CONTROL LAW DESIGN AND PARAMETERS ESTIMATION OF A SATELLITE ATTITUDE CONTROL SYSTEM SIMULATOR

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Abstract. *Future space missions will involve satellites with stringent pointing precision requiring from the Attitude Control Systems (ACS) better performance than before. On the other hand, there are in the literature a variety of attitude control techniques, dealing with stabilization, identification, estimation and robustness that need to be validated experimentally in order to improve ACS performance. One drawback for developing experimental ACS test is to obtain zero gravity and torque free conditions similar to what ACS experiences in space. This paper presents the development of a Satellite Attitude Control System Simulator (SACSS) model, which allows experimental verification of fundamental aspects of the satellite attitude dynamics and control, conjugated with parameters identification process. The model describes the satelite equations of motions in 3-D with three reaction wheel as actuator and three gyro as sensor. Although the simulator has the possibility of implementing different control algorithm, its efficiency is initially investigated designing a PD control law using the pole allocation method.Three sets of poles are chosen to demonstrate the ACS dynamic behavior with total state feedback. The first one is closer to the imaginary axis than the second set, and the third one is far from the imaginary axis. Simulation has shown that the ACS has the undesirable behavior of being less damped and very oscillatory for the first set while the reaction wheel achieves saturation for the third set. The second set of poles has good control properties, such as setting time, moderate overshoot, and gains values that keeps the reaction wheel rotation below saturation level. The parameters estimation has been done initially by the least squares algorithm in regressive form. This approach is not very efficient, since it is needed to obtain too much data. Alternatively, a recursive least squares estimation is done to avoid the explicit calculation of the pseudo inverse matrix. Both methods have estimated the parameters with errors below the requirements. However, the recursive least squares is more suitable for the SACSS objectives. These results show that the SACSS model is reliable and it is ready to implement and test other attitude control strategies.*

Keywords:*Satellite attitude control system, parameters identification*

1. INTRODUCTION

There are several methodologies to investigate the satellite ACS performance, depending on the investigation objectives computer simulation cannot be the appropriate one. The use of experimental platforms have the important advantage of allowing the satellite dynamics representation in laboratory and once validated this platform, it is possible to accomplish experiments associated with simulations to evaluate satellites ACS with simple rigid dynamics as well as complex configurations involving flexible components. The preference for using experimental test is associated to the possibility of introducing more realism than the simulation, however, it has the difficulty of reproducing zero gravity and torque free space condition, extremely relevant for satellites with complex dynamics and ACS with great degree of precision. Experimental platforms with rotation around three axes are more complicated assemblies than rotation around one axis, but the first is more representative. Examples of the use of experimental platforms for investigating different aspects of the satellite dynamic and control system can be found in Hall et al. (2202) and Berry et al. (2003). A classic case of a phenomenon no investigated experimentally before launch was the dissipation energy effect that has altered the satellite Explorer I rotation Kaplan (1976). A pioneering experiment to study energy dissipation effect has been done by Peterson (1976). Important aspects that are possible to investigate through experimental platforms are the satellite ACS performance and inertia parameters identification. An initial way to estimates inertia parameters is using CAD software. The results obtained by CAD can be compared to the results obtained through estimation techniques. The least square method with batch processing was used with success by the satellite simulator developed by Ahmed (1988) and Tanygin and Williams (1997). For platforms where the dynamic has variation in time, like, mass center and inertia parameters variation, the application of parameters estimation methods in real time becomes more appropriate. In Lee and Wertz (2002), has developed an algorithm based on the least square method to identify mass parameters of a space vehicle in rotation during attitude maneuvers. Methods with the same objectives, but based on Kaman filter theory were used by Bergmann and Dzielski (1990). Experimental platforms can also be used for familiarizing with modeling aspect and controllers design for complex space structures (Dichmann and Sedlak, 1998). An experimental apparatus was used in Cannon and Rosenthal, (1984) to investigate the dynamics and the control laws for a satellite composed of rigid and flexible parts. The results showed that the control of the flexible structure is extremely sensitive to the variation of the parameters of the system, indicating the need of more robust control strategies (Souza,1992) in order to improve the controller performance.

2 PLATFORM DESCRIPTION

The simulator consists of a platform in disk form that is supported by film of air. Over this platform is possible to accommodate several satellite attitude control system components with their respective interfaces and connections. Usually the main equipments are sensor, actuators, computers, interfaces, batteries and etc. Figure 1 shows the coordinates system (x, y, z) that is fixed to the platform with origin in its rotation center. The coordinates systems $(x, y, z)_{1,2,3}$ are fixed to the reaction wheels 1, 2 and 3 with origin in their respective mass centers and aligned with their rotation axes. The vectors $\vec{R}_{1,2,3}$ indicate the reaction wheels position and the vector $\vec{r}$ locates the platform elements of mass dm with respect to (xyz). The vectors $\vec{\rho}_{1,2,3}$ locates the reaction wheels elements of mass $dm_{1,2,3}$ with respect to the coordinates $(x,y,z)_{1,2,3}$. The platform angular velocity is given by

$$\vec{W} = p\vec{i} + q\vec{j} + r\vec{k} \tag{1}$$

and the reaction wheels angular velocity are given by $\vec{w}_1$, $\vec{w}_2$ and $\vec{w}_3$.

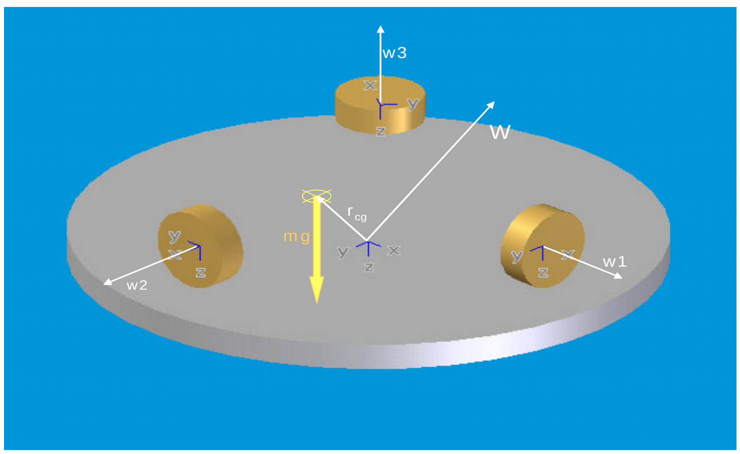

Figure 1 Satellite Attitude Control System Simulator Platform

3 EQUATIONS OF MOTION AND CONTROL LAW

The platform total angular moment is the sun of the base and reaction wheels angular moment given by

$$\vec{H} = \int_B (\vec{r} \times \vec{v}) dm + \sum_{i=1}^{3} \int_{RW} (\vec{r} \times \vec{v}) dm \tag{2}$$

where the angular velocity of the base is $\vec{v} = \vec{W} \times \vec{r}$ and the reaction wheels is $\vec{v} = \vec{W} \times \vec{R}_i + \vec{w}_i \times \vec{\rho}_i$. After manipulating Eq.(2) is

$$\vec{H} = \int_{B+RW} \vec{r} \times (\vec{W} \times \vec{r}) dm + \sum_{i=1}^{3} R \int_{RW} \vec{\rho}_i \times (\vec{w}_i \times \vec{\rho}_i) dm \tag{3}$$

which in compact form is given by

$$\vec{H} = \vec{h} + \sum_{i=1}^{3} \vec{h}_i \tag{4}$$

The equations of motion of the platform is obtained deriving the total angular moment and equalizing to the external torques acting on the platform given by

$$\vec{r}_{cg} \times (m\vec{g}) = \frac{d\vec{H}}{dt} = (\dot{\vec{h}})_r + \vec{W} \times \vec{h} + \sum_{i=1}^{3} (\dot{\vec{h}}_i)_r + \vec{W} \times \left(\sum_{i=1}^{3} \vec{h}_i \right) \tag{5}$$

where $\vec{r}_{cg}$ is the center of gravity location, where the gravitational force (mg) acts.

Applying the same principle, the reaction wheels equations of motion are given by

$$\begin{aligned} T_1 &= I_1 \left[\dot{w}_1 + \dot{p} \right] \\ T_2 &= I_2 \left[\dot{w}_2 + \dot{q} \right] \\ T_3 &= I_3 \left[\dot{w}_3 + \dot{r} \right] \end{aligned} \tag{6}$$

where $T_{1,2,3}$ and $I_{1,2,3}$ are the control input and inertia moments of the reaction wheels, respectively.

The cinematic equations describing the temporal variations of the Euler angles (ϕ, θ, ψ) in the sequence 3-2-1 are

$$\begin{aligned} \dot{\phi} &= p + \tan(\theta) \left[q \sin(\phi) + r \cos(\phi) \right] \\ \dot{\theta} &= q \cos(\phi) - r \sin(\phi) \\ \dot{\psi} &= \sec(\theta) \left[q \sin(\phi) + r \cos(\phi) \right] \end{aligned} \tag{7}$$

Putting Eq. (6) and Eq. (7) together with respect the same coordination system one has

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & 0 & 0 & 0 & I_1 & 0 & 0 \\ I_{xy} & I_{yy} & I_{yz} & 0 & 0 & 0 & 0 & I_2 & 0 \\ I_{xz} & I_{yz} & I_{zz} & 0 & 0 & 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{Bmatrix} = \left\{ \begin{array}{c} (I_{xx} - I_{zz})(qr) + I_{xy}(pr) - I_{xz}(pq) + I_{yz}(r^2 - q^2) + I_2(w_2 r) + \\ - I_3(w_3 q) + mgr_y \cos(\phi)\cos(\theta) - mgr_z sin(\phi)\cos(\theta) \\ (I_{zz} - I_{xx})(pr) + I_{yz}(pq) - I_{xy}(qr) + I_{xz}(p^2 - r^2) - I_1(w_1 r) + \\ + I_3(w_3 p) - mgr_x \cos(\phi)\cos(\theta) - mgr_z sin(\theta) \\ (I_{xx} - I_{yy})(pq) + I_{xz}(qr) - I_{yz}(pr) + I_{xy}(q^2 - p^2) + I_1(w_1 q) + \\ - I_2(w_2 p) + mgr_x sin(\phi)\cos(\theta) + mgr_y sin(\theta) \\ p + \tan(\theta)\left[q sin(\phi) + r\cos(\phi) \right] \\ q\cos(\phi) - r sin(\phi) \\ \frac{1}{\cos(\theta)}\left[q sin(\phi) + r\cos(\phi) \right] \\ \frac{T_1}{I_1} \\ \frac{T_2}{I_2} \\ \frac{T_3}{I_3} \end{array} \right\} \tag{8}$$

which in compact form is given by

$$[M]\{\dot{X}\} = \{f(X)\} \tag{9}$$

where M represent the mass matrix, X the state vector of the system and F(x) the function matrix given by the right hand side elements of Eq.(8).

In order to design the control law, Eq.(9) needs to be linear. Therefore, assuming small angular displacements the equations of motion for design purpose is

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \\ \phi \\ \theta \\ \psi \end{Bmatrix} + \begin{bmatrix} \frac{1}{I_1 - I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_2 - I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_3 - I_{zz}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} \tag{10}$$

which in state space form is given by

$$\begin{aligned} \dot{X} &= AX + Bu \\ Y &= CX \end{aligned} \tag{11}$$

where Y represents the system outputs {p, q, r, ϕ, θ, ψ , w_1 , w_2 e w_3 } and C the sensor location matrix. Using a proportional control law given by

$$u = -[K]\{X\} \tag{12}$$

with X = {p, q, r, ϕ, θ e ψ}, once the reaction wheel rotation is not used in the control law for controlling the system . The control law gains are obtained applying the pole allocation methods as described in Ogata (1998).

The equations of motion obtained previous are in general form and can be applied for different satellite attitude system. The identification process employed uses the equations of motion in its linear matrix form. From now on the vector X consists of the parameters to be identified, here the elements of the inertia matrix and the terms that give the location of the platform center of gravity. The elements of vector Y contain terms associated to the sensor measures and terms of the reaction wheels inertia matrix, therefore totally known. The identification problem can be solved applying an algorithm based on the least square method, in the form

$$[G]\{X\} = \{Y\} \tag{13}$$

where the matrix G contains the measured of angles, angular velocities and angular accelerations. Therefore, it is function of the types of sensor used in the experiment.

Using the notation where the matrixes $[G_K]$ and $\{Y_K\}$ represent the values of G and Y in the instant (or step) k, the previously matrix vector can write

$$[\overline{G}_K] = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_K \end{bmatrix} \qquad \{\overline{Y}_K\} = \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_K \end{Bmatrix} \tag{14}$$

Applying the regression least square method, the vector X that minimizes the error given by

$$\left\| \overline{G}_K X - \overline{Y}_K \right\| \tag{15}$$

is obtained by

$$\{X_K\} = \left(\left([\overline{G}_K]^T [\overline{G}_K] \right)^{-1} [\overline{G}_K]^T \right) \{\overline{Y}_K\} \tag{16}$$

Although the identification based on the least square regression method is relatively simple, it is important to verify the effects of the numeric errors introduced when calculating the pseudo-inverse matrix in Eq. 16, due to the need of processing a great number of points to obtain good results.

In order to avoid the calculation of the pseudo-inverse matrix one can use the recursive form of the least square method. The main difference of the regression algorithm is that the value for the matrix X at instant t(k) (step k) is done with measures obtained at instant t(k-1). Therefore, considering

$$
\begin{aligned}
&[P_0] = \left([G_0][G_0]^T\right)^{-1} \\
&\{X_0\} = [P_0][G_0]^T\{Y_0\}
\end{aligned}
\tag{17}
$$

the recursive form of the least square method needs to satisfy the following recursive equations

$$
\begin{aligned}
&[L_K] = [P_{K-1}][G_K]^T\left([I] + [G_K][P_{K-1}][G_K]^T\right)^{-1} \\
&[P_K] = ([I] - [L_K][G_K])[P_{K-1}] \\
&\{X_K\} = \{X_{K-1}\} + [L_K](\{Y_K\} - [G_K]\{X_{K-1}\})
\end{aligned}
\tag{18}
$$

It is observed, that the estimation of $\{X_K\}$ is obtained adding a correction to the estimation done previously in the step (k-1). The correction term is proportional to the difference between the measured values of Y_k and the measures based on the estimative done in the previous step, given by $[G_K]\{\hat{X}_{K-1}\}$

The components of the vector $\{L_k\}$ are weight factors (gain) that gives information about how the correction and the previous estimative should be combined. The matrix $[P_k]$ is only defined when the matrix $[\overline{G}_K]^T[\overline{G}_K]$ is no singular. To avoid singularities the recursive process must be initiated with a big matrix $[P_0]$ positive define.

4 SIMULATIONS RESULTS

The parameters are estimated applying both methods described in the previously section. The measures are obtainer by integration of the nonlinear equations of motion. The platform and reaction wheel inertia data, and the term of the external torque are show in Table 1 in international system unit.

TABLE 1 – System data used in the simulation in international system unit.

Platform	Platform	Reaction wheel	External torque
I_{xx} = 1.1667	I_{xy} = 0.0107	I_1 = 0.001792	Mgr_x =0.0101
I_{yy} = 1.1671	I_{xz} = -0.0159	I_2 = 0.001792	Mgr_y =0.0323
I_{zz} = 2.1291	I_{yz} = 0.0159	I_2 = 0.001792	Mgr_z =0.7630

The parameters estimation by the least square regression methods has been done taking the measure of the matrix $[A_K]$ and the vector $\{T_K\}$ for time period of 3, 7 and 20 in a simulation of 20s. Table 2 shows the parameters estimation values and they respective error with respect they real values

Table 2 shows that the parameters estimation performed by the least square regression method is quite precise for measures took in time period greater than 3. From shadow columns one observes that the estimation for time period of 20s is more precise. However, some instability in calculating the pseud-inverse has appears, when it has many elements near zero.

Table 2 - Parameters estimation and error

time	20s	7s	3s	20s	7s	3s
	estimation	*estimation*	*estimation*	*error*	*error*	*error*
I_{xx}	1.1665	1.1666	-3.134	0.00021283	8.1393e-5	4.3007
I_{yy}	1.1681	1.1686	-8.4305	0.001016	0.001483	9.5976
I_{zz}	2.1295	2.1292	2.0531	0.00036696	7.3107e-5	0.076038
I_{xy}	0.0098343	0.010854	0.14886	0.00086572	0.00015386	0.13816
I_{yz}	0.016812	0.016213	-0.58618	0.00091155	0.00031343	0.60208
I_{xz}	-0.018679	-0.018476	-0.11682	0.00017878	2.4408e-5	0.098317
mgr_x	0.010133	0.010102	0.015778	3.2726e-5	1.6989e-6	0.056778
mgr_y	0.032319	0.032298	0.040902	1.9404e-5	2.3561e-6	0.086024
mgr_z	0.76187	0.76231	4.7281	0.0011303	0.00069486	0.039651

In the parameters estimation applying the least square recursive method the measures are took in time interval of 5s for a simulation period of 20s. Figures 2, 3 and 4 show the platform inertias and external torques parameters estimations, respectively.

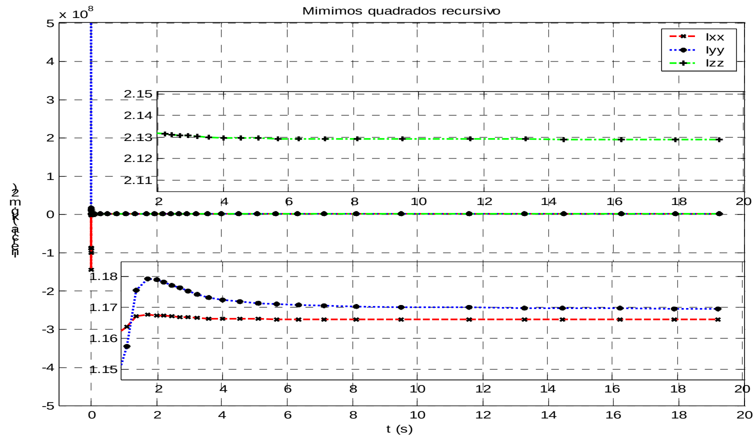

Figure 2 – Platform principal inertia parameters estimation

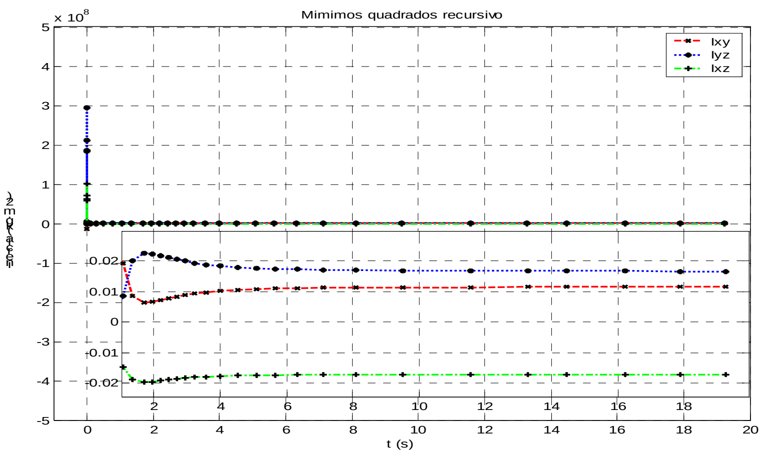

Figure 3 – Platform cross inertia parameters estimation

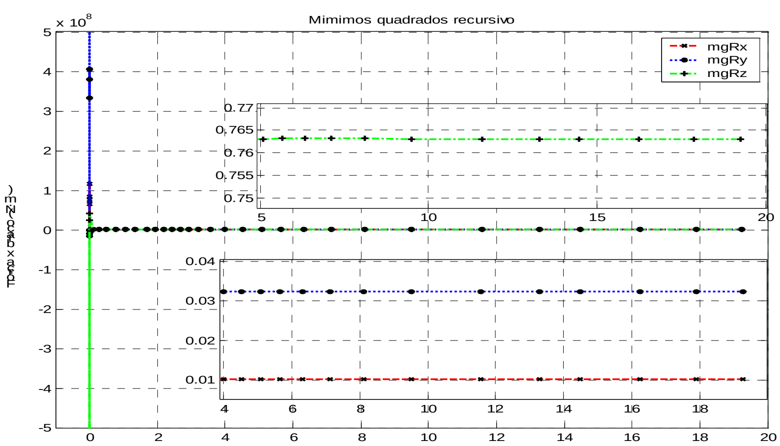

Figure 4 – External torque parameters estimation

From figures 2, 3 and 4 one observes that although it is necessary a relatively great numbers of interactions to obtain an accurately estimative, the recursive procedure are more stable, once the errors tend to decrease with time. That behavior shows that the recursive least square method is more reliable than the regression to estimate the system parameters.

5 CONCLUSIONS

In this paper a mathematical model of a platform that simulates the satellite ACS is developed and presented. The model consists of the satellite equation of motion in three axes, with three reactions wheel as actuators plus three gyro as sensors. The control system is based on a PD control law and its efficiency is investigated and implementing using the poles allocation method. The three sets of poles are choose to demonstrate the ACS performance and the simulator dynamic behavior. The first set poles are closer to the imaginary axis than the second set, and the third one is far from the imaginary axis. Simulation has shown that the ACS has the undesirable behavior of being less damped out and very oscillatory for the first set and taking the reaction wheel to saturation due to great gains for the third set. On the other hand, the second set of poles has shown good control properties, like setting time, moderate overshoot, and gains values that keep the reaction wheel rotation below saturation level. The parameters estimation was initially done applying the least square regression method. In the sequence, the parameters were estimated applying the least square recursive method. Both methods have estimated the system parameters with small error. However, the least square recursive methods had performance more adequate for the SACSS objectives. As a result, one can say that the simulator with the platform model developed is reliable and it is ready to implement and test others satellites configurations with more complex structures like flexible panels and different control strategies.

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