PASSIVE ATTENUATION OF VIBRATING STRUCTURES BY OPTIMIZATION METHODS

Olavo M. Silva, olavo@lva.ufsc.br
GTVA - Federal University of Santa Catarina - Florianopolis - Brazil

Miguel M. Neves, maneves@dem.ist.utl.pt
Leonel M. Fernandes, leonel@dem.ist.utl.pt
IDMEC - IST - Technical University of Lisbon - Lisbon - Portugal

Abstract. *Structural optimization has shown to be a very effective procedure in vibration control. In this paper, optimization methods for passive vibration attenuation are implemented and investigated by two different applications. The first application presents results for the case of structures made of two different solid materials which are optimized to maximize the separation of two adjacent eigenfrequencies. In the second application, we investigate the procedure of minimizing the frequency response displacement in a frequency range of interest and proposed a modified methodology to get a wider attenuation region in that range. A numerical method combining finite element analysis (FEA) and optimization techniques is used. The assessment of adding proportional damping to an optimized undamped structure is discussed with respect to the frequency response. Some results are presented to illustrate the potential of these applications.*

Keywords*: passive attenuation, damping, structural optimization, finite element analysis, vibration.*

1. INTRODUCTION

In mechanical component design, the use of dissipative mechanisms and active control is usually expensive and should be avoided when passive solutions are feasible. Passive vibration control is achieved by adjusting adequately the stiffness, damping and mass distributions and has the main advantage and limitation in its simplicity. In some cases, finding an optimal distribution of stiffness, damping and mass requires numerical methods which engineers and scientific researchers are still reviewing and trying to improve (see, e.g., Olhoff and Parbery, 1984, Mead, 1996, Sigmund and Jensen, 2003, Du and Olhoff, 2005, Hussein *el al*, 2006, Silva et al, 2007a).

The present work is motivated by our need to apply existing finite element tools to improve passive attenuation of vibrations in industrial applications. It is our purpose to achieve considerable vibration attenuation firstly without considering damping effects. At the end, the effect of damping on such optimized structures is then analyzed. When damping is considered, components subjected to alternated strains dissipate energy. In the literature, there are descriptions of several ways of including the damping effects (see, e.g., Nashif *et al*, 1985) in the structural analysis, as well as some methodologies to optimize structures for vibration reduction using damping materials (see, e.g., Eschenauer *et al*, 1993).

Techniques to obtain reduction of vibration depend on how far (or near) from an eigenfrequency is the specific structure being excited. Near the resonance peaks the damping effect is more pronounced, but it can be a choice when shift of eigenfrequencies is no more an alternative. Far away of a resonance peak, the decrease in amplitudes can be achieved by shifting eigenfrequencies as a consequence of changes in mass and stiffness distributions. It is also of interest here to verify how the behavior of an optimized structure without damping considerations is affected when a proportional damping is added.

Structural optimization techniques use the obtained results from structural finite element analysis (*FEA*) to design optimized structures for a specific objective and constraints. Two different objective functions are considered of interest here, specifically: i) the difference between two adjacent eigenfrequencies, to be maximized; and ii) the sum of nodal displacements at a certain point of the structure, to be minimized for a discrete number of frequencies of interest. It can be seen as related with the minimization of resonance responses in a given frequency interval.

For simplicity, the models presented here were limited to the elastic bar (i.e., link or rod elements) and to Euler-Bernoulli beams, but extensions to shells and solid models are also applicable as well.

The numerical implementation technique used in present work combines commercial code ANSYS for finite element analysis with optimization algorithms running at MatLab environment.

Tests done validated that systematic separation of two adjacent eigenfrequencies can be achieved by these methods.

For longitudinal vibrations, a topology optimization technique (see Diaz and Kikuchi, 1992, Ma *et al*, 1995, Min *et al*, 1999, Pedersen, 2000, Allaire *et al*, 2001, Jog, 2002, Tcherniak, 2002 and Jensen, 2003) can be formulated to allow a distribution of two materials where a size optimization technique is not applicable. This formulation follows the proposed by Pedersen and Jensen, 2006.

For transversal vibrations, we can minimize directly the peaks of displacement (resonances) in the frequency response range of interest as an alternative way to get attenuation. When no damping effects (or little damping) are taken into account, the minimization of displacement peaks within a frequency range of interest can present some difficulties related to the meaning of the numerical peak value.

We found that, for optimization purposes, the peak location seems to be more meaningful than numerical peak values and this fact is used here. We notice that problems of separation of eigenfrequencies and of minimization of peak displacement at frequency response are not independent. Indeed, one way of minimizing displacement peaks for a given frequency or frequency range is to increase the separation of the two adjacent eigenfrequencies.

Motivated by these observations on the minimization problem of the maximum displacement at a certain point of the structure we then pursue an alternative optimization methodology. It is hereafter called the modified algorithm to contrast with the original algorithm. As the original formulation tends to move both peaks to the right, it would be desirable to investigate if it is possible to move right peak rightwards and left peak leftwards.

The idea of this modified algorithm yields surprisingly good results inside the region of interest and better than those obtained with the original algorithm. The sensitivities were changed in the sign to decrease the lower frequencies within the range of interest and kept the same for the ones that increase the higher frequencies inside the mentioned frequency interval. Current work is being undertaken to develop a new formulation that considers naturally the presented idea.

2. MODEL FOR STRUCTURAL ANALYSIS AND OPTIMIZATION

2.1. Structural response via finite element analysis

The mathematical model used here to characterize the structural responses is restricted to the elastic bar and to the Euler-Bernoulli beam model. The structural analysis in terms of the finite element method is expressed by a modal problem (Equation 1) to obtain the eigenvalues (eigenfrequencies) or by a harmonic analysis (Equation 2) to obtain the frequency response:

$$\left([K] + j\omega_i[C] - \omega_i^2[M]\right)\{\phi_i\} = \{0\}, \tag{1}$$

$$\left([K] + j\omega_{ap}[C] - \omega_{ap}^2[M]\right)\{U\} = \{F_\omega\}, \tag{2}$$

where a proportional (Rayleigh) damping is used. The damping matrix $\boldsymbol{C}$ is usually expressed as a linear combination of the global Stiffness matrix $\boldsymbol{K}$ and the global Mass matrix $\boldsymbol{M}$ ($[C] = \beta[K] + \alpha[M]$), but only β-damping is used in this work, i.e., C is simply defined by:

$$[C] = \beta[K]. \tag{3}$$

In the above expressions, ω is the eigenfrequency, $\boldsymbol{\phi}$ the eigenmode, ω_{ap} the applied force excitation frequency, $\boldsymbol{U}$ the respective maximum displacement and $\boldsymbol{F}_\omega$ the magnitude of the applied force.

Next, we introduce the two optimization problem formulations used here to maximize the attenuation of vibrations at desired points of a given harmonically excited structure. The first one is the maximization of the separation between two specific adjacent eigenfrequencies, while the second problem is a minimization of displacements (peak and intermediate points) inside of the given frequency range of interest.

2.2. Maximization of the separation of two adjacent eigenfrequencies by a distribution of two materials in a uniform section rod: a topology optimization's problem.

In this section, the objective is to find an optimized distribution of two different material components along a given rod, as the distribution illustrated by Figure 1. The distribution should maximize the separation of two specified adjacent eigenfrequencies ω_{i+1} and ω_i , i.e. for a given 'i' (i=1, 2, 3, 4, 5,…).

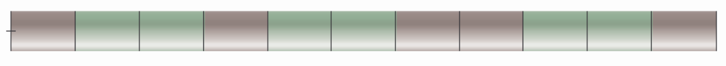


Figure 1. Example of a distribution of two material components along the rod.

A topology optimization method is used and formulated without damping effects as:

$$\begin{aligned} &\max_{t}\ (\omega_{i+1} - \omega_i) \\ &\text{subject to:} \\ &\left([K] - \omega^2[M]\right)\{\phi\} = \{0\}, \\ &0 \le t_j \le 1 \qquad j = 1, 2, \ldots, NDes. \end{aligned} \tag{4}$$

After obtaining the optimized solution, it will be studied with damping addition. The formulation follows the work presented by Jensen and Pedersen (2006) where the material interpolation proposed for this problem is different from the usual SIMP technique described by Bendsøe and Sigmund (2003). It is expressed by the following relations:

$$E = \frac{E_1}{1 + t * (E_1/E_2 - 1)}, \tag{5}$$

$$\rho = \rho_1 + t * (\rho_2 - \rho_1). \tag{6}$$

The design variable for each element 'e' is t_e and it has one value in the interval between $t_e = 0$ (where 0 means only material 1 is present, in our case only epoxy) and $t_e = 1$ (only material 2, in our case only Aluminium). To relax the problem for a continuum variation of the parameter t, mixtures between these materials are allowed and represented by intermediate values of t. Material properties are then computed by (5-6).

Each finite element is associated with one value of t_e although each design variable can be assigned to several consecutive finite elements. With these, the optimization is done according to the iterative procedure described in Section 3. The modal problem used does not include the damping effect and the shifting of the eigenvalues is done under this assumption. The optimized solution is afterwards analyzed by harmonic FEA (for specific boundary conditions in displacement and applied force) according to Equation (2) and (3) for different Rayleigh damping parameter β values.

2.3. Size optimization to minimize the sum of nodal displacements in a given frequency range.

This problem consists on minimizing the sum of $u^T(\omega_i)Lu(\omega_i)$ that is a displacement positive measure at certain points of the structure for a discrete number of frequencies inside a given frequency range of interest [$\omega_{initial}$, ω_{final}]. The matrix *L* locates the corresponding degrees of freedom. The optimization problem is set for a distribution of external diameters D_e (see Fig. 2) where the number of variables used is equal to the number of elements.

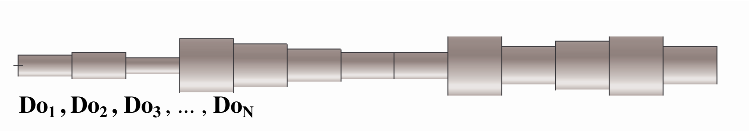


Figure 2. Example of a distribution of diameters along the beam.

The optimization problem of finding optimal diameters *D* is formulated without considering damping effects as:

$$\begin{aligned} &\min_{D} \sum_{i=1}^{Nfreq} u^T(\omega_i)Lu(\omega_i) \\ &subject \;\; to: \\ &([K] - \omega_{ap}^2[M])\{U\} = \{F_\omega\}, \\ &D_{min} \le D_e \le D_{max} \qquad e = 1, 2, ..., Nelem. \end{aligned} \tag{7}$$

After obtaining the optimized solution, it will be studied with damping addition. The number of frequencies *Nfreq* within the range of interest is calculated at each iteration with the initial, final and interior peak corresponding frequencies (ω_i), as illustrated in the following figure.

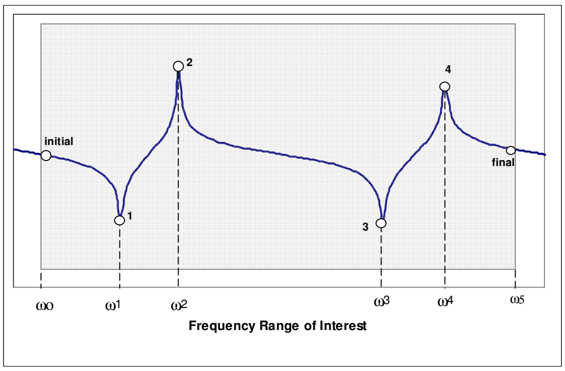


Figure 3. Schematic representation of the points used to set *Nfreq* value based on the frequency response.

In the particular case of the Fig. 3, *Nfreq* is equal to six (i=0, 1, …, 5) but, depending on the frequency response, this number can be larger or smaller. We kept this value fixed during the optimization process.
As mentioned before, this problem is related with the minimization problem of resonance responses as described by Eschenauer *et al* (1993). They use a minmax formulation that minimizes with respect to design variables the maximum vibration response (a resonance) inside the given frequency range of interest [$\omega_{initial}$, ω_{final}]. There are two main differences: 1) in our work we consider the objective function as the sum of $u^T(\omega_i)Lu(\omega_i)$ that is a positive measure related with the frequency range of interest; and 2) we are not considering damping optimization which is the main purpose of the work reported by Eschenauer *et al* (1993). Other interesting difference is that our approximation deals with a fixed number of displacements in the interval of frequencies, while Eschenauer *et al* (1993) involves an approximation based upon modal identification to get the maximum displacement in the interval. A detailed discussion on the comparison between these two methodologies is subject of future work.

For the optimization, an iterative procedure is described in Section 3.

3. METHOD OF SOLUTION

The implementation of the first optimization problem introduced in Section 2.2 uses the *fmincon* function available in the Optimization Toolbox of MatLab. This function finds a constrained minimum of a scalar function of several variables starting from an initial guess and without requiring any sensitivity analysis from the user. This requires some programming effort to build the interface between the structural analysis code (*Ansys*) and the optimization routine (*MatLab*).

The used medium-scale optimization routine solves a sequential quadratic programming *(SQP),* i.e. a quadratic programming *(QP)* sub problem at each iteration. The routine computes a quasi-Newton approximation to the Hessian of the Lagrangian at *x*. The objective function and restrictions are calculated in the commercial finite element software *ANSYS*, while the optimization process is performed within *MatLab* (see Carvalho *et al*, 2006).

For the optimizations of section 2.2 we defined an iterative procedure with the following steps:

1) Initialize by giving initial design values and the order 'i' of the lowest eigenvalue to separate;

2) Run a harmonic analysis for the initial design;

3) Start the optimization loop in MatLab by calling *fmincon*. After writing the values of the current iteration design variables for a file it will call *ANSYS* program in batch mode and will return to *fmincon* the current calculated values of eigenfrequencies ω_{i+1} and ω_i;

4) After satisfying the stopping criteria, run a harmonic analysis for the final design this time using damping properties.

The second optimization problem, introduced in section 2.3, uses the *MatLab* version of Method of Moving Asymptotes (*MMA*) developed by Svanberg (1987). The original problem could be, in principle, done with the same algorithm as the previous problem, but we decided to use an idea that requires sensitivities instead. That is the reason why we decided to use the *MMA* optimizer.

An interface was built to manage the reading and writing of the relevant data from (and to) files for use in both the FEA software and the *MMA* optimizer (for a more detailed description, see Carvalho *el al*, 2006). The calculation of the objective and constraint function analytical sensitivities (*MMA* requires sensitivities) is made in the interface.

The iterative procedure uses *MMA* with the following steps:

1) Initialize by supplying the initial design values, generally, whole structure with uniform external diameter.

2) Start the optimization loop at *MatLab* by calling *MMA* procedure. It will call *ANSYS* (after writing the current iteration values of D_e to a file) to run a harmonic analysis without damping effects in batch mode and will return to *MMA* procedure the values of displacement for the frequency range of interest [$\omega_{initial}$, ω_{final}]. Next, calculate sensitivities in the interface module written in *MatLab*.

3) When the stopping criterion is satisfied, plot the final design.

4) Run a harmonic analysis for the final design this time using damping properties.

4. NUMERICAL RESULTS

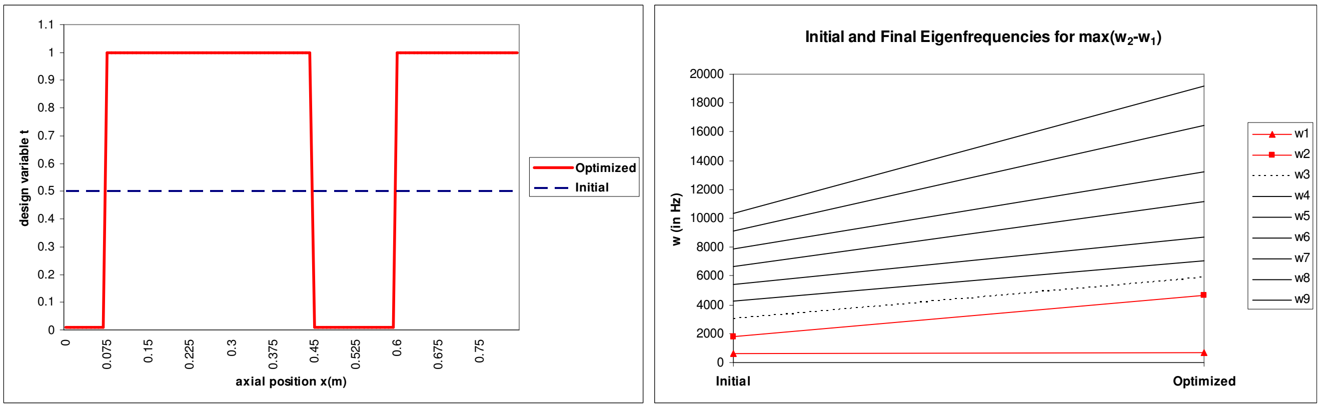
4.1. Results from the maximization of the separation of two adjacent eigenfrequencies ω_{i+1} and ω_i by a distribution of two different materials in a uniform section rod.

Here, a given rod is optimized for the formulation of section 2.2 (to maximize the separation of two adjacent eigenfrequencies ω_{i+1} and ω_i, where three cases i=1, 5 or 7 are considered). We use an Epoxy (EP) for material 1 and an Aluminium (AL) for the material 2. The rod has a 0.01 m uniform diameter, a total length of *L*=0.825m and the material properties are: $E_{AL} = 70.0\,\text{GPa}$, $\rho_{AL} = 2870\ \text{kg/m}^3$ and $E_{EP} = 4.1\,\text{GPa}$, $\rho_{EP} = 1142\ \text{kg/m}^3$.

The number of design variables 't_e' is eleven, all with the same length, and bar finite elements (element LINK1 in *ANSYS*) were used for each design variable. The number of finite elements should be adjusted between the highest frequency in analysis and the finite element length. A modal analysis is done by *ANSYS* with no transversal displacements, Uy=0, for all nodes and Ux=0 at x=L.

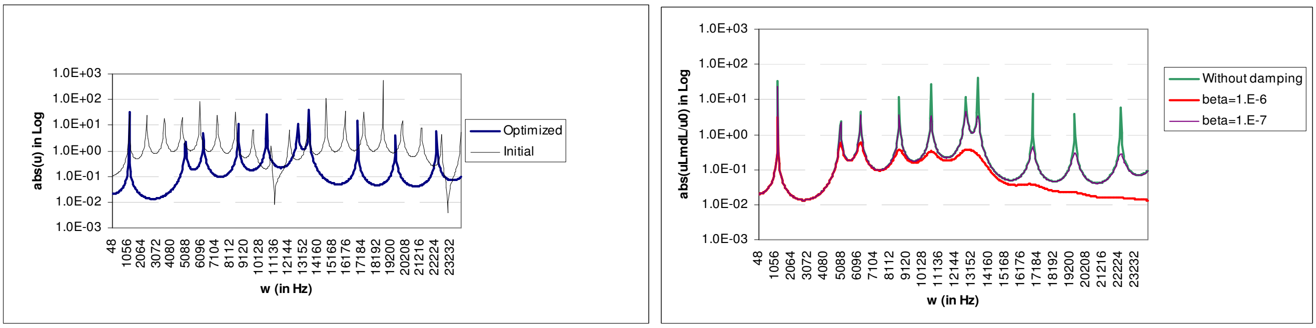
The initial design consists in all design variables t_e being 0.5, as indicated by a blue line at the figures 4 a), 6 a) and 8 a). Using the optimization procedure described in Section 3, the optimized designs were obtained for a specific 'i' value and presented at mentioned figures (The red line in figure 4 a) indicates a sequence of materials: EP/AL/EP/AL). Improvement in the separation of two adjacent frequencies ('i' and 'i+1'), relatively to the initial design, i.e. from initial separation Δ_{ini} to optimized final value Δ_{fin} were obtained as shown by the values in figures 4 b), 6 b) and 8 b), for 'i'=1, 5 and 7, respectively. The frequency response of displacement at node x=(L-(L/11)) – the node that separates left 10 variables from the last design variable - is presented at Figures 4 b), 6 b) and 8 b) for an applied axial force with 200 N magnitude at x=0 m.

These separations obtained are evident when comparing frequency response curves from initial design with corresponding curves of optimized designs (see figures 5 a), 7 a) and 9 a) for 'i'=1, 5 and 7, respectively). The effect of introducing damping in the harmonic analysis of the optimized structure is the expected i.e. damping effect is localized around resonances as presented at figures 5 b), 7 b) and 9 b).



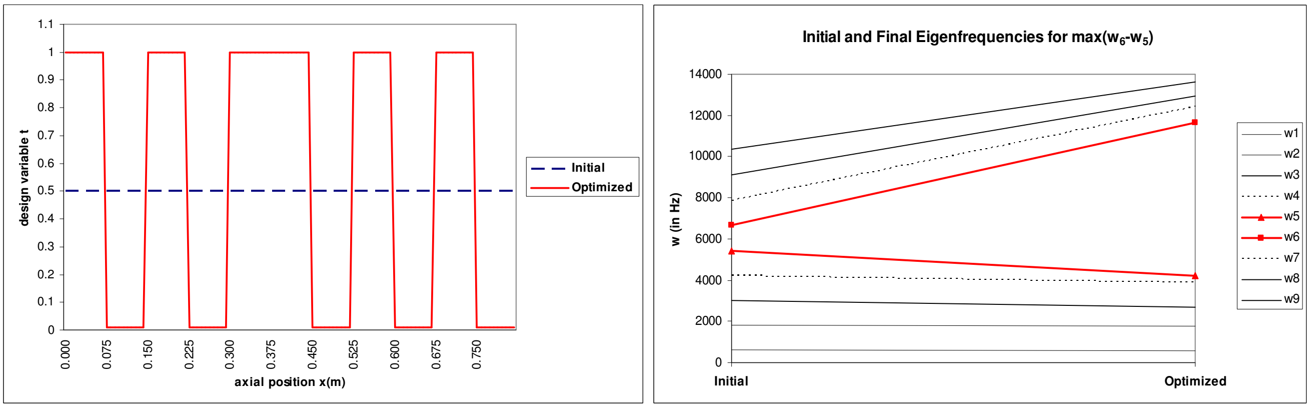
a) b)

Figure 4. Results obtained with 'i' = 1. a) Material distributions along the rod longitudinal axis. b) Changes in eigenvalues due optimization, from Δ_{ini} = 1205.1 to Δ_{fin} = 3969.6, initial and final eigenvalue separation, respectively.



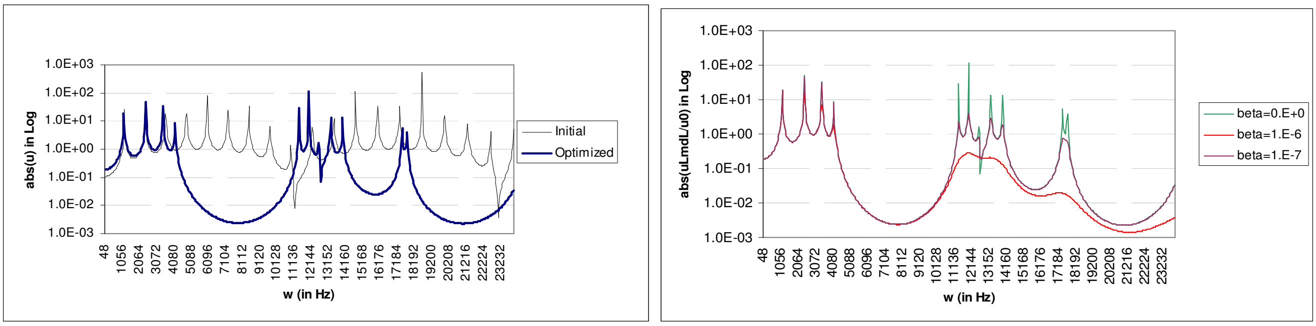
a) b)

Figure 5. Results obtained with 'i' = 1. a) Axial displacement frequency response at x=L-L/11. b) Transmissibility between x=0 and at x=L-L/11 for different β damping parameters.



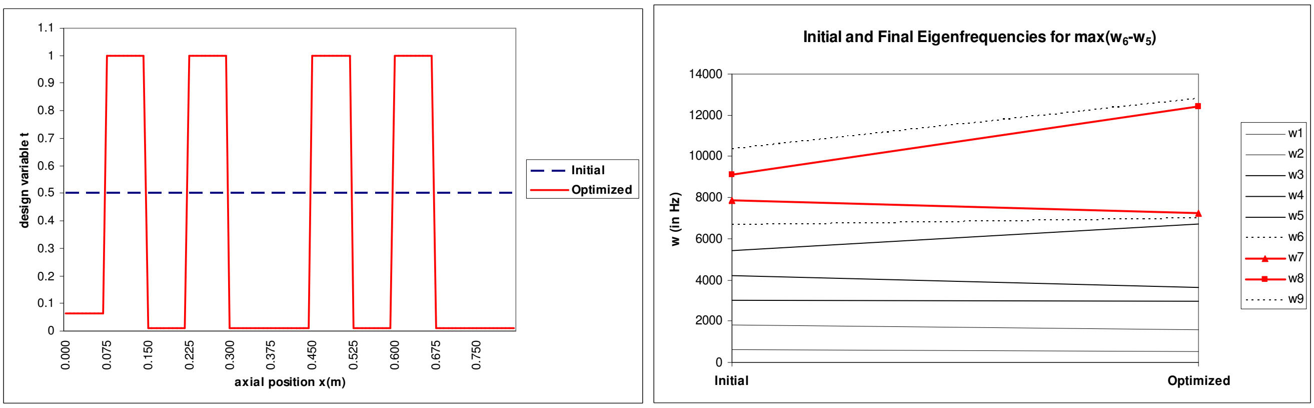
a) b)

Figure 6. Results obtained with 'i' = 5. a) Material distributions along the rod longitudinal axis. b) Changes due optimization, from Δ_{ini} = 1226.8 and Δ_{fin} = 7445.1, initial and final eigenvalue separation, respectively.



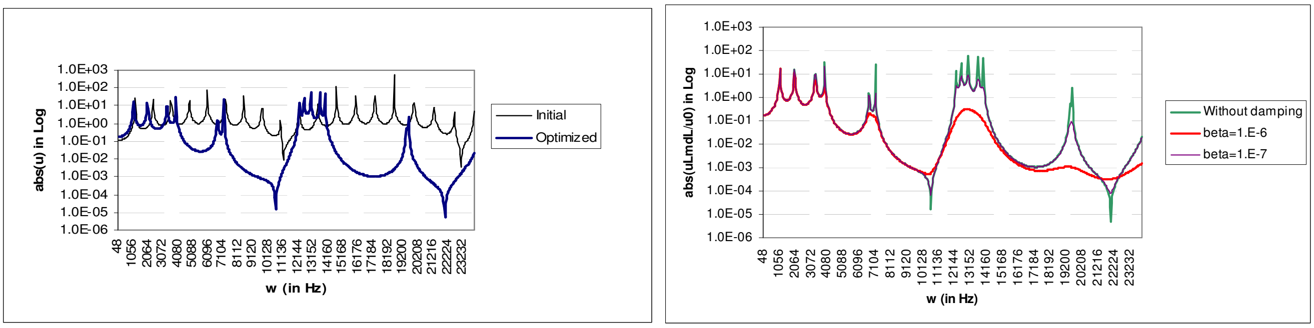
a) b)

Figure 7. Results obtained with ‘i’ = 5. a) Axial displacement frequency response at x=L-L/11. b) Transmissibility between x=0 and at x=L-L/11 for different β damping parameters.



a) b)

Figure 8. Results obtained with ‘i’ = 7. a) Material distributions along the rod longitudinal axis. b) Changes due optimization, from Δ_{ini} = 1228.7 and Δ_{fin} = 5155.9, initial and final eigenvalue separation, respectively.



a) b)

Figure 9. Results obtained with ‘i’ = 7. a) Axial displacement frequency response at x=L-L/11. Right: Transmissibility between x=0 and at x=L-L/11 for different β damping parameters.

While optimization cases with ‘i’= 5 and 7 run with small number of finite element analysis (FEA), the same was not the case for ‘i’= 1. Indeed, after an initial significant improvement a very high number of FEA calls was done due a slow convergence to the presented design (see figure 4 a).

We notice that for high frequencies the damping model chosen is not (physically) the most adequate. Anyway, our purpose of testing damping effect on the optimized solution was achieved. The optimization procedure can also work with damping but at the expense of computational running time efficiency. Additional issues are to be addressed on future work.

4.2. Results from size optimization to minimize the sum of nodal displacement $u^T Lu$ for a given frequency range [$\omega_{initial}$, ω_{final}]

A steel hollow shaft is considered with 210 GPa of Young modulus and material density of 7800 kg/m^3, a total length of 0.5 m is fixed, internal diameter 0.002m is fixed and a design variable initialized with 0.008m external diameter. Each element diameter, in this case, is an optimization design variable.

The optimization function is the sum of quadratic displacement $u^T L u$ for a specific node and a frequency range given by the interval of [3000, 6000] Hz. Matrix L is a zero matrix with ones at the diagonal elements corresponding to the displacement degrees of freedom of the node where displacement is to be minimized. The sensitivities of this objective function can be found by the adjoint method as e.g. in Sigmund and Jensen (2003) and Silva *et al* (2007b).

The optimization results are shown in Figure 10.

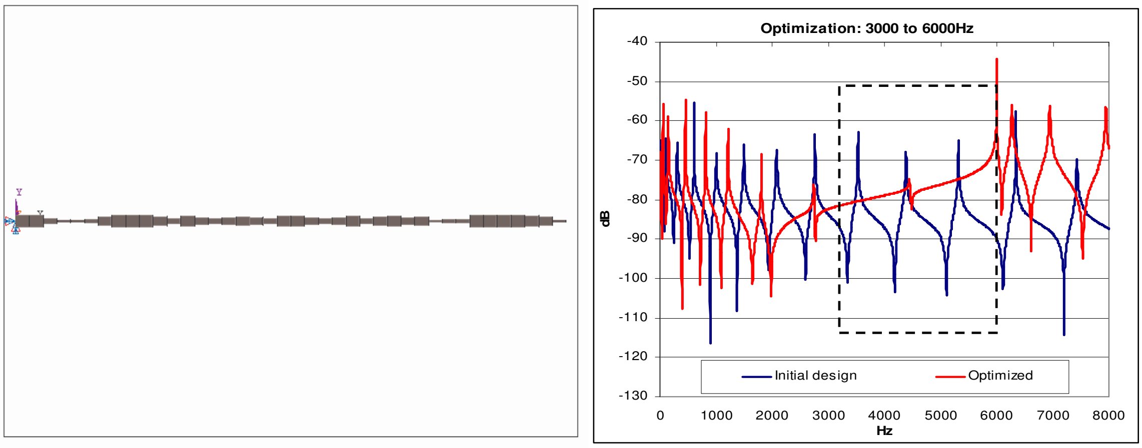


Figure 10. Results obtained with the original formulation. On the left hand side is the optimized geometry with a clamped node at left end and vertical force of magnitude 0.01 N at right end of the beam. On the right are for comparison the displacement frequency responses at node of applied force: blue line for the initial design and red for the optimized undamped structure.

The standard formulation tends to move both peaks (ω_i) to the right (inside a frequency band of interest), while it would be desirable to investigate if it is possible to move right peak to right and left peak to left (according to a middle frequency ω_m). We implemented the idea of changing sign of the derivatives for the left peaks according to the arrow directions indicated in Figure 11.

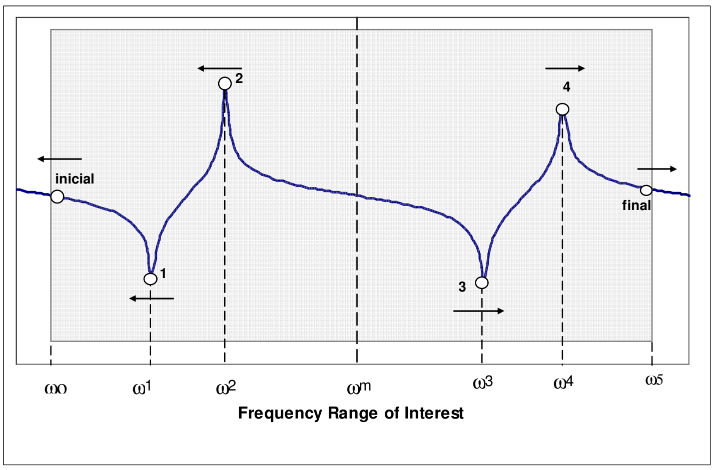


Figure 11. Illustration of the modification proposed to move right peak rightwards and left peak leftwards.

For the tested examples, the new algorithm gave surprisingly good results, better than those obtained with the original formulation (see Fig. 12).

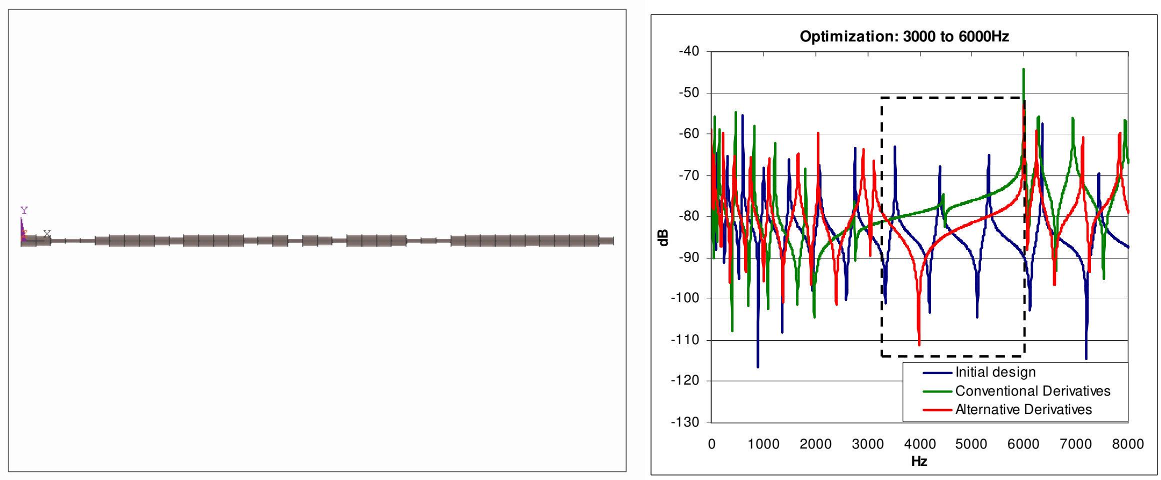


Figure 12. On the left hand side is an illustration of the optimized geometry obtained with the modified algorithm for a clamped node at left and a vertical force of magnitude 0.01 N at right end. On the right are for comparison the displacement frequency responses at node of applied force given by: blue line for the initial design (original), green line for the optimized using the original formulation and red line for the optimized structure using the modified algorithm.

Physically, to minimize the target displacement the original optimization procedure tends to increase the value of the external diameter (increasing both mass and stiffness). One can observe that the stiffness effect dominates over the mass effect and consequently in general peaks move rightwards. Based on this, we decided for the left frequency peaks inside the region of interest (see Fig. 11) to impose a change in the "direction" of the design change step by inverting the sign of the objective function derivatives. Current work is being done to develop a formulation that considers naturally the idea of the modified algorithm.

Finally, we run a harmonic analysis this time using damping properties and the results are presented in Fig. 13 for the initial design, in Fig. 14 for the optimized structure and in Fig. 15 for the modified formulation.

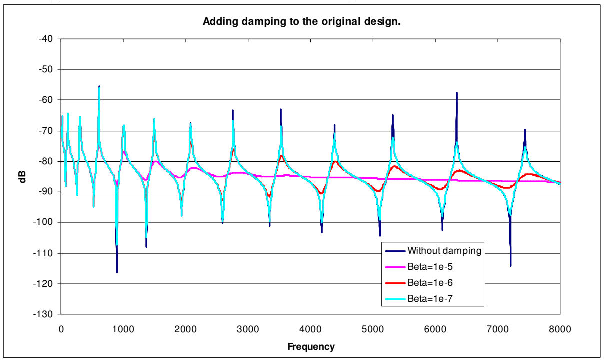


Figure 13. The transversal displacement frequency responses for the initial design at node of applied transversal force from the harmonic analysis with different β damping values.

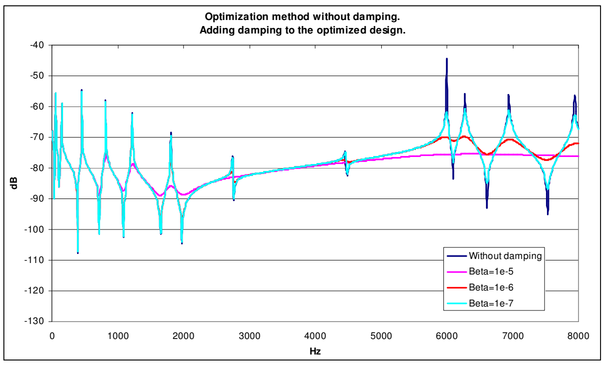


Figure 14. The transversal displacement frequency responses for the optimized design by original formulation at node of applied transversal force with different β damping values.

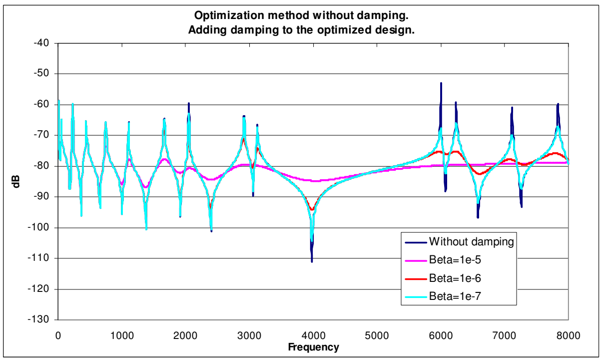


Figure 15. The transversal displacement frequency responses for the optimized design by modified formulation at node of applied transversal force with different β damping values.

This optimization procedure can also work with damped model but again it loses computational running time efficiency.

5. CONCLUSIONS

Two structural optimization problems for the passive vibration attenuation and respective testing examples were presented. Well known optimization methodologies were reviewed. Based on a critical analysis of the results from the first problem we proposed a new algorithm for the second problem and presented improved results obtained by this way.

The advantage of the proposed optimization strategy results from moving relatively to the centre of the frequency range of interest the right peaks to right direction and the left peaks to left. It was implemented by changing sign of the derivatives for the left peaks according to the arrow directions indicated in Fig. 11. This idea of changing sign of certain terms is not straightforward and for this reason work is being done to present a mathematical formulation that considers naturally the modified algorithm. As expected, damping effect is localized around resonances, but for high frequencies the damping model chosen is not physically the most adequate.

For the sake of simplicity, the models were limited to the elastic bar (i.e., link or rod elements) or to Euler-Bernoulli beams, but the algorithms used can be readily applied to shells and solids as well.

We investigated damping influence on the optimized structures and concluded that the added proportional damping to the optimized structures did not significantly changed the main behavior obtained by optimization, as expected. It supports the methodology of doing optimization without damping followed by a check on the effect of damping addition to the optimized structure.

Current work is being done to test extension of these approaches to 3D examples of industrial interest.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Allaire G., Aubry S., Jouve F., "Eigenfrequency optimization in optimal design". *Comp Methods Appl Mech Eng*; 190 (28), p.3565–3579 (2001).

Bendsøe M.P. and Sigmund O., *Topology Optimization - Theory, Methods and Applications*, Springer Verlag, Berlin Heidelberg, XIV+370 pp (2003).

Carvalho M., Vale J.L., Pinheiro L.T., Neves M.M., *Techniques Adopted in Integrating Structural Analysis With Numerical Optimization*, in CD of *Proceedings of 5th International Conference on Mechanics and Materials in Design*, Porto, Refª A0204.0215 (2005).

Diaz A.R. and Kikuchi N., "Solutions to shape and topology eigenvalue optimization problems using a homogenization method". *Int. J. Num. Meth. Engng.*; **35** (7) p. 1487-1502 (1992).

Du J., Olhoff N., "*Topology optimization of continuum structures with respect to simple and multiple eigenfrequencies*". In Proceedings of the Sixth World Congress of Structural and Multidisciplinary Optimization. Rio de Janeiro, International Society for Structural and Multidisciplinary Optimization (ISSMO), p. 9 (2005).

Eschenauer H.A. and Wodtke H.W., *Optimal design with damping materials for vibration reduction*, in *OPTIMAL DESIGN WITH ADVANCED MATERIALS*, Proceedings of the IUTAM Symposium, Lyngby, Denmark, 18-20

August 1992, edited by P. Pedersen, ELSEVIER, ISBN: 0-444-89869-7, 528 pages, (1993).
Hussein M.I., Hulbert G.M., Scott R.A., "Dispersive elastodynamics of 1D banded materials and structures: analysis". *J Sound Vib* (in press 2006).
Jensen J.S. and Pedersen N.L, "On maximal eigenfrequency separation in two-material structures: The 1D and 2D scalar cases". *Journal of Sound and Vibration*; **289** (4-5); p. 967-986 (2006).
Jensen J.S., "Phononic band gaps and vibrations in one- and two-dimensional mass-spring structures.", *J Sound Vib* **266**(5), 1053–1078, (2003).
Jog C.S., "Topology design of structures subjected to periodic loading". *J Sound Vib*; **253**(3), p. 687-709 (2002).
Ma Z.D., Kikuchi N. and Cheng H.C., "Topological design for vibrating structures". *Comput. Methods. Appl. Mech. Engrg.*; **121**(1-4) pp. 259-280 (1995).
Mead D.J., "Wave propagation in continuous periodic structures: research contributions from Southampton: 1964-1995". *J Sound Vib*, **190**(3): 495-524 (1996).
Min S., Kikuchi N., Park Y.C., Kim S. and Chang S., "Optimal topology design of structures under dynamic loads". *Struct. Optim.,* **17** (2-3) p.208-218 (1999).
Nashif A., Jones D. and Henderson J., *Vibration Damping*, JohnWiley & Sons, New York (1985).
Olhoff N. and Parbery R., "Designing vibrating beams and rotating shafts for maximum difference between adjacent natural frequencies", *Int. J. Solids Structures*, **20** (1) pp. 63-75 (1984).
Pedersen N., "Maximization of eigenvalues using topology optimization" *Struct. Multidisc. Optim.*, **20** (1) p. 2-11 (2000).
Sigmund O., Jensen J.S., "Systematic design of phononic band-gap materials and structures by topology optimization". *Phil. Trans. R. Soc. Lond.* **A 361**; p.1001–1019 (2003).
Silva O.M., Neves M.M., Fernandes J.L.M., *On finite element analysis and optimization of structural properties for a maximum passive vibration attenuation*, in Proceedings of *International Conference on Modeling and Optimization of Structures, Processes and Systems*, ICMOSPS'07, 22 to 24.01.2007, Durban, South Africa Cd of Proceedings (2007a)
Silva O.M., Neves M.M., Lenzi, A., "Structural optimization technique for power flow problems in steady state vibrations", Submitted proceeding to 7th World Congress on Structural and Multidisciplinary Optimization, WCSMO7, 21 to 25.05.2007, Seoul, Korea (2007b).
Svanberg K., "The method of moving asymptotes – a new method for structural optimization". *International Journal for Numerical Methods in Engineering*; **24** (2) p. 359–373 (1987).
Tcherniak D., "Topology optimization of resonating structures using SIMP method". *Int. J. Numer. Meth. Engng*. **54** (11), p.1605-1622 (2002).

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