AN H-ADAPTIVE PANEL METHOD FOR SOLVING A TRIDIMENSIONAL INVISCID FLOW AROUND ARBITRARY BODY SHAPES

Rodrigo Guimarães Moraes,
Raul B.V. Pessolani, raul@vm.uff.br
Universidade Federal Fluminense - Depto. de Eng. Mecânica
Rua Passos da Patria, 156 – Niterói – Rio de Janerio

Abstract. *An H-Adaptive panel method is proposed for investigating the 3D incompressible and inviscid flow around arbitrary body shapes. The Panel method is based on a combination of distribution of sources and doublets applied over the boundary and using Dirichlet boundary condition to form the final system of equations. The singularity distributions are constants, and the integrals are computed analytically. The Adaptive strategy is based on the subdivision of the elements. The local error estimator which compares the solution of two successive iterations is present. Also, a global error estimator is used to prevent the possibility of non-convergence. Finally, based on four examples, for hydro and aerodynamics, the applicability of the method is shown.*

Keywords*: Adaptive procedures, Panel Method, Potential flow.*

1. INTRODUCTION

The Panel Method is a numerical Technique to compute the flow analysis over an arbitrary 3D body, immersed in a inviscid, incompressible and irrotational fluid with a given boundary condition. The Fluid Flow is simulated by distribution of singularities that can be sources, doublets and vortices, applied to the boundary. It's not necessary to refine the entire volume, since the formulation has only boundary terms. The refinement is made only on the boundary, which considerably reduces the system order. Velocity at any point of the flow is computed using the calculated boundary terms.

The self-adaptive formulations represent an area of great interest for researchers and analysts in the Boundary Element Method (BEM) field. Those techniques can be seen as ways of automation of calculation procedures, seeking for and minimizing errors. The user provides the minimum necessary information to define geometry, loads and boundary conditions, and the program tries to generate the best element mesh that gives the most accurate calculation within a certain tolerance. This procedure has the advantage of reducing processing time, also considerably reducing the dimension of the equation system and resulting in great computational economy. At the same time, the adaptive strategy helps the engineering in the problem analysis, avoiding occasional evaluation errors in mesh distribution. There are three types of Adaptive Formulations: The P Adaptive Technique increases the degree of the interpolation function in the critical areas, whereas H refines the mesh of elements, and HP is a combination of H and P techniques. The manager of the adaptive process is the error indicator that provides information about size and error distribution. If it is local, this information is called indicator. The estimator shows the measure of the global error.

Formerly, the authors have developed a 2D Adaptive using a hierarchical formulation. (Pessolani e Moraes(2005)). This paper is an extension, applying an H formulation on a Source and Doublet Panel Method to treat three dimensional bodies. A posteriori error indicator and estimator of Rencis e Mullen (1986) was adopted. It compares the change on the element solution between two successive meshes and verifies if another refinement is needed. The formulation is not hierarchical, since it doesn't accumulate the information between iterations.

2. THE PANEL SOURCE AND DOUBLET METHOD

According to Anderson (1994), the fundamental formulation to develop a solution for the equation, which determines the potential in any flow, is based on considering a physical structure with the boundary known submerged in a potential flow. The possibility of the flow in subject to be in an incompressible ($\nabla \cdot \mathbf{V} = 0$, where $\mathbf{V}$ is the vector velocity) and irrotational field is justified by the fact that the thickness of the boundary layer is very small. Therefore, the viscous effects has no influence on the calculation of the pressure and lifting coefficients on the surface. So, the continuity's Laplace equation is given by Eq. (1):

(1)

The method uses analytical solutions through a combination of elementary flows. In these solutions, there are the doublets, sources or vortex distributions that defined the body boundary (panels), as shown in Fig. 1.

Due to influence of the uniform flow (free current V_{oo}), the desired potential solution can be obtained. According to the potential obtained we calculate the velocity in some direction, deriving the potential on a right direction.

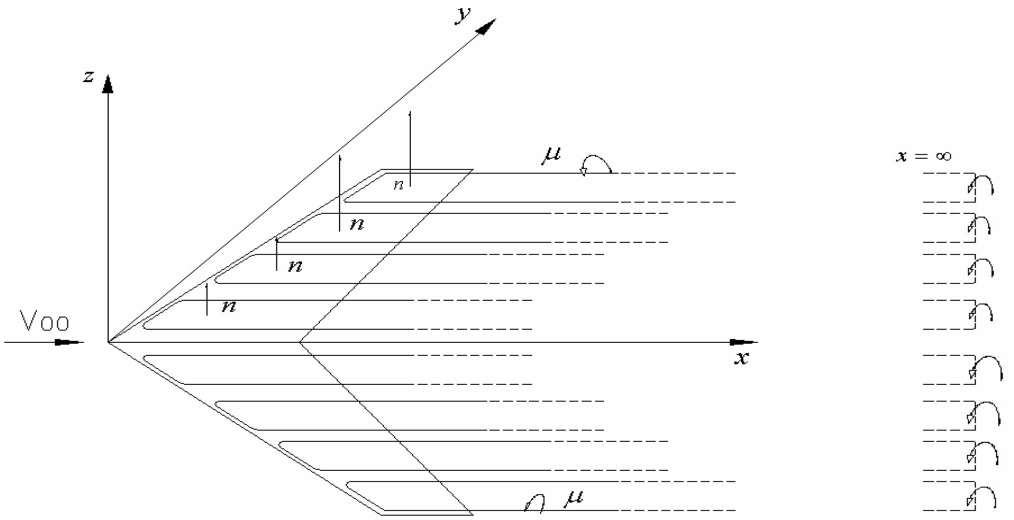

Figure 1 – Example of structures on the surface of the wing and singularity elements

Using the Green identity, it's possible to find the Potential general solution at points (x, y, z) in the domain. This solution is given by the sum of a combined local distribution of doublets μ and sources σ, on each panel:

$$\Phi^*(x,y,z) = \frac{1}{4\pi}\int_{S+w} \mu n \cdot \nabla\left(\frac{1}{r}\right) dS - \frac{1}{4\pi}\int_S \sigma\left(\frac{1}{r}\right) dS + \Phi_\infty \tag{2}$$

where **S** is the boundary structure, **w** is the wake, **r** is the vector between the point P(x,y,z) and the panel analyzed, and **n** is the extern normal vector of the panel.

To solve this equation numerically, the structure is divided in N panels each one with its singularity elements as shown in the figure 1. Assuming an incompressible and irrotacional flow, then the wake panels must be parallel in the free current, also assuming that the velocity within the geometry is zero, Dirichlet's boundary condition, it's possible to say that the inner potential of the aerodynamic structure must be constant:

$$\Phi_i^* = (\Phi + \Phi_\infty)_i = const \tag{3}$$

where the inner potential is the sum of the free current (ϕ_{oo}) and the disturbed flow (ϕ).

Putting a point P(x, y, z) inside of the tridimensional structure and using the Dirichlet's boundary condition, it can be said that the inner potential, in terms of the singularity combination on the surface, is given by:

$$\Phi_i^*(x,y,z) = \frac{1}{4\pi}\int_{S+w} \mu \cdot \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS - \frac{1}{4\pi}\int_S \sigma\left(\frac{1}{r}\right) dS + \Phi_\infty = cons \tag{4}$$

For constant strength singularity elements on each panel, the influence of panel j at a point P is given by

$$C_k \equiv -\frac{1}{2\pi}\int_S \frac{\partial}{\partial n}(\ln r) dS|_k \tag{5}$$

When a doublet and source distribution exist on each panel, the influence of panel j at a point P is given by:

$$B_k \equiv \frac{1}{2\pi}\int_S (\ln r) dS|_k \tag{6}$$

After the calculations of those integrating the boundary condition in some point next to surface, becomes:

$$\Phi_i^* = \sum_{k=1}^{N} B_k \sigma_k + \sum_{k=1}^{N} C_k \mu_k + \sum_{l=1}^{Nw} C_l \mu_l + \Phi_\infty = const \tag{7}$$

where **N** is the number of elements (panels) of the structure, $\boldsymbol{\mu}$ is the doublet which will be calculated and $\boldsymbol{\sigma}$ is the source which will be calculated by Eq. (8)

The Eq. (7) shows a non-penetration condition on the surface, as it being the velocity a derivate of the potential flow and constant, it will be zero inside of the structure.

Then, to determine the singularity distribution of each panel it's established that the sources on the surface boundary be known through Eq. (8):

$$\sigma_k = n_k \cdot Q_\infty \tag{8}$$

where Q_{oo} is the free current and n_k is the normal vector calculated by Eq. (9).

$$n_k = \frac{A_k \; x \; B_k}{| A_k \; x \; B_k |} \tag{9}$$

where A_k and B_k are defined by Fig. 2

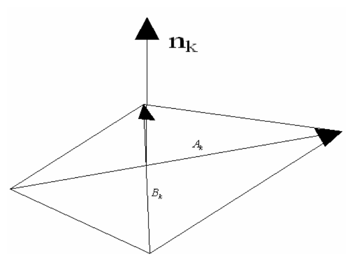

Figure 2 – Parallel vectors on the panel A_k, B_k and normal vector n_k

where ϕ_i^* is arbitrary constant. So it's possible to do $\phi_i^* = \phi_\infty$. The Eq. (7) becomes:

$$\sum_{k=1}^{N} B_k \sigma_k + \sum_{k=1}^{N} C_k \mu_k + \sum_{l=1}^{Nw} C_l \mu_l = 0 \tag{10}$$

The coefficients B_k, C_k e C_l can be calculated using analytical or numeric integration. Katz e Plotkin (2002) developed analytical expressions to the coefficients B and C.

$$B_k = \frac{1}{4\pi} \Big\{ (x - x_1) \ln \left[(x - x_1)^2 + z^2 \right] - (x - x_2) \ln \left[(x - x_2)^2 + z^2 \right] - 2(x_2 - x_1) + 2z \left(\tan^{-1} \frac{z}{x - x_2} - \tan^{-1} \frac{z}{x - x_1} \right) \Big\} \tag{11}$$

$$C_{k,l} = \frac{1}{2\pi} \left[\tan^{-1} \frac{z}{x - x_2} - \tan^{-1} \frac{z}{x - x_1} \right] \tag{12}$$

where x_1, x_2 e z are local coordinates as shown in Fig. 3.

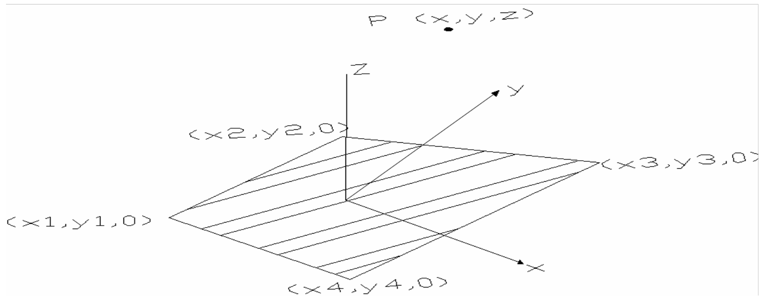

Figure 3 – Local coordinates x_1, x_2, z at each panel

Looking at the Eq. (10) we ended the flow on the collocation point of each panel where it is disturbed by others N panels.

Considering the Kutta's condition, the circulation at the trailing edge must be zero. The doublet in the panel of the wake added to the difference between the doublets of the first and last panels must be zero. The equation that represents this is given by:

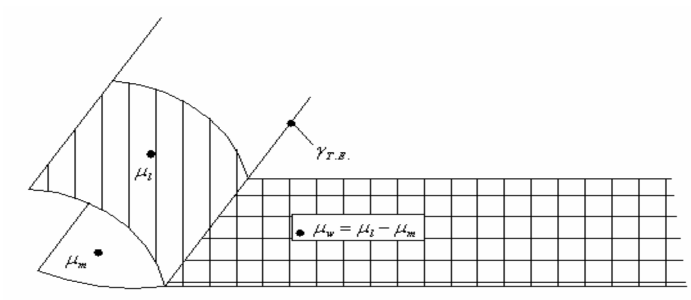

Figure 4 – Kutta's condition

The equation of first collocation point is:

$$C_{1l}\mu_1 + ... + C_{1l}\mu_l + ... + C_{1m}\mu_m + ... + C_{1N}\mu_N + \sum_{p=1}^{N_w} C_{1p}\mu_p + \sum_{k=1}^{N} B_{1k}\sigma_k = 0 \tag{13}$$

The elements c_{ii} of doublet matrix must be equal at 0.5, because they represent the performance strength of the doublet of the panel by itself. So, it's possible to reduce the order of the matrix C by relating the doublet in the wake panel to the doublet in the first and last panels. This relation is given by:

$$\begin{aligned} a_{ij} &= c_{ij}, \quad j \neq 1,N \\ a_{i1} &= c_{i1} - c_{iw}, \quad j=1 \\ a_{iN} &= c_{iN} + c_{iw}, \quad j = N \end{aligned} \tag{14}$$

where N is the last element matrix. The matrix system finally becomes:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{pmatrix} = 0 \tag{15}$$

Once the strength of the doublets μ_j is known, the potential outside the surface can be calculated. Through the boundary condition proposed by Dirichlet, we arrive at the following equation for the external potential, as shown in the beginning.

$$\Phi_u = \Phi_i - \mu \tag{16}$$

The first local external tangential velocity component above each collocation point can be calculated by differentiating the velocity potential along this tangential direction, that is:

$$q_c = \frac{\partial \Phi_u^*}{\partial l_c} \; ou \; q_l = \frac{1}{2\Delta l_c}(\mu_{l-1} - \mu_{l+1}) \tag{17}$$

where Δl_c is the length of the panel along the tangential direction.

On the same way, the second local external tangential velocity component will be:

$$q_e = \frac{\partial \Phi_u^*}{\partial l_e} \quad ou \quad q_m = \frac{1}{2\Delta l_e}(\mu_{m-1} - \mu_{m+1}) \tag{18}$$

where Δl_e is the length of the panel along the tangential direction.

The local external normal velocity component is given by:

$$q_n = \sigma \tag{19}$$

The total velocity above each collocation point is given by the free current component plus the disturbed flow component as shown below.

$$Q_k = (Q_{\infty c}, Q_{\infty e}, Q_{\infty n})_k + (q_c, q_e, q_n)_k \tag{20}$$

The pressure coefficient can be calculated by the following equation:

$$C_{pk} = 1 - \frac{Q_k^2}{Q_\infty^2} \tag{21}$$

3. THE ADAPTIVE TECHNIQUE

To automatize the potential computation, an adaptive routine which includes an error indicator and estimator was introduced. The error indicator has the function of identifying which elements need the refinement and then subdivide it. The global error estimator is a way of controlling the evolution error and avoid numerical errors due to the refinement.

The indicator error used is based on the work of Rencis and Mullen (1986) developed for the Laplace equation for the Boundary Element Method. It compares two successive solutions on the element, and then computes the number of elements necessary for the next iteration.

For 3-D bodies, the number of subdivisions necessary on the element j for the iteration (i+1) is given by:

$$(n_j^{i+1})^2 = \frac{\Delta \gamma_j}{\left(1 - \frac{(S_j^i)^2}{(S_j^{i-1})^2}\right) \cdot \varepsilon \cdot (\gamma^i)^2} \tag{22}$$

Where ε is a pre-fixed tolerance, h is the area of panel j on the i^{th} and the $(i-1)^{th}$ and γ is the doublet computed on each panel j by the expression:

$$\gamma^2 = \sum_{j=1}^{N} \gamma_j^{\,2} \cdot S_j \tag{23}$$

The comparison of the two solutions is given by the term:

$$\Delta\gamma_j^i = (\gamma_j^{i-1} - \gamma_j^{i-1}) \cdot S_j^i \tag{24}$$

Based on the number of elements, the program refines the mesh and performs the next iteration, and then the error estimator is applied again. The program executes as many iteration as necessary until a satisfactory convergence is found.

4. APPLICATIONS

To validate the program two generic structures were executed: the first structure was a Van de Vooren's wing while the second was a structure representing a tube with a valve.

4.1 Rectangular wing with a Van de Vooren section

We studied the behavior of the incompressible and uniform flow around a rectangular wing with a Van de Vooren section. We used a wing with a 10m span, 20° angle of attack and submerged in a free current (Q_{oo}) with 1m/s.

The wing described is shown below:

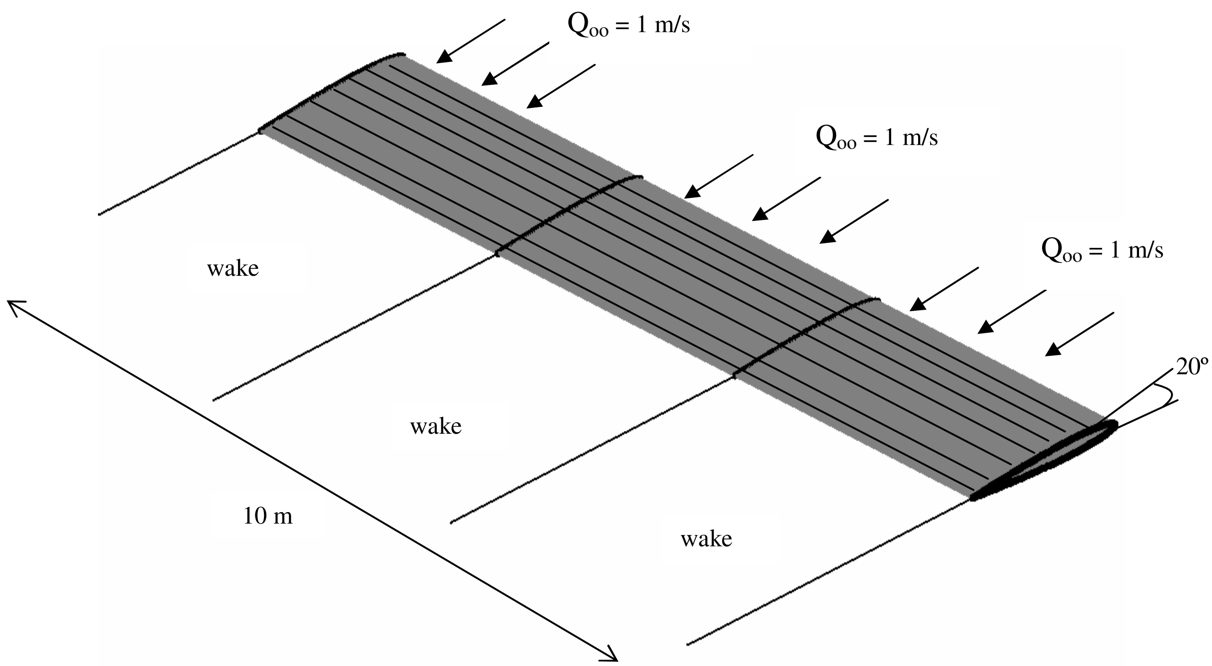

Figure 5 – A non discretized wing submerged on an uniform potential flow

Being the constant panel based on a (17), (18), (20) and (21), the necessary expressions to calculate the tangential velocity span wise, the tangential velocity chord wise and the pressure coefficient on a wing surface are give by:

$$q_c = Q_{oo} * COS(\alpha) * \vec{n}_3 - Q_{oo} * SEN(\alpha) * \vec{n}_1 + \frac{(\mu_2 - \mu_1)}{\Delta l_c} \quad (26)$$

$$q_e = \frac{(\mu_2 - \mu_1)}{\Delta l_e} \quad (27)$$

$$C_p = 1 - \frac{(V_l^2 + V_c^2)}{Q_{oo}^2} \quad (28)$$

where α is the angle of attack in radians, Q_{oo} is the free curren $\vec{n}$ is the unitary normal vector , μ is the doublet, Δl_c and Δl_e the length of the panels chord-and-span wise, respectively.

First, the wing section was defined with 26 points, as shown on the Fig. 6, with 3stripes placed on the span and 3 panels on the wake. It was a total of 78 panels.

We can see on Fig. 6 a concentration of panels on the leading edge. In fact, this occurs because at that part the geometry is grocery.

The tolerance used was 0.02 and the maximum aspect ratio was 40 by element.

After 3 iterations the H-adaptive strategy generates 59 points on the normal section, 60 stripes of panels on the chord totaling 3540 panels. Also the wake was discretized and generates 60 panels totaling 3600 panels on the surface of the wing.

The section discretized is shown on the Fig. 7. It can be seen that the discretization is mainly greater at the leading edge.

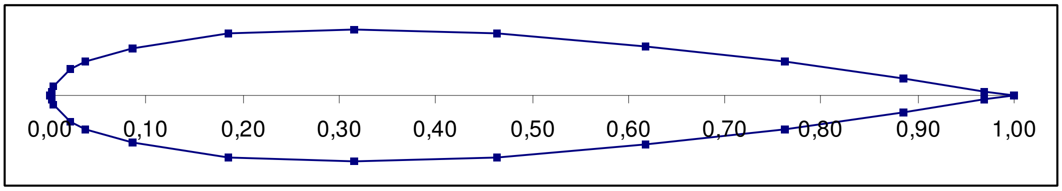

Figure 6 – First iteration after discretization of the wing

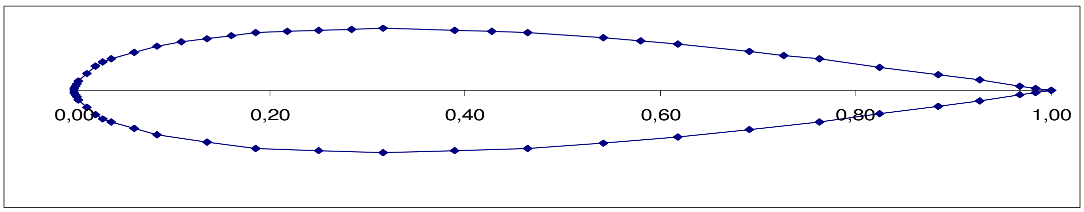

Figure 7 – Last iteration after discretization of the wing.

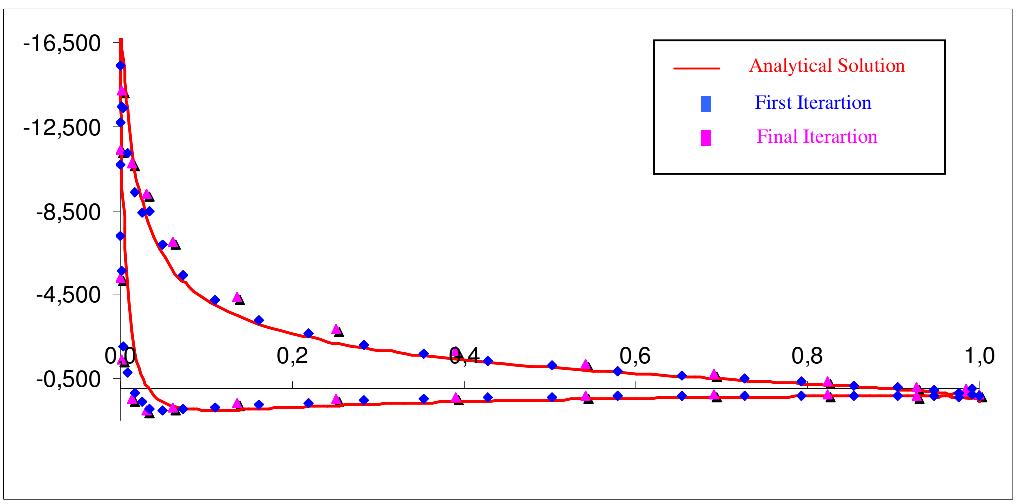

Figure 8 – Distribution of pressure (C_P) on the surface of the wing after last iteration.

In the lack of an analytical solution for a 3D wing, a comparison was performed of the pressure coefficients between the results obtained by the program and the results of 2D Van de Vooren's wing.

These data are compared with the results of the central stripe which have a type of flow the same as 2D. This can be seen in Fig. 8.

It's possible to see in Fig. 8 that the last iteration and the 2D analytical results are very close, mainly on the leading edge where the gradient of the pressure coefficient is steeper and where the elements are more refined.

The error indicator detected necessitated the refinement on the leading edge of the bad panels and shows the good results of the H-adaptive strategy.

4.2 Immersed pipe line supported by a cube.

As a second application, we have considered an oil pipe submerged and anchored at the bottom of the sea by a block, submitted to a uniform flow of 1m/s normal to the section pipe.

In Fig. 9 one can see the discretization over the pipe and the block surface with 40 elements. The cube has 1 element on each side, and the pipe has 8 nodes per section defining the geometry. It's a very poor mesh, and the adaptive strategy will act to improve it. On the curved boundary, the program uses a geometric processor developed by Moller and Pessolani (2006) that tend to round the sharp corners between linear elements. Each pipe is 3 meters long and the cube has a 2m side.

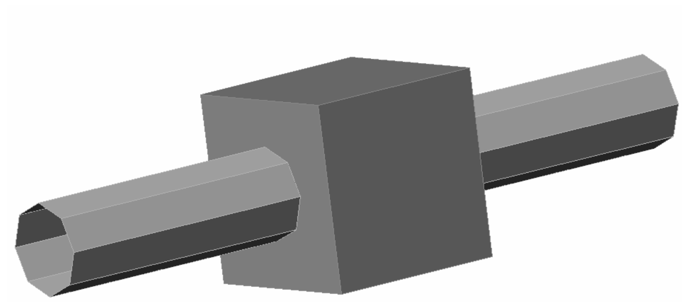

Figure 9 – Initial Mesh with 22 trapezoidal elements

The final discretization is shown on figure 10. At the pipe region it can be seen the effect done by the geometric processor, smoothing the boundary and approximating it to a circle. The refinement was concentrated on the critical points of the pipe: on the "leading edge" and on the top, where the velocity gradient is steeper. The cube had an uniform mesh distribution.

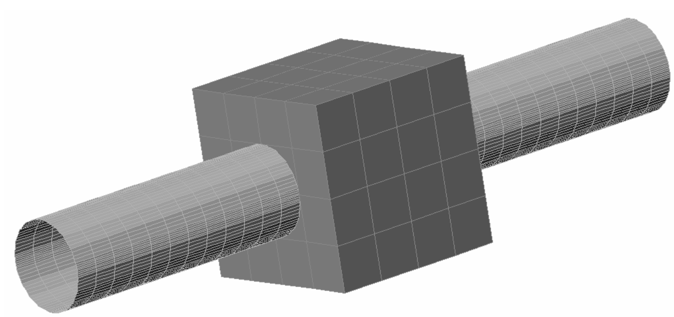

Figure 10 – Final Discretization with 2400 elements

Finally, on Fig. 11 one can see the velocities range on tube normal section. The critical points can be seen, at the "leading edge" of the pipe, and on the top. In these regions the program made a better refinement to capture this effect.

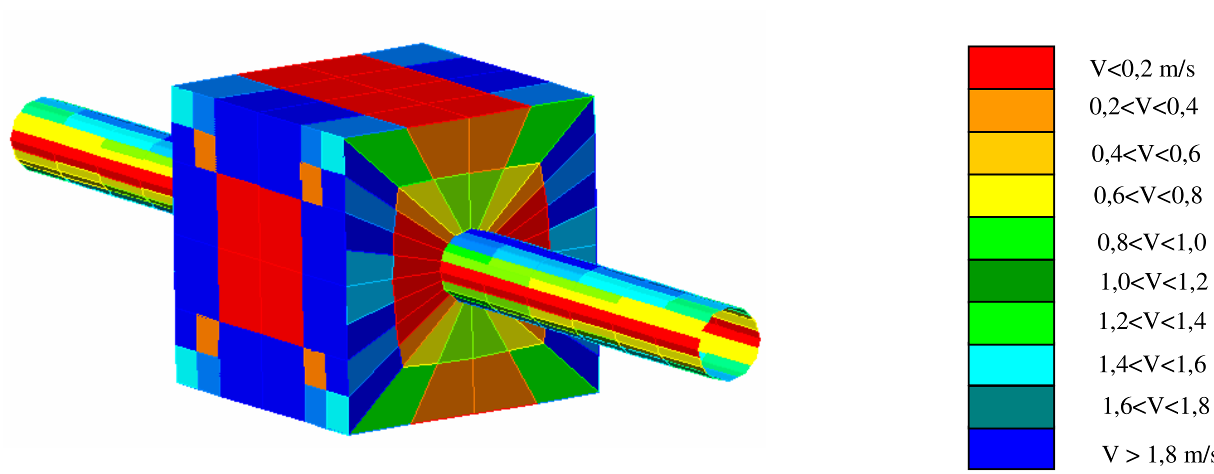

Figure 11 – Range on Tangential velocity distribution on the pipe surface after last iteration.

4.3 An airplane at flying at low Reynolds number

As a third example, consider a little airplane flying in laminar regime submitted to a uniform flow of 1m/s normal. It could any complex structure submitted to air or water like an oil platform or a ship.

In Fig. 12 one can see the velocity distribution over the airplane with 2560 elements and 2230 panels after 4 iterations. It can be observed a detail of the approximation on the wing and on the horizontal tail. Also, the error indicator tried to refine the mesh on the critical areas.

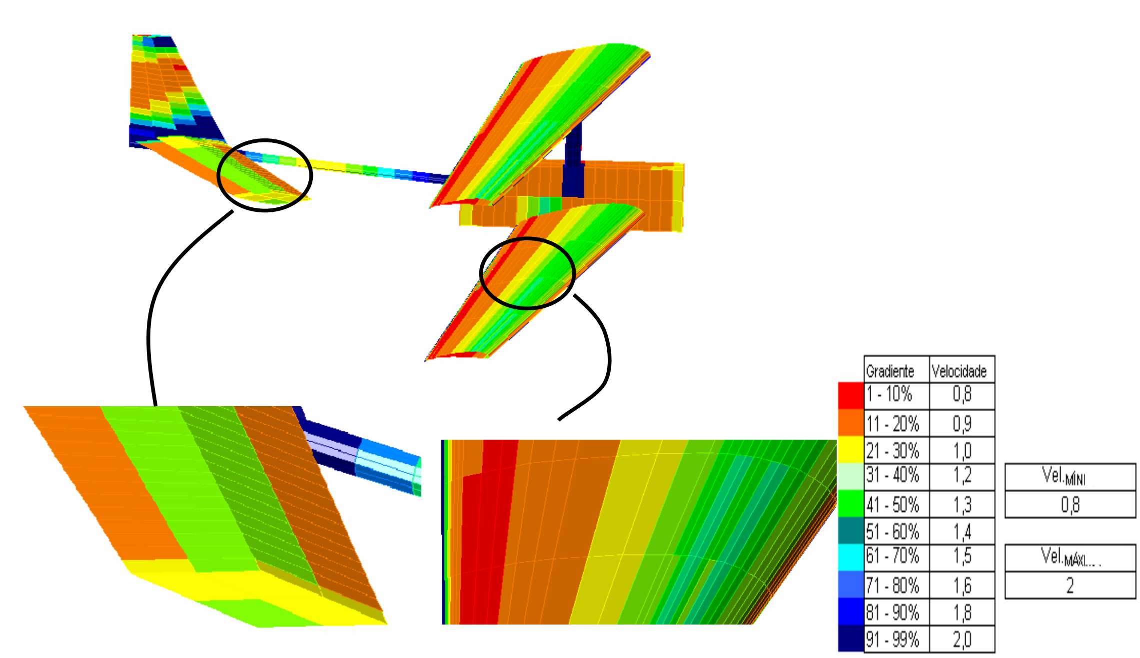

Figure 12 – Velocity distribution on the airplane surface

4.4 Uniform Flow acting on a platform jacket

At last, consider a jacket of a Petroleum Platform Structure, submitted by an uniform unitary flow from down to up on the 45º direction on the XZ Plane.

On Fig. 13 one can see the velocity percent gradient varying from 0% (below 0.9m/s) to 100% (higher that 2,0m/s). On the horizontal tubes the flux makes an static load. It can be seen that the maximum is at the front of the tube ("Leading edge"), where the flux change its direction, while the minimum is at the "shoulder" of the tube. These results are consistent with the theory.

If one wants to consider the dynamics loads, it must take in account the waves at the surface, or the vortices that are loosing of the tubes.

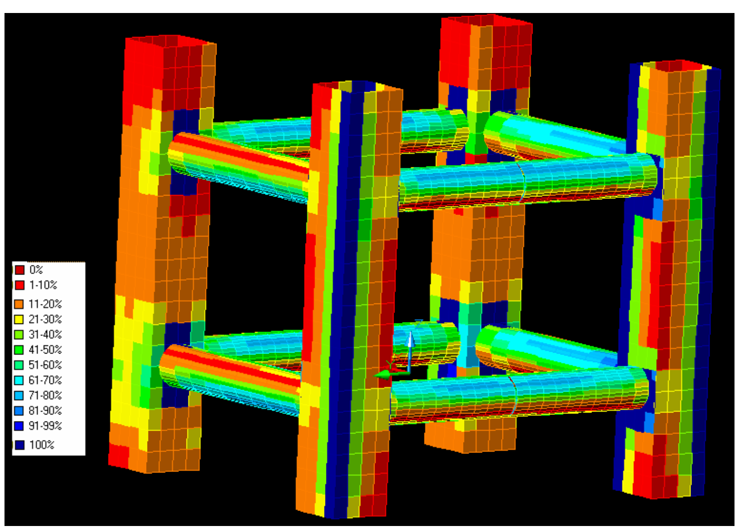

Figure 13 – Percentage of the velocity on the panels of a platform jacket acting at 45º on the XZ Plane.

5. CONCLUSIONS

An Adaptive H program based on Panel Source and Dirichlet Method for solving a potential flow over a generic 3D Body was shown.

The wing example showed the accuracy response. The second example showed the applicability of these kind of programs in maritime structures like immersed pipes and offshore structures. Those programs compute the velocity and the pressure over a boundary surface submitted to a flow.

The adaptive programs seem to be very useful. Given an initial poor mesh, only necessary to describe the geometry of the problem, the program generates the other meshes necessary to give good results. The use of an adaptive geometric pre-processor was fundamental to refine smooth boundaries. Using it, engineers have their woks so much simplified and those programs can be executed in personal computers of small capacity, since it requests less memory and computational effort than the conventional analysis.

The applications showed that the error estimator works well, refining the mesh on the critical points.

It's also important to notice the advantages of boundary formulations over domain ones, especially on infinite domains. This formulations don't need to refine the entire volume and can solve fluid problems with a considerable reduction of the system dimension and the input data.

As the subject of future works, this formulation can be extended to the analysis of wave propagation and fluid-structure iteration, using the Boundary Element Method.

8. ACKNOWLEDGEMENTS

The authors acknowledge CNPq and FAPERJ for the financial support in part of this work.

9. REFERENCES

Anderson, 1994. *Theory of Aerodynamics*. 2nd Edition, Springer Verlag.

Katz, J., & Plotkin, A., 2002, *Low Speed Aerodynamics*. Cambridge University Press, 2nd Edition.

Moller, D.F.M. & Pessolani, R.B.V., Análise Aeroelástica de Aerofólios em 2D com Painéis lineares utilizando o Método dos Elementos de Contorno, in XXVII Iberian Latin-American Congress on Computational Methods in Engineering – CILAMCE 2006, Belém, Pará, Brasil

Pessolani, R.B.V., & Moraes, R.G., 2005. An H Adaptive Hierarchical Panel Constant Method for computing a bidimensional subsonic flow, *COBEM 18th International Congress of Mechanical Engineering*, 6 to 11 November 2005, Ouro Preto, Minas Gerais

Rencis, J.J. e Mullen, R.L., 1986. *Solution of Elasticity problems by a self-adaptive mesh refinement technique for the Boundary Element Computation,* International Journal Num. Meth. Engng*, 23, 1509-1527.*

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.