# Adaptive Elevation Control of a Three Degrees-of-Freedom Model Helicopter Using Neural Networks by State and Output Feedback

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***Abstract.*** *This work presents computer simulations and practical results concerning adaptive control of a class of nonlinear dynamical systems using neural networks. Two recently developed adaptive nonlinear control techniques are tested and compared on a three degrees-of-freedom under-actuated helicopter model with both parametric uncertainty and unmodeled dynamics. Computer simulations compare these methodologies in performance with a classic linear controller when parametric perturbations are induced during the tests to show how these techniques can deal with such a situation and how they overcome typical drawbacks encountered when only linear controllers are used. Moreover, the two approaches considered, including state and output feedback using two classes of linearly parameterized neural networks, illustrate the effects of previous knowledge about the plant dynamics, network architecture, information available by sensors and choice of such methodology in control system performance and design. Some practical aspects, that are crucial to real implementations, are also discussed during the design involving both approaches.*

***Keywords:*** *Adaptive control, neural networks, output feedback, state feedback, under-actuated systems.*

## 1. INTRODUCTION

In spite of the existing degree of maturity that can be encountered in linear dynamical systems theory, which can be easily checked in works like (Ogata, 1997), the same can not be said for a wide class of nonlinear dynamical systems. Consequently, many nonlinear control systems still continue to challenge the scientific community. Additionally, the growing interest in the use of novel nonlinear actuator devices and the inexistence of systematic constructive design procedures applicable to many nonlinear systems motivate the research in nonlinear control theory and design. To deal with such systems many techniques have been proposed (Kokotović and Arcak, 2001). Among them, nonlinear adaptive control using neural networks (NN) has gained attention specially in the last two decades.

The first works that used neural networks to control nonlinear plants date back to the 1980s and early 1990s. A pioneer article in this field, published by Narendra and Parthasarathy (1990), presented many NN structures for adaptive control and identification. Two years later, Sanner and Slotine (1992) used radial basis functions (RBF) NN to control affine in control nonlinear systems by state feedback. In contrast to Narendra and Parthasarathy's work, which does not have any stability proof, Sanner and Slotine used a Lyapunov based method to show the stability of the closed loop system. Furthermore, Sanner and Slotine explicitly used NN combined with a linear proportional-derivative (PD) controller and a nonlinear sliding control (Khalil, 2002) and no previous training phase was required for the NN. Following a similar approach, Lewis, Liu and Yesildirek (1995) used linearly parameterized NN to control a planar robotic manipulator.

All the works that were mentioned so far are a kind of assisted NN control. A different approach that uses NN as the main centralized controller can be found in (Rovithakis, 1999; Ge, Hang and Zhang, 1999). Despite being very interesting mathematical results, in practical applications like aircraft control (Stevens and Lewis, 2003), these methodologies have a drawback: the centralized NN control architecture can lead to undesirable or intolerable transient response.

For practical applications it is important to have a control architecture which is simple to implement, able to use the previous information available and at the same time that is less demanding of these information to have a reasonable performance. An approach that shares these characteristics may be the one explored in the works of Calise, Hovakimyan and Idan (2001), Hovakimyan, Nardi, Calise and Kim (2002), Hovakimyan, Calise and Kim (2004), Yang and Calise (2005) and Rysdyk and Calise (2005). In the works (Calise et al., 2001; Hovakimyan et al., 2002; Hovakimyan et al., 2004), dynamic inversion is used in conjunction with NN in a model reference adaptive control (MRAC) scheme to regulate the output of a non-affine in control nonlinear system where only the output is available whilst in the works (Yang and Calise, 2005; Rysdyk and Calise, 2005) a similar approach is used to control nonlinear plants by state feedback.

Most of works in adaptive control end with computational simulations and rarely compare the studied approaches. An example is the work of Kutay, Calise, Idan and Hovakimyan (2005). Even though this paper illustrates some experimental results regarding the approach explored in (Calise et al., 2001), the work is not compared to any one and many practical questions are not touched upon. Another example of practical application of NN based adaptive control scheme is that reported in (Yang, Hovakimyan, Calise and Craig, 2003), where an output feedback augmenting approach was used to control a 3-disk torsional pendulum and an inverted pendulum.

In this work we apply the techniques presented in the works of Calise et al. (2001) and Rysdyk and Calise (2005) in the control of the elevation angle of a three degrees-of-freedom (3DOF) model helicopter (Quanser, 2005). We compare both techniques regarding many practical issues that are fundamental for successful implementations and illustrate how these points can affect the control system performance. Additionally, the controllers are compared with an linear controller to illustrate how the inclusion of an adaptive element can improve the system performance. Moreover, two linearly parameterized NN architectures are used to test each approach.

## 2. PROBLEM STATEMENT

Consider a SISO, continuous-time and observable nonlinear dynamical system given by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u), \tag{1}$$

$$y = h(\boldsymbol{x}), \tag{2}$$

where $\boldsymbol{x} \in \Omega \subset \mathcal{R}^n$ is the state, $u \in \mathcal{R}$ is the input and $y \in \mathcal{R}$ is the output. In (1)-(2) $\boldsymbol{f}(.,.) : (\mathcal{R}^n \times \mathcal{R}) \mapsto \mathcal{R}$ and $h(.) : \mathcal{R}^n \mapsto \mathcal{R}$ are both continuous and differentiables. In this work, a MRAC scheme is used in the context of state and output feedback, that is, if the state is available and given a stable reference model which have a desired state trajectory given by $\boldsymbol{x}_d(t)$, generated by a bounded input signal $r(t)$, the objective is to design a control law $u(t)$ such that

$$\lim_{t\to\infty} ||\boldsymbol{x}(t) - \boldsymbol{x}_d(t)|| \leq \epsilon, \tag{3}$$

for a prespecified $\epsilon$. In (3), ||.|| stands for the Euclidean norm. When only the output measurements are available, in (3) $\boldsymbol{x}(t)$ and $\boldsymbol{x}_d(t)$ are replaced by $y(t)$ and $y_d(t)$, respectively, where $y_d(t)$ is the desired system output. It must be pointed out that in the case of output regulation it must be assured that the states are also stable.

The MRAC approach uses dynamic inversion (Khalil, 2002) and NN. The two methodologies that are used are briefly reviewed in section 3.

## 3. SUMMARY OF THE TWO ADAPTIVE NONLINEAR CONTROL METHODS

### 3.1 Neural network control of nonlinear systems by state feedback

Consider a nonlinear system given by (1). By defining $\boldsymbol{f}(\boldsymbol{x}, u) = \boldsymbol{v}$ and supposing the existence of an inverse transformation with respect to the second argument, we get

$$\dot{\boldsymbol{x}} = \boldsymbol{v}. \tag{4}$$

The use of an approximate system model $\hat{\boldsymbol{f}}(\boldsymbol{x}, u)$ furnishes an approximate linearizing control law given by

$$u = \hat{f}^{-1}(\boldsymbol{x}, v). \tag{5}$$

The inversion error is then defined as

$$\boldsymbol{\Delta}_{inv} = \boldsymbol{f}(\boldsymbol{x}, u) - \hat{\boldsymbol{f}}(\boldsymbol{x}, u). \tag{6}$$

And to cancel $\boldsymbol{\Delta}_{inv}$ and properly control the system, $\boldsymbol{v}$ is chosen as

$$\boldsymbol{v} = \boldsymbol{v}_0 - \boldsymbol{v}_{ad}, \tag{7}$$

where $\boldsymbol{v}_0$ is the output of an linear controller and $\boldsymbol{v}_{ad}$ is the output of an linearly parameterized NN, given by

$$\boldsymbol{v}_{ad} = \hat{\boldsymbol{W}}^T \boldsymbol{\beta}(\bar{\boldsymbol{x}}). \tag{8}$$

The update law for $\hat{\boldsymbol{W}}$ is obtained via Lyapunov stability analysis (Rysdyk and Calise, 2005) and can be analytically expressed as

$$\dot{\hat{\boldsymbol{W}}} = -\gamma \left[ \zeta \boldsymbol{\beta} + \lambda |\zeta| (\hat{\boldsymbol{W}} - \hat{\boldsymbol{W}}_0) \right] \tag{9}$$

where

$$\zeta = \mathbf{e}^T \mathbf{P} \mathbf{b} \tag{10}$$

In Eq. (8-10), $\mathbf{P}$ is the positive definite unique solution of $\mathbf{A}_n^T \mathbf{P} + \mathbf{P} \mathbf{A}_n + \mathbf{Q} = 0$, $\mathbf{A}_n$ and $\mathbf{b}$ are the matrices representing the error dynamics in canonical form, $\mathbf{Q} = \mathbf{I}_n$ is the identity matrix of order $n$, $\gamma$ and $\lambda$ are positive adaptation gains, $\hat{\mathbf{W}}_0$ are the initial weights, $\boldsymbol{\beta}(.)$ is a vector nonlinear function, $\bar{\mathbf{x}}$ is the vector input to the NN and $\mathbf{e}$ is the state error.

Additionally, to assure that a contraction mapping exists between $\boldsymbol{v}_{ad}$ and $\boldsymbol{\Delta}_{inv}$, it is necessary that

$$i) \qquad sgn(\partial \boldsymbol{f}/\partial u) = sgn(\partial \hat{\boldsymbol{f}}/\partial u)$$

$$ii) \qquad ||\partial \hat{\boldsymbol{f}}/\partial u|| > \frac{||\partial \boldsymbol{f}/\partial u||}{2} > 0$$

In condition (*i*), $sgn(.)$ is the signal function.

### 3.2 Neural network control of nonlinear systems by output feedback

For the output feedback case, input-output linearization (Khalil, 2002) and NN are used. Considering again the model (1-2) and differentiating the output with respect to time $r$ times gives

$$y^{(r)} = h_r(\boldsymbol{x}, u), \tag{11}$$

where $h_r \triangleq d^r h/dt^r$. To perform an approximate feedback linearization a *pseudo control* variable is defined as

$$v = \hat{h}_r(y, u). \tag{12}$$

In this case, the error due to approximate inversion is given by

$$\Delta_{inv} = h_r(\boldsymbol{x}, u) - \hat{h}_r(y, u). \tag{13}$$

Similar to the state feedback case, invertibility of $\hat{h}_r(y, u)$ with respect to the second argument is also assumed and

$$u = \hat{h}_r^{-1}(y, v). \tag{14}$$

The controller design is performed as if there were $r$ pure integrators between $v$ and $y$ and $v$ is designed as

$$v = y_d^{(r)} + v_{dc} - v_{ad}, \tag{15}$$

where $y_d^{(r)}$ is the $rth$ time derivative of the output of the reference model (RM), $v_{ad}$ is the output of an NN designed to cancel $\Delta_{inv}$ and $v_{dc}$ is the first output of a linear single-input multi-output compensator that is given by

$$\begin{bmatrix} v_{dc}(s) \\ \tilde{y}_{ad}(s) \end{bmatrix} = \frac{1}{D_{dc}(s)} \begin{bmatrix} N_{dc}(s) \\ N_{ad}(s) \end{bmatrix} \tilde{y}(s), \tag{16}$$

where $\tilde{y} = y_d - y$.

To warrant the existence and uniqueness of $v_{ad}$, it is also assumed a condition similar to (*i*) and (*ii*), where $\boldsymbol{f}$ and $\hat{\boldsymbol{f}}$ are replaced by by $h_r$ and $\hat{h_r}$, respectively. The parameter update law is given by (Calise et al., 2001)

$$\dot{\hat{\boldsymbol{W}}} = -\boldsymbol{F}\left[\tilde{y}_{ad}\boldsymbol{\beta}_f + \lambda \hat{\boldsymbol{W}}\right], \tag{17}$$

where $\hat{\boldsymbol{W}}$ are the network parameter estimates, $\boldsymbol{F}$ is a positive definite gain matrix, $\tilde{y}_{ad}$ is the second output of the linear compensator, $\boldsymbol{\beta}_f$ is the filtered output of the vector function $\boldsymbol{\beta}(\bar{\boldsymbol{x}})$ and $\lambda$ is the adaptation gain.

## 4. NONLINEAR SYSTEM DYNAMICS AND CONTROL SYSTEM DESIGN

### 4.1 Nonlinear system dynamics

Figure 1 shows the 3DOF model helicopter used to test the two adaptive control system approaches previously described and Eq. (18-19) correspond to the nonlinear dynamic model that was used to perform simulations (Lopes, 2007).

$$\begin{array}{l}
\dot{x_1} = x_2 \\
\dot{x_2} = K_1(u_1^2 - u_2^2) + K_2(u_1 - u_2) \\
\dot{x_3} = x_4 \\
\dot{x_4} = K_3cos(\alpha - x_3) + K_4cos(\beta - x_3) + K_5cos(\gamma - x_3) + K_6cos(\lambda - x_3) \\
\qquad +x_6^2[K_7cos(\alpha - x_3)sin(\alpha - x_3) + K_8cos(\beta - x_3)sin(\beta - x_3) + K_9cos(\gamma - x_3)sin(\gamma - x_3) \\
\qquad +K_{10}cos(\lambda - x_3)sin(\lambda - x_3)] + [K_{11}(u_1^2 + u_2^2) + K_{12}(u_1 + u_2)]cos(x_1) \\
\dot{x_5} = x_6 \\
\dot{x_6} = \frac{1}{f(x_3)}\{sin(x_1)[K_{13}(u_1^2 + u_2^2) \\
\qquad +K_{14}(u_1 + u_2)] + x_4x_6[K_{15}cos(\alpha - x_3)sin(\alpha - x_3) + K_{16}cos(\beta - x_3)sin(\beta - x_3) \\
\qquad +K_{17}cos(\gamma - x_3)sin(\gamma - x_3) + K_{18}cos(\lambda - x_3)sin(\lambda - x_3)]\}
\end{array}, \tag{18}$$

$$f(x_3) = K_{19}cos^2(\beta - x_3) + K_{20}cos^2(\alpha - x_3) + K_{21}cos^2(\lambda - x_3) + K_{22}cos^2(\gamma - x_3). \tag{19}$$

In Eqs. (18-19), $[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T = [\phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}]^T$. $K_i$, $\alpha$, $\beta$, $\gamma$, $\lambda$ are experimentally determined system parameters and $u_1$ and $u_2$ are voltages applied to the motors.

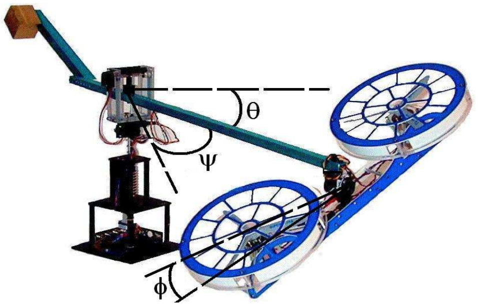


Figure 1. 3DOF model helicopter.

### 4.2 Linear control system design

To perform the control of $x_3$ it is necessary to get the required voltage to keep $x = x_{op} = [0\ 0\ 0\ 0\ 0\ 0]^T$. This is done by solving

$$2K_{11}u^2 + 2K_{12}u + K_3cos(\alpha) + K_4cos(\beta) + K_5cos(\gamma) + K_6cos(\lambda) = 0, \tag{20}$$

which is got by making $u_1 = u_2 = u$ and that have a positive approximate solution given by $u_0 = 2.9727$ volts . After that, a linearized elevation dynamics can be written in terms of small deviations with respect to the operating point

$$\begin{aligned} \dot{x}_{3op} &= x_{4op} \\ \dot{x}_{4op} &= -1.0389x_{3op} + 0.8333U_L \end{aligned} \tag{21}$$

where $U_L = u_{1L} + u_{2L}$ and $u_{1L}$ and $u_{2L}$ are voltages that should be added to $u_0$.

For the state feedback case, the approximate linearizing control can be written in terms of the pseudo control $v$ as

$$U_L = \frac{v + 1.0389x_{3op}}{0.8333}. \tag{22}$$

In the output feedback design, if we consider $x_3$ as the output, $r$ is 2 and the control project can be made with $U_L = v$.

*Remark 1*: If the model represented the true system perfectly, the exact linearizing control law would be those obtained by solving

$$2cos(x_1)\left[K_{11}u^2 + K_{12}u\right] + K_3cos(\alpha - x_3) + K_4cos(\beta - x_3) + K_5cos(\gamma - x_3) + K_6cos(\lambda - x_3) - v = 0, \tag{23}$$

which does not have a solution for $x_1 = \pi/2 \pm k\pi$, k = 0, 1, 2, ... . Additionally, it is easy to see for the output feedback case that the control effectiveness, $\partial h_r(x,u)/\partial u$, changes its sign when $x_1 > \pi/2$ or $x_1 < -\pi/2$.

At first, the linear control system design was carried out with a lead-lag compensator $G_c(s) = \frac{8s+6}{s+5}$ for the output feedback case and with similar PD controller, given by $G_{cPD}(s) = 1.2890 + 1.5538s$ for the state feedback case. In both cases, the outputs of the nominal controllers are summed to the second derivative of the RM output, that is given by

$$\ddot{y}_d + \sqrt{2}\dot{y}_d + y_d = u. \tag{24}$$

These control schemes allocate the poles of the error dynamics in $p_1 = -3$, $p_2 = -1 + j$ and $p_3 = -1 - j$. To improve the steady state performance, an additional proportional-integral (PI) action was added and the nominal controller becomes $G_c(s) = \frac{22.99s^2+29.98s+18.03}{s^2+8s+0.0079}$. To use a faster controller pseudo control hedging (Kutay et al., 2005) might be required.

*Remark 2*: In the output feedback technique the transfer function $G_c(s)$ must be stable, therefore a pure integrator can not be added. This problem is solved using an approximate PI controller given by $G_{PI}(s) = K_p + \frac{K_i}{s+c}$, where $K_p$ and $K_i$ are proportional and integral gains, respectively, and $c$ is a small positive constant, in this case $c = 0.01$.

*Remark 3*: Despite the fact that the addition of an approximate integral control action can be done easily for the output feedback case, this situation does not apply for the state feedback case because an integral action does not allow to write the error dynamics in a state space realization that does not rely on the time derivative of $\Delta_{inv}$ and $v_{ad}$.

### 4.3 Adaptive control signal design

Radial basis function and sigma-pi neural networks were used to generate the adaptive control signal.

For RBF nets, the output of each unit is given by

$$\beta_i(\bar{\boldsymbol{x}}) = e^{-\frac{(\bar{x}-x_{ci})^T(\bar{x}-x_{ci})}{\sigma^2}}, \tag{25}$$

where $\beta_i(\bar{\boldsymbol{x}})$ is the $i$th component of $\boldsymbol{\beta}(\bar{\boldsymbol{x}})$, $\boldsymbol{x}_{ci}$ are the centers of the unit and $\sigma$ is the standard deviation of the Gaussian function. It is important to note that function centers should be placed inside the interval in which the approximation is required. In this work they were uniformly distributed over the interval $[-0.18, 0.18]$ while $\sigma$ was made equal to $\sqrt{2}$.

Sigma-pi NN, on the other hand, provide polynomial approximation. In this case, the output of units are given by

$$\boldsymbol{\beta}(\bar{\boldsymbol{x}}) = [\bar{x}_1\bar{x}_1 \quad \bar{x}_1\bar{x}_2 \quad ... \quad \bar{x}_1\bar{x}_n \quad ... \quad \bar{x}_n\bar{x}_n]^T. \tag{26}$$

Despite the fact that RBF neural networks are squashing mappings, sigma-pi NN do not share this property and if $v_{ad}$ is one component of $\bar{\boldsymbol{x}}$, it must be fed through a squashing function. Moreover, while in the case of RBF networks the number of units can be freely chosen, for sigma-pi NN the number of units is $g^2$, with $g = dim(\bar{\mathbf{x}})$. A reduction could be made in the number of sigma-pi network free parameters, but this cut in the network is only justifiable when there are enough information about the structure of reconstruction error, which may not be a realistic assumption.

Neural network inputs are selected based in the functional dependence of the $\Delta_{inv}$ with respect to involved variables. In the state feedback case, the inputs are **e**, $y_c$, $\dot{y}_c$, $v_{ad}$ and a *bias* term. For the output feedback case, the network inputs are $v_{ad}$, $y(t)$, $y(t-d)$, $y(t-2d)$, $y(t-3d)$ and a *bias* term, with $d = 0.025s$ and $bias = 1$.

Moreover, output feedback requires a stable low pass filter $T^{-1}(s)$, and $N_{ad}(s)$, needed to bring $\beta_f(\bar{\mathbf{x}})$ and $\tilde{y}_{ad}$, respectivelly. In our project, when no integral action is used $\mathbf{Q} = 6.43\mathbf{I}_3$, $T^{-1}(s) = \frac{1}{s+0.6353}$ and $N_{ad}(s) = s + 0.9070$. To include an integral control action we set $\mathbf{Q} = \mathbf{I}_4$, which makes $\mathbf{C}_{c1} = [0.0854 \;\; 0.1835 \;\; 0.1480 \;\; 0.0278]$, $T^{-1}(s) = \frac{1}{s+0.2627}$ and $N_{ad}(s) = s^2 + 1.8847s + 1.2377$.

*Remark 4*: In (Calise et al., 2001) there is an error which was carried out throughout the stability proof. The corrected condition that restricts the choice of **Q** is $Q_m > 2||C_{c1}||^2$, where $Q_m$ is the minimum eigenvalue of **Q**. It is an important point and the user must pay attention because an arbitrary increase in $Q_m$ can lead to loss of stability.

It is important to stress that both approaches were used to control only the elevation angle and that the design was performed considering $u_1 = u_2$. In spite of the fact that this procedure can be considered correct from a theoretical point of view, this can not be done in a real environment because any small perturbation or system asymmetry will make $x_1$ and $x_5$ grow until a physical limit be reached. To avoid such a situation a stabilizing control loop must be designed for $x_1$ and $x_4$ can be let free to roll around the axis perpendicular to the table (in practice $x_4$ may not swivel because friction forces will counteract). In other words, while the adaptive scheme calculates $U = u_1 + u_2$, the $x_1$ stabilizing loop furnishes $V = u_1 - u_2$, thereafter the real control signals are calculated as $u_1 = \frac{U+V}{2}$ and $u_2 = \frac{U-V}{2}$.

To stabilize $x_1$, a PID controller was designed with the following approximate model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \bar{K}V \end{aligned} \tag{27}$$

where $\bar{K} = 2K_1u_0 + K_2$. The PID controller has $K_p = 17.63$, $K_i = 18.89$ and $K_d = 5.67$, calculated with $\bar{K} = 1.59$.

It must be pointed out that the aim was not to get the best achievable performance when the design was carried out. The main issue addressed was performance comparison for adaptive NN controllers assisted by similar linear controllers.

## 5. COMPUTER SIMULATION RESULTS

All the simulations were carried out considering that the system has to follow a square wave reference input of amplitude $\pm 10^o (\pm 0.1745 rad)$ and period equal to $30s$. The initial condition was set equal to $x_0 = [0.1 \;\; 0.05 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$ to show the need for a stabilizing loop for $x_1$. The integration step was $0.01s$ and the Euler's method was used.

Additionally, two tests were performed do simulate a system failure by means of parameter variation. In the first test, $20\%$ of the voltage applied to one propeller is lost and in the second the counter weight has it mass diminished by $5\%$. In both cases the parameter variation occurs at $10s$ and does not violate the contraction mapping assumption.

To compare control system performance the following performance indices were used

$$\bar{E} = \left\{ \frac{1}{T} \int_0^T [y_d(\tau) - y(\tau)]^2 d\tau \right\}^{1/2}, \tag{28}$$

$$\eta(\%) = \frac{\bar{E}_{nnoff} - \bar{E}_{ad}}{\bar{E}_{nnoff}} \times 100\%, \tag{29}$$

where $\bar{E}_{nnoff}$ is $\bar{E}$ calculated for the system only with the linear controller and $\bar{E}_{ad}$ is the same index calculated when the adaptive element is inserted.

### 5.1 Test with asymmetrical parameter variation

This section shows simulation results for the case where one propeller has its applied voltage reduced by $20\%$.

In the graphs $y_d$ is the RM output, $y_{of}$ is the response with adaptive output feedback, $y_{sf}$ is the output for the state feedback case and $y_{nnoff}$ is the output when the NN is turned off and $G_c(s)$ with PI is used. The graphs shown in Fig. 2(a) exhibit only simulations performed with sigma-pi NN. Results with Gaussian NN are summarized in Tab. 1.

Table 1. Performance indices for state and output feedback approaches and asymmetric parameter variation

| Technique | Network Type | Number of Neurons | Adaptation Gains | $\bar{E}_{ad}$ | $\eta(\%)$ |
|---|---|---|---|---|---|
| State Feedback | Gaussian | 6 | $\gamma = 15$, $\lambda = 0.2$ | 0.65 | 90.79 |
| Output Feedback | Gaussian | 6 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.05$ | 2.36 | 69.11 |
| Output Feedback + PI | Gaussian | 6 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.05$ | 0.37 | 82.22 |
| State Feedback | Sigma-pi | 36 | $\gamma = 15$, $\lambda = 0.2$ | 0.33 | 95.24 |
| Output Feedback | Sigma-pi | 36 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.08$ | 3.01 | 60.71 |
| Output Feedback + PI | Sigma-pi | 36 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.08$ | 0.43 | 79.43 |

In all the simulations it is clear that linear control design alone can not reach the performance requirements associated to the RM. When parametric changes occurs the system performance without the adaptive element is affected significantly. Inclusion of adaptive element has made the system robust to parameter variation in both approaches. It is important to stress that the linear controller alone can stabilize the system and thus can also be regarded as a “backup” controller.

Comparing the two adaptive approaches, when there is no integral control action the state feedback method performs better than the output feedback case. During the steady state phase, state feedback approach and output feedback with integral action perform almost the same. Concerning the network architecture selection, there was no significant difference in performance when sigma-pi NN were used in place of Gaussian networks.

### 5.2 Test with symmetrical parameter variation

In this section a symmetric parameter variation is simulated by changing the counter weight mass by $-5\%$.

Figure 2(b) shows the system performance for state feedback and output feedback when sigma-pi neural networks are used. Simulation results for the cases with Gaussian neural networks are summarized in Tab. 2.

Table 2. Performance indices for state and output feedback approaches and symmetric parameter variation

| Technique | Network Type | Number of Neurons | Adaptation Gains | $\bar{E}_{ad}$ | $\eta(\%)$ |
|---|---|---|---|---|---|
| State Feedback | Gaussian | 6 | $\gamma = 15$, $\lambda = 0.2$ | 1.37 | 91.17 |
| Output Feedback | Gaussian | 6 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.05$ | 5.89 | 56.15 |
| Output Feedback + PI | Gaussian | 6 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.05$ | 1.01 | 57.97 |
| State Feedback | Sigma-pi | 36 | $\gamma = 15$, $\lambda = 0.2$ | 0.35 | 97.74 |
| Output Feedback | Sigma-pi | 36 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.08$ | 4.25 | 68.36 |
| Output Feedback + PI | Sigma-pi | 36 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.08$ | 0.30 | 87.36 |

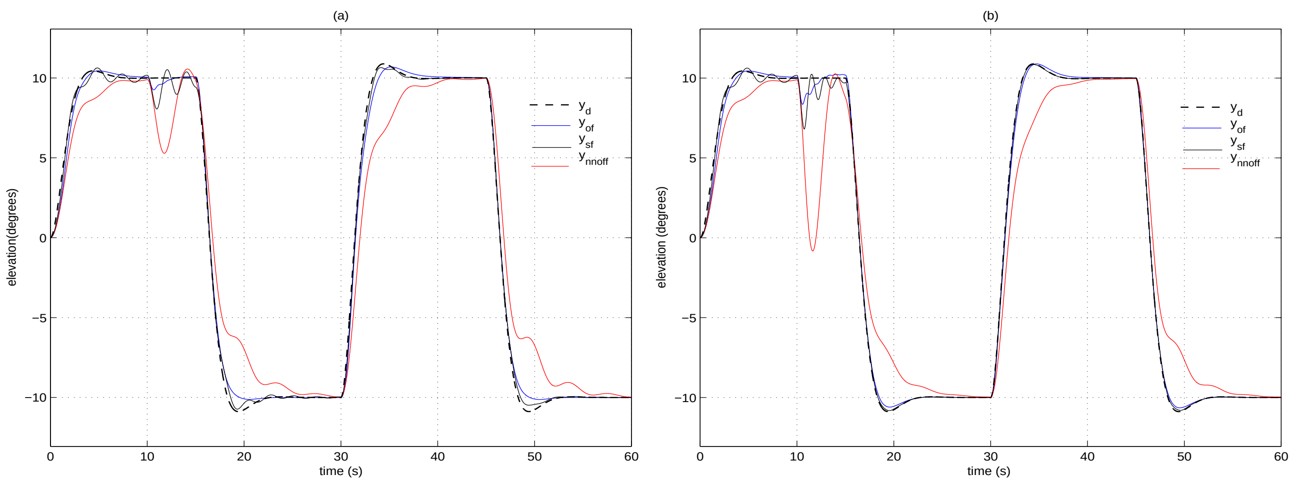


Figure 2. System performance for asymmetrical (a) and asymmetrical (b) parameter variation.

As in the previous simulations, the state feedback approach performs better than the output feedback when no integral control action is used. Inclusion of PI controller improves the output feedback controller performance significantly. The proposed parameter variation also has made the plant experience a more wide change in the elevation and all the simulations involving sigma-pi networks outperformed the Gaussian ones. It is an indicator that the number of Gaussian nodes should be increased or that the centers should be selected over a new range.

## 6. EXPERIMENTAL RESULTS

This section shows experimental results when the two approaches are used to control the 3DOF model helicopter.

The $\theta$ and $\phi$ angles are measured by a 12 bits encoder while $\psi$ has a 13 bits encoder. There are no sensors to measure the angular rates nor accelerations. To carry out the state feedback approach calculations, a filter $\frac{50s}{s+50}$ was used to estimate the required derivatives. Moreover, to keep $x_5 = \psi = 0$, $x_1 = \phi$ should be kept about $5.45^o$.

The input to the reference model was the same as in the simulations. This range does not cause actuator saturation, which occurs at $+5V$ and $0V$. Moreover, an additional pole $p = -3$ was added to the RM to make $\ddot{y}_d$ continuous.

Figure 3(a) illustrates the system performance for the state feedback and sigma-pi NN. Figure 3(b) illustrates system output for output feedback and Gaussian NN. In these figures $y_{SF}$, $y_{OF}$, $y_{PD}$ and $y_{Gc}$ are the system response for adaptive state feedback, output feedback technique, PD controller only and $G_c(s)$ controller only, respectively. Figure 4 shows NN output and $x_1$ for the output feedback case. Table 3 summarizes experimental results.

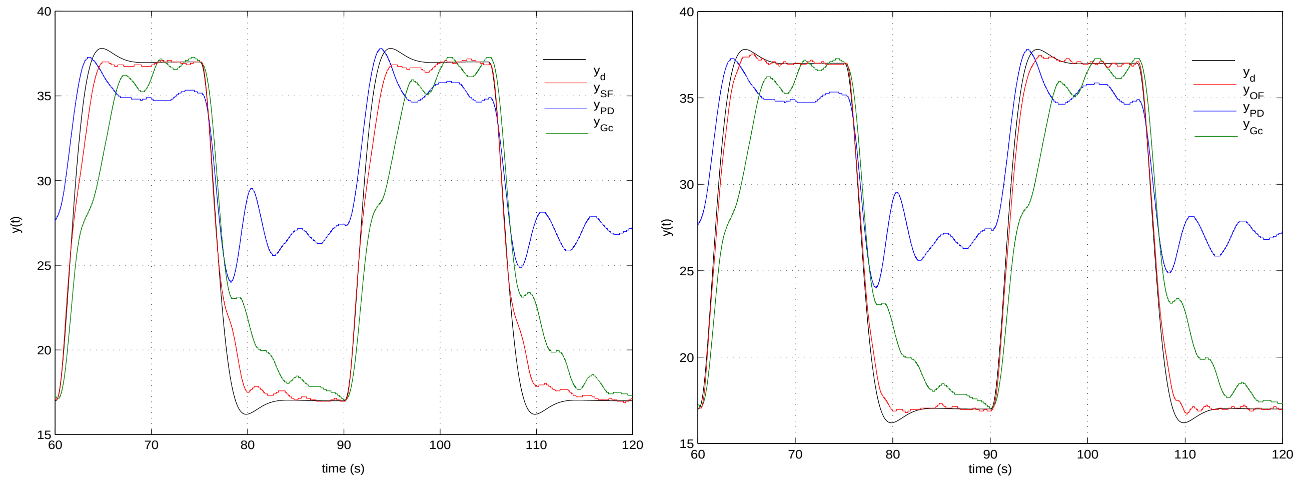


Figure 3. System performance for the adaptive state feedback (a) and output feedback (b) approaches.

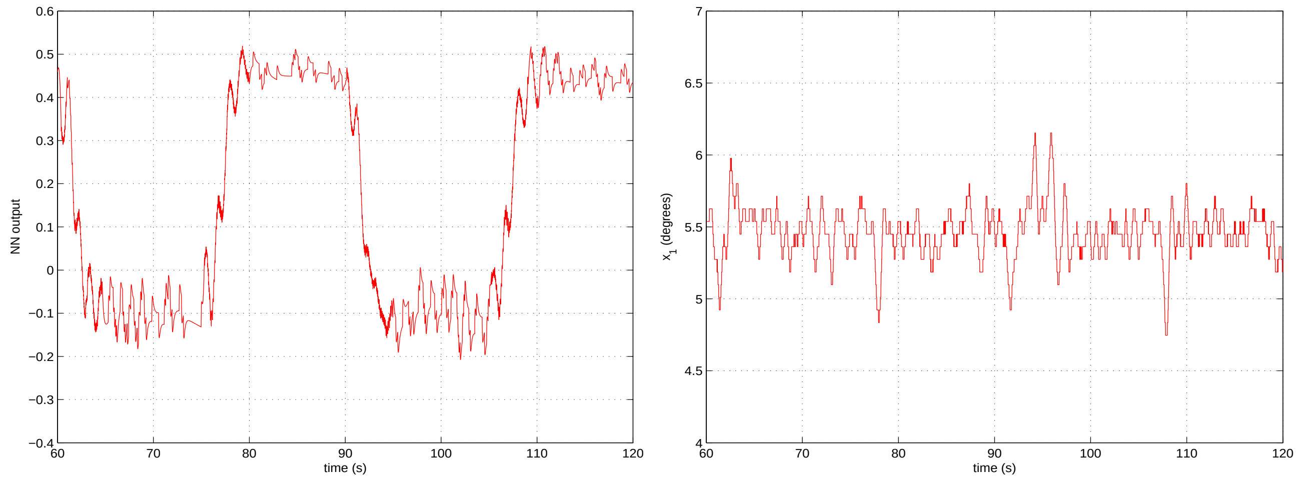


Figure 4. Neural network output (a) and $x_1$ angle (b) for output feedback approach

As in the simulations, both state and output feedback techniques seem to be effective with respect to error cancellation.

Concerning the network architecture selection, sigma-pi neural networks and RBF NN showed equivalent performance. It must be pointed out , however, that the same algorithm may require different adaptation gains when the NN architecture is changed.

Table 3. Performance indices for state and output feedback approaches

| Technique | Network Type | Number of Neurons | Adaptation Gains | $E_{ad}$ | $\eta(\%)$ |
|---|---|---|---|---|---|
| State Feedback | Gaussian | 6 | $\gamma = 4$, $\lambda = 0.5$ | 1.50 | 78.20 |
| OutputFeedback + PI | Gaussian | 6 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.12$ | 0.52 | 85.19 |
| State Feedback | Sigma-pi | 36 | $\gamma = 4$, $\lambda = 0.5$ | 1.09 | 84.23 |
| Output Feedback + PI | Sigma-pi | 36 | $\boldsymbol{F} = 250\boldsymbol{I}_6$, $\lambda = 0.12$ | 0.90 | 74.33 |

## 7. CONCLUDING REMARKS

In this work two nonlinear adaptive control techniques were experimentally validated and compared in a non-affine in control 3DOF model helicopter. In both cases the adaptive NN controller seemed to be effective in nonlinear dynamic compensation. This assertion can be seen in significant improvements in tracking capability, showed in the simulation and experimental results. Moreover, simulation results show that both techniques share robustness to parameter variation.

Nonlinear adaptive output feedback approach with PI control has showed performance similar to the state feedback one. In addition to the fact that the output feedback approach does not rely on state measurement or estimation, computational and experimental results suggest that another advantage of this approach over the state feedback one is the freedom left for the linear control system design, which significantly affects the overall system performance, specially in the transient phase. It is important to stress, however, that the state feedback approach performed well even when a rough state estimate was used in place of real variables measurements.

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## 9. RESPONSIBILITY NOTICE

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