}}\frac{T_0}{\tau_t}\frac{P_0^{design}}{p_{TS}}\frac{\tau_{TS}}{T_0}, \tag{16}$$

since $\rho = \frac{p}{RT}$. The variable ρ is density, T is stagnation temperature, τ is static temperature, $P_0^{design} = P_{TS}^{design}$ is stagnation pressure defined in the design of the nozzle, p is static pressure and the subscripts 0, *TS*, and *t* are related to settling chamber, test section and throat of nozzle, respectively. It is well known that (Kuethe, 1998):

$$\frac{p_t}{P_0^{design}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}, \quad \text{and} \tag{17}$$

$$\frac{T_0}{\tau_t} = \frac{\gamma+1}{2} . \tag{18}$$

It is possible to determine $\frac{P_0^{design}}{p_{TS}}$ in Eq. (16) from the isentropic relations and Eq. (14), namely:

$$\frac{P_0^{design}}{p_{TS}} = \frac{P_{TS}^{design}}{P_{TS}} \frac{P_{TS}}{p_{TS}} . \tag{19}$$

where:

$$\frac{P_{TS}}{p_{TS}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} , \tag{20}$$

$$\frac{P_{TS}^{design}}{P_{TS}} = \frac{1}{\alpha} . \tag{21}$$

Then:

$$\frac{\rho_t}{\rho_{TS}} = \frac{1}{\alpha}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \frac{\gamma + 1}{2}\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \frac{\tau_{TS}}{T_0} . \tag{22}$$

But:

$$\frac{u_t}{u_{TS}} = \frac{\sqrt{\gamma R \tau_t}}{u_{TS}} = \sqrt{\frac{\gamma R \tau_{TS}}{u_{TS}^2} \frac{\tau_t}{T_0} \frac{T_0}{\tau_{TS}}} , \tag{23}$$

that is:

$$\frac{u_t}{u_{TS}} = \frac{1}{M}\left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}} \left(\frac{T_0}{\tau_{TS}}\right)^{\frac{1}{2}} . \tag{24}$$

Substituting Eq. (22) and Eq. (24) in Eq. (15), it has:

$$\frac{A_{TS}}{A_t} = \frac{1}{M}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \frac{1}{\alpha}\left(\frac{\tau_{TS}}{T_{TS}}\right)^{\frac{1}{2}} , \tag{25}$$

since $T_0 = T_{TS}$ (isentropic flow). But:

$$\frac{T_{TS}}{\tau_{TS}} = 1 + \frac{\gamma - 1}{2} M^2 , \tag{26}$$

Therefore:

$$M\left(\frac{\frac{\gamma + 1}{2}}{1 + \frac{\gamma - 1}{2} M^2}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} = \frac{A_t}{A_{TS}} \frac{1}{\alpha} , \tag{27}$$

In Eq. (27) it is possible to identify two terms: term of evolution in time, $\frac{1}{\alpha}$, and Mach number function, which is:

$$f(M) = \frac{A_{TS}}{A_t} M\left(\frac{\frac{\gamma + 1}{2}}{1 + \frac{\gamma - 1}{2} M^2}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} , \tag{28}$$

Equation (27) will be used to determine Mach number in test section since the temporal evolution of the pressure in test section (implicit in variable α) was defined in Eq.(14).

2.5. Diffuser

The diffuser captures the flow from the test section. Supersonic tunnels, in which a diverging diffuser after the test section would produce a further increase in Mach number, are equipped with a second throat at the end of the test section: the first throat is the one upstream of the test section through which the flow accelerates through the speed of sound. In the converging section leading to the second throat the flow is decelerated to slightly above sonic speed (obeying the one-dimensional inviscid compressible flow equations to a first approximation); in the diverging section downstream of the throat the Mach number rises again, until a shock wave or waves produce a reduction to subsonic speed. It may be shown that a shock wave in the converging portion of the second throat would be unstable, and in practice the second-throat Mach number is chosen large enough for the breakdown shock system to be located downstream of the throat, to ensure stability under all operating conditions. Consider the case shown in Fig. 3. The required condition at test section are: Mach = M , stagnation pressure of P_{TS}^{design} and temperature of T_{TS}^{design} . The cross section in area of test section is A_{TS} . The area of nozzle's throat (A_t) is obtained by use of Eq. (11). The design area A_{dif} to reach the requirements can be obtained by use of isentropic flow relations and continuity equation, namely:

$$\dot{m}_{dif} = \dot{m}_t , \tag{29}$$

then, from Eq.(8):

$$P_0^{design} A_t = P_{TS}^{design} A_{dif} , \qquad \text{or}$$

$$A_{dif} = A_t . \tag{30}$$

Analogous to the Eq. (8), the mass flow thought diffuser is:

$$\dot{m}_{dif} = P_{TS} A_{dif} \left(\frac{\gamma}{RT_T} \right)^{\frac{1}{2}} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} . \tag{31}$$

It can be noted that the Eq.(31) will be used here considering the wind tunnel operating at design conditions. In this way the nozzle and diffuser will be in critical condition. From this moment, the mathematical model for the mass flow thought diffuser was approximately by:

$$\dot{m}_{dif} = \left(1 + \frac{V_{TS}}{V_0} \right) \dot{m}_t - \frac{V_{TS}}{V_0} \dot{m}_v . \tag{32}$$

This relation was obtained from the equations Eq.(7) and Eq.(13).

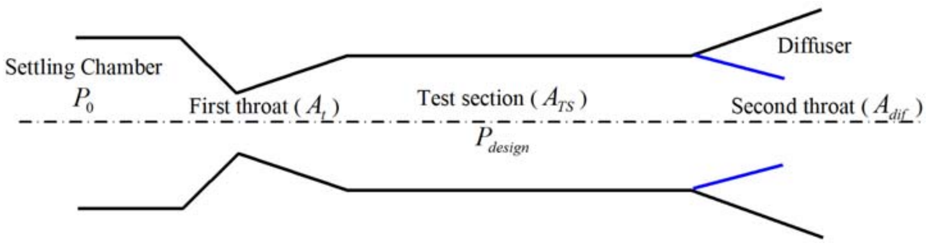

Figure 3. Geometrical configuration of test section

3. Sensitivity

The Mach number determination in any flow can be obtained experimentally. However, these experimental measurements have some intrinsic errors. The uncertainties can be reduced toward the precision of the instrument by performing ensemble averages. The non-deterministic portion of the error is reduced with the square of the number of averaged data. The deterministic part of the error is often related to some physical effect that causes a repeatable offset in the measurements. This type of error can be easily recognized in a calibration procedure. Despite the techniques of error reduction, the experimental measurement of the Mach number is susceptible to many sources of uncertainties. Assuming that the instruments and their read-outs are perfect, errors will still arise in two ways. First, the pressure, density or other property measured by the instrument may differ physically from the quantity, which is used in the formula to deduce the Mach number. For example, a static pressure probe may correctly read the pressure at the orifice; but due to boundary layer or some effect of the presence of the probe on the stream, this may not actually be the free stream static pressure. Other errors may arise because the gas does not actually follow the assumed process between two measured states, or may not obey the perfect gas equation, or the computation of Mach number based on non-viscous laws may be affected by viscous effects.

The purpose of this topic is to present the sensitivity of any measurable parameter due to deviations in Mach number. In the determination of functional relation between Mach number and sensitivity, it was assumed that the one-dimensional flow relations might be applied, that the instrument correctly measures the required property, and that this measurement may be read with "infinite" precision. Since the formulas for the parameters and their sensitivities are derived on the assumptions of a prescribed process in a non-viscous perfect gas, the violation of these assumptions will lead to errors in the calculation of Mach number. However, the results from the formulation above mentioned, can be used to help in the choice of the best experimental procedure to measure the test section Mach number.

Consider the parameter Θ_i as a particular combination of measurable quantities from which the Mach number may be deduced. Then, the sensitivity S_i is defined as the fractional change in Mach number, which results for a unit fractional change in this measured parameter. For small increments, the effect of sensitivity may be written in the differential form:

$$\frac{dM}{M} = S_i \frac{d\Theta_i}{\Theta_i} . \tag{33}$$

A parameter that is sensitive to Mach number, and hence provides a good basis for measurement, will have a small value for S_i.

3.1 Pressure

Experimentally, the Mach number in the test section can be determined in many ways. It can be given by the combination between the stagnation pressure at the settling chamber and the static pressure in the test section. Another method is similar to subsonic measurements and consists of the combination between the static and the stagnation pressures at the test section. In Fig. 4 the pitot tube measures the stagnation pressure (P_2) behind the shock. In this diagram, p_1 corresponds to the static pressure in front of the shock. The expression (34), known as the "Rayleigh Pitot Relation" (Matsumoto, 2000), is usually solved recursively to obtain the Mach number once p_1 and P_2 are known. In supersonic wind tunnel this measurement is obtained through scavenger scoop. The scavenger scoop is a duct in the diffuser, which is used to support probes or to vacuum the air out of an engine model in the starting process (Matsumoto, 2000).

$$\frac{P_2}{p_1} = \left(\frac{\gamma+1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma+1}M^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{-1}{\gamma-1}} . \tag{34}$$

The ratio of stagnation to static pressure can be used as parameter Θ_p. Then, the correspondent sensitivity will be:

$$S_P = \frac{M^2 - \frac{\gamma-1}{2\gamma}}{2M^2 - 1} . \tag{35}$$

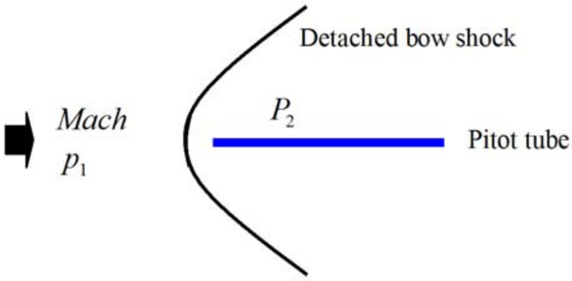

Figure 4. Pitot tube in supersonic flow

3.2 Temperature

One of the most elusive thermodynamic properties is the temperature. At not high-speed flow, it is relatively easy to measure the stagnation temperature (thermocouple or ordinary thermometer); but at high speed only a so called "recovery temperature" can be determined directly. Due to the combined effects of viscosity, thermal conductivity, and radiation, a temperature probe measures some quantity intermediate between free stream static temperature, and stagnation temperature. The fraction of stagnation temperature rise "recovered" by the probe is defined by the recovery factor r:

$$r = \frac{T_r - \tau_{TS}}{T_{TS} - \tau_{TS}}. \tag{36}$$

Note that, if r =1, the temperature $T_r = T_{TS}$; while if r =0, then $T_r = \tau_{TS}$. The ratio of stagnation to static temperature can be written as a function of Mach number with Eq.(26). In terms of T_r, this relation gives for the parameter Θ_T:

$$\Theta_T = \frac{T_{TS}}{T_r} = \frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}rM^2}, \tag{37}$$

with sensitivity:

$$S_T = \frac{\left(1+\frac{\gamma-1}{2}M^2\right)\left(1+\frac{\gamma-1}{2}rM^2\right)}{(\gamma-1)M^2(1-r)}, \tag{38}$$

3.3 Density

Density measurements are, of course, limited to compressible gas flows and principally make use of optical or radiation techniques. One method of deducing Mach number from the density is to obtain the experimental ratio $\frac{\rho_{TS}}{\rho_{TS}^{static}}$, where ρ_{TS}^{static} is the static density in test section. The density of air at rest may be deduced from measurements of stagnation pressure and temperature by means of the equation of state. If the air is approximately at rest, such as upstream of the WT nozzle then stagnation pressure and temperature are easily measured. The most common instrument for to measure the local density is the interferometer. Considering the density parameter as the ratio of stagnation and static density:

$$\Theta_\rho = \frac{\rho_{TS}}{\rho_{TS}^{static}} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}. \tag{39}$$

From Eq. (33), the correspondent sensitivity will be:

$$S_\rho = \frac{1+\frac{\gamma-1}{2}M^2}{M^2}. \tag{40}$$

4. Control Problem

The primary reason for installing a good controller to a wind tunnel is to significantly improve flow quality in the test section. The required flow steadiness may vary with the type of tunnel. For a typical airplane test, criteria such as less than 1.0 percent of error in *Cd* and *Cp* are usually sufficient. To meet those criteria, the Mach number steadiness in the test section must stay close to ± 0.3 percent at M = 3.0 (Marvin, 1987). According to Eq.(27) this control can be obtained from different ways. The first option is to control just the stagnation pressure of settling chamber in order to keep the nozzle throat (A_t) chocked at the design conditions. Another option is to control the difference of stagnation pressure in test section and settling chamber. It will be necessary different reference parameter (set point) in both cases.

The objective in setting up the controller parameters for the valve is to minimize the initial transient duration to obtain as long steady run time as possible. The control process needs a model of the pressure transmitter, the digital valve controller and the automatic ball valve to perform the SWT's control. The stagnation pressure is converted to current signal by a pressure transmitter located upstream the nozzle. Then this signal feeds the digital valve controller. The controller has two parameters that can be changed to maintain a steady settling pressure, a proportional gain (K_p) and an integral gain (K_i). The digital valve controller compares the stagnation pressure with a set pressure and derives a corrective output signal according to the setting of these two parameters. These parameters may be modified to increase the process performance. Typically, the transfer function of the PI controller is:

$$G(s) = \frac{\theta(s)}{E(s)} = K_p\left(1 + \frac{1}{K_i s}\right), \tag{41}$$

where $\theta(s)$ is the valve opening position and $E(s) = P_{stagnation}^{setpoint} - P_{stagnation}(s)$ is the error signal between the reference input $P_{stagnation}^{setpoint}$ (desired stagnation pressure), and the output of the system $P_{stagnation}(s)$ which represent the actual pressure measured. Applying the inverse Laplace transform, the differential relationship between the input $E(s) = P_{stagnation}^{setpoint} - P_{stagnation}(s)$ and output $\theta(t)$ of the PI controller is:

$$\frac{d\theta(t)}{dt} = -K_p \frac{dP_{stagnation}(t)}{dt} + \frac{K_p}{K_i}\left(P_{stagnation}^{setpoint} - P_{stagnation}(t)\right), \tag{42}$$

5. Numerical Implementation

From the preceding discussion, expressions were obtained which describe the behavior of the SWT and the control systems. These are summarized here (keeping the original reference numbers):

- SUPERSONIC WIND TUNNEL:

Storage Tank:

$$\frac{d\rho_T}{dt} = -\frac{1}{V_T}\dot{m}_v, \tag{1}$$

$$\frac{dP_T}{dt} = -\left(\frac{\gamma R T_T}{V_T}\right)\dot{m}_v, \tag{3}$$

Control Valve:

$$\dot{m}_v = \frac{2.295810^{-8}}{\sqrt{T_T}} C_g P_T \sin\left(2.71\sqrt{\frac{\Delta P}{P_T}}\right), \tag{4}$$

Settling Chamber:

$$\frac{dP_0}{dt} = \left(\frac{\gamma R T_T}{V_0}\right)(\dot{m}_v - \dot{m}_t), \tag{7}$$

Nozzle:

$$\dot{m}_t = P_0 A_t \left(\frac{\gamma}{R T_T}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, \tag{8}$$

Test Section:

$$\frac{dP_{TS}}{dt} = \left(\frac{\gamma R T_T}{V_{TS}}\right)(\dot{m}_t - \dot{m}_{dif}), \tag{13}$$

Diffuser:

$$\dot{m}_{dif} = P_{TS} A_{dif} \left(\frac{\gamma}{R T_T}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}. \tag{32}$$

- CONTROL DEVICES:

Valve Angle:

$$\frac{d\theta(t)}{dt} = -K_p \frac{dP_{stagnation}(t)}{dt} + \frac{K_p}{K_i}\left(P_{stagnation}^{setpoint} - P_{stagnation}(t)\right). \tag{42}$$

The above equations compose a system of five first-order nonlinear differential equations, in the time, with four state variables: ρ_T, P_T, P_0, P_{TS} and one control variable: $\theta(t)$. The inputs of this system are: geometrical configuration, design conditions for nozzle (first throat) and diffuser (second throat); the relation $\theta(C_g)$, which determines the control valve behavior, according to changes in C_g and initial conditions for all subsystems. In reference Silva et al. (2006) it is possible to find the block diagram used for stagnation pressure control system at settling chamber. This work uses another option for control, Fig. 5.

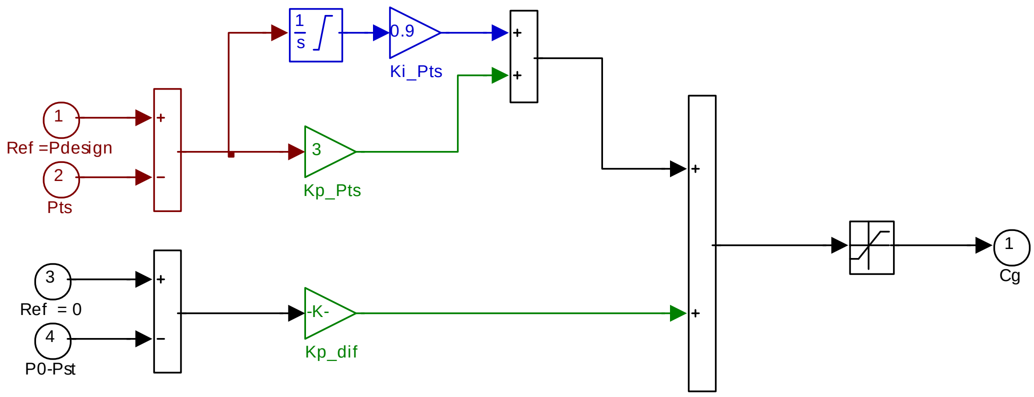

Figure 5 – Control-loop based on stagnation pressure at test section and settling chamber

6. Results

6. 1. Configuration of the Simulation

In order to compare the experimental results those from the mathematical model simulation, it was established the same conditions adopted by Fung (1987) for the present case, as shown in Fig. 6 (SI units). The research of Fung (1987) deals with the solution of the stagnation pressure control problem at the settling chamber in the SWT. So, the mathematical model just included the tank, valve, settling chamber, nozzle and controller. However, it is interesting to note that the experimental facility had a supersonic diffuser, Fung (1988, Fig. 1). This reference case is a good test to evaluate the concordance among different mathematical models.

By adding a controller in a feedback loop to the wind tunnel plant, the mathematical model for the closed-loop system is established. The geometrical design parameters and proportional (K_p) and integral (K_i) gains for the control parameters are shown in Fig. 6. These values were adopted in the simulations with stagnation pressure control at the settling chamber.

6. 2. Evaluation of Test Configuration

Table 2 shows the results from experimental data and the present work during a tunnel run. It can be seen that the performance of the real wind tunnel is even better than the simulation. Fung (1987) obtained similar results. The reason is the assumption of an adiabatic process in the simulation. In reality, heat transfer takes place particularly through the large tank surface during the test. While the tank temperature decreases during the test, a finite amount of heat is transferred from the tank walls to the inner air. This leads to a higher tank temperature as well as a higher tank pressure than predicted by the model, Fung (1987). In Fig.7 is shown the behavior of the system at Mach 2.5. The results are expressed in terms of stagnation pressure and stagnation temperature at the settling chamber ($PP_0 = \frac{P_0}{P_0^{design}}$, $TT_0 = \frac{T_0}{T_0^{design}}$ respectively) and stagnation pressure and Mach number at the test section ($PPst = \frac{P_{ST}}{P_{ST}^{design}}$, $MM = \frac{M}{M^{design}}$, respectively). It was used the stagnation pressure control at the settling chamber, Silva et al. (2006). It can be conclude that the control system based on the stagnation pressure at the settling chamber was found to be satisfactory. Curiously, for this particular configuration, it was not find significantly variation in angle of valve. Thus, this control would be run manually. Finally, it can be observed that the constant average controller parameters found above are effective at all Mach number (2.5 to 4.0) in obtaining a response with a minimum steady-state error and overshoot with a minimum settling time.

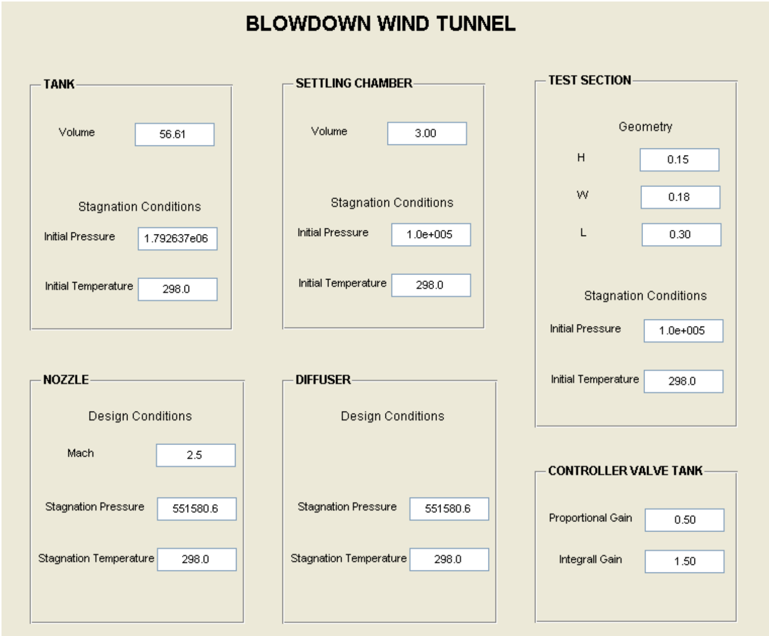

Figure 6 – Initial and required conditions (SI units)

Table 2 – Comparison of results from simulation and experimental data (P_T = 260 psia)

Mach	P_0 [Psia]	Run Time Experimental [s]	Run Time Present Work [s]
2.5	80	55	49
3.0	110	50	45
3.5	160	40	32

Another interesting simulation is shown in Fig. 8. In this case, the controller was configured to correct the stagnation pressure at the test section. Analogously to the simulation with control of stagnation pressure of settling chamber it was possible to obtain the desirable conditions (set point). However, it was necessary to use the control loop illustrated in Fig. 5. Note that, in order to reach the desirable conditions stagnation it is necessary to control stagnation pressure at the test section and the pressure relation $\frac{P_0}{P_{TS}}$, since all the mathematical formulation was based on isentropic process. In this case it was used another group of gains (Fig.5), which is: branch P_{TS} : K_{p_TS} = 3.0 and K_{i_TS} = 0.9, branch $(P_0 - P_{TS})$: K_{p_dif} = 0.015. It is important to say that all analysis of the Mach number was run only during the period in that the nozzle was chocked. Obviously, depending on the values adopted to the gains, it will occur different delays and overshoots in the initial response. Another question, since the Mach number is related to the stagnation pressure at the test section, stagnation pressure at the settling chamber and stagnation temperature of system, it is not easy work to find the correct gain values in the control loop. Thus, it is preferable to choose a control system with minimum possible number of parameters for Mach number control gain.

From the preceding results obtained with the isentropic approach, it is only necessary the control of stagnation pressure at the settling chamber in order to control the Mach number at the test section. According to Pope and Goin (1965), there are two ways in which blow-down WT are customarily operated: with stagnation pressure constant or with constant mass flow. For constant mass runs the stagnation temperature must be held constant and either a heater or a thermal mass external to the tank is required. For constant stagnation pressure (settling chamber), the only control necessary is a pressure regulator that holds the stagnation pressure constant. Thus, this mathematical model is an interesting tool to analyze configurations of test, which require Mach number control.

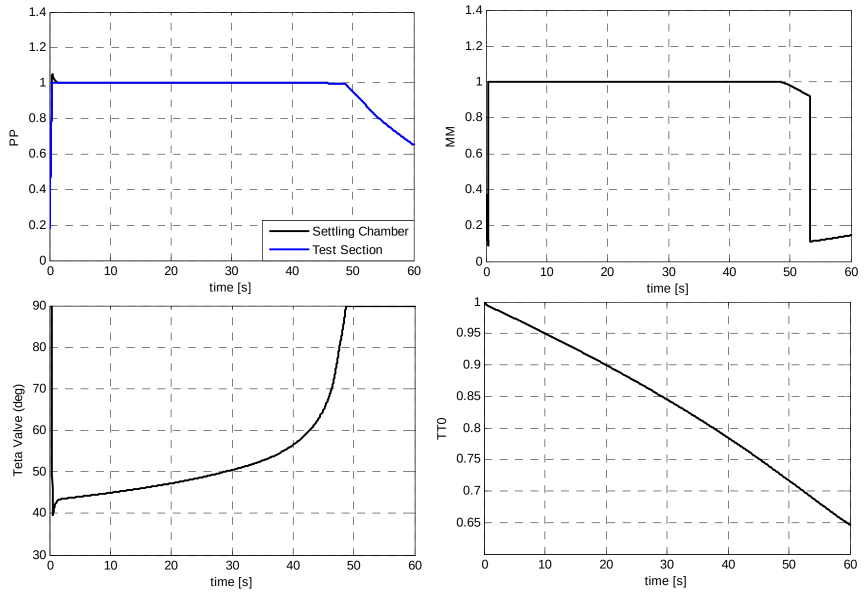

Figure 7. Results obtained with control of stagnation pressure at the settling chamber

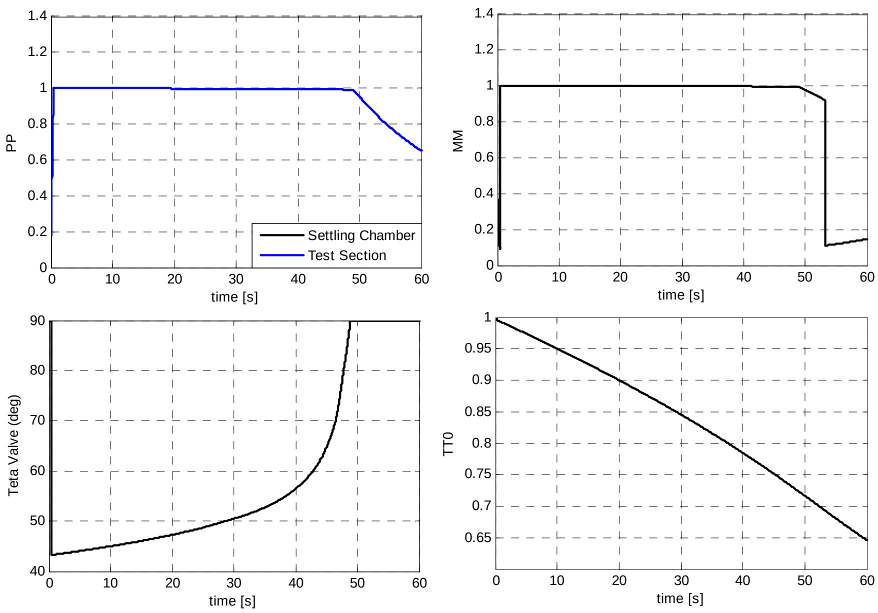

Figure 8. Results obtained with control of stagnation pressure at the test section

6. 3. Sensitivity Analysis

In Fig. 9 is shown the sensitivity analysis for the design condition (M = 2.5). Among all the characteristics of this analysis it is important to observe the recovery factor r. Depending on this value, and therefore on the uncertainty of measuring instruments of this value, the results from the analysis of data change significantly. Figure 9 can be used to help the choice of experimental methodology “pressure method” and “density method” that must be used to estimate the test section Mach number, under this simulated condition. Now, consider a sinusoidal perturbation in stagnation pressure at the test section with amplitude 0.05 and frequency 2π rad/sec, namely: $P_{TS}^{new} = P_{TS}\left[1 + 0.05Sin(time)\right]$. The variations of test section Mach number and stagnation pressure are shown in Fig.10. It is worth noting here that the system presents different amplitudes for P_{TS} and Mach number. This result is corroborated when analyzing Eq. (28). Additionally, the variation in P_{TS} does not affect the run time. Since the simulation is configured to analysis only when the nozzle is chocked, the variations in P_{TS} do not affect the settling chamber conditions, and consequently, the run time. Should be note that this methodology can be applied to verify the limits of stability of SWT in a future work. Figure 11 shows that the sensitivity coefficient does not change significantly during quasi-steady flow. Therefore, it can be concluded that the "pressure method" and "density method" are more indicated for this conditions of test, since the variations in the values of stagnation pressure at the test section is in the range of ±5%.

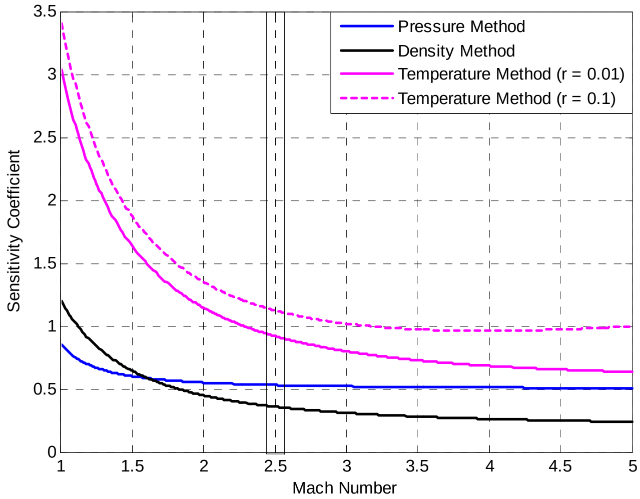

Figure 9. Sensitivity coefficient for design condition

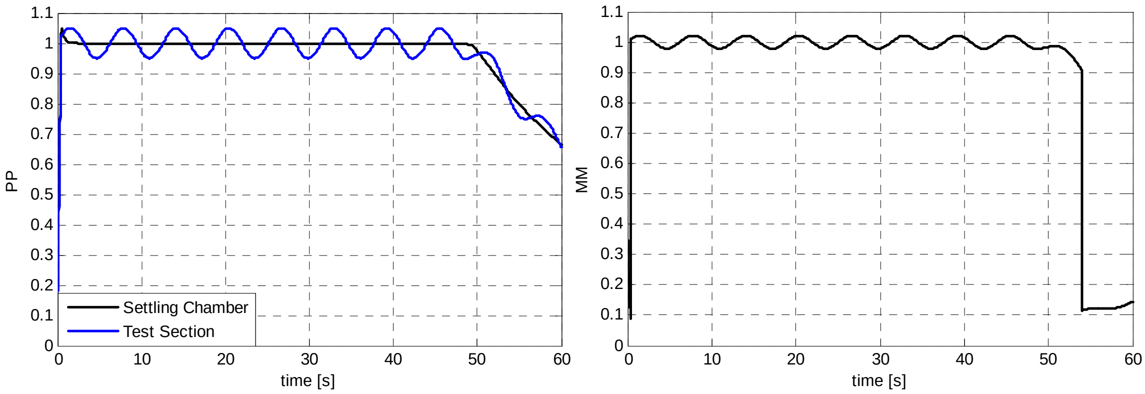

Figure 10. Thermodynamic conditions at the test section for sinusoidal variation of P_{TS}

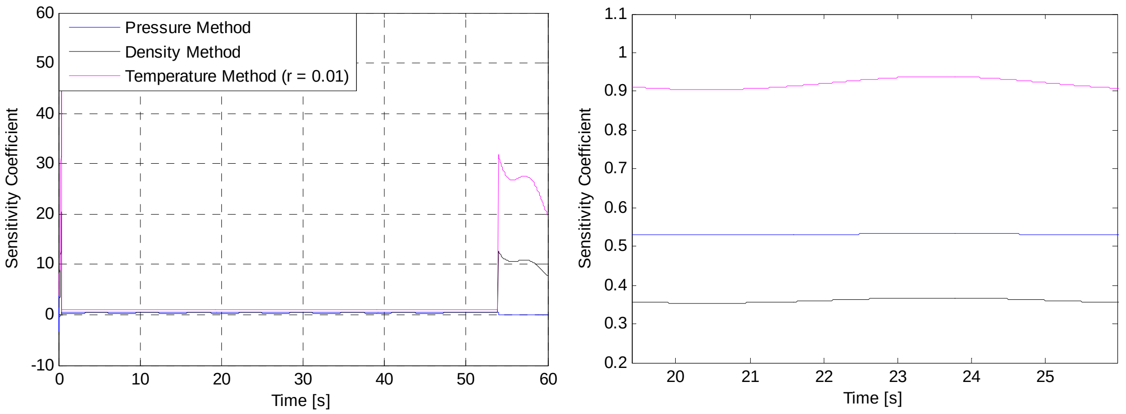

Figure 11. Sensitivity coefficient for sinusoidal variation of P_{TS}

7. Conclusions

The problem of flow in blow down wind tunnel test has been studied. The variation of Mach number at the test section has been investigated using isentropic approach. Many aspects of this formulation have been discussed and the positive and negative aspects were presented. The following conclusions were drawn from the results of simulations:

(i) Isentropic approach can be used for preliminary project of control system based on stagnation pressure at the settling chamber or test section. According to the single-loop adopted in these analyses, the former is more indicate since it uses only two gains;

(ii) Sensitivity analysis coupled with isentropic approach shows that it is important to help in the choice of measuring instrument;

(iii) The mathematic formulation presented to the Mach number can be an interesting tool to be used in analysis of stability of SWT.

Finally, it is interesting to improve this mathematical model in order to provide a design tool of blow down wind tunnel test. Among these improvements it should be implemented:

(i) Isentropic approach by parts in order to include loss factors;

(ii) Linear analysis of controller in order to automate the gain searching process;

(iii) Optimization tool for design of SWT. Coupled analysis that use design parameters (geometrical configuration, stagnation pressure-settling chamber, run time and Mach number) in a definition of geometric and thermodynamic characteristics of SWT.

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