# A ANALYTICAL MODEL FOR CONICAL INJECTOR FLOWS 

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Abstract. This study presents an analysis for conical injector flow characteristics and analytical expressions to determining its main parameters: discharge coefficient, spray cone angle and mean drop diameter. The major interest of this type of injector on propulsion is due to its good atomization characteristics with a low spray angle. This work was complemented with experimental results with drop diameter measurements evaluated using a diffracted laser beam technique. The drop diameter predictions are not far from the experimental obtained values. The analytical model prediction for the discharge coefficient and spray cone angle approximates well the empirical distributions of Radcliffe, Couto and Jasuja and also the experimental measured spray both indicating the reliability of that analysis.

Keywords: injectors, pressure swirl injectors, atomization, sprays

## 1. INTRODUCTION

In the context of the jet propulsion that equips the aircrafts that generates approximately $90 \%$ of the revenue of the national aeronautical industry, there is a series of fields that still could be explored in Brazil: technology of materials, compressors, turbines, combustion chambers, among other possibilities. In the case peculiar of jet engines, considerable amount of knowledge doesn't still meet in the books and it is very dispersed in scientific articles. Methodologies of project in use in the exterior by great manufacturers of engines are restricted to their own project offices. That is the reality in the case of the combustion chambers, in matter of the kerosene injectors. In terms of complexity of the flow inside the conical type injector is between the classical centrifugal tangential and the duplex injector (more complex), which demand is conditioned in two regimes of flows to assist to a wide and varied operation of the gas turbines. This work intends to present a methodology of preliminary project of a specific case of a simplex type injector: the conical injector and to compare the model with the experimental data of a manufactured injector, looking for validation and verification of the hypotheses of the proposed method.

## 2. MODELING A CONICAL INJECTOR

The liquid flow in the conical injector is resulted of the movement in spiral determined by the geometry of the vortex chamber, detailed in Figure 1. Assuming the hypothesis of incompressible flow, the Bernoulli equation can be used from the entrance of the fuel in the injector until its exit inside the combustion chamber.

$$
\begin{equation*}
P_{0-1}=P_{e x t}+\frac{\rho V_{e n t}^{2}}{2}+\Delta P_{1-2}=P+\frac{\rho V_{T}^{2}}{2}+\frac{\rho V_{a}^{2}}{2}+\frac{\rho V_{R}^{2}}{2}+\Delta P_{1-3} \tag{1}
\end{equation*}
$$

Where $P_{0-1}$ is the stagnation pressure at the inlet, while the others pressures are static. The radial component of the velocity in the above equation was retained for sake of complete representation of the flow around a central cone, although its value is less than the axial component $V_{A}$ and also less than the tangential velocity $V_{T}$. For the continuity equation and the angular moment conservation, it has:

$$
\begin{equation*}
\dot{m}=\rho S_{\text {exit }}\left|\overrightarrow{V_{A}} \| \vec{n}\right|=\rho S_{\text {channel }} V_{\text {ent }} \cos \gamma_{1} \sin \theta \tag{2}
\end{equation*}
$$

From the equation of conservation of the angular momentum:

$$
\begin{equation*}
V_{T} \cdot R_{i n}=V_{T m} \cdot r_{m} \tag{3}
\end{equation*}
$$

As can be seen in Eq. (3), diminishing the nucleus radius of gas (related to the line center of the injector) configures increasing velocities. Theoretically in the limit this would represent an almost infinite velocity with an almost null pressure of the fluid in the central axis, what in fact doesn't happen. On contrary it will have in the center the formation of a liquid film of internal radius $\boldsymbol{r}_{\boldsymbol{m}}$ for which the velocity at the surface balances the pressures in the nozzle exit. Starting with ratio of the annular area of liquid to the nozzle area of the injector, the coefficient $\phi$ can be defined as:

$$
\begin{equation*}
\varphi=\frac{S_{\text {liquid }}}{S_{\text {exit }}}=1-\frac{r_{m}^{2}}{r_{c}^{2}} \tag{4}
\end{equation*}
$$

Essentially, $\phi$ represents the total portion of nozzle area filled out by the liquid.


Figure 1. a) Schematic design of the conical injector; b) Entrance of the vortex chamber
Here the dynamic pressure (relative to the velocity $V_{A}$ ) is modeled, taking on Bernoulli equation adapted to represent any point in the annular area of liquid, delimited by the border between the liquid and the gas and the internal wall of the injector.

$$
\begin{equation*}
\Delta P_{i n j}=\frac{1}{2} \rho\left(V_{A}^{2}+V_{T}^{2}+V_{R}^{2}+\xi_{i n j} V_{e n t}^{2}\right) \tag{5}
\end{equation*}
$$

The first three terms (eq.5) represent the components of dynamic pressures and the last term refers to the pressure losses relative to the entrance channel, quite significant in the case of injectors of gas turbine engines. Those losses are due to the viscous friction between the fluid and the injector and the load losses referring to the geometry of the entrance channels of small dimensions. The characteristic coefficients of those losses are described by the following equation:

$$
\begin{equation*}
\xi_{i n j}=\xi_{e n t}+\xi_{i} \tag{6}
\end{equation*}
$$

Considering this group of losses, assuming a uniform axial velocity according to:

$$
\begin{equation*}
V_{A}=\sqrt{\frac{2}{\rho} \Delta P_{i n j}-\xi_{i n j} V_{e n t}{ }^{2}-V_{T m}{ }^{2}-V_{R m}{ }^{2}} \tag{7}
\end{equation*}
$$

With the hypothesis of uniform axial velocity $V_{A}$, one can meet the expression for the total flow of the injector:

$$
\begin{equation*}
\dot{m}=\pi r_{c}{ }^{2} \varphi \sqrt{2 \rho \Delta P_{i n j}-\xi_{i n j} \rho^{2} V_{e n t}{ }^{2}-\rho^{2} V_{T m}{ }^{2}-\rho^{2} V_{R m}{ }^{2}} \tag{8}
\end{equation*}
$$

where: $V_{\text {ent }}=\dot{m} / \rho \pi r^{2} n$
Here $V_{\text {ent }}$ is the entrance velocity and with $\gamma_{1}=\operatorname{arctg}(\tan \gamma \sin \theta)$. The velocity components tangential (on the border area between the liquid film and the gas core) $V_{T m}$ and radial $V_{R m}$ are given by Eqs. (9) and (10) respectively.

$$
\begin{align*}
& V_{T m}=K V_{e n t} R_{i n j} \cos \gamma_{1} \cos \theta\left(1-\sin \theta \cos \gamma_{1}\right) / r_{m}  \tag{9}\\
& V_{R m}=V_{e n t} \sin \gamma_{1} \tag{10}
\end{align*}
$$

Where $K$ represents the loss coefficient of angular moment, linked to the viscosity of the liquid fuel.
A parameter is introduced that represents the geometry of the conical injector denoted by $A$, similar to those defined as geometric parameters of Lefebvre (1989) in the case of the classical pressure-swirl injector according to Eq. (11)

$$
\begin{equation*}
A=\frac{R_{i n} \cdot r_{c}}{n \cdot r_{e n t}{ }^{2}} \tag{11}
\end{equation*}
$$

It is obtained new expressions for $V_{T m}$ e $V_{R m}$ respectively:

$$
\begin{align*}
& V_{T m}=\frac{A K \dot{m} \cos \gamma_{1} \cos \theta}{\rho \pi r_{c}^{2} \sqrt{1-\varphi}}  \tag{12}\\
& V_{R m}=\frac{\dot{m} \sin \gamma_{1}}{\rho \pi r^{2} n} \tag{13}
\end{align*}
$$

Finally, substituting Eq. (11) in Eq. (9) a new expression for the entrance speed in function of $A$ is found, and inserting the resulting expression now together with Eq. (12) in Eq. (8), it follows the final expression of the total flux in function of the geometric parameters of the injector, of the pressure difference and of the density of the fuel, represented by:

$$
\begin{equation*}
\dot{m}=\frac{\pi r_{c}^{2} \sqrt{2 \rho \Delta P_{i n j}}}{\sqrt{\frac{1}{\varphi^{2}}+\xi_{i n j} \frac{A^{2}}{C^{2}}+\frac{A^{2} K^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta}{1-\varphi}+\sin ^{2} \gamma_{1} \frac{A^{2}}{C^{2}}}} \tag{14}
\end{equation*}
$$

Of the total flux in Eq. (14) - is related to the discharge coefficient function of the loss coefficient $\xi_{i n j}$ and of the geometry of the conical injector:

$$
\begin{equation*}
C_{D}=\frac{\dot{m}_{s}}{\pi r_{c}^{2} \sqrt{2 \rho \Delta P_{i n j}}}=\frac{1}{\sqrt{\frac{1}{\varphi^{2}}+\xi_{i n j} \frac{A^{2}}{C^{2}}+\frac{A^{2} K^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta}{1-\varphi}+\sin ^{2} \gamma_{1} \frac{A^{2}}{C^{2}}}} \tag{15}
\end{equation*}
$$

### 2.1. Modeling of the conical injector with the hypothesis of ideal fluid:

The hypothesis of ideal fluid particularized to the case of this analysis simplifies the modeling starting from the absence of the losses $\xi_{i n j}$ and assuming $K=1$, in the case of null viscosity. Starting from that, Eq. (15) is worked then results:

$$
\begin{equation*}
C_{D}=\frac{1}{\sqrt{\frac{1}{\varphi^{2}}+\frac{A^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta}{1-\varphi}+\sin ^{2} \gamma_{1} \frac{A^{2}}{C^{2}}}} \tag{16}
\end{equation*}
$$

Equation (16) provides a relationship among the dimensions of the injector, of the annular area of liquid and of the discharge coefficient. Simplifying the analysis, it can be seen that for each $\phi$ coefficient will correspond a geometric parameter $A$, which gives a maximum of discharge coefficient. Being like this, differentiating Eq. (16) in relation to that $\phi$ coefficient, it follows:

$$
\begin{equation*}
\frac{d C_{D}}{d \varphi}=0 \rightarrow \frac{2}{\varphi^{3}}=\frac{A^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta}{(1-\varphi)^{2}} \tag{17}
\end{equation*}
$$

In exposing $A$, it has:

$$
\begin{equation*}
A=\frac{(1-\varphi) \sqrt{2}}{\varphi \sqrt{\varphi} \cos \gamma_{1} \cos \theta} \tag{18}
\end{equation*}
$$

Equation (18), when being substituted in Eq. (16) it results Eq. (19), which represents the discharge coefficient that is directly proportional to the total flux and is expressed in function of the parameter $\phi$.

$$
\begin{equation*}
C_{D}=\frac{1}{\sqrt{\frac{2 \sin ^{2} \gamma_{1}}{C^{2}} \frac{(1-\varphi)^{2}}{\varphi^{3} \cos ^{2} \gamma_{1} \cos ^{2} \theta\left(1-\sin \theta \cos \gamma_{1}\right)}+\frac{2-\varphi}{\varphi^{3}}}} \tag{19}
\end{equation*}
$$

For an ideal fluid, starting from Eqs. (12) and (14), arrives:

$$
\left\{\begin{array}{l}
V_{T m}=\frac{A C_{D} \cos \gamma_{1} \cos \theta\left(1-\sin \theta \cos \gamma_{1}\right)}{\sqrt{1-\varphi}} \sqrt{\frac{2 \Delta P_{i n j}}{\rho}}  \tag{20}\\
V_{R m}=\frac{A}{C} C_{D} \sin \gamma_{1} \sqrt{\frac{2 \Delta P_{i n j}}{\rho}}
\end{array}\right.
$$

The term inside of the root of the previous equation it is same to the total speed $\quad V=\sqrt{2 \Delta P_{i n j} / \rho}$ approximated for occasion of the deduction of Eq. (7). Substituting $V$ in Eq. (20) and using the definition of the parameter of the annular layer $\phi$, in agreement with Eq. (4), it is obtained:

$$
\left\{\begin{array}{l}
\frac{V_{T m}}{V}=\frac{A C_{D} \cos \gamma_{1} \cos \theta\left(1-\sin \theta \cos \gamma_{1}\right)}{\sqrt{1-\varphi}}  \tag{21}\\
\frac{V_{R}}{V}=\frac{A}{C} C_{D} \sin \gamma_{1}
\end{array}\right.
$$

Considering the thin film of liquid, especially in the case of the injector of low flux, it will be taken as radius in Eq. (21) a mean value between the internal radius of the annular portion and the radius of the orifice of the injector, as displayed by:

$$
\begin{equation*}
r_{m}=r_{c} \sqrt{1-\varphi} \tag{22}
\end{equation*}
$$

Of the own definition of the parameter $\phi$, according to Eq. (4), it follows the relationship below Eq. (23):
$r=\frac{r_{m}+r_{c}}{2}=\frac{r_{c}}{2}(1+\sqrt{1-\varphi})$
which substituted in Eq. (21) it generates Eq. (24):

$$
\begin{equation*}
\frac{V_{T}}{V}=\frac{2 A C_{D} \cos \gamma_{1} \cos \theta}{1+\sqrt{1-\varphi}} \tag{24}
\end{equation*}
$$

### 2.2. Spray cone angle.

The results of several authors show that spray angle is influenced by nozzle dimensions, liquid properties, and the density of the medium into which the liquid is sprayed. In this work the last factor will not be taken in account. The sinus of the spray cone half-angle corresponds to the relation among the velocity components $V_{T}$ and $V_{R}$ and the total velocity $V$, as it can see in Eq. (25). Using the relationships that define such velocities, it results the possibility to find the half-angle in function of $\phi$ :

$$
\begin{equation*}
\sin \alpha=\frac{\sqrt{V_{T}{ }^{2}+V_{R m}{ }^{2}}}{V} \tag{25}
\end{equation*}
$$

Finally,

$$
\begin{equation*}
\alpha=\sin ^{-1} \sqrt{\frac{4 A^{2} C_{D}^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta\left(1-\sin \theta \cos \gamma_{1}\right)^{2}}{(1+\sqrt{1-\varphi})^{2}}+\frac{A^{2}}{C^{2}} C_{D}{ }^{2} \sin ^{2} \gamma_{1}} \tag{26}
\end{equation*}
$$



Figure 2. a) Conical injector schema and b) test rig for experimental tests.

## 3. EXPERIMENTAL VALIDATION OF THE MODEL

Starting from the modeling equations seen in the previous section, this must be compared with some existent results in the sense of evaluating its reliability and discrepancies between the theory and the experimentation.

The analysis of that confrontation allows that, in a subsequent instant, adjust the model correcting it to adapt to reality. Evidently that in any work of this nature the empiric correction depends on a series of experiments that allow to vary the several involved parameters, and in this sense this article doesn't intend to give a finished correction implemented of the model here developed, however the points to attack and to get better they are pointed and eventual measures to this respect are suggested.

### 3.1. Description of the injector and the accomplished tests

The simplex conical injector whose results served as validation of the model is shown in Fig.2a. Its principal characteristics are described in the Table 1.

Table 1. Construction data for the Conical Injector

| $2 r_{\text {ent }}$ | 0.45 mm | $r_{c}$ | 1.0 mm |
| :---: | :---: | :---: | :---: |
| $L$ | 3.3 mm | $R_{k}$ | 3.2 mm |
| $\theta$ | $37^{0}$ | $L_{k}$ | 2.0 mm |
| $R_{\text {in }}=R_{k}-r_{\text {ent }}$ | 2.98 mm | $C$ (adim) | 3. |

A conical injector was built in brass, for the readiness and production easiness. Besides, cold tests don't demand special materials. The entrance in four channels, for constructive easiness, had rectangular profile, whose total area: $0.20 \mathrm{~mm}^{2}$ and the radius of the equivalent circle are referred by $r_{\text {ent }}$ (see Table 1). Water was used as test fluid. The main parameters of the injector - discharge coefficient, cone angle and drop diameter were measured and the data consist of the Table 2.

Table 2. Conical injector test data and liquid properties

| Total flux | $1.2 \mathrm{E}-02 \mathrm{~kg} / \mathrm{s}$ | Liquid density | $1.0 \mathrm{E}+03 \mathrm{~kg} / \mathrm{m} 3$ |
| :---: | :---: | :---: | :---: |
| injection pressure | $8.0 \mathrm{E}+05 \mathrm{MPa}$ | Kinematic viscosity | $0.85 \mathrm{E}-06 \mathrm{~m} 2 / \mathrm{s}$ |
| discharge coefficient | $0.10+/-0.01$ | Surface tension | $7.34 \mathrm{E}-02 \mathrm{~kg} / \mathrm{s} 2$ |
| Spray cone angle $(2 \alpha)$ | $123^{\circ}+/-1^{\circ}$ | Ambient density (air) | $1.0 \mathrm{~kg} / \mathrm{m} 3$ |

Once defined the parameters of the injector, it becomes necessary to define the conditions of tests. In the Table 2, we have the conditions of the tests for which all the main parameters that characterize the injector - discharge coefficient, cone angle and drop diameter were measured.

An outline of the implement of tests is sketched in Fig. 2b. The stippled line establishes the borders of the control volume. There isn't a pressure tap just at the entrance of the injector, but in the tank. The control volume here adopted it is shown reasonable, once it is tiny the pressure difference among those points.

In the peculiar case of the spray cone angle, digital pictures were taken during the test. In the pictures, it was traced lines to delimit the limits of the spray that aided in the obtaining this angle, which value can be seen in the Table 2 as well other parameters.

### 3.2. Comparisons with the developed model

Starting of the geometric data of the injector and using the relationship (11) we have $A$; and $\varphi$ will be given by Eq. (18). It is obtained: $A=15.8$, and the value of $\varphi=0.357$. Finally, taking Eq. (16) and (26), respectively the value of the discharge coefficient is obtained: $C d=0.16$ and of the angle of cone $2 \alpha=117^{\circ}$.

### 3.2.1. Influence of the viscosity of the liquid

A new geometric parameter is defined $A_{e q}$ that takes viscosity of the fluid in account:

$$
\begin{equation*}
A_{e q}=\frac{\mathrm{R}_{i n} r_{c}}{n r_{e n t}^{2}+\frac{\lambda}{2} R_{i n}\left(R_{i n}-r_{c}\right)} \tag{27}
\end{equation*}
$$

In that methodology all the employed relationships for ideal fluid are enlarged to real fluid so the parameter $A$ is substituted for $A_{e q}$. For the calculation of $A_{e q}$ the following relationship is used:

$$
\begin{equation*}
A_{e q}=A \cdot K \tag{28}
\end{equation*}
$$

Where $\lambda$ is the friction coefficient of Blasius, and is correlated by $\lambda=0.3164$. $R e^{-0.25}$. The Reynolds number of the flow is defined in function of the area passage of the entrance channels in the vortex chamber:

$$
\begin{equation*}
\operatorname{Re}=\frac{2 \dot{m}}{\rho v \pi r_{e n t} \sqrt{n}} \tag{29}
\end{equation*}
$$

In this sequence, Blasius friction coefficient is obtained: $\lambda=0.011$. Then it follows the value $A_{e q}=8.36$
consequently, $K=0.73$.

### 3.2.2. Loss of the channels entrance

Those losses are calculated in function of the angle between the channel axis and the vortex bottom plan chamber. In the case of that injector: $\beta=58^{\circ}$. For that value, it is found on graphics in Kessaev and Kupantekov (1997) the corresponding loss is $\xi_{e n t}=0.7$. The sum of the losses in the entrance is given by Eq.(6), where $\xi_{i}=\lambda . L / 2 r_{e n t}$ the total loss will be: $\xi_{i n j}=0.78$. Finally, updating the discharge coefficients considering those losses, it was found: $C d=0.14$.

### 3.2.3. Injection angle

As Eq. (26), the half-angle corresponds to the $\arcsin \left(V_{T} / V\right)$ what supplies the value of the half-angle spray cone:

$$
\begin{equation*}
\alpha=\sin ^{-1}\left\{\frac{\sqrt{C_{D}{ }^{2} \sin ^{2} \gamma_{1} \frac{A^{2}}{C^{2}}+\frac{4 A^{2} K^{2} C_{D}{ }^{2} \cos ^{2} \gamma_{1} \cos ^{2} \theta\left(1-\sin \theta \cos \gamma_{1}\right)^{2}}{(1+\sqrt{1-\varphi})^{2}}}}{\sqrt{1-\frac{A^{2}}{C^{2}} C_{D}{ }^{2} \xi_{i n j}}}\right\} \tag{30}
\end{equation*}
$$

A clear peculiarity of both expressions for spray angle is they consider only the influence of geometric parameters of the injector in their formulations.

### 3.2.4. Theoretical and Experimental Results

In order to establish a comparison between experimental values and those obtained with theoretical calculations in the case of an ideal liquid with and without losses, the results is summarized in the following table.

Table 3 - Experimental and Theoretical Results for the parameters of the injector

| Parameters | Theoretical <br> without losses | Theoretical with <br> losses | Experimental |
| :--- | :---: | :---: | :---: |
| Discharge Coefficients Cd | 0.16 | 0.14 | $0.10+/-0.01$ |
| Total Spray Cone $2 \alpha$ | $117^{\circ}$ | $108^{\circ}$ | $123^{\circ}+/-1^{\circ}$ |

It is noticed in the values demonstrated in the Table 3: the experimental values (of $C d$ and spray half cone angle $\alpha$ ), they locate near the theoretical values with and without losses. Observing the experimental data measured for a conical injector, the averaged deviation is $50 \%$ for the predicted $C d$. In the same comparison base, the deviation in the angle from experimental results was of the order of $8.5 \%$. With this verification it is possible that (considering a more precise experimental data) the model of losses in the entrance channels is not conservative for the conical case, where the flow suffers fewer losses until reaching the exit hole, leading to over predicted discharge coefficients.

Results have certain order of coherence with those found in the model, showing although corrections in the hypotheses can still be made; the experimental part must be more worked. It should still be observed certain constructive imprecision. Discrepancies between the nominal dimensions and the indeed implemented might have led deviations on results of same order. In the continuation of that research it is foreseen to build at least five injectors supposedly with the same dimensions, and to compare them experimentally.

### 3.3. Models analysis for SMD Determination.

In the consulted literature, there are models for the cone angle, discharge coefficient, and also for the SMD (Sauter Mean Diameter) predictions. In that work, unlike the first two parameters, wasn't intended develop any model type to foresee SMD, what doesn't exclude the need of approach of that important parameter in a conical injector in future studies. There are such a variety of atomization processes that hinders obtaining of a general solution for this problem.

## Radcliffe's model (1960)

$$
\begin{equation*}
S M D=7.3 \sigma^{0.6} v^{0.2} \stackrel{\bullet}{m}^{0.25} \Delta P^{-0.4} \tag{31}
\end{equation*}
$$

The Radcliffe empiric model is one of the oldest and more used expressions to obtain SMD diameter. This model uses the basic variables and of more influence in the phenomenon of the formation of the drops: surface tension,
kinematic viscosity, total flux and drop pressure in the injector. A peculiarity to be noticed is that geometric parameters of the injector, as the nozzle diameter, was not taken in account, unlike what is observed in more current methods.

## Jasuja model (1979)

As one can see, it is similar to the model of Radcliffe, however with different coefficients.

$$
\begin{equation*}
S M D=4.4 \sigma^{0.6} v^{0.16} \dot{m}^{0.22} \Delta P^{-0.43} \tag{32}
\end{equation*}
$$

## Couto, Carvalho and Bastos-Netto model (1997)

This model gives the ligament diameter for a rotating conical sheet and including the Rayleigh's model for the primary diameter $d_{d}$ one may write:

$$
\begin{equation*}
\mathrm{d}_{\mathrm{d}}=1,8172 \cdot \cos \theta\left(\frac{h_{0}^{4} \sigma^{2} \cos ^{2} \theta}{\rho_{a} \rho_{L} \mathrm{U}_{0}^{4}}\right)^{\frac{1}{6}}\left(1+2,6 \mu_{L} \cos \theta \sqrt[3]{\frac{h_{0}^{2} \rho_{a}^{4} \mathrm{U}_{0}^{7}}{72 \rho_{\mathrm{L}}^{2} \sigma^{5} \cos ^{8} \theta}}\right)^{\frac{1}{5}} \tag{33}
\end{equation*}
$$

where $h_{0}$ represents the thickness of the liquid film.
It extrapolates the work of Dombrovski and Johns (1963), that it falls back upon the number of Ohnesorge and the studies of Rayleigh relative to the drop size. This model consists of applying the method mentioned to the leaf of liquid of a hollow cone, as produced by the centrifugal injectors, under the hypothesis that the wavelength of any disturbance in the liquid leaf should be much smaller than its characteristic length.

### 3.4. Choice between the SMD models and the experimental result

Starting from the physical parameters used in the experiment with the conical injetor, it was obtained a prediction of the theoretical SMD. Table 5 shows those results and the corresponding experiment.

Table 4 - Predicted and measured SMD mean diameter

| drop size model <br> $(\mu \mathrm{m})$ | Radcliffe | Jasuja | Couto et al. ${ }^{1}$ | Couto et al. ${ }^{2}$ | This work <br> $($ experimental $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta \mathrm{P}=4 \mathrm{bar}$ | 245 | 148 | 108 | 123 | $166+/-5$ |
| $\delta \mathrm{P}=6 \mathrm{bar}$ | 207 | 125 | 94 | 106 | $147+/-4$ |
| $\delta \mathrm{P}=7 \mathrm{bar}$ | 194 | 117 | 88 | 101 | $126+/-4$ |
| $\delta \mathrm{P}=8$ bar | 183 | 110 | 85 | 96 | $126+/-4$ |
| $\%$ deviation | $47 \%$ | $11 \%$ | $33 \%$ | $24 \%$ | -- |

Designations ()$^{1}$ and ()$^{2}$ refers to the injector model respectively without and with internal losses. Through this table, it is immediate to notice that the experimental results approach more of the model of Jasuja, especially for the lowest pressure, and of the model of Couto et al. ${ }^{2}$, for the one of larger pressure. Radcliffe model presented the largest deviation compared to results of the injector tests.

## 4. CONCLUSIONS

The present work presented a modeling of a conical injetor being used equations of mass conservation and of angular moment and longitudinal momentum besides the conservation of kinetic energy. Soon after the predictions obtained with that model were compared with experimental results of spray angles, discharge coefficient and SMD mean drop diameter of a manufactured injector. With that formulation it was possible to forecast a acceptable discharge coefficient $\left(\mathrm{C}_{\mathrm{d}}\right)$. Besides, as experimental angle of the spray cone of the conical injetor was visibly greater than the expected with the present model. As the forecast of drops size, in spite of a good success margin in our applications, the approach still allows improvements in their base of hypotheses. That theoretical model showed satisfactory results, in spite of its non-linear aspects. That modeling for project applications, assisted well the expectations, pointing a way for the development and improvements for a new injector design.

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