ENGINEERING SYSTEM DESIGN WITH MULTI-OBJECTIVE DIFFERENTIAL EVOLUTION

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Abstract. *Realistic problems are usually characterized by the presence of constraints and several conflicting objectives to be taken into account simultaneously. The solution to these problems, differently from single objective optimization, consists in obtaining non-dominated solutions thus forming the so-called Curve of Pareto or Pareto Optimal Front. Two approaches exist for obtaining the Curve of Pareto: the Deterministic approach that makes use of the Variational Calculus and the Non-Deterministic one that it is based on the process of natural selection, i.e., in the genetics of the populations or in purely structural methodologies. The use of the Non-Deterministic approach is getting increasing attention in the last decade, mainly due to the fact that they do not use derivatives and they can be easily implemented. The algorithm known as Differential Evolution (DE), which is considered as a structural approach, has shown to be a viable alternative for handling realistic problems. This work is dedicated to the extension of DE for cases in which multiple objectives are considered. For this purpose, two operators are used to obtain the optimal solutions, namely the ranking of the Pareto-optimal solutions and the exploration of neighborhood potential solutions. The algorithm was tested in classic problems found in the literature. The obtained results have shown promising results as compared with other evolutionary strategies such as the one based on genetic algorithms.*

Keywords: *multi-objective optimization, differential evolution, neighborhood exploring evolution.*

1. INTRODUCTION

Most real-world problems involve the simultaneous optimization of two or more (often conflicting) objectives. The solution of such problems (called multi-criteria or multi-objective optimization problem - MOOP) is different from that of a single-objective optimization problem. The main difference is that multi-objective optimization problems normally have not one but a set of solutions which are all equally good (Stadler, 1984).

The development of specific methodologies for the treatment of multiple objectives allows the formulation of the optimization problem such that various objectives can be taken into account simultaneously. As a number of optimal solutions are found, it is possible to explore these solutions according to the practical application studied (Deb, 2001).

There are several methods available in the literature for solving MOOP (Deb, 2001). These methods follow a preference-based approach, in which a relative preference vector is used to scalarize multiple objectives. Since classical searching and optimization methods use a point-by-point approach, at which the solution is successively modified, the outcome of this classical optimization method is a single optimized solution. However, Evolutionary Algorithms (EA) can find multiple optimal solutions in one single simulation run due to their population-based search approach. Thus, EA are ideally suited for multi-objective optimization problems. A detailed account of multi-objective optimization using EA and some of the applications using genetic algorithms can be widely found in the literature (Zitzler and Thiele, 1999; Knowles and Corne, 2000; Deb, 2001).

The main goal of this paper is to introduce a systematic methodology for the solution of multi-objective optimization problems by using the Differential Evolution Algorithm. This work is presented as follows. Section 2 shows the basic concepts on Multi-Objective Optimization, emphasizing Differential Evolution. In Section 3 and Section 4 the proposed methodology is introduced and some case studies are presented. Finally, the conclusions are outlined in Section 5.

2. MULTI-OBJECTIVE OPTIMIZATION

When dealing with MOOP, the notion of optimality needs to be extended. The most common one in the current literature is that originally proposed by Edgeworth (Edgeworth, 1881) and later generalized by Pareto (Pareto, 1896). This notion is called Edgeworth-Pareto optimality, or simply Pareto optimality, and refers to finding good tradeoffs among all the objectives. This definition leads us to find a set of solutions that is called the Pareto optimal set, whose corresponding elements are called nondominated or noninferior.

The concept of optimality in single objective is not directly applicable in MOOPs. For this reason a classification of the solutions is introduced in terms of Pareto optimality, according to the following definitions (Deb, 2001):

Definition 1 - The Multi-objective Optimization Problem (MOOP) can be defined as:

$$Minimize/Maximize \qquad y = f(x) = (f_1(x), f_2(x), ..., f_m(x)) \qquad m = 1, ..., M \tag{1}$$

subject to

$$g(x) = (g_1(x), g_2(x), ..., g_j(x)) \geq 0 \qquad j = 1, ..., J \tag{2}$$

$$x = (x_1, x_2, x_n) \in X \qquad n = 1, ..., N \tag{3}$$

where x is the vector of decision (or design) variables, y is the vector of objective functions and X is denoted as the decision (or design) space. The constraints $g(x)\geq 0$ determine the feasible region.

Definition 2 - *Pareto Dominance*: For any two decision vectors **u** and **v**, **u** is said to dominate **v**, if **u** is no worse than **v** in all objectives and **u** is strictly better than **v** in at least one objective.

Definition 3 - *Pareto Optimality*: When the set *P* is the entire search space, or *P* = *S*, the resulting non-dominated set *P'* is called the Pareto-optimal set. Like global and local optimal solutions in the case of single-objective optimization, there could be global and local Pareto-optimal sets in multi-objective optimization.

Definition 4 - *Non-dominated Set*: Among a set of solutions *P*, the non-dominated set of solutions *P'* are those that are not dominated by any member of the set *P*.

2.1 Multiple-Objective Evolutionary Algorithms (MOEAs)

Multiple-Objective Evolutionary Algorithms (MOEAs) is the term employed in the evolutionary multi-criteria optimization field to refer to as a group of evolutionary algorithms formulated to deal with MOOP. This group of algorithms conjugates the basic concepts of dominance described in the later section with the general characteristics of evolutionary algorithms. MOEAS are able to deal with non-continuous, non-convex and/or non-linear spaces, as well as problems whose objective functions are not explicitly known (Deb, 2001).

Since the first MOEA (VEGA (Schaffer, 1985)), the development of MOEAS has successfully evolved, producing better and more efficient algorithms. The existing MOEAS are classified in two groups (Zitzler and Thiele, 1999; Deb, 2001), according to its characteristics and efficiency. On the one hand there is a first group known as first-generation, which includes all the early MOEAS (Weighted Sum, VEGA (Schaffer, 1985), NPGA (Horn and Nafpliotis, 1993), NSGA (Srinivas and Deb, 1994). On the other hand there is a second group named second-generation MOEAS, which comprises very efficient optimizers like SPEA (Zitzler and Thiele, 1999)/SPEA2 (Zitzler *et al.*, 2001) and NSGA-II (Deb *et al.*, 2002), among others. Basically, the main features that distinguish second generation MOEAS from the first-generation group are:

- *Mechanism of adaptation assignment in terms of dominance*: between one non-dominated solution and another dominated, the algorithm will favor the nondominated one. Moreover, when both solutions are equivalent in dominance, the one located in a less crowded area will be favored. Finally, the extreme points, (i.e. the solutions that have the best value in one particular objective) of the non-dominated population are preserved and their adaptation is better than any other non-dominated point, to allow maximum front expansion;

- *Incorporation of elitism*: the elitism is commonly implemented using a secondary population of nondominated solutions previously stored. When performing recombination (selection-crossover-mutation), parents are taken from this archive in order to produce the offspring.

2.2 Multi-Objective Differential Evolution

Differential Evolution (DE) (Price and Storn, 1997) is an improved version of the Goldberg's Genetic Algorithm (GA) (Goldberg, 1989) for faster optimization. Unlike simple GA that uses binary coding for representing problem parameters, DE is a simple yet powerful population based, direct search algorithm for globally optimizing functions with real valued parameters. Among the DES advantages are its simple structure, easiness of use, speed and robustness (Babu and Gaurav, 2000; Babu *et al.*, 2001; Babu *et al.*, 2005; Price *et al.*, 2005). Price and Storn (1997) gave the working principle of DE with single strategy. The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. The key parameters of control in DE are: N the population size, p_c the crossover constant, and F the weight applied to random differential (scaling factor). Price and Storn (1997) have given some simple rules for choosing key parameters of DE for any given application. Normally, N should be about 5 to 10 times the dimension (number of parameters in a vector) of the problem. As for F, it lies in the range 0.4 to 1.0. Initially F = 0.5 can be tried then F and/or N is increased if the population converges prematurely.

DE has been successfully applied in various fields. Some of the successful applications of DE include: digital filter design (Storn, 1995), batch fermentation process (Chiou and Wang, 1999), estimation of heat transfer parameters in trickle bed reactor (Babu and Sastry, 1999), synthesis and optimization of heat integrated distillation system (Babu and Singh, 2000), optimization of an alkylation reaction (Babu and Gaurav, 2000), optimization of thermal cracker operation (Babu and Angira, 2001), solution of multi-objective optimal control problems with index fluctuation applied to determine the switching times (events) and time of operation of the fermentation process (Lobato *et al.*; 2007), and other applications (Price *et al.*, 2005).

Recently, several attempts to extend the DE to solve multi-objective problems can be found in the literature. The most representative of them are briefly described below:

- PDE (Abbass, 2002): It handles only one (main) population. Reproduction is undertaken only among nondominated solutions, and offspring are placed into the population if they dominate the main parent. A distance metric relationship is used to maintain diversity;
- PDEA (Madavan, 2002): It combines DE with key elements from the NSGA-II (Deb *et al.*, 2002) such as its nondominated sorting and ranking selection procedure;
- MODE (Xue *et al.*, 2003): It uses a variant of the original DE, in which the best individual is adopted to create the offspring. Also, the authors adopt Pareto ranking and crowding distance in order to produce and maintain well-distributed solutions;
- VEDE (Parsopoulos *et al.*, 2004): It is a parallel, multi-population DE approach, which is based on the Vector Evaluated Genetic Algorithm (VEGA) (Schaffer, 1985);
- NSDE (Iorio and Li , 2004): It is a simple modification to the NSGA-II (Deb *et al.*, 2002) where the real-coded crossover and mutation operators of the NSGA-II are replaced with the DE scheme.
- DEMO (Robič and Filipič , 2005): It combines the advantages of DE with the mechanisms of Pareto-based ranking and crowding distances sorting. In DEMO, the newly created candidates immediately take part in the creation of the subsequent candidates.
- MODE (Babu *et al.*, 2005): algorithm based on DE and in the concept of Pareto. The authors applied the proposed algorithm to optimize industrial adiabatic styrene reactor considering productivity, selectivity and yield as the main objectives.

3. METHODOLOGY

The proposed algorithm in this work for MOOP using DE has the following structure: an initial population of size N is randomly generated. All dominated solutions are removed from the population through the operator Fast Non-Dominated Sorting (Deb, 2001). In this way, the population is sorted into non-dominated fronts F_j (sets of vectors that are non-dominated with respect to each other). This procedure is repeated until each vector is member of a front. Three parents are selected at random in the population. A child is generated from the three parents (this process continues until *N* children are generated). Starting from population P_1 of size 2*N*, neighbors are generated each one of the individuals of the population in the following way (Hu *et al*, 2006):

$$\chi(x) = [x - D_k(g)/2, x + D_k(g)/2] \tag{4}$$

where

$$D_k(g) = \frac{k}{R}[U - L] \tag{5}$$

D_K(g) is a vector in $\mathbb{R}^n$ and a function of the generation counter g. R is the number of pseudo fronts defined by the user and the initial maximum neighborhood size in a population is D_K(0)=[U-L], where L and U represent the lower and upper bounds of the variables. The pre-defined number of individuals in each pseudo front is given by (Hu *et al*, 2006):

$$n_k = rn_{k-1} \qquad k = 2, ..., R \tag{6}$$

where n_k is the number of individuals in the k-th front and r (<1) is the reduction rate. For a given population with N individuals, n_k can be calculated as

$$n_k = N\frac{1-r}{1-r^R}r^{k-1} \tag{7}$$

According to Hu *et al.* (2006), if $r < 1$, the number of individuals in the first pseudo front is the highest and each pseudo front has an exponentially reducing number of solutions, this emphasizing a local search. On the contrary, a greater r results in more solutions in the last pseudo front and hence emphasizes the global search.

This way, the neighbors generated are classified according to the dominance criterion and only the neighbors non-dominated (P_2) will be put together with P_1 to form P_3. The population P_3 is then classified according to the dominance criterion. If the number of individuals of the population P_3 is larger than a number defined by the user, it is truncated according to the criterion the Crowding Distance (Deb, 2001). The Crowding Distance describes the density of solutions surrounding a vector. To compute the Crowding Distance for a set of population members the vectors are sorted according to their objective function value for each Objective Function. To the vectors with the smallest or largest values an infinite Crowding Distance (or an arbitrarily large number for practical purposes) are assigned. For all other vectors the Crowding Distance is calculated according to:

$$dist_{x_i} = \sum_{m-1}^{j=0} \frac{f_{j,i+1} - f_{j,i-1}}{|f_{j,max} - f_{j,min}|} \tag{8}$$

where f_j corresponds to the j-th objective function and m equals the number of Objective Functions.

In this work, the treatment of constraints is made through of the Static Penalization Method (Vanderplaats, 1999). In the previous mentioned reference, the author affirms that the difficulty in the choice of the parameters constitutes the main drawback in this method, because no general rule can be applied to determine these parameters. To overcome this disadvantage, Castro (2001) proposed an approach that consists of the attribution of limit values to each objective to play the role of penalization parameters. According to this author, it is guaranteed that any nondominated solution dominates any solution that violates at least one constraint. In the same way, any solution that violates one constraint will dominate any solution that presents two constraint violations, and so on. This way, layers of solutions are obtained, and consequently the number of constraint violations corresponds to the rank of the solution. For a constrained problem the vector containing the objective functions to be accounted for is given by:

$$f(x) \equiv f(x) + r_p n_{viol} \tag{9}$$

where $f(x)$ it is the vector of objective functions, r_p it is the vector of penalty parameters that depends on the type of problem considered, and n_{viol} is the number of violated constraints.

4. ILLUSTRATIVE EXAMPLES

In this section two cases to demonstrate the efficiency of the proposed methodology are studied.

4.1 Beam with Section I

The objective of this problem is the determination of the curve of Pareto for the multi-objective optimization of the beam with section I, as presented in Fig. 1 (Castro, 2001):

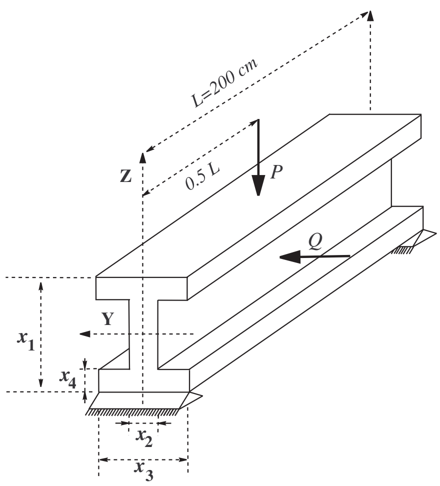

Figure 1. Beam with section I.

The properties of the beam and the values of the loads are (Castro, 2001):

- Young Modulus $(E) = 2 \times 10^4$ kN/cm^2;
- Yielding stress of the beam $(\sigma) = 16$ kN/cm^2;
- Vertical load (P) and horizontal load (Q), applied in the mid point of the beam: 600 KN and 50 kN, respectively.

The design variables, x_1, x_2, x_3 and x_4, are the dimensions of the beam (given in centimeters), whose magnitudes should not violate the following constraints: $10 \le x_1 \le 80$, $10 \le x_2 \le 50$, $0.9 \le x_3 \le 5$ e $0.9 \le x_4 \le 5$. Besides, the problem presents the following design constraint:

$$\frac{M_{\mathbf{Y}}}{W_{\mathbf{Y}}} + \frac{M_{\mathbf{Z}}}{W_{\mathbf{Z}}} \le \sigma \quad (10)$$

where M_y (30000 kN cm) and M_z (25000 kN cm) are the maximum moments in the directions **Y** and **Z**; $W_{\mathbf{Y}}$ and $W_{\mathbf{Z}}$ are the resistance modules of the section in the directions **Y** and **Z**, and σ it is the stress limits established. The shear modules are calculated by the following expressions:

$$W_{\mathbf{Y}} = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4(4x_4^2 + 3x_1(x_1 - 2x_4))}{6x_1} \quad (11)$$

$$W_{\mathbf{Z}} = \frac{(x_1 - 2x_4)x_3^3 + 2x_4x_2^3}{6x_2} \quad (12)$$

The objectives of this problem are the area of the cross section (in cm^2) and the maximum static displacement (in cm), which are both to be minimized, according, respectively, to equation 13:

$$\begin{cases} \min f_1 = 2x_2x_4 + x_3(x_1 - 2x_4) \\ \min f_2 = \dfrac{PL^3}{48EI} \end{cases} \quad (13)$$

where the moment of inertia I is calculated by equation (14):

$$I = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4(4x_4^2 + 3x_1(x_1 - 2x_4))}{12} \quad (14)$$

The parameters used by PMOGA and by MODE are presented in Tab. 1.

Table 1. Parameters used to solve the I-beam problem.

Parameter	PMOGA	MODE
$Ngen$	500	**50**
N	50	**30**
p_c	0.85	0.85
p_m	0.05	-
F	-	0.50
R	-	10
r	-	0.90
r_{p_1}	1000	1000
r_{p_2}	10	10

The initial population and the solution obtained by MODE, compared with those obtained by PMOGA, are presented in Fig. 2 and in Fig. 3, respectively.

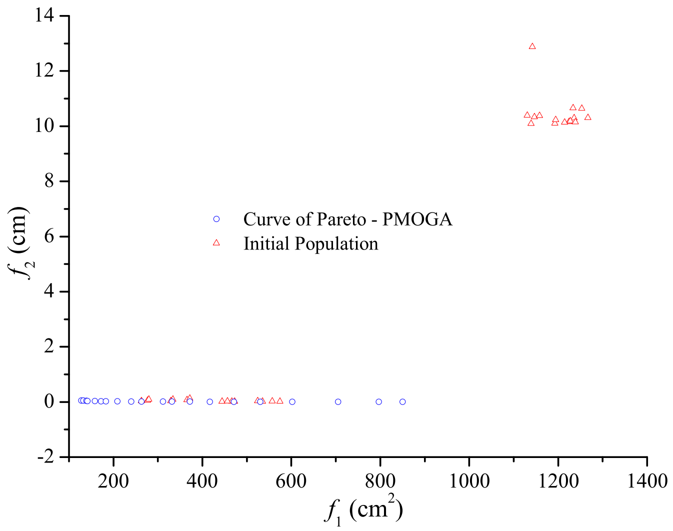

Figure 2. Initial population for the I-beam problem.

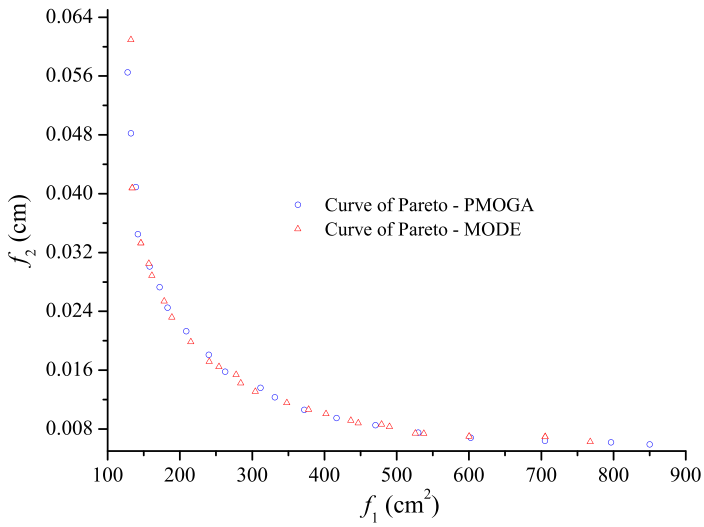

Figure 3. Results obtained for the I-beam problem.

4.2 Welded Beam

The problem consists of a beam submitted to a force F applied to one of its ends that is to be welded to another structural component satisfying the conditions of stability and the design limitations. The four design variables, thickness of the weld (h), length of the weld (l), width of the beam (t) and thickness of the beam (b), they are suitable in the Fig. 4 (Castro, 2001).

The two conflicting objective functions to be minimized are the cost of the beam and the displacement of the free end of the beam:

$$\begin{cases} \min f_1 = 1.10471h^2l + 0.04811tb(14 + l) \\ \min f_2 = \dfrac{2.1952}{t^3b} \end{cases} \tag{15}$$

subject to the following constraints:

$$\tau - \tau_{max} \leq 0 \tag{16}$$

$$\sigma - \sigma_{max} \leq 0 \tag{17}$$

$$F - P_c \leq 0 \tag{18}$$

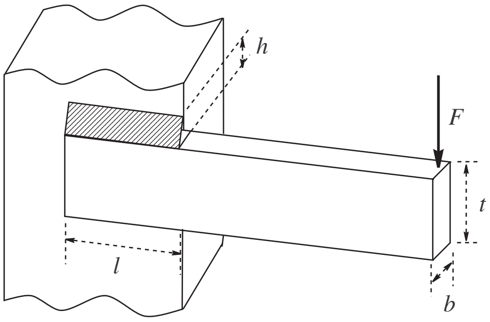

Figure 4. Welded beam.

$$\frac{2.1952}{t^3 b} - u_{max} \leq 0 \quad (19)$$

$$h - b \leq 0 \quad (20)$$

for $0.125 \leq h, b \leq 5, l \geq 0.1, t \leq 10$.

The first two constraints guarantee that the shear stress and the normal stress developed along the support of the beam are respectively smaller than the permissible shear stress (τ_{max}) and normal stress (σ_{max}) of the material. The third constraint guarantees that the resistant effort (along the direction of t) of the beam end is larger than the applied load F. The fourth constraint is a maximum limit (u_{max}) for the displacement in the end of the beam. Finally, the fifth constraint guarantees that the thickness of the beam is not smaller than the thickness of the weld.

The stress and the terms of the equations (16-18) are given by:

$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + \frac{l\tau_1\tau_2}{\sqrt{0.25(l^2 + (h+t)^2)}}} \quad (21)$$

$$\tau_1 = \frac{6000}{\sqrt{2}hl} \quad (22)$$

$$\tau_2 = \frac{6000(14 + 0.5l)\sqrt{0.25(l^2 + (h+t)^2)}}{2(0.707hl(\frac{l^2}{12} + 0.25(h+t)^2))} \quad (23)$$

$$\sigma = \frac{504000}{t^2 b} \quad (24)$$

$$P_c = 64746.022(1 - 0.0282346t)tb^3 \quad (25)$$

The adopted data for this problem are the following (Castro, 2001): F = 6000 lb, τ_{max} = 13600 psi, E = 30 × 10^6 psi, σ_{max} = 30000 psi, G = 12 × 10^6 psi, u_{max} = 0,25 in and L = 14 in.

The parameters used by PMOGA and by MODE are presented in Tab. 2.

Table 2. Parameters used to solve the problem of the welded beam.

Parameter	PMOGA	MODE
Ngen	500	**50**
N	200	**100**
p_c	0.85	0.85
p_m	0.05	-
F	-	0.50
R	-	10
r	-	0.90
r_{p_1}	100	100
r_{p_2}	0.01	0.01

The initial population and the solution obtained by MODE are presented in Fig. 5 and in Fig. 6, respectively.

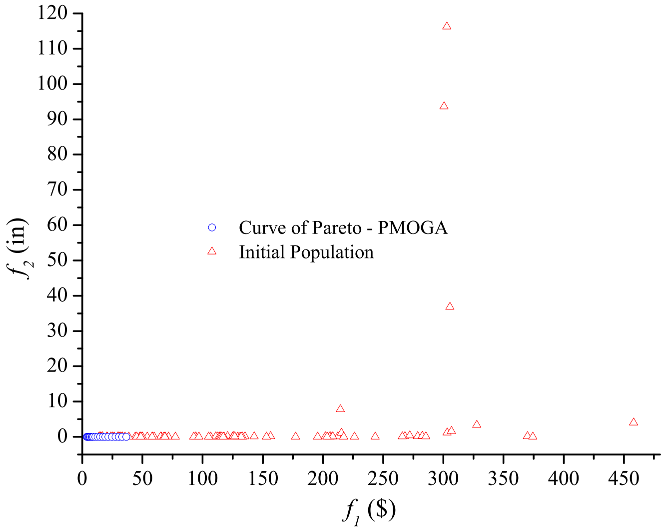

Figure 5. Initial population for the problem of the welded beam.

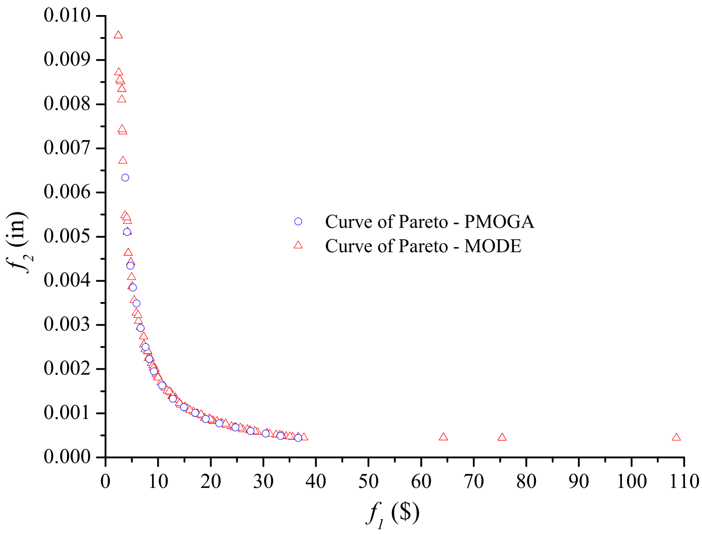

Figure 6. Results obtained for the problem of the welded beam.

5. DISCUSSIONS AND CONCLUSIONS

In this paper, a new differential evolution approach is presented for dealing with multi-objective optimization problems. This methodology consists in the extension of the Algorithm of Differential Evolution to problems with multiple objectives, through the incorporation of two operators to the original algorithm, namely the mechanisms of rank ordering and the exploration of the neighborhood potential solution candidates.

The proposed algorithm is applied to two classical problems of engineering. In the resolution of the two cases studies, the parameters used by MODE led to good results as compared to those found in the literature, however with a smaller number of individuals in the population and a smaller number of generations. The computational time for the two applications studied was about 30 seconds for the first case and 160 seconds for second case, using a PENTIUM IV microcomputer with 3.2 GHz and 2 GB RAM.

Finally, the results show that the proposed algorithm represents an interesting alternative for the treatment of optimization problems with conflicting objectives.

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7. REFERENCES

Abbass, H. A., 2002, "The Self-Adaptive Pareto Differential Evolution Algorithm". In Congress on Evolutionary Computation (CEC 2002), volume 1, pages 831-836, IEEE Service Center.

Babu, B. V. and Angira, R., 2001, "Optimization of Thermal Cracker Operation using Differential Evolution," in Proceedings of International Symposium and 54th Annual Session of IIChE (CHEMCON-2001).

Babu, B. V., Chakole, P. G. and Mubeen, J. H. S., 2005, "Multiobjective Differential Evolution (MODE) for Optimization of Adiabatic Styrene Reactor". Chemical Engineering Science. Vol 60, pp. 4822-4837, 2005.

Babu, B. V. and Gaurav, C., 2000, "Evolutionary Computation Strategy for Optimization of an Alkylation Reaction," in Proceedings of International Symposium and 53rd Annual Session of IIChE (CHEMCON-2000).

Babu, B. V. and Sastry, K. K. N., 1999, "Estimation of Heat-transfer Parameters in a Trickle-bed Reactor using Differential Evolution and Orthogonal Collocation," Computers & Chemical Engineering, Vol. 23, pp. 327-339.

Babu, B. V. and Singh, R. P., 2000, "Synthesis and Optimization of Heat Integrated Distillation Systems Using Differential Evolution," in Proceedings of All-India seminar on Chemical Engineering Progress on Resource Development: A Vision 2010 and Beyond, IE (I).

Castro, R. E., 2001, "Optimization of Structures with Multi-objective through Genetic Algorithms". Tese de D.Sc. (in portuguese), COPPE/UFRJ, Rio de Janeiro, Brazil.

Chiou, J. P. and Wang, F. S., 1999, "Hybrid Method of Evolutionary Algorithms for Static and Dynamic Optimization Problems with Application to a Fed-batch Fermentation Process," Computers & Chemical Engineering, vol. 23, pp. 1277-1291.

Deb, K., 2001, "Multi-objective Optimization using Evolutionary Algorithms", John Wiley & Sons, Chichester-UK, ISBN 0-471-87339-X.

Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T., 2002, "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II". IEEE Transactions on Evolutionary Computation, 6(2):182-197.

Edgeworth, F. Y., 1881, "Mathematical Physics", P. Keagan, London, England.

Goldberg, D. E., 1989, "Genetic Algorithms in Search, Optimization, and Machine Learning," MA: Addison-Wesley.

Horn J. and Nafpliotis, N., 1993, "Multiobjective Optimization using the Niched Pareto Genetic Algorithm". IlliGAL Report 93005, Illinois Genetic Algorithms Laboratory, University of Illinois, Champaign, IL.

Hu, X., Coello Coello, C. A. and Huang, Z., 2006, "A New Multi-objective Evolutionary Algorithm: Neighborhood Exploring Evolution Strategy", http://www.lania.mx/∼ccoello/EMOO, accessed in 15/07/2006.

Iorio, A. W. and Li, X., 2004, "Solving Rotated Multi-objective Optimization Problems using Differential Evolution". In AI 2004: Advances in Artificial Intelligence, Proceedings, pages 861-872. Springer-Verlag, LNAI Vol. 3339, 2004.

Knowles, J. D. and Corne, D. W., 2000, "Approximating the Non-dominated Front using the Pareto Archived Evolution Strategy". Evolutionary Computation, Vol 8, No. 2, pp. 149-172.

Lobato, F. S., Murata, V. V., Oliveira-Lopes, L. C., Steffen, V., 2007, "Solution of Multi-objective Optimal Control Problems with Index Fluctuation using Differential Evolution". In CD Rom of 6^o Brazilian Conference on Dynamics, Control and Their Applications. UNESP, Campus de São José do Rio Preto, SP, Brazil.

Madavan, N. K., 2002, "Multiobjective Optimization Using a Pareto Differential Evolution Approach". In CEC 2002, volume 2, pages 1145-1150, IEEE Service Center.

Pareto, V., 1896, "Cours D'Economie Politique", Vol. I and II, F. Rouge, Lausanne.

Parsopoulos, K., Taoulis, D., Pavlidis, N., Plagianakos, V. and Vrahatis, M., 2004, "Vector Evaluated Differential Evolution for Multiobjective Optimization". In CEC 2004, volume 1, pages 204-211, IEEE Service Center.

Price, K. and Storn, R., 1997, "Differential Evolution - A Simple Evolution Strategy for Fast Optimization", Dr.Dobb's Journal, vol. 22, n. 4, pp. 18-24.

Price, K. V. and Storn, R. M. and Lampinen, J. A., 2005, "Differential Evolution, A Pratical Approach to Global Optimization", Springer-Verlag Berlin Heidelberg Media.

Robič, T. and Filipič, B., 2005, "DEMO: Differential Evolution for Multiobjective Optimization". In C. A. Coello Coello, *et al.*, editors, Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005, pp. 520-533, Springer, LNCS Vol. 3410.

Schaffer, J. D., 1985, "Multiple Objective Optimization with Vector Evaluated Genetic Algorithms". In Proceedings of the First International Conference on Genetic Algorithms, pages 93-100. Lawrence Erlbaum, 1985.

Srinivas, N. and Deb, K., 1994, "Multiobjective Optimization using Nondominated Sorting in Genetic Algorithms". Evolutionary Computational, Vol 2(3), pp. 221-248.

Stadler, W., 1984, "A Survey of Multicriteria Optimization or the Vector Maximum Problem, Part I", Journal of Optimization Theory and Applications, Vol 29, No. 1, pp. 1776-1960.

Storn, R., 1995, "Differential Evolution Design of an IIR-Filter with Requirements for Magnitude and Group Delay," International Computer Science Institute, TR-95-026.

Vanderplaats, G. N., "Numerical Optimization Techniques for Engineering Design", VR D INC. Colorado Springs, CO, USA, Third Edition, pp. 441.

Xue, F., Sanderson, A. C. and Graves, R. J., 2003, "Pareto-based Multi-Objective Differential Evolution". In Proceedings of the 2003 Congress on Evolutionary Computation (CEC 2003), Vol. 2, pp. 862-869, IEEE Press.

Yan, L. 1998. "New algorithm of Global Optimization for Process System Line-up Competition Algorithm". Ph. D. Thesis, Beijing University of Chemical Technology, Beijing (in Chinese).

Zitzler, E., Laummans, M. and Thiele, L., 2001, "SPEA2: Improving the Strength Pareto Evolutionary Algorithm". TIK Report No. 103, Swiss Federal Institute of Technology (ETH), Computer Engineering and Networks Laboratory (TIK).

Zitzler, E. and Thiele, L., 1999, "Multi-objective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach". IEEE Trans. on Evolutionary Computation, Vol 3, No. 4, pp. 257-271.

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