INVERSE DYNAMICS OF REDUNDANTLY ACTUATED FOUR-BAR MECHANISM USING AN OPTIMAL CONTROL FORMULATION

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Abstract. *Closed kinematic chain mechanisms are extensively used in several applications, from machine tools to biomechanical models. A few works however address the mechanics of these systems with redundant actuation, i.e., with more actuators than degrees of freedom. The inverse dynamics analysis of this kind of system does not possess an unique solution, and therefore optimization procedures should be applied to estimate net joint torques when a kinematics is previously imposed. Literature presents some methods to solve this undetermined problem, based on multi-body system dynamics approach. In this paper the inverse dynamics problem is formulated as an Optimal Control Problem (OCP): to find a set of controls that minimizes an integral state and control variables cost function, subjected to endpoint, trajectory, and control constraints, both equality and inequality. Some possible sets of constraints are explored to force a 3-link open chain system dynamics behave like a four bar mechanism with a crank rotating at a constant velocity. The controls calculated by OCP are assumed to be the input joint torques. The regular case with one torque actuator is solved and compared to the two and three actuators case.*

Keywords: *Optimal Control, Inverse Dynamics, Closed-Chain Mechanisms, Redundant Mechanisms*

1. INTRODUCTION

There are a plenty of mechanisms with closed kinematic chain, like 4-bar, quick return, Peaucellier's, Altmann's, Bennett's mechanisms, etc (Doughty, 1988; Gao et al., 2001; Stejskal and Valasek,1996). They have been studied for more than a century and a half and played an important role in industrial revolution. Some closed kinematic chain mechanisms have been revisited in the last decades, in the context of biomechanics, because they are suitable to describe human body motions like in cycling, rowing, and stance phase of gait (Hull and Jorge, 1985; Lee et al., 2005; Pandy and Berme, 1998). These biomechanical models are characterized by redundancy because the number of control inputs (muscle forces) are fairly bigger than the number of degrees of freedom (DOF). Even if simplified models are considered, where the effect of muscles are all "packed" into net joint moments, this redundancy is usually still present. This redundancy introduces a multiplicity of solutions when a regular inverse dynamics calculation is attempted, because the number of unknowns exceeds the number of equations.

On the other hand, some authors realized that closed kinematic chain structures could be advantageous in industrial robots and machine tools because they have several advantages over their serial counterparts, like: high mechanical stiffness (Miller, 2001), high trajectory accuracy (Nakamura and Ghodoussi, 1989), positioning accuracy, high load capacity (Dasgupta and Mruthyunjaya, 1998) and small mobile mass (Miller, 2001). However, the existence of kinematical singularities is a key problem in the analysis of closed chain mechanism. Liu et al. (2001), Cheng et al. (2003) and Valasek et al. (2004), suggest the use of redundant actuation to circumvent this problem. Some methods where proposed by Nakamura and Ghodoussi(1989), Cheng et al. (2003) and Lee et al. (2002) to solve the inverse dynamics of redundant actuation closed kinematics mechanisms, but they are all based on the minimization of a Euclidean 2-norm (Nakamura and Ghodoussi 1989) or the use of Moore-Penrose pseudo inverse matrix (Cheng et al., 2003 and Lee et al., 2002).

Optimal control theory offers a new perspective over this problem. Mainly because the quantity that is desired to be minimized (or maximized) can be freely chosen. It is also a promising field in the investigation of optimal open-loop control (Terceiro and Fleury, 2004) and optimal trajectory design (Betts, 2000). Specifically in the field of biomechanics it is been used to estimate muscle forces (Menegaldo et al., 2006; Kaplan and Heegaard, 2001) and to compute net joint torques trajectories (Ashby and Delp, 2006; Koh and Jennings, 2003). Although optimal control theory has been well established since the 60's, it's increasing popularity is partially due to the availability of computer packages - like RIOTS (Schwartz, 1996) and MISER (Goh and Teo, 1988) - that are able to deal with large scale problems, avoiding complicated non-linear equations that are untreatable algebraically.

In this paper an optimal control formulation is presented to solve a redundantly actuated 4-bar mechanism. The obtained torque actuator trajectories are "*optimal control functions*" ($\mathbf{u}^*(t)$), rather than "*optimal control laws*" ($\mathbf{u}^*(\mathbf{x}(t), t)$). The 4-bar mechanism was chosen because it is one of the simplest closed chain mechanisms known. Therefore it could be used as a test bed to experiment different formulations of the OCP and to choose the appropriate values of model and simulation parameters. The main objectives of this paper are: (i) to calculate optimal control torque functions for the redundantly actuated 4-bar mechanism (with 2 and 3 actuators); (ii) to calculate optimal control torque functions for a single actuated 4-bar mechanism and compare it with a regular inverse dynamics analysis; (iii) to gain insight and experience in the formulation and computational implementation of the OCP to further use it in the implementation of more complicated biomechanical problems.

2. METHODS

2.1 Generic Optimal Control Formulation

An optimal control problem (OCP) can be formulated, (Citron, 1969; Menegaldo, 2001; Bottasso and Croce, 2004), as to determine the optimal states $\mathbf{x}^*$, the optimal controls $\mathbf{u}^*$ and possibly the final time T that minimize a cost function G_o (Eq.1a). The minimization problem is subjected to the state equations, Eq. (1b) that describe the dynamics of the system, and various possible additional constraints as required by the problem at hand, such as for example, trajectory constraints Eq. (1f), boundary conditions Eqs. (1c) and Eq. (1d), and control upper/lower bounds Eq. (1e).

$$\text{min: } G_o(\mathbf{x},\mathbf{u},t) = g_o(\mathbf{x},\mathbf{u},t) + \int_0^T f_o(\mathbf{x}(t),\mathbf{u}(t),t)dt \qquad \text{Objective Funtion} \qquad (1a)$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x},\mathbf{u},t) \qquad \text{Equation of Motion} \qquad (1b)$$

$$\mathbf{x}(0) = \mathbf{x}^0 \qquad \text{Inicial Condition} \qquad (1c)$$

$$\mathbf{x}(T) = \mathbf{x}^T \qquad \text{Final Condition} \qquad (1d)$$

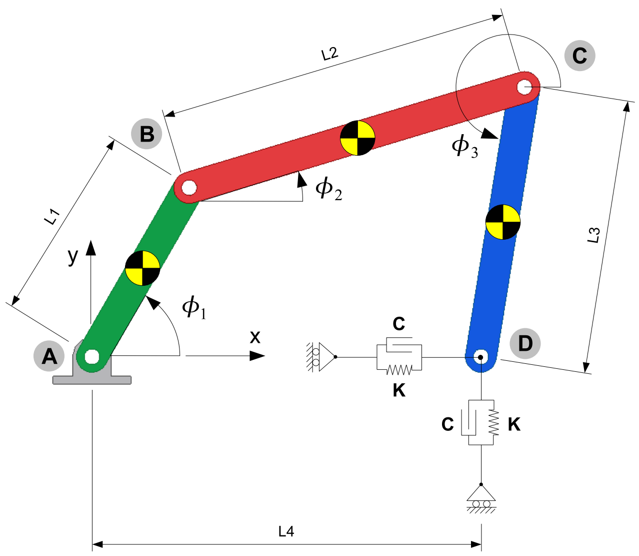
$$\mathbf{u}^{min} \leq \mathbf{u}(t) \leq \mathbf{u}^{max} \qquad \text{Control Bounds} \qquad (1e)$$

$$\mathbf{x}^{min} \leq \mathbf{x}(t) \leq \mathbf{x}^{max} \qquad \text{Trajectory Constraint} \qquad (1f)$$

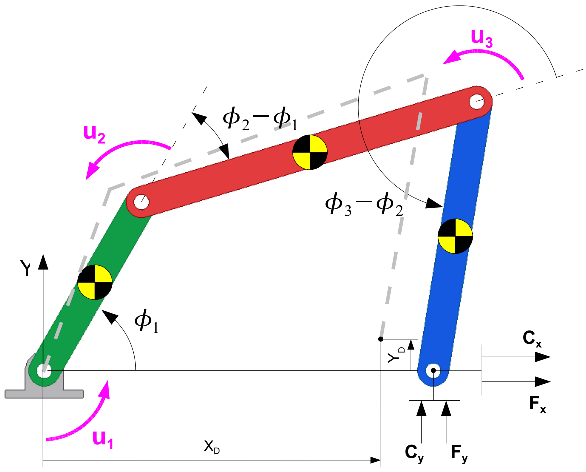
2.2 Dynamical Model

Equation (1b) represents the equations of motion of the dynamical system. The system considered is a planar, frictionless pin-joined, triple pendulum, with joint torque actuators (controls) that may, or may not be under external spring and damper forces at its most distal extremity, as depicted in Figure (1). Designing the system this way, it can represent a triple pendulum if stiffness (K) and damping (C) constants are set to zero. On the other hand, if K and/or C are large enough, then an external horizontal and vertical force will be applied such as to maintain Point D fixed. By doing this way, 2 DOFs of the system are restricted, forcing it to behave approximately like a 4-bar mechanism.

This "fictional" 4-bar mechanism was modeled this way because closed-chain mechanisms usually require the incorporation of "positions loop" constraints into the equations of motion, which leads to a system of Differential Algebraic Equations (DAE) (Brenan et al.,1989). Although DAEs can be converted into a state space form, suitable to be used by well established first order integrators, it suffers from constraint stabilization problems (Yu and Chen, 2000; Pennestrì and Vita, 2004). This condition is not desired because DAE requires more sophisticated integrators which are not present at current version of RIOTS'95.



(a) Nomenclature and coordinates



(b) Control forces and restriction forces

Figure 1. Dynamical Model Diagram

The left hand side of triple pendulum's equations of motion can be easily derived, by means of Lagrange's Equations Eq.(2), since the positions of the centers of mass of each bar can be described as function of independent angular coordinates ϕ_1, ϕ_2, ϕ_3, as well as the potential and kinetic energy of the hole system.

$$\frac{d}{dt}\frac{\partial L}{\partial\{\dot{\phi}\}} - \frac{\partial L}{\partial\{\phi\}} = \{Q\} \tag{2}$$

The resulting system of equations takes the form of Eq. 3, where $[\mathbf{A}]$ is the mass matrix, $[\mathbf{B}]$ is the centripetal terms matrix, $[\mathbf{C}]$ is the vector gravitational terms, $[\mathbf{D}]$ is a matrix of ones and zeros that relates the torque actuators to the coordinates where they act upon and $[\mathbf{E}]$ is a vector the encompasses the spring and damper forces. The right hand side of Eq. 3 is equal to $\{Q\}$ in Eq.(2) and is usually termed as "Generalized Force Vector". Due to space limitations, only the elements of matrix $[\mathbf{E}]$ are shown at Eqs.(4a) to (4c)

$$\begin{bmatrix} & & \\ & \mathbf{A} & \\ & & \end{bmatrix}\begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \end{bmatrix} + \begin{bmatrix} & & \\ & \mathbf{B} & \\ & & \end{bmatrix}\begin{bmatrix} \dot{\phi}_1^2 \\ \dot{\phi}_2^2 \\ \dot{\phi}_3^2 \end{bmatrix} + \begin{bmatrix} \\ \mathbf{C} \\ \\ \end{bmatrix} = \begin{bmatrix} & & \\ & \mathbf{D} & \\ & & \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} + \begin{bmatrix} \\ \mathbf{E} \\ \\ \end{bmatrix} \tag{3}$$

$$\begin{aligned} E_1 = &- L_1(C(L_1\dot{\phi}_1 + L_2\dot{\phi}_2\cos(\phi_1 - \phi_2) + L_3\dot{\phi}_3\cos(\phi_1 - \phi_3)) \\ &+ K(L_4\sin(\phi_1) - L_2\sin(\phi_1 - \phi_2) + L_3\sin(\phi_1 - \phi_3))) \end{aligned} \tag{4a}$$

$$\begin{aligned} E_2 = &- L_2(C(L_2\dot{\phi}_2 + L_1\dot{\phi}_1\cos(\phi_1 - \phi_2) + L_3\dot{\phi}_3\cos(\phi_2 - \phi_3)) \\ &+ K(L_1\sin(\phi_1 - \phi_2) + L_4\sin(\phi_2) - L_3\sin(\phi_2 - \phi_3))) \end{aligned} \tag{4b}$$

$$\begin{aligned} E_3 = &- L_3(C(L_3\dot{\phi}_3 + L_1\dot{\phi}_1\cos(\phi_1 - \phi_3) + L_2\dot{\phi}_2\cos(\phi_2 - \phi_3)) \\ &+ K(L_1\sin(\phi_1 - \phi_3) + L_2\sin(\phi_2 - \phi_3) + L_4\sin(\phi_3))) \end{aligned} \tag{4c}$$

Equation (3) was transformed to state space form with a Matlab routine developed by the authors named EOM2SS, which stands for *Equation of Motion to State Space*, (Silva and Menegaldo, 2007). It was designed to make the coordinates transformation shown below in order to transform 2^{nd} order ODE's into an equivalent set of 1^{st} order ODE's.

$$\begin{aligned} x_1 &= \phi_1 \xrightarrow{d/dt} \dot{x}_1 = \dot{\phi}_1 = x_4 \\ x_2 &= \phi_2 \xrightarrow{d/dt} \dot{x}_2 = \dot{\phi}_1 = x_5 \\ x_3 &= \phi_3 \xrightarrow{d/dt} \dot{x}_3 = \dot{\phi}_1 = x_6 \\ x_4 &= \dot{\phi}_1 \xrightarrow{d/dt} \dot{x}_4 = \ddot{\phi}_1 \\ x_5 &= \dot{\phi}_2 \xrightarrow{d/dt} \dot{x}_5 = \ddot{\phi}_2 \\ x_6 &= \dot{\phi}_3 \xrightarrow{d/dt} \dot{x}_6 = \ddot{\phi}_3 \end{aligned}$$

where:

$$\begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \end{bmatrix} = [\mathbf{A}]^{-1}\left([\mathbf{D}]\,\mathbf{u} + [\mathbf{E}] - ([\mathbf{B}]\,\dot{\phi}^2 + [\mathbf{C}])\right)$$

2.3 Characterization of Studied Cases

Once the equations of motion were obtained in state space form, 7 different cases were identified, which can be classified into 2 main groups: Unrestricted and Restricted. In the Unrestricted group we consider that the triple pendulum has no force acting on Point D. That is, no force restricts the movement of the triple pendulum except the control forces. In the Restricted group a spring force or a spring and damper force acts in order to maintain Point D fixed, and they increase as Point D deviates from its original position.

In Case 1, triple pendulum has 3 torque actuators, one at each joint, and no restriction force, like a robotic arm. In Case 2, a spring force is introduced in horizontal and vertical directions. In Case 3 we consider the unrestricted triple pendulum with a sole actuator at joint A. The Case 4, is just like Case 3 with a spring force restriction. Case 5 differs from Case 4, in that a damper force is introduced. The 2 actuators case is considered in Case 6 with spring and damper restriction. Finally, in Case 7, we return to the 3 actuators case, now considering a restriction of spring and damper force. This is all shown at Tab. 1.

Table 1. Overview of Studied Cases

# Actuators	Unrestricted	Restricted	
		Spring	Spring + Damper
1	CASE 3	CASE 4	CASE 5
2	X	X	CASE 6
3	CASE 1	CASE 2	CASE 7

The different cases were enumerated this way because it was the order the simulations were performed. We decided to maintain this numbering scheme because, as we'll see, at each case there was a step we have learned that certain parameters and/or strategies were better or not for the numerical convergence of the problem, and this knowledge was subsequently applied to the following step.

2.4 Specific Optimal Control Formulation

As it was shown at Section 2.1 an optimal control problem is defined by a minimization of an objective function subject to the equations of motion and may contain additional trajectory, control bounds, initial and final condition constraints. The right choice of objective function and constraints in the formulation of OCP is relevant for the convergence of the simulation. In this work we focused in the minimization of the squared actuator torques, as shown in Eq. 5. The parameters $w_1 \dots w_3$ are ones/zeros factors that adequate the objective function to the case studied. Their values are shown at Tab. 2.

Another important issue was the formulation of the OCP's constraints. The most noticeable one is that the triple pendulum must obey the position loop equations of the 4-bar mechanism. But this sole condition did not prove to be sufficient to impose a 4-bar mechanism behavior to the triple pendulum. Several attempts with different constraints were made. The one which best presented convergence properties is described in Eqs. 8a to 8e. It represents the mechanism position loop equations, its derivative and an additional equation to impose that $\dot{\phi}_1$ should be constant. They were transformed into a Trajectory Inequality Constraint squaring and summing it all into a single equation (Eq. 7) that should be smaller than as small parameter named $EPSNEQ$, whose value varied typically from 10^{-1} to 10^{-3}.

$$\text{min: } f(\mathbf{u}) = \int_0^{tf} w_1 {U_1}^2 + w_2 {U_2}^2 + w_3 {U_3}^2 dt \tag{5}$$

subject to:

Equation of Motion:

$$\{\dot{\mathbf{x}}\} = \mathbf{g}(\mathbf{x}, \mathbf{u}) \tag{6}$$

Trajectory Inequality Constraint:

$$(f_1(\mathbf{x}, \mathbf{u}))^2 + (f_2(\mathbf{x}, \mathbf{u}))^2 + (f_3(\mathbf{x}, \mathbf{u}))^2 + (f_4(\mathbf{x}, \mathbf{u}))^2 + (f_5(\mathbf{x}, \mathbf{u}))^2 \leq EPSNEQ \tag{7}$$

where:

$$f_1(\mathbf{x}, \mathbf{u}) = L_1 \cos(\phi_1) + L_2 \cos(\phi_2) + L_3 \cos(\phi_3) - L_4 \tag{8a}$$

$$f_2(\mathbf{x}, \mathbf{u}) = L_1 \sin(\phi_1) + L_2 \sin(\phi_2) + L_3 \sin(\phi_3) \tag{8b}$$

$$f_3(\mathbf{x}, \mathbf{u}) = -L_1 \dot{\phi}_1 \cos(\phi_1) - L_2 \dot{\phi}_2 \cos(\phi_2) - L_3 \dot{\phi}_3 \cos(\phi_3) \tag{8c}$$

$$f_4(\mathbf{x}, \mathbf{u}) = -L_1 \dot{\phi}_1 \sin(\phi_1) - L_2 \dot{\phi}_2 \sin(\phi_2) - L_3 \dot{\phi}_3 \sin(\phi_3) \tag{8d}$$

$$f_5(\mathbf{x}, \mathbf{u}) = \dot{\phi}_1 - 2\pi \tag{8e}$$

The OCP was implemented and solved with RIOTS'95 (Recursive Integration Optimal Trajectory Solver) which is a group of programs and utilities, written mostly in C and designed as a toolbox for Matlab, that provides an interactive environment for solving a very broad class of optimal control problems. The numerical methods used by RIOTS'95 are based at Consistent Approximations Theory. According to Schwartz et al, (1997) "a solution is obtained as an accumulation point of the solutions to a sequence of discrete-time optimal control problems that are, in a specific sense, consistent approximations to the original continuous-time, optimal control problem. The discrete-time optimal control problems are constructed by discretizing the system dynamics with one of four fixed step-size Runge-Kutta integration methods and by representing the controls as finitedimensional B-splines. The integration proceeds on a (possibly non-uniform) mesh that specifies the spline breakpoints." More information regarding to RIOTS'95 can be found at Schwartz (1996).

In all cases, the simulations tried to reproduce 1 cycle of a 4-bar mechanism working steady-state regimen, with a constant crank angular velocity of 2π rad/s. Thus, to complete 1 cycle it is necessary $tf = 1$ s of simulation, with initial

Table 2. Model parameters / outputs

	[D] Eq. (3)	**Stiffness Factor - K** [N/m]	**Damping Factor - C** [N.s/m]	**Final Obj. Func. Value**	**CPU time** (min)	**Obj. Func. Weighting Parameters** Eq.(5)
CASE 1	$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$	0	0	75079	46	$w_1 = 1, w_2 = 1, w_3 = 1$
CASE 2	Idem CASE 1	$1,0 \cdot 10^3$	0	43644	43	$w_1 = 1, w_2 = 1, w_3 = 1$
CASE 3	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	0	0	-	-	$w_1 = 1, w_2 = 0, w_3 = 0$
CASE 4	Idem CASE 3	$1,0 \cdot 10^7$	0	10721	301	$w_1 = 1, w_2 = 0, w_3 = 0$
CASE 5	Idem CASE 3	$5,0 \cdot 10^6$	$1,0 \cdot 10^6$	9694	14	$w_1 = 1, w_2 = 0, w_3 = 0$
CASE 6	$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$5,0 \cdot 10^7$	$1,0 \cdot 10^7$	4172	8	$w_1 = 1, w_2 = 1, w_3 = 0$
CASE 7	Idem CASE 1	$1,0 \cdot 10^5$	$1,0 \cdot 10^4$	22058	455	$w_1 = 1, w_2 = 1, w_3 = 1$

conditions of: $\phi_1 = 1.0472$ rad (60.0^o), $\phi_2 = 0.2907$ rad (16.5^o), $\phi_3 = 4.5513$ rad (260.7^o), $\dot{\phi}_1 = 6.2832$ rad/s (360^o/s), $\dot{\phi}_2 = -1.3760$ rad/s (78.8^o/s) and $\dot{\phi}_3 = 3.4240$ rad/s (196.1^o/s). It was adopted a time discretization mesh of 200 points. It means that, as the solution of the optimal control problem represented by a finite dimensional B-spline, that spline would have $200 + \rho - 1$ breakpoints. In all cases, a cubic ($\rho = 4$) spline was used. Finally geometric/mass parameters used were: $L_1 = 0.5$ m, $L_2 = 0.9$ m, $L_3 = 0.7$ m, $L_4 = 1.0$ m, $m_1 = 6.59$ kg, $m_2 = 11.55$ kg, $m_3 = 9.07$ kg. Centers of mass are located in the middle of each bar and its respective moment of inertia was computed with $I = \frac{1}{12}mL^2$.

3. RESULTS

Before proceeding the optimal control simulations, a regular kinematic and inverse dynamics analysis was performed for the 4-bar mechanism, according to the theory presented by Haug (1989). The trajectories of angular coordinates as well as its velocities are shown at Figures 2a and 2b. All simulations were carried out in an Intel Core 2 Duo E6600, 2.40 GHz desktop computer.

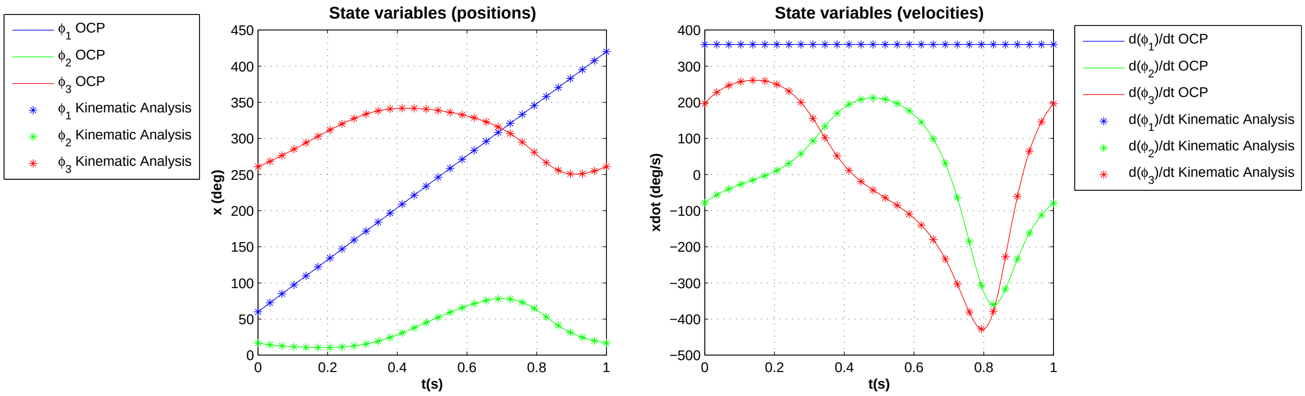


Figure 2. Comparison of state variables trajectories for CASE 1 with kinematic analysis of 4-bar mechanism.

CASE 1: In case 1, a triple pendulum with 3 torque actuators was expected to perform the kinematics of a 4-bar mechanism without any restriction at Point D. This situation could be also interpreted as a movement of a 3-link planar robotic arm that the first link should perform a complete revolution but its end-effector should remain fixed.

It was not possible to obtain an acceptable solution (the algorithm didn't converge) with all conditions imposed. The strategy used was to decrease gravitational acceleration to $g = 4$ m/s^2 (Menegaldo et al., 2004), and decrease final time (for to complete about 80% of the cycle) trying to achieve a normal termination, with a zero vector as initial guess, for the optimal control. Then, the solution obtained with $tf = 0.8$s was used as an initial control guess for the next simulation with $tf = 0.9$s. This process was repeated once more to $tf = 1.0$s with $g = 4$ m/s^2 fixed. Then, the same strategy was used to vary the gravitational acceleration, increasing g by 1m/s^2 at time, up to $g = 9.81$ m/s^2.

The value of $EPSNEQ$, which somehow controls the violation of the constraints (Eq. 7) that force the triple pendulum to behave like a 4-bar mechanism, was initially set to 10^{-3}. A final simulation, using $ESPNEQ = 10^{-5}$ and as initial control guess the optimal control for the trial with $ESPNEQ = 10^{-3}$ and $g = 9.81$ m/s^2; was performed which resulted in a very good reproduction of the states trajectories. This result can be verified at Figure 2 where state variables obtained in the best trial of CASE 1 was plotted over the angular coordinates position and velocity curves obtained with the regular kinematic analysis.

CASE 2: In this case, a horizontal and vertical springs were introduced as shown in Figure 1 to force Point D remain fixed. Although the displacement of Point D in CASE 1 was very small, specially when using $ESPNEQ = 10^{-5}$, it means that if a real 4-bar mechanism were impelled to move with those three torque curves, it would have almost no reaction forces on joint D. At first sight it may sounds positive, but it could be argued that torque actuators are acting against gravity. Therefore, reaction force at Point D would play a role to support part of the weight of the bars, thus relieving torque actuators to do all this job. This hypothesis was confirmed comparing Figures 3a and 3b. Peak forces in all torque curves in CASE 2 were smaller than in CASE 1, and the value of the objective function in CASE 2 was almost 42% smaller than in CASE 1. In fact, the objective funtion in CASE 1 was the highest in all cases studied.

CASE 3: Six attempts were made, with different conditions (for example, $g = 0$ m/s^2, or $tf = 0.05$s), to simulate the system with only one actuator at joint A and no other external forces; but all have failed. In fact, it was already expected because this system is uncontrollable, i.e, the number of actuators is smaller than the number of DOF. Thus, when RIOTS tries to estimate a control, this control cannot take all state variables to the desired state (i.e. respecting all constraints).

CASE 4: In CASE 4, a horizontal and vertical springs were introduced at Point D, with the triple pendulum impelled by only one torque actuator at joint A. With a stiff spring acting in order to maintain Point D fixed, a physical constraint is introduced in the 3-DOF such that it becomes, as a matter of fact, a 1-DOF system. In doing so, we are actually doing a regular inverse dynamics analysis by means of an optimal control formulation. That is, the expected torque curve should be similar to the one found with a regular inverse dynamics analysis.

In this case, a normal termination of RIOTS was very hard to find. In fact, only 1 (which is plotted in Figure 3c) in 25 trials. The strategy adopted in CASE 1 (to begin the simulations with low gravitational acceleration and low final time) didn't worked this time. Nevertheless, even in the failed trials, the kinematics were reasonably reproduced, but control forces were excessively high. This was possibly due to the oscillatory nature of the spring force whose frequency increases as the spring stiffness gets bigger. Up to this moment, a 4th order, fixed step-size Runge-Kutta integrator had been used. So an attempt was made changing Runge-Kutta integrator to a variable step-size LSODA integrator (Radhakrishnan and Hindmarsh, 1993; Petzold, 1983), which is also adequate to integrate “stiff” EDO's[1]. The only one successful trial was obtained with this integrator, using $g = 0$ m/s^2 and $tf = 0.05$s, therefore, the curve at Figure 3c could not be compared to the others because they do not represent the system under a normal acceleration of gravity.

CASE 5: What we have learned from CASE 4 was that the simulations were not converging because of integrators limitations, but probably due to a high frequency excitation force, introduced by the spring that was not being damped. So, the modification introduced in CASE 5 was the damping force acting in parallel with the springs, which in general facilitated the convergence of the problem. We started the trials using fixed step-size Runge-Kutta integrator because RK have shown to be much faster than LSODA. The strategy to begin the simulations with low g and tf was once more employed. Beginning with $g = 0$ m/s^2 held fixed and increasing from $tf = 0.2$s, using the control obtained in the previous trial as initial guess of the next trial, successful terminations were obtained up to $tf = 0.6$s. Changing to LSODA integrator, it was realized that a successful result could be obtained with a zero initial control guess vector, and $tf = 1$s. From now on, 10 successful trials followed each one adopting a bigger gravitational acceleration.

Looking at Figure 3d, it is possible to see that the optimal control solution was quite similar to the torque calculated by a regular inverse dynamics analysis. With this in mind, it is possible to conclude that the model of a triple pendulum with stiff springs and dampers acting at is most distal point is the model that best represents the 4-bar mechanism in this context.

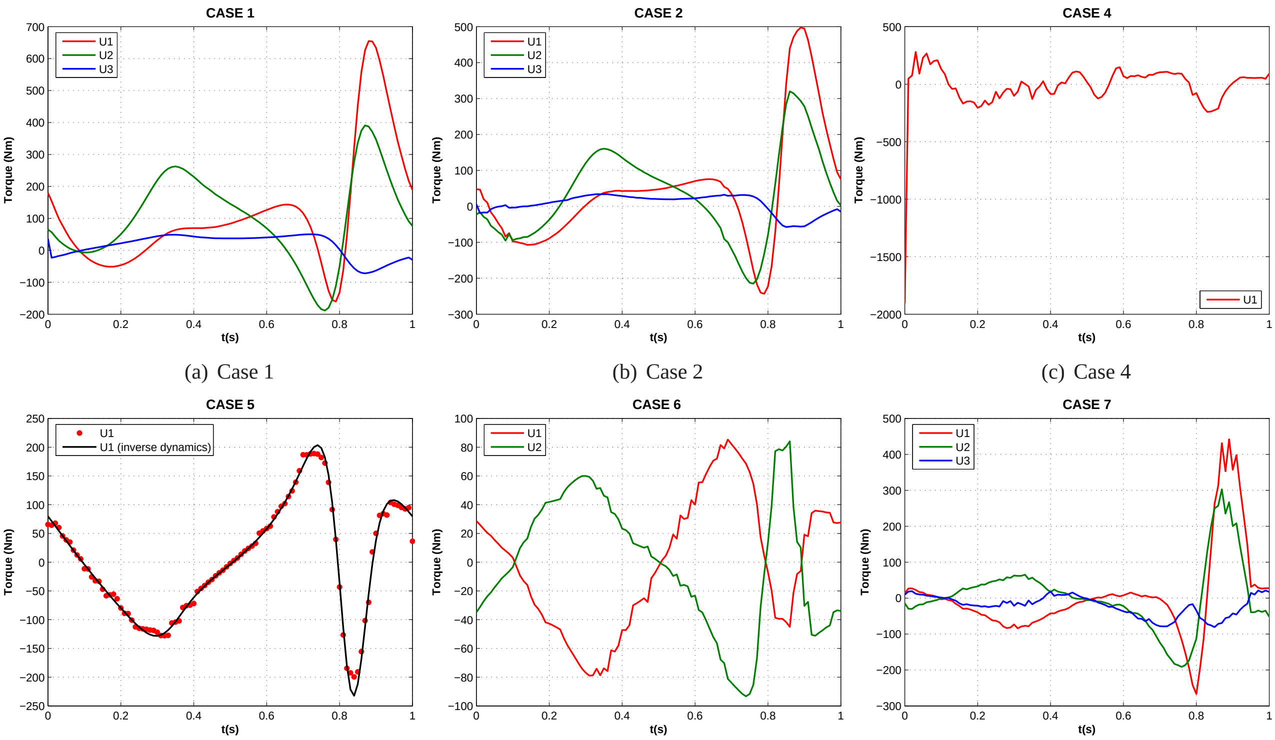
CASE 6: As the introduction of the spring and damper proved to be the best model to represent a single actuated 4 bar mechanism, in CASE 6, this model was employed to calculate torque curves of a 4-bar mechanism with 2 actuators at joints A and B (see Figure 1a). All that was learned from the previous cases was used in the first trial of CASE 6. The consequence was that the first trial had a normal termination using $g = 9.81$ m/s^2, $tf = 1.0$s, $K = 5,0 \cdot 10^4$,

[1] LSODA, as well some other Runge-Kutta integrators, are functions built in RIOTS.

$C = 1,0 \cdot 10^4$, $EPSNEQ = 10^{-1}$, and also LSODA integrator with a zero initial guess control vector. A smoother curve was obtained using higher stiffness and damping constants (shown at Table 2).

Looking at Figure 3e, it is possible to see that the peaks of the curves were significantly smaller than in CASE 5, although it were less smooth. In fact, this was the case where the objective function had its smallest value (see Table 2). This result suggests that a 4-bar mechanism with 2 actuators could use smaller motors to perform the same task as the traditional one.

CASE 7: Finally in CASE 7 we returned to the 3 actuators case, but now considering both spring and damper forces which presented good results in cases 5 and 6. Normal terminations were also easy to find in CASE 7 using either fixed or variable step-size integrator, and no intermediate steps were necessary to get a normal termination with $g = 9.81$ m/s^2 and $tf = 1.0$s. But, as it is possible to see at Figure 3f torque curves were not as smooth as in CASES 1 and 2. On the other hand, the peak of the curves were even smaller than in CASE 2 and so were the value of the objective function.



(a) Case 1 (b) Case 2 (c) Case 4

(d) Case 5 (e) Case 6 (f) Case 7

Figure 3. Optimal controls from the best trials of each case

4. CONCLUSIONS

A method for solving the inverse dynamics of redundantly actuated 4-bar mechanism, formulated as an OCP has been presented. The method is based on a modification of the dynamical model of the system that transforms the closed kinematic chain into an equivalent open kinematic chain with spring and damper forces that preserves the kinematics of the original closed kinematic chain system. As this principle is quite general, it could be applied to solve similar problem of others closed kinematic chain mechanisms. The main finding of this work was that the 4-bar mechanism with 2 actuators (CASE 6) requires less maximum torque effort to perform the same task when compared to a 1-actuated (CASE 5) and 3-actuated (CASE 1 and CASE 2) counterparts. Other aspects related to the computational implementation of this problem were also presented, and can be promptly used for further implementation of similar problems.

5. ACKNOWLEDGEMENTS

This work used hardware bought with FAPERJ's resources and is supported by a MSc scholarship from CAPES.

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