A PROCEDURE TO OPTIMIZE DYNAMIC OBSERVERS BY TUNING PARAMETERS

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Abstract. *The use of dynamic observers based on Green's functions was developed to be applied in multidimensional heat transfer inverse problems. It can be observed that to apply this technique three parameters must be specified: filter order, ripple and cut off frequency The cut off frequency and the chebyshev filter order are the tuning parameters of the inverse algorithm and must be specified depending on the noise level of the experimental data and the desired resolution. This work presents a new procedure based on optimization techniques to identify these tuning parameters. Simulated and experimental cases are investigated and the optimum configuration for the chebyshev filter is obtained for each case.*

Keywords: *inverse problem, dynamic observers, Green's function, simulated annealing, heat flux.*

1. INTRODUCTION

The inverse problem can be found in a large area of science and engineering and can be applied in different ways (Beck and Blackwell, 1985). The main characteristic of this kind of problems is that the physical problem solution is obtained indirectly. For example, the determination of a surface temperature without directs access or even the diagnostic of diseases using computerized tomography. In both cases, the boundaries are unknown and inaccessible. Thus, these problems can be solved using information obtained from sensors located in accessible positions.

Inverse heat conduction problems require the use of experimental data to obtain the solution of a thermal problem. The problem can be then the estimation of thermal properties, the estimation of an unknown heat flux surface, the estimation of an internal heat source, or even the estimation of a surface temperature of an inaccessible wall.

Recently techniques based on filters such as the use of dynamic observers (Blum and Marquardt, 1997), have also been employed for the solution of the IHCP. Works employing observers (Blum and Marquardt, 1997), (Sousa *et al.*, 2006) have demonstrated its flexibility and efficiency to solve one-dimensional problems. Sousa (2006) presents a technique of dynamic observers based on Green's functions that can be applied directly to solve multidimensional problems. This work proposes to enhance this technique using an optimization heuristic method. The method, called simulated annealing technique (Metropolis *et al.*, 1953), is used to obtain the optimum tuning parameters of the inverse algorithm that means to establish the Chebyshev filter configuration.

2. FUNDAMENTALS

The inverse problem solution technique of dynamic observers based on Green's functions, Sousa (2006) can be divided in two distinct steps: i) the obtaining of the transfer function model G_H; ii) the obtaining of the heat transfer functions G_Q and G_N and the building algorithm identification. The transfer function model, G_H, is obtained from the equivalent dynamic systems theory and using Green's functions. The G_Q and G_N are obtained by following the procedure presented by Blum and Marquardt (1997). As ever mentioned, the novelty presented here is the obtaining of tuning parameter that defines the Chebyshev filter by using an optimization technique instead of the Blum and Marquardt procedure.

2.1. Dynamic Observers Based on Green's Function applied to Inverse Problems.

2.1.1. Original 3D problem

The 3D-transient thermal problem shown in Fig. 1 can be described by diffusion equation as

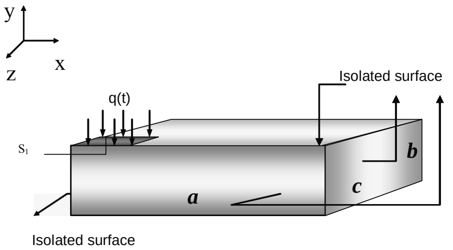


Figure 1. 3D transient thermal model

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1a}$$

In the region R (0 < x < a, 0 < y < b, 0 < z < c) and t> 0, subjected to the boundary conditions

$$\begin{aligned} -k\frac{\partial T}{\partial y}\bigg|_{y=b} &= q(t) \text{ on } S_1 \ \left(0 \le x \le x_H\,,\ 0 \le z \le z_H\right) \\ -k\frac{\partial T}{\partial y}\bigg|_{y=b} &= 0 \text{ on } S_2 \ \left(x,z \in S \,\middle|\, (x,z) \notin S_1\right) \end{aligned} \tag{1b}$$

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = \frac{\partial T}{\partial x}\bigg|_{x=a} = \frac{\partial T}{\partial z}\bigg|_{z=o} = \frac{\partial T}{\partial z}\bigg|_{z=c} = \frac{\partial T}{\partial y}\bigg|_{y=b} = 0 \tag{1c}$$

and initial condition

$$T(x,y,z,0) = T_0 \tag{1d}$$

where S is defined by $(0 \le x \le a, 0 \le z \le c)$ and x_H and z_H are the boundary of S_1 where the heat flux is applied.

The solution of Eqs. (1) can be given in terms of Green's function as in Beck *et al.* (1992)

$$T(x,y,z,t) = \int_{\tau=0}^{t}\left[G_H(x,y,z,t/\tau)\ q(\tau)\right]d\tau \tag{2}$$

where

$$G_H(x,y,z,t/\tau) = \frac{\alpha}{k}\int_0^{x_h}\int_0^{z_h} G(x,y,z,t-\tau)\big|_{y'=0}\,dx'\,dz' \tag{3}$$

and $G(x,y,z,t-\tau)$ represents the Green function of the thermal problem given by Eq.(1).

The Green's function is available for the homogeneous version associated to the problem defined by Eqs. (1). Although the analytical Green's function is available and exists, Beck *et al.* (1992), it will not be used in this work. By the contrary, the solution of the problem defined by Eqs. (1) will be performed numerically.

Equation (2) reveals that an equivalent thermal model can be associated with a dynamic model. It means, a response of the input/output system can be associated to Eq. (2) in the Laplace domain as the convolution product (Özisik, 1993)

$$T(x,y,z,t) = G_H(x,y,z,t-\tau) * q(\tau) \tag{4}$$

This dynamic system can be represented as shown in Fig. (2). Equation (4) can also be evaluated in the Laplace domain as a single product

$$\overline{T}(x,y,z,s) = \overline{G}_H(x,y,z,s)\ \overline{q}(s) \tag{5}$$

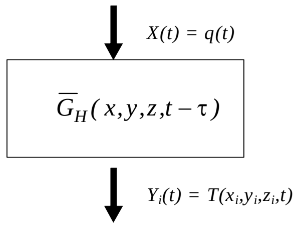


Figure 2. Dynamic thermal model system

The heat transfer function $\overline{G}_H(x,y,z,s)$ can, then, be obtained through the auxiliary problem which is a homogenous version of the problem defined by Eq.(1) for the same region with a zero initial temperature and unit impulsive source located at the region of the original heating. It means, the auxiliary thermal problem can be described as

$$\frac{\partial^2 T^+}{\partial x^{+2}} + \frac{\partial^2 T^+}{\partial y^{+2}} + \frac{\partial^2 T^+}{\partial z^{+2}} = \frac{\partial T^+}{\partial t^+} \tag{6a}$$

in the region R and $t^+ > 0$, subjected to the boundary condition

$$-k\frac{\partial T^+}{\partial y^+}\bigg|_{y=0} = 1 \ on \ S_1 \qquad and \qquad -k\frac{\partial T^+}{\partial y^+}\bigg|_{y=0} = 0 \ on \ S_2 \tag{6b}$$

$$\frac{\partial T^+}{\partial x^+}\bigg|_{x^+=0} = \frac{\partial T^+}{\partial x^+}\bigg|_{x^+=1} = \frac{\partial T^+}{\partial z^+}\bigg|_{z^+=o} = \frac{\partial T^+}{\partial z^+}\bigg|_{z^+=1} = \frac{\partial T^+}{\partial y^+}\bigg|_{y^+=1} = 0 \tag{6c}$$

and the initial condition

$$T^+\left(x^+,y^+,z^+,0\right) = 0 \tag{6d}$$

Similarly, the auxiliary thermal problem solution can be derived using Green function and the convolution properties as

$$T^+\left(x^+,y^+,z^+,t^+\right) = G_H(x^+,y^+,z^+,t^+-\tau^+) * 1 \tag{7}$$

Once the Laplace transform of 1 is

$$L[1] = \frac{1}{s} \text{, then} \tag{8}$$

$$\overline{T}^+\left(x^+,y^+,z^+,s\right) = \overline{G}_H(x^+,y^+,z^+,s)\ \frac{1}{s} \tag{9}$$

If the dynamic system is linear, invariant, and physically invariable, the response function $\overline{G}_H(x,y,z,s)$ is the same, independently of the pairs input/output and can be obtained by

$$\overline{G}_H(x,y,z,s) = s\ \overline{T}^+\left(x,y,z,s\right) \tag{10}$$

In order to complete the $\overline{G}_h(x,y,z,s)$ identification, the $\overline{T}^+(x,y,z,s)$ must be obtained at a specific position $r_i = (x_i, y_i, z_i)$.

A simple and efficient procedure is proposed here to obtain $T^+(r_i,s)$: if Eq.(9) represents a cross correlation function of the two functions of stationary random process ***s*** and $\overline{T}^+\left(x,y,z,s\right)$, then $\overline{G}_H(x,y,z,s)$ will be independent of the absolute time ***t*** and will depend only on the time separation ***t_a.***

In this case, provided that the function $T^{+}(r_i,s)$ can be fitted by a polynomials function in the sampled interval [0, t_a] as

$$T^{+}(r_i,t) = a_1\,t + a_2\,t^2 + a_3\,t^3 + \cdots \tag{11}$$

where a_i are the polynomial coefficients. Then, taking the Laplace transform of Eq. (11) gives

$$\bar{T}^{+}(r_i,s) = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{2s^3} + \frac{a_4}{6s^4} + \cdots \tag{12}$$

Thus, from Eq. (21) G_H can be written as

$$\bar{G}_H(r_i,s) = s\,\bar{T}^{+}(r_i,s) = s\left[\frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{2s^3} + \frac{a_4}{6s^4} + \cdots +\right] \tag{13}$$

or $$\bar{G}_H(r_i,s) = a_1 + \frac{a_2}{s} + \frac{a_3}{2s^2} + \frac{a_4}{6s^3} + \cdots + \tag{14}$$

It can be observed that from the theory of partial fractions, if $\bar{G}_H(r_i,s)$ is expressible in partial fractions as in Eq.(14) its inversion is readily obtained by using the Laplace transform table. Since, Eq. (14) does not present any pole for s>0 then its inversion is stable. This fact guarantees robustness to the inverse algorithm given by Eqs. (10) and (11). Another advantage of this procedure is that the same procedure can be used indistinctly by one, two or three-dimensional models, provided the only active boundary condition is the unknown heat source. At this point, the G_Q and G_N functions need to be obtained in order to complete the inverse algorithm.

2.1.2. Obtaining of the estimators G_Q and G_N

The thermal model can be represented by a dynamic system given by a block diagram shown in Fig. 3 (Blum and Marquardt, 1997):

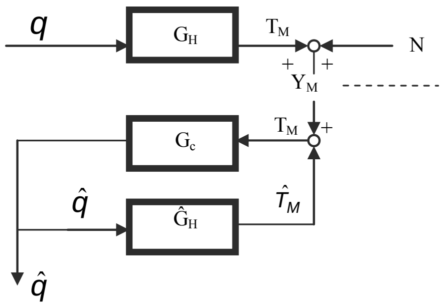


Figure 3. Frequency-domain block diagram (Blum and Marquardt, 1997)

It can be observed from the block diagram that:

i) The unknown heat flux *q(s)* is applied to the conductor (reference model), G_H, and results in a measurement signal Y^*_M corrupted by noise N,

$$Y^*_M = T_M + N = G_H\;\hat{q} + N \tag{15}$$

ii) The estimate value $\hat{q}$ is computed from the input data Y^*_M. Thus, the estimator can be represented in a closed-loop transfer function of the feedback loop as

$$\hat{q} = \frac{G_C}{1 + G_C G_H} Y_M \tag{16}$$

Eq. (16) characterizes the behavior of the solution algorithm. The signal and noise transfer function G_Q and G_N can, then, be found by combining Eqs.(15) and (16) as

$$\hat{q} = \underbrace{\frac{G_C G_H}{1+G_c G_H}}_{G_Q} q + \underbrace{\frac{G_C}{1+G_c G_H}}_{G_N} N \tag{17}$$

Thus, from Eq. (17) G_N can be written as

$$G_N = G_Q G_H^{-1} \text{ or } \left|G_N(j\omega)\right| = \frac{\left|G_Q(j\omega)\right|}{\left|G_H(j\omega)\right|} \tag{18}$$

It can also be observed in Eq. (17) that if the algorithm estimates the heat flux correctly, G_Q is equal to unity, $G_Q = 1$, and the frequency w is within the pass band. In this case, the noise transfer function G_N is equal to the inverse transfer function of the heat conductor (G_H^{-1}). From Eq. (16) the resulting algorithm can then be given by

$$\hat{q}(s) = G_N(s) \times Y_M(s) \tag{19}$$

According to Blum and Marquardt (1997) the observer is essentially an on-line scheme. In this case, estimation of the heat flux at the current time step is based on current and past temperature measurements only. In this case, any on-line estimator involves a phase shift or lag. To remove this lag Blum and Marquardt proposes a filtering procedure that can be resumed in the use of two discrete-time difference equations

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_i Y_M(k-i) - \sum_{i=1}^{n_n} a_i \hat{q}(k-i) \tag{20}$$

and

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_i q(k-i) - \sum_{i=1}^{n_n} a_i \hat{q}(k-i) \tag{21}$$

In Eqs.(20) and (21) a_i and b_i are coefficients obtained using Eqs.(17) and (18). The transfer function G_Q is chosen to have the behavior of type I Chebychev filter and its frequency response magnitude assume the form

$$G_Q(s) = \frac{k_{cheb}}{(s - s_{Cheb,1})(s - s_{Cheb,2})\cdots(s - s_{Cheb,n_Q})} \tag{22}$$

The poles $s_{cheb,I\ldots}$ are computed using MATLAB® package software. Thus, the dynamic observer technique can then be implemented using Eqs.(20) and (21) and the unknown heat flux can be estimated once G_H and Chebyshev filter (identification of G_Q) are obtained.

Despite of its importance, the choice of Chebyshev filter configuration is normally done by using a tentative method. Next section presents a procedure to automate this choice by optimizing some filter tuning parameters.

2.2. Optimization technique for filter configuration

It can be mentioned that the Chebyshev filter presents the required behavior for the inverse technique (section 2.1.2). One of the main characteristic of this filter is that the frequency response magnitude fall monotonically beyond the cutoff frequency. In addition, while the modulus of G_Q is allowed to fluctuate within a certain tolerance around its ideal value of 1 up to the cutoff frequency – referred to as the “passband ripple” – the roll-off beyond the cutoff frequency is monotonic and maximally steep (Blum and Marquardt, 1997).

In order to configure a Chebyshev filter three parameters must be chosen: the cut off frequency (ω_c), the ripple, and the Chebyshev polynomials order (dp). While the ripple can be chosen based on the noise level of the measurements signals the cut off frequency and the polynomial order are parameters more general and its identification can be optimized.

Simulated Annealing is derived from an analogy with the annealing process of material physics. In the process of annealing, the metal is heated up to a high temperature, causing the atoms to shake violently. Providing that the cooling is slow enough, the metal will eventually stabilize into an orderly structure. Otherwise, unstable atom structure is found. There is plenty of information on the current literature, so that in depth description of this technique is out of this work scope .

Simulated Annealing can be performed in optimization by randomly perturbing the decision variable and keeping track of the best objective function value for each randomized set of variables. After many trials, the set that produced the best objective function value is designed to be the center, over which perturbation will take place for the next temperature. The temperature, that in this technique is the standard deviation of the random number generator, is then reduced, and news trials performed.

Since the parameter ω_c and *dp* are the parameters to be estimated, a suitable objective function can, then, be defined as the least-square residuals, F, between the computed heat flux $q(i)$ and the real $\hat{q}$ (i), it means

$$F=\sum_{i=1}^{nt}\left(q(i)-\hat{q}(i)\right)^2 \tag{23}$$

were j represents the index for the thermocouple position and i the index for measurement time.

Let each configuration be defined by the set of atom positions where *E* represents the energy of the configuration and T is the temperature. In each step of the configuration, an atom is given a small random displacement and the resulting change, ΔE, in the energy of the system is computed. When the processes generating new states, this states is either accepted or rejected, according to the metropolis criterion (Metropolis et al., 1953): if ΔE ≤ 0, the displacement is accepted and this configuration is used as the starting point of the next step. If ΔE ≥ 0, the probability that the configuration is accepted is given by the following equation:

$$P(\Delta E)=e^{(-E/K_bT)} \tag{24}$$

where K_b is the Boltzmann´s constant. The choice of the probability function given by Eq. (24) has the consequence that the system evolves, according to a Boltzmann's distribution. The inverse algorithm can be summarized in 8 steps.

1. Obtain the thermal model and identification of G_H.
2. Initially, guess the values of the cut off frequency *(ω_c)* and Chebyshev polynomials degree *(dp)*.
3. Formulate of transfer function of a Chebyshev low-pass filter G_Q *(s)*, Eq. (22).
4. Obtaining of G_N by using Eq. (18)
5. Obtain of the heat flux by using Eq. (20) and (21).
6. Optimize of the objective function *F*, Eq. (23), with relation the design variables ω_c and *dp* using the simulated annealing technique.
7. Return to step 3.
8 Repeat the process is until the value of objective function be minimized (ε less than a small number)

Figure 4 and 5 present, respectively, the inverse and simulated annealing algorithm schemes.

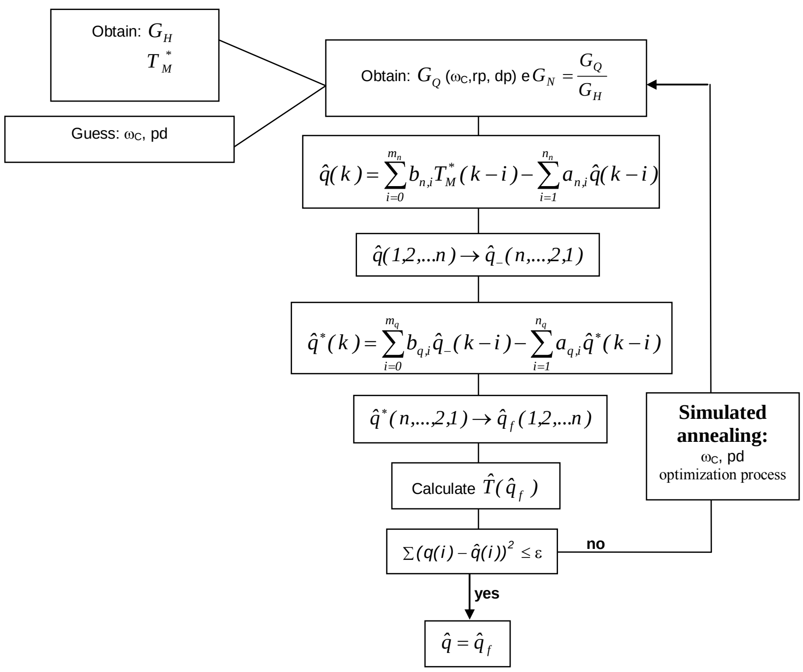


Figure 4. Inverse algorithm scheme

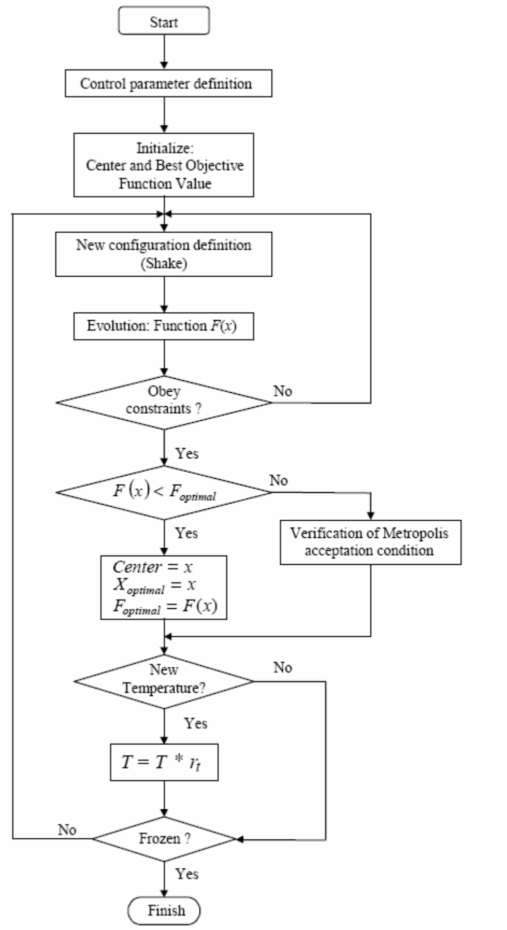


Figure 5. Simulated annealing algorithm scheme (Saramago et al., 1999)

3. RESULTS AND DISCUSSION

3.1. Three dimensional simulated problem optimization

The three-dimensional case described in Fig. (1) is analyzed in this section. Temperature distribution for the direct problem are generated using the solution of Eq. (1) considering a known heat flux evolution q(t). Random errors, ε_j , are then added to these temperatures. The temperatures with error are then used in the inverse algorithm to reconstruct the imposed heat flux. The parameter ε_j assumes values of 0, ± 0.5°C (1.5% of the maximum temperature).

The simulated temperature are calculated from the following equation

$$Y(L,t) = T(L,t) + \varepsilon_j \, . \tag{23}$$

The case simulates a copper sample with dimensions 30x30x20mm, thermal conductivity of k=401 W/mK and thermal diffusivity of α= 117 10^{-6}m^2/s submitted a sinusoidal heat flux. Figure 6 presents simulated temperature sensor locations and Figures 7 present the simulated temperature at the opposite face to the heat fluxes.

Different guess for values of ω_c and *dp* are used in order to verify the simulated annealing convergence. Table 1 shows these tests. It can be observed that SA converges to the minimum values independently of the initial guess. .

A small variation in the cut off frequency values can also be observed. In this case the final value is assumed the average values from estimated frequencies. Figure 8 shows a comparison between the real and estimated heat flux input.

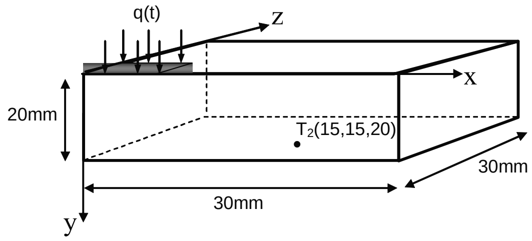


Figure 6. 3D thermal model test with simulated temperature sensor locations.

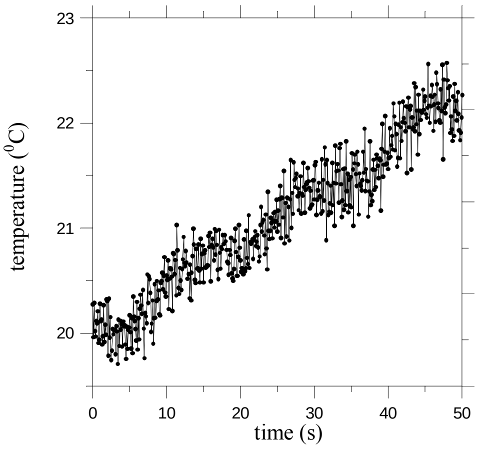


Figure 7. Simulated temperature at the sensor position with $\delta_j = \pm 0.5°C$

Table1. Optimum values of tuning parameters obtained by using simulated annealing

Initial guess		Optimum values (Simulated annealing)	
ω_C	**p.d**	**ω_C**	**p.d.**
0.1	6	0.35693	4
0.5	7	0.35721	4
1.2	3	0.35663	4
3	5	0.28958	4
4.5	7	0.28992	4

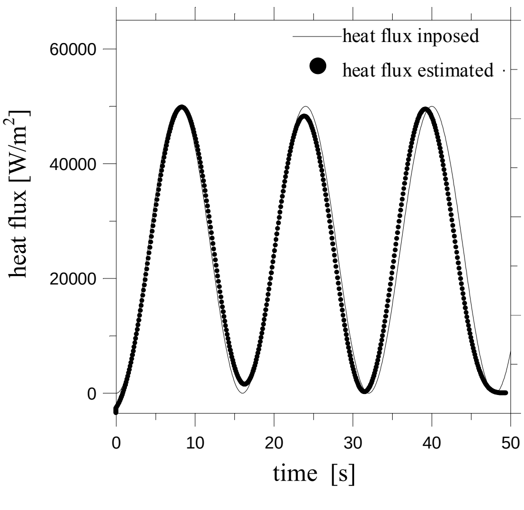
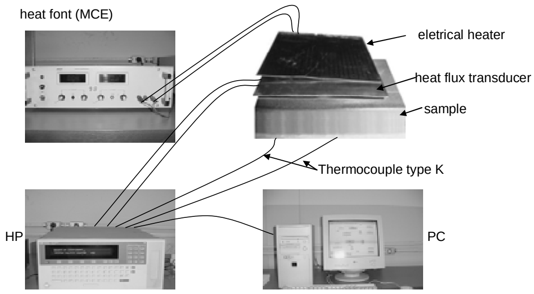


Figure 8. Imposed and estimated heat flux

3.2. Experimental three dimensional problem optimization

An experimental test was carried out in order to analyze the algorithm efficiency. Fig. 9 a shows the apparatus. An AISI304 stainless steel samples with dimensions of 127 x 127x47 mm is used in this test. The sample initially in thermal equilibrium at T_0 is then submitted to a unidirectional and uniform heat flux. The heat flux is supplied by a 318 Ω electrical resistance heater, covered with silicone rubber, and this heat flux is acquired by a transducer both with lateral dimensions of 100 x 100 mm. The temperatures are measured using surface thermocouples (type k). The signals of heat flux and temperatures are acquired by a data acquisition system HP Series 75000 with voltmeter E1326B controlled by a computer. One thermocouple was brazed on the bottom surface as shown in Fig. 9 b. Figures 10 a and b shows the experimental results: imposed heat flux and temperature evolution measured at thermocouple position.



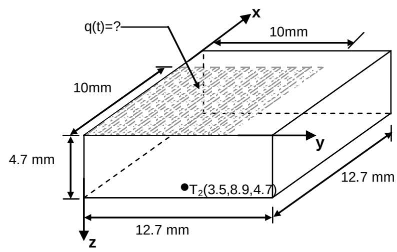
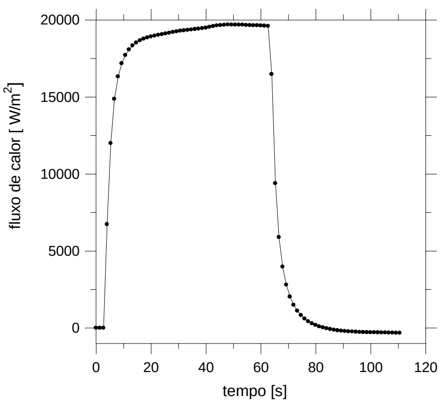


Figura 9. a) Experimental apparatus. b). Sensor location and geometric sample description



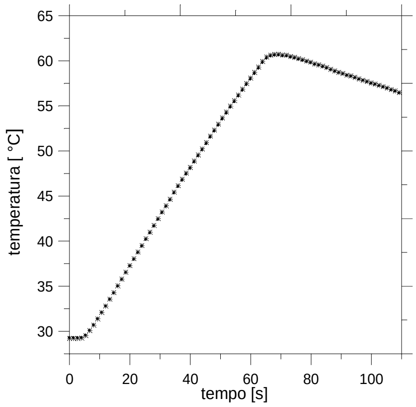


Figure 10. a) Heat flux imposed b) Temperature evolution

Using experimental temperature, Fig 10 b, and a Chebyshev filter initial configuration the optimization problem is solved. Table 2 shows initial guess the best values computed of the parameters using simulated annealing. The optimum filter is configured using cut off frequencies and Chebyshev's polynomial degrees average values. The IHCP is solved using the experimental temperature and the optimum Chebyshev filter configuration. Figure 11a shows the heat flux estimated in this work and presents a comparison with the result computed by Sousa (2006). In the Sousa's work, the Chebyshev filter has been configured by a tentative method without the optimization technique. Figure 11 b presents the absolute error between the imposed and estimated heat flux imposed using results from this work and from the Sousa's work. Although the results seems to be a similar behavior the technical proposed here presents the great advantage of obtaining of optimal configuration minimizing the time costs and the user strong dependence.

Table 2. Optimum values of tuning parameters obtained using simulated annealing

Initial guess		Optimum values (Simulated annealing)	
ω_C	**pd**	ω_C	**pd**
0.5	8	0.9974	13
0.1	6	0.8218	12
1.2	10	1.0939	13
1	3	08995	12
0.1	2	0.8795	12
1	14	1.0743	14

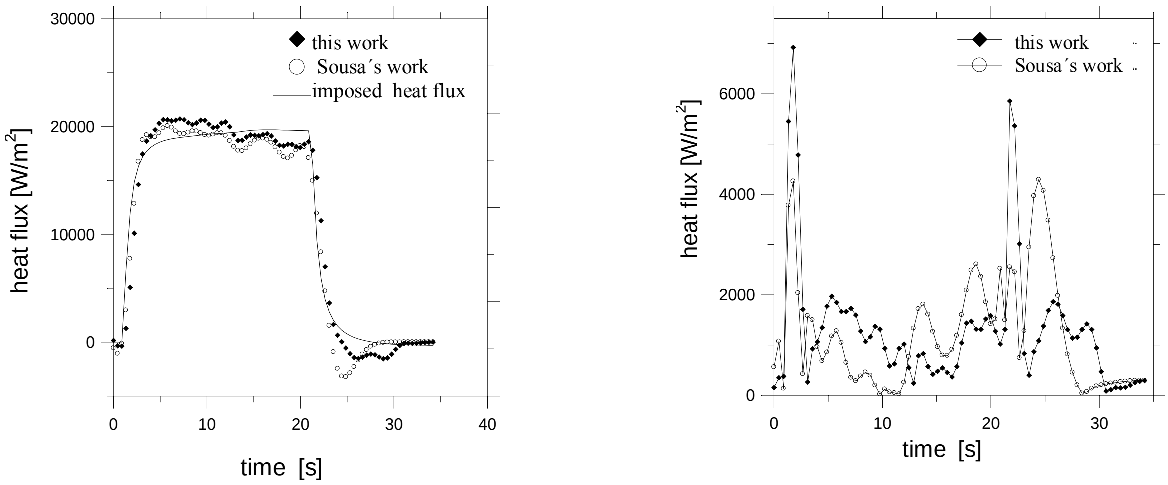


Figure 11. a) Imposed and estimated eat flux b) Absolute Error

4. CONCLUSION

This work has proposed a new procedure to configure the Chebyshev filter used in the inverse technique called dynamic observers based on Green's functions, early presented by Sousa (2006). The basic idea here is to minimize the great user dependence that is present in the dynamic observers technique. This work has show that the determination of tuning parameter like the cut off frequency *(ω_c)* and Chebyshev polynomials degree (dp) can be the way to obtain the optimum configuration with a low time cost and using an automatic process.

The efficiency of the optimization process can be observed when comparing with the estimated and experimental imposed heat flux heat flux values obtaining estimations error less than 2%. It also can be observed that although a direct comparison with the Sousa´s work do not present estimation results advantage the user dependence was greatly reduced.

5. ACKNOWLEDGEMENT

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