FUNDAMENTAL SOLUTION IN THERMALLY DEVELOPING FLOW THROUGH MICROCHANNELS

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Abstract. The need of accurate solutions in microchannels is important since they are employed in various fields, such as in electronic industry, microfabrication technologies, biomedical engineering, etc. In this work, attention is focused on the hydrodynamically developed, thermally developing single phase flow in microchannels. The boundary conditions studied are those associated with uniform and non-uniform temperature on the channel walls. These mathematical models are solved through integral transform techniques furnishing benchmark solutions for all the mentioned situations studied in this contribution, which deal with an exponential behaviour of the temperature at the solid boundaries. Quantities of practical interest such as temperature and Nusselt Number distributions are presented in tabular form and an assessment is made regarding the importance of the slip flow condition and the Knudsen Number. In general, it was found that the solution procedure exhibits a very fast convergence rate and the heat transfer coefficients are sensitive to the variation of the Knudsen Number, increasing for higher values of rarefaction.

***Keywords**: Microchannels, Knudsen Number, Integral Transform Technique, Heat and Mass Diffusion.*

1. INTRODUCTION

The advances of aerospacial, medical and electronics industries have required deep investigations on the behavior of heat transfer in small dimension structures or in rarefaction conditions. Hence, a clear understanding of the phenomenon has become necessary. One of the biggest challenges that the engineers have nowadays is to project and build equipments that have become even smaller, in order of being adapted to also smaller structures.

There are many points that must be taken into consideration when we deal with microchannels. The main differences, compared to the macro scale, lays in boundary conditions of the momentum equation. There must be considered a non-zero velocity of the fluid near the wall (Eckert and Drake, 1959). This effect is due to the fact that, in micro or rarefied flow, the caracteristic dimensions of the channel are comparable with the mean free path of the molecules of the fluid. Because of that, the continuum condition can not be considered. In order to quantify how strong the non-continuum condition is, a special relation between the mean free path length of the molecules of the fluid and a representative physical length of the channel is introduced. This dimensionless relation is called Knudsen Number (Kn) in honor to the the Danish physicist Martin Hans Christian Knudsen and specifies the degree of rarefaction in a flow. Usually, four regimes are considered (Barron et al., 1997):

continuum flow	$\mathrm{Kn} < 10^{-3}$
slip-flow	$10^{-3} \leq \mathrm{Kn} < 10^{-1}$
transition flow	$10^{-1} \leq \mathrm{Kn} < 10$
free molecular flow	$10 \leq \mathrm{Kn}$

For a flow in a microchannel, the slip-flow regime should be used. It is known that the fluid next to the solid surface, in microchannels, does not have the same velocity as the surface. In other words, there is a non-zero relative velocity between the fluid next to the wall and the wall. Eckert and Drake (1959), based on the Kinetic Theory of Gases, formulated an expression for this velocity jump, that is stated below in rectangular coordinates:

$$u_s = -\frac{2-F_v}{F_v}\lambda\left(\frac{du}{dy}\right)_{y=a} \tag{1}$$

where:

u_s = flow velocity next to the solid surface
F_v = tangential momentum accomodation coefficient

λ = molecular mean free path of the fluid
u = flow velocity
y = rectangular coordinate, relative to the height of the channel, with its origin at the middle point between the lower and the upper plates
a = half-height of the channel

The coefficient F_v represents the fraction of the incident molecules of the fluid that are spreadly reflected by the solid surface, and for most engineering applications has unitary value.

2. ANALYSIS

In the present study, a flow between two parallel plates is analysed. It is considered to be a slip-flow, hence the Knudsen Numbers are between 0.001 and 0.1. The basic characteristics of this flow are: laminar, incompressible, steady state, hidrodynamically developed flow, symmetrical velocity relative to the origin of the ordinate axis; constant thermophisical properties, negligible field forces, as well as viscous dissipation and heat diffusion along the length of the channel; uniform inlet temperature, temperature symmetry relative to the origin of the ordinate axis, prescribed boundary temperature, with exponential decrease or increase, according to the exponent signal in the proper equation.

Hence, the hidrodynamic part of the problem is:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx} \tag{2}$$

$$\left.\frac{du}{dy}\right|_{y=0} = 0 \tag{3}$$

$$u(a) = -\frac{2-F_v}{F_v}\lambda\left.\frac{du}{dy}\right|_{y=a} \tag{4}$$

where:

μ = dynamic viscosity
p = pressure
x = rectangular coordinate, relative to the length of the channel, with its origin at the origin of the channel

Equation (2), with its boundary conditions represented by Eq. (3) and Eq. (4), is solved and presents the following solution:

$$u(y) = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + \frac{1}{\mu}\frac{dp}{dx}\left(-\frac{2-F_v}{F_v}\lambda a - \frac{a^2}{2}\right) \tag{5}$$

In order to make Eq. (5) easier to deal, it is trasformed on these following variables:

$$Y = \frac{y}{a} \tag{6}$$

$$\beta_v = \frac{2-F_v}{F_v} \tag{7}$$

$$Kn = \frac{\lambda}{2a} \tag{8}$$

$$U(Y) = \frac{u(y)}{u_{av}} \tag{9}$$

where β_v is called tangential momentum accomodation constant and u_{av} is the mean fluid velocity, relative to the total developed velocity profile. It can be noticed in Eq. (8) that Kn actually represents the relation between the mean free path of the molecules of the fluid and a representative physical length of the channel, in this case the distance between the two parallel plates. Therefore, Eq. (5) takes the following dimensionless form:

$$U(Y) = \frac{\frac{3}{2}\left(1-Y^2\right)+6\beta_v Kn}{1+6\beta_v Kn} \tag{10}$$

Equation (10) is in accordance with the literature (Aydin and Avci, 2007). Once the hidrodynamic part of the problem is solved, the thermal part can be stated:

$$u(y)\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{11}$$

$$T(y,0) = T_i \tag{12}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \tag{13}$$

$$T(a,x) = T_i + \left(T_w - T_i\right)e^{-\beta x} \tag{14}$$

where:

T = temperature
α = thermal diffusion
T_i = inlet temperature
T_w = wall temperature
β = arbitrary exponent

Equations (11-14) are now transformed into a dimensionless set of equations, based on the following defined dimensionless variables:

$$\theta = \frac{T - T_i}{T_w - T_i} \tag{15}$$

$$X = \frac{\alpha x}{u_{av} a^2} \tag{16}$$

Therefore, Eqs. (11-14) take the following dimensionless form:

$$U(Y)\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} \tag{17}$$

$$\theta(Y,0) = 0 \tag{18}$$

$$\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} = 0 \tag{19}$$

$$\theta(1,X) = e^{-\left(\frac{\beta u_{av} a^2}{\alpha}\right)X} \tag{20}$$

Equation (17) together with its boundary conditions, Eqs. (18-20), can be solved as a particular case of the so-called Class I problem in heat and mass diffusion (Mikhailov and Özişik, 1994). This particular problem can be further generalized to include viscous dissipation effects, a non-uniform inlet temperature, a parallel plate or tube geometry as well an outer wall generalized boundary condition. This more complete problem related to the thermal entry in microchannels can be stated as:

$$w(x)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left[k(x)\frac{\partial T}{\partial x}\right]+q(x) \tag{21}$$

$$T(x,0)=f(x) \tag{22}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0 \tag{23}$$

$$\alpha T(1,t)+\beta\left.\frac{\partial T}{\partial x}\right|_{x=1}k(1)=\phi_0+ke^{-\gamma t} \tag{24}$$

where all the variables above are generic, which mean that the letters that represent them are not necessarily the same of Eqs. (17-20). This metodology was adopted to state a more general case, that can be used to include aspects not present in this work.

Therefore, Eq. (21), with its boundary conditions, is solved and the corresponding variables of Eqs. (17-20) can then be related in order to recast the original problem as a particular application. If the dimensional variables are desired, they can be found by direct substitution in Eqs. (6-9) and Eqs. (15-16). Since we are dealing with an intrinsic non-homogenous boundary condition, Eq. (24), the Split-Up procedure should be used. The new variables that appear next are dimensionless, as are in this case the ones of Eqs. (21-24). Accordingly, the temperature field shall be split into a sum of two variables, as described below:

$$T(x,t)=T_{ss}(x)+\theta(x,t) \tag{25}$$

The problem for $T_{ss}(x)$ is:

$$\frac{d}{dx}\left[k(x)\frac{dT_{ss}}{dx}\right]+q(x)=0 \tag{26}$$

$$\left.\frac{dT_{ss}}{dx}\right|_{x=0}=0 \tag{27}$$

$$\alpha T_{ss}(1)+\beta k(1)\left.\frac{dT_{ss}}{dx}\right|_{x=1}=\phi_0 \tag{28}$$

The problem for $\theta(x,t)$ is the following:

$$w(x)\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial x}\left[k(x)\frac{\partial \theta}{\partial x}\right] \tag{29}$$

$$\theta(x,0)=f(x)-T_{ss}(x)=g(x) \tag{30}$$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0}=0 \tag{31}$$

$$\alpha\theta(1,t)+\beta k(1)\left.\frac{\partial \theta}{\partial x}\right|_{x=1}=ke^{-\gamma t} \tag{32}$$

The solution procedure for Eq. (29), its initial and boundary conditions, is to transform $\theta(x,t)$ into a sum of two terms, as follows:

$$\theta(x,t) = \theta_{aux}(x)e^{-\gamma t} + \theta_h(x,t) \tag{33}$$

Hence, the problem for $\theta_{aux}(x)$ is:

$$\frac{d}{dx}\left[k(x)\frac{d\theta_{aux}}{dx}\right] + w(x)\gamma\theta_{aux}(x) = 0 \tag{34}$$

$$\left.\frac{d\theta_{aux}}{dx}\right|_{x=0} = 0 \tag{35}$$

$$\alpha\theta_{aux}(1) + \beta k(1)\left.\frac{d\theta_{aux}}{dx}\right|_{x=1} = k \tag{36}$$

The problem for $\theta_h(x,t)$ is:

$$w(x)\frac{\partial\theta_h}{\partial t} = \frac{\partial}{\partial x}\left[k(x)\frac{\partial\theta_h}{\partial x}\right] \tag{37}$$

$$\theta_h(x,0) = f(x) - T_{ss}(x) - \theta_{aux}(x) = g(x) - \theta_{aux}(x) = h(x) \tag{38}$$

$$\left.\frac{\partial\theta_h}{\partial x}\right|_{x=0} = 0 \tag{39}$$

$$\alpha\theta_h(1,t) + \beta k(1)\left.\frac{\partial\theta_h}{\partial x}\right|_{x=1} = 0 \tag{40}$$

The Integral Transform Technique is applied to solve Eq. (37). Therefore, $\theta_h(x,t)$ must be represented as an eigenfunction expression in the form:

$$\theta_h(x,t) = \sum_{i=1}^{\infty} A_i(t)\psi_i(x) \tag{41}$$

The auxiliary problem related to the determination of the eigenfunctions $\psi_i(x)$ is chosen to be:

$$\frac{d}{dx}\left[k(x)\frac{d\psi_i}{dx}\right] + \mu_i^2 w(x)\psi_i(x) = 0 \tag{42}$$

$$\left.\frac{d\psi_i}{dx}\right|_{x=0} = 0 \tag{43}$$

$$\alpha\psi_i(1) + \beta k(1)\left.\frac{d\psi_i}{dx}\right|_{x=1} = 0 \tag{44}$$

Equation (42) together with its two boundary conditions is immediately recognized as a particular case of the standard Sturm-Liouville Problem and efficient algorithms, such as the Sign-Count Method (Mikhailov and Özişik, 1994), can be utilized in order to precisely evaluate the eigenvalues μ_i, its related eigenfunctions $\psi_i(x)$ and other desired eigenquantities such as, for example, the norm N_i. Furthermore, by applying the well established orthogonality property of this classical eigenvalue problem, it is possible to determine $A_i(t)$ such as:

$$A_i(t) = \frac{1}{N_i}\int_0^1 w(x)\psi_i(x)\theta_h(x,t)dx \tag{45}$$

By inserting the above result in Eq. (41), it is a simple matter to determine the following inverse-transform pair:

$$\bar{\theta}_{h_i}(t) = \frac{1}{N_i^{\frac{1}{2}}}\int_0^1 w(x)\psi_i(x)\theta_h(x,t)dx \tag{46}$$

$$\theta_h(x,t) = \sum_{i=1}^{\infty}\frac{1}{N_i^{\frac{1}{2}}}\psi_i(x)\bar{\theta}_{h_i}(t) \tag{47}$$

The next step is to rewrite the original problem in terms of the transformed potential $\bar{\theta}_{h_i}(t)$. This task is accomplished through a series of mathematical operations which are briefly discussed now. Initially, the governing partial differential equation is multiplied by $\frac{1}{N_i^{\frac{1}{2}}}\psi_i(x)$ and then integrated over the definition domain [0,1]. Secondly, the Sturm-Liouville Equation is multiplied by $\frac{1}{N_i^{\frac{1}{2}}}\theta_h(x,t)$ and also integrated over the definition domain. The results are added and the boundary conditions of both the original and auxiliary problems are employed to furnish:

$$\frac{d\bar{\theta}_{h_i}}{dt} + \mu_i^2\bar{\theta}_{h_i}(t) = 0 \tag{48}$$

$$\bar{\theta}_{h_i}(0) = \frac{1}{N_i^{\frac{1}{2}}}\int_0^1 w(x)\psi_i(x)h(x)dx = \bar{h}_i \tag{49}$$

Equation (48) and its initial condition, Eq. (49), has the following solution:

$$\bar{\theta}_{h_i}(t) = \bar{h}_i e^{-\mu_i^2 t} \tag{50}$$

Therefore, Eq. (47) has the following solution:

$$\theta_h(x,t) = \sum_{i=1}^{\infty}\frac{1}{N_i}\left[\int_0^1 w(x)\psi_i(x)h(x)dx\right]\psi_i(x)e^{-\mu_i^2 t} \tag{51}$$

Finally, it is possible to state the solution of Eq. (25):

$$T(x,t) = T_{ss}(x) + e^{-\gamma t}\theta_{aux}(x) + \sum_{i=1}^{\infty}\frac{1}{N_i}\left[\int_0^1 w(x)\psi_i(x)h(x)dx\right]\psi_i(x)e^{-\mu_i^2 t} \tag{52}$$

Naturally, it is necessary to first solve Eq. (26) and its boundary conditons, Eqs. (27-28), and Eq. (34), with its boundary conditions, Eqs. (35-36).

Once Eq. (52) is solved, it is possible to find quantities of practical interest such as the axial distribution of the mean fluid temperature and the local heat transfer across the channel outer wall. Also of interest is the local Nusselt number distribution which, for the specific case of the parallel plate channel, is defined as:

$$Nu(x) = \frac{4 \left. \frac{\partial \theta}{\partial X} \right|_{X=1}}{\theta(1, X) - \theta_{av}(X)} \quad (53)$$

3. RESULTS

The methodology outlined in the previous section was employed to solve the problem defined by Eqs. (17)-(20) for some values of the Knudsen Number and for the cases of mild and sharp surface temperature decay represented by the coefficient γ. For all the results presented in this contribution it was found that for $X < 10^{-3}$, the variations of both the average temperature of the fluid and that of the wall surface were negligible. Consequently, most of the results depicted in this contribution address the thermal entry problem at regions $X \geq 10^{-3}$.

Table 1. Nusselt number for different truncation orders (N) in the series of Eq. (52), for $\gamma = 0.5$ and Kn = 0.1.

X	N = 10	N = 30	N = 50	N = 80	N = 100
1E-3	1.6313	49.0139	49.0148	49.0148	49.0148
2E-3	0.7318	36.0429	36.0429	36.0429	36.0429
3E-3	0.1501	30.2621	30.2621	30.2621	30.2621
4E-3	-0.2667	26.8006	26.8006	26.8006	26.8006
5E-3	-0.6303	24.4298	24.4298	24.4298	24.4298
6E-3	-1.0411	22.6746	22.6746	22.6746	22.6746
7E-3	-1.6431	21.3069	21.3069	21.3069	21.3069
8E-3	-2.8036	20.2022	20.2022	20.2022	20.2022
9E-3	-6.3890	19.2854	19.2854	19.2854	19.2854
1E-2	95.8995	18.5087	18.5087	18.5087	18.5087
2E-2	1.1059	14.2859	14.2859	14.2859	14.2859
3E-2	0.7654	12.4145	12.4145	12.4145	12.4145
4E-2	0.6364	11.3081	11.3081	11.3081	11.3081
5E-2	0.5585	10.5640	10.5640	10.5640	10.5640
6E-2	0.4991	10.0257	10.0257	10.0257	10.0257
7E-2	0.4475	9.6180	9.6180	9.6180	9.6180
8E-2	0.3996	9.2993	9.2993	9.2993	9.2993
9E-2	0.3539	9.0450	9.0450	9.0450	9.0450
1E-1	0.3100	8.8388	8.8388	8.8388	8.8388
2E-1	0.0466	7.9722	7.9722	7.9722	7.9722
3E-1	0.0064	7.7341	7.7341	7.7341	7.7341
4E-1	0.0028	7.5399	7.5399	7.5399	7.5399
5E-1	0.0025	7.2669	7.2669	7.2669	7.2669

Perhaps, the first aspect to be addressed is the rate of convergence of the proposed methodology. Table 1 presents a convergence study for the local Nusselt Number at various locations of the parallel plate channel when subjected to a prescribed exponentially decaying outer wall temperature in a situation where the Knudsen Number is equal to 0.1. In general terms, a careful assessment of this results indicates that an expansion term up to 50 terms is enough to ~~warrant~~ guarantee a 4 digit accuracy at the $0.001 < X < 0.5$ region.

The effects of a prescribed exponentially varying temperature at the channel wall with respect to the overall heat transfer performance can be inspected in Tab. 2. Here a comparison is made between the fully converged simulations for $\gamma = 0.5$ with respect to the classical uniform temperature boundary condition for three distinct values of the Knudsen Number. These tabular results reveal that for a fixed Knudsen Number, the local Nusselt Number for the uniform temperature boundary condition is consistently higher when compared to the varying wall temperature situation. This effect is naturally expected when one keeps in mind that the boundary condition adopted in this work will add less heat to the fluid inside the channel when compared to the uniform temperature case. At this point it is worth mentioning that, consistent with previous findings (Castilho, 2002), Tab. 2 also shows that the heat transfer to the fluid inside the channel increases as the slip flow effect becomes more severe. As a result, the local Nusselt Numbers become higher when the Knudsen number is increased.

Table 2. Converged local Nusselt number for some values of the Knudsen number.

X	γ	Kn = 0	Kn = 0.04	Kn = 0.1
1E-3	0	31.0052	40.4727	49.0370
	0.5	30.9974	40.4563	49.0148
2E-3	0	24.6882	30.5561	36.0747
	0.5	24.6758	30.5325	36.0429
3E-3	0	21.6357	26.0589	30.3013
	0.5	21.6195	26.0295	30.2620
4E-3	0	19.7179	23.3341	26.8462
	0.5	19.6982	23.2997	26.8006
5E-3	0	18.3590	21.4512	24.4812
	0.5	18.3361	21.4123	24.4298
6E-3	0	17.3265	20.0472	22.7312
	0.5	17.3007	20.0042	22.6746
7E-3	0	16.5051	18.9468	21.3684
	0.5	16.4764	18.9000	21.3069
8E-3	0	15.8300	18.0535	20.2682
	0.5	15.7986	18.0030	20.2022
9E-3	0	15.2616	17.3090	19.3558
	0.5	15.2276	17.2551	19.2854
1E-2	0	14.7738	16.6759	18.5831
	0.5	14.7373	16.6186	18.5087
2E-2	0	12.0145	13.1936	14.3950
	0.5	11.9562	13.1081	14.2859
3E-2	0	10.7288	11.6289	12.5516
	0.5	10.6520	11.5202	12.4145
4E-2	0	9.9507	10.6999	11.4700
	0.5	9.8574	10.5705	11.3081
5E-2	0	9.4212	10.0752	10.7484
	0.5	9.3125	9.9270	10.5640
6E-2	0	9.0360	9.6247	10.2311
	0.5	8.9131	9.4590	10.0257
7E-2	0	8.7441	9.2854	9.8430
	0.5	8.6077	9.1031	9.6180
8E-2	0	8.5166	9.0222	9.5431
	0.5	8.3675	8.8243	9.2993
9E-2	0	8.3362	8.8142	9.3066
	0.5	8.1750	8.6014	9.0450
1E-1	0	8.1913	8.6476	9.1175
	0.5	8.0185	8.4205	8.8388
2E-1	0	7.6322	8.0087	8.3952
	0.5	7.3659	7.6635	7.9722
3E-1	0	7.5537	7.9198	8.2954
	0.5	7.2058	7.4660	7.7341
4E-1	0	7.5425	7.9074	8.2815
	0.5	7.0946	7.3157	7.5399
5E-1	0	7.5410	7.9056	8.2796
	0.5	6.9499	7.1118	7.2669

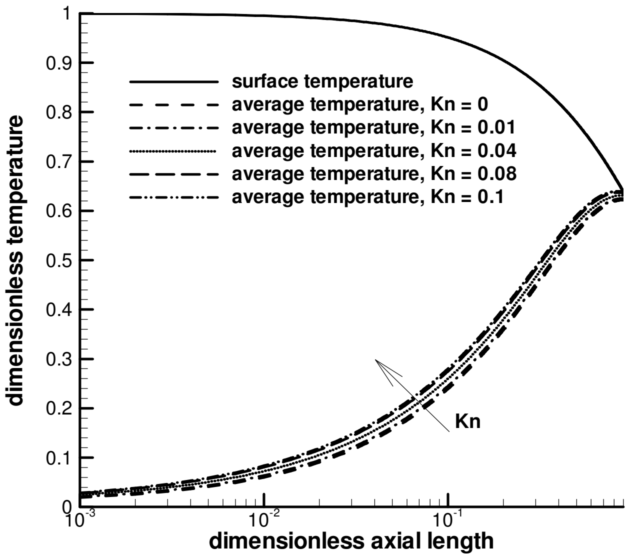

Figure 1. Axial distributions of the channel surface temperature and average fluid temperature for $\gamma = 0.5$ and different values of Kn.

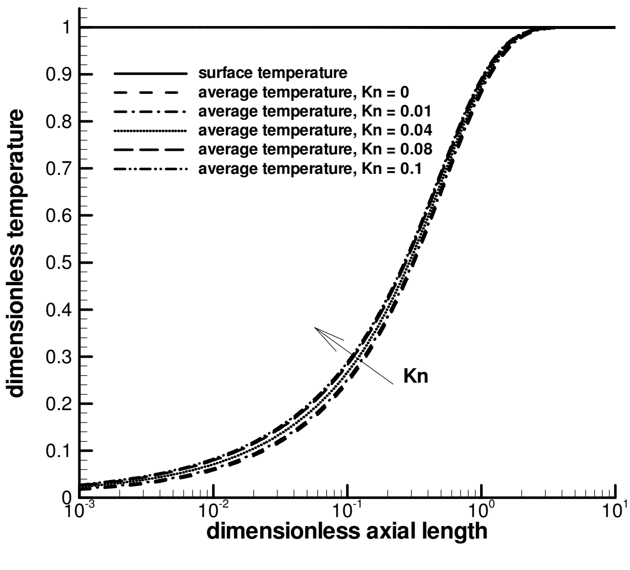

Figure 2. Axial distributions of channel surface temperature and average fluid temperature for $\gamma = 0$ and different values of Kn.

Figures 1 and 2 present the fluid average temperature distribution for the uniform and non-uniform wall temperature boundary conditions studied in this contribution. These results also suggest that the uniform temperature boundary condition will yield higher mean fluid temperature values. For example, Fig. 1 indicates that an equilibrium condition is attained at $X = 0.9$ resulting in a temperature around *0.63* for the higher value of the Knudsen number. Alternatively, Fig.2 reveals that at the same location, the fluid average temperature is about *0.86*, suggesting that this latter boundary condition is indeed more effective for the heat transfer process.

Finally, Fig. 3 shows the axial distribution of the mean fluid temperature in a situation where the wall temperature experiences an exponential increase. An inspection of these results points that in the $X < 0.1$ region the physical problem addressed here is essentially similar to that related to an uniform wall temperature. However, due to the particular nature of the boundary condition employed in this simulation, for the $X > 0.1$ region the fluid inside the channel experiences a steep rise reaching a dimensionless temperature value around unity at approximately $X = 1$.

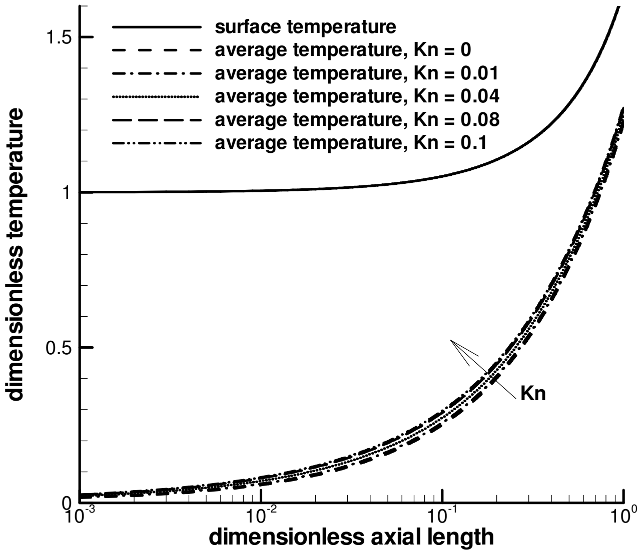

Figure 3. Axial distributions of channel surface temperature and average fluid temperature for γ = -0.5 and different values of Kn.

4. CONCLUSIONS

The main purpose of this contribution is to address a generalized thermal entry problem in microchannels where the effect of slip flow is noticeable. More specifically, a mathematical formulation was devised in order to incorporate an exponentially varying boundary condition at the outer wall of the channel. This problem was solved by means of integral transforms and a case related to a parallel plate channel with an exponentially varying wall temperature was examined. The simulations revealed that the proposed methodology is able to furnish precise results for the quantities of practical interest such as the average fluid temperature and local Nusselt Number with a relatively low expansion term resulting in a low computational effort.

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