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A HYBRID ELECTROMECHANICAL-VISCOELASTIC DYNAMIC VIBRATION NEUTRALIZER: A NEW MODEL AND ANALISYS

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Abstract. *Vibration absorbers are mechanical devices for attaching to another mechanical system or structure (called the primary system) to control or reduce vibration and sound radiation from machines or structural surfaces. The cheapest and easiest way to construct a vibration absorber is by incorporating a viscoelastic material as both the resilient and energy-dissipating element. A major problem in the analysis and design of such absorbers is that, when applied to a structure, they result in equations with coefficients that are frequency dependent. This difficult problem was solved efficiently by the PISA-CNPq group with the introduction of a new concept of generalized quantities for the absorber and a new approach to the optimal design of viscoelastic absorber systems. This kind of neutralizer, although achieving optimal vibration reduction, can cause detuning as temperature varies. Electromechanical vibration neutralizers are another class of vibration neutralizer that use the interaction between a magnetic field and the displacement of a coil to generate an EMF in a resonant RLC circuit. Such neutralizers can be configured as passive/active control devices and also add less mass to the primary system. However, practical difficulties are caused by the need for them to be installed with an auxiliary structure to support the magnetic field generator. In this study, a new kind of vibration neutralizer that combines the benefits of optimal vibration reduction and active vibration control with minimal power consumption and the ability to function without an auxiliary structure is introduced. Then, a model for a hybrid electromechanical-viscoelastic dynamic neutralizer is described and its equivalent generalized parameters are introduced and analyzed. The theoretical mathematical basis developed by the PISA-CNpq group is applied in this study to the optimal design of a hybrid device. An example of vibration control in a one-degree-of-freedom system is introduced and the results are discussed.*

Keywords: *Vibration Control, Electromechanical Device, Optimization, Dynamic Neutralizers, Viscoelastic Material.*

1. INTRODUCTION

Vibration absorbers, as they are commonly known, but which should more appropriately be called vibration neutralizers, are mechanical devices for attaching to another mechanical system or structure (called the primary system) to control or reduce vibration and sound radiation from machines or structural surfaces. Vibration neutralizers were first used to reduce rolling motions of ships (Frahm, 1909). Since then, many publications on the subject have been demonstrating their efficiency in reducing vibrations and sound radiation in many kinds of structures and machines (Den Hartog, 1956).

Using viscoelastic materials, which can be manufactured to meet design specifications, vibration neutralizers had became easy to make and apply to almost any complex structure (Bavastri, 1997; Snowdon, 1968).

In recent times, a great deal of effort has been done to generalize vibration neutralizers theory, applied to more complex structures than the undamped single degree of freedom model, tackled by Ormondroyd & Den Hartog (1928). In the work of Espíndola and Silva (1992), a general theory for optimum design of neutralizer systems, when applied to generic structures, was derived. This approach has been applied to many types of viscoelastic neutralizers (Espíndola & Silva, 1992; Freitas & Espíndola, 1993). The theory is based on the concept of equivalent generalized quantities for the neutralizers. With this concept, it is possible to write down the composite system (primary plus absorbers) movement equations in terms of the generalized coordinates (degrees of freedom), previously chosen to describe the primary system alone, despite the fact that the composite system has additional degrees of freedom (Espíndola & Bavastri, 1999).

A nonlinear optimization technique can be used to design the neutralizer system to be optimum (in a certain sense) over a specific frequency band.

The concept of fractional derivative is applied to the construction of a parametric model for the viscoelastic material (Espíndola *et al*., 2004). Viscoelastic materials are both frequency and temperature dependent. Thus, a disadvantage for the use of such material is that vibration neutralizers designed to optimally work in a specific frequency range, when exposed to temperature variations, can cause detuning.

Electromechanical vibration neutralizers are another class of dynamic vibration neutralizers that use the interaction between a magnetic field and the displacement of a coil to generate an electromotive force in a resonant RLC electrical circuit. The resulting circuit current, when appropriately setting RLC parameters, develops a counter-electromotive force. This force can reduce the primary system vibration (Bavastri, 2001; Abu-Akeel, 1967; Nagem *et al*., 1995). Such neutralizers can be set as passive or active control devices by varying RLC parameters. This kind of neutralizer also adds less mass to the primary system, in comparison with viscoelastic ones. However, practical difficulties are caused by the need for them to be installed with an auxiliary structure to support the magnetic field generator.

In this study, a new kind of vibration neutralizer that combines the benefits of a viscoelastic and an electromechanical neutralizer is presented. This hybrid neutralizer can achieve optimal vibration reduction and act as an active vibration control device by changing the electrical circuit parameters. This characteristic can be applied to retune the neutralizer if it is exposed to temperature variation. Besides, the hybrid neutralizer active control configuration consume minimal power comparing to others active vibration control configurations such as an adaptive filter active noise reduction one, that needs to use an exciter to impose a cancel force to the primary system. Additionally, the hybrid configuration does not need to be installed with an auxiliary structure to support the magnetic field generator. Thus, the proposed configuration is extremely versatile.

2. FRACTIONAL DERIVATIVE MODEL TO VISCOELASTIC MATERIALS

To obtain a precise modeling of the viscoelastic material and, thus, of the control device, it was employed the fractional derivative model. This model was firstly introduced by Nutting (1921), modeling the relaxation of tension in viscoelastic materials by means of fractional powers of time. After that, Gemant (1936) observed that the elasticity and damping of viscoelastic materials were proportional to fractional powers of frequency. In Bagley and Torvik (1986), the description of the viscoelastic behavior by fractional calculus was tackled. In that work, it was shown that the fractional model is closely related to the molecular theory which describes the microscopic behavior of most viscoelastic materials.

The constitutive relationship in shear regarding the four parameter fractional derivative model is given by:

$$\tau(t)+\varphi_0 \frac{d^{\beta}\tau(t)}{dt^{\beta}}=G_L\gamma(t)+G_H\varphi_0 \frac{d^{\beta}\gamma(t)}{dt^{\beta}} \tag{1}$$

where $\tau(t)$ and $\gamma(t)$ are the stress and strain time histories, respectively, and φ_0, G_L, G_H *e* β are the four parameters to be experimentally determined. The fractional derivative model given by Eq. (1) describes the linear behavior of thermorheologically simple viscoelastic materials (Bagley & Torvik, 1986; Pritz, 1996). These materials present a complex modulus of elasticity, where the real part accounts for the storage of energy (spring effect) and the imaginary part for the dissipation of energy (damping effect).

In the frequency domain, the complex shear modulus is given by Lopes (1998):

$$\bar{G}(\Omega,T)=\frac{G_L+G_H\varphi_0\left[i\alpha_T(T)\Omega\right]^{\beta}}{1+\varphi_0\left[i\alpha_T(T)\Omega\right]^{\beta}} \tag{2}$$

The shift factor α_T is given by:

$$\log_{10}\alpha_T(T)=\frac{-\theta_1(T-T_0)}{\theta_2+(T-T_0)} \tag{3}$$

where Ω is the circular frequency [rad/s]; T is the absolute temperature [K]; T_0 is the reference temperature [K]; θ_1 and θ_2 are parameters experimentally determined. Once the shear modulus of a viscoelastic material is known, it is possible to determine the corresponding stiffness of any simple system made of this material. From now on, the temperature will be regarded as constant and, therefore, omitted in the shear modulus. In Fig. 1 it is shown a typical plot of a neoprene type viscoelastic material used in sound radiation and vibration control. This material was used in the present work. Material parameters were measured by the PISA-CNPq group in LVA-PISA , being both temperature and frequency dependent.

3. HYBRID ELECTROMECHANICAL-VISCOELASTIC DYNAMIC NEUTRALIZER (HEVDN) MODEL

To combine benefits of both viscoelastic and electromechanical dynamic vibration neutralizers, it is presented a new model of hybrid viscoelastic-electromechanical vibration neutralizer that can be used for vibration and sound radiation control. As shown in Fig. 2, this model is made of two resonant systems: one mechanical and one electromechanical. The former is made of a tuning mass and a viscoelastic material. The viscoelastic material holds together the tuning mass to the shell that is attached to the primary system. The shell also holds the magnet in which magnetic field lies the tuning mass. Around the tuning mass there is a coil that is linked to a resonant RLC electric circuit. Thus, when there is relative displacement between the coil around the tuning mass and the magnetic field, an electromotive force is generated in the electric circuit.

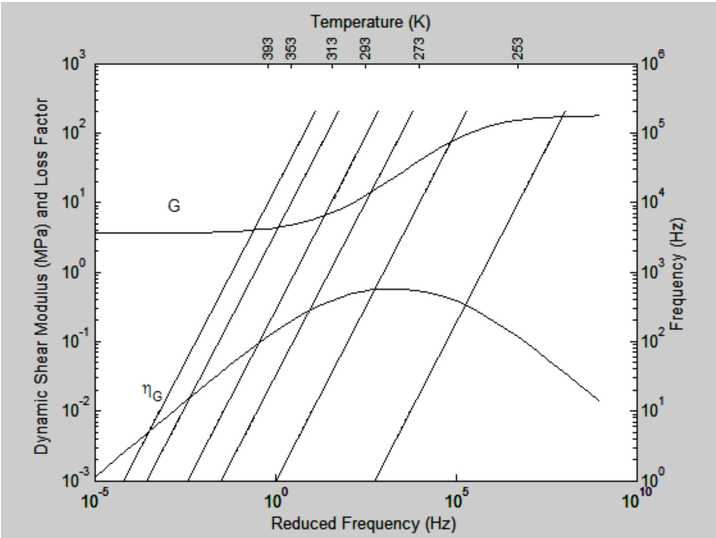

Figure 1. Monogram of the viscoelastic material used in this study:
$\varphi_{0},=2,46\ 10^{-3}$; $G_{L},= 3,57\ 10^{6}$ $G_H =1,79\ 10^{8}$e $\beta =0,435$

Conceptually, dynamic neutralizer's goal is to offer to the vibrating system high mechanical impedance in a certain frequency range, in which the system has low mechanical impedance. It is shown that, in this range, there are one or more natural frequencies to be controlled, and, for this reason, system mechanical impedance is low.

Often, when a dynamic neutralizer is designed to open-loop control, it can reduce amplitude vibration to acceptable levels. Depending on the operating region, the viscoelastic material can be highly temperature-frequency dependent. Thus, small temperature variation implies in big shear modulus variation, what can make the neutralizer natural frequency vary considerably. This situation can lead to control detuning and thus to a non-optimum performance.

Electromechanical dynamic neutralizers are resonant systems that can also be used in vibration passive control. They are weather variation independent and do not add mass to the primary system. Besides, they can be adaptively set, in case the characteristics of the primary system change. However, they need an auxiliary structure to fix the magnet and can have high power consumption.

The hybrid electromechanical-viscoelastic dynamic neutralizer does not need any auxiliary structure and is adaptable. Thus, if detuning caused by temperature variation increase vibration amplitude, the electrical circuit can be reset to retune the viscoelastic neutralizer. Therefore, the hybrid neutralizer can always work in an optimum way.

To design this device it is necessary to mathematically model its dynamical behavior.

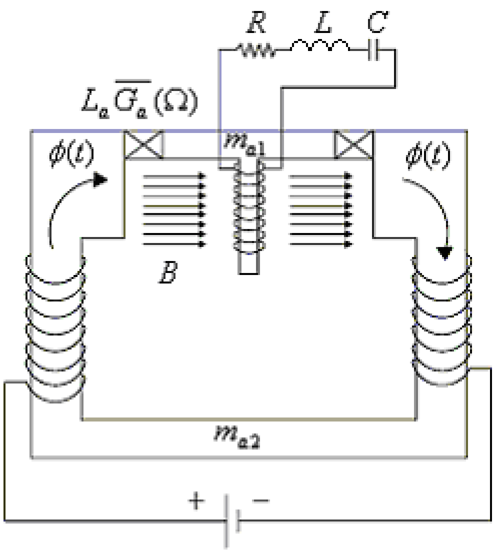

Figure 2. HEVDN physical configuration

3.1. Electromechanical model

Figure 3 shows the electromechanical resonant system. The system is made of a coil exposed to a magnetic field intensity *B*. A RLC circuit is attached to the coil. Applying Faraday's Law of Induction and Kirchhoff's Voltage Law it is possible to mathematically describe the system.

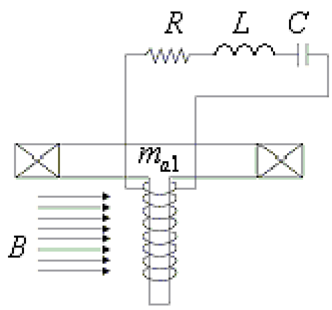

Figure 3. HEVDN electromechanical system

From Fig. 3 circuit, applying Kirchhoff ´s Voltage Law and considering *q(t)* the electrical charge it is shown that:

$$L\frac{d^2q(t)}{dt^2}+R\frac{dq(t)}{dt}+\frac{1}{C}q(t)=EMF \tag{4}$$

The Electromotive force – EMF is generated by magnetic flux and coil relative displacement. Structure vibration causes the HEVDN to move. The suspended mass m_{a1} moves in a relative way along with the magnet attached to the primary system because of the viscoelastic material, where $x(t)$ is the primary system displacement and $x_a(t)$ the tuning mass displacement.

$$\left|EMF\right|=\frac{d\phi(t)}{dt}=2\pi r_b nB\frac{dx(t)}{dt}=\Theta\frac{d(x(t)-x_a(t))}{dt} \tag{5}$$

$$\phi(t)=2\pi r_b nBx(t) \tag{6}$$

In which:

n	Coil number of turns,
r_b	Coil radius,
$n2\pi r_b$	Coil total length,
$\phi(t)$	Magnetic flux intensity.
$\Theta=n2\pi r_b B$	

Thus:

$$L\frac{d^2q(t)}{dt^2}+R\frac{dq(t)}{dt}+\frac{1}{C}q(t)=\Theta\frac{d(x(t)-x_a(t))}{dt} \tag{7}$$

Obtaining the derivative:

$$L\frac{d^3q(t)}{dt^3}+R\frac{d^2q(t)}{dt^2}+\frac{1}{C}\frac{dq(t)}{dt}=\Theta\frac{d^2(x(t)-x_a(t))}{dt^2} \tag{8}$$

As $i(t) = \dfrac{dq(t)}{dt}$ the differential equation becomes:

$$L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = \Theta\frac{d^2(x(t) - x_a(t))}{dt^2} \tag{9}$$

Applying Fourier Transform:

$$I(\Omega)\left[-L\Omega^2 + iR\Omega + 1/C\right] = -\Theta\Omega^2[X(\Omega) - X_a(\Omega)] \tag{10}$$

It is obtained:

$$I(\Omega) = \frac{-\Theta\Omega^2[X(\Omega) - X_a(\Omega)]}{(-L\Omega^2 + i\Omega R + 1/C)} \tag{11}$$

Equation (11) is the RLC circuit inducted current equation.

3.2. HEVDN Stiffness, Impedance and Dynamic Mass Calculus

To obtain the neutralizer's equation, the free-body diagram shown below is analyzed.

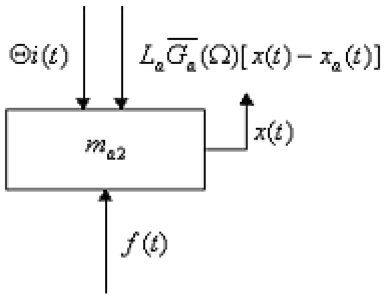

Figure 4. HEVDN free-body diagram for m_{a2}

From Fig. 4, applying Newton's Second Law, where $\Theta i(t)$ is the generated counter-electromotive force:

$$\sum F = m\ddot{x}$$
$$f(t) - L_a\overline{G_a}(\Omega)[x(t) - x_a(t)] - \Theta i(t) = m_{a2}\ddot{x}(t) \tag{12}$$

Equation (12) is valid only to harmonic excitation of frequency Ω. Applying the Fourier Transform to Eq. (12):

$$-\Omega^2 X(\Omega)m_{a2} + L_a\overline{G_a}(\Omega)\left[X(\Omega) - X_a(\Omega)\right] + \Theta I(\Omega) = F(\Omega) \tag{13}$$

and isolating $X_a(\Omega)$,

$$X_a(\Omega) = \frac{[-\Omega^2 m_{a2} + L_a\overline{G_a}(\Omega)]X(\Omega) - F(\Omega) + \Theta I(\Omega)}{L_a\overline{G_a}(\Omega)} \tag{14}$$

The free-body diagram for m_{a1} is analyzed in an analogous way. From Fig. 5, applying Newton's Second Law:

$$\Theta i(t) + L_a\overline{G_a}(\Omega)\left[x(t) - x_a(t)\right] = m_{a1}\ddot{x}_a(t) \tag{15}$$

Applying the Fourier Transform to Eq. (15):

$$\Theta I(\Omega) + L_a \overline{G_a}(\Omega) X(\Omega) = \left[-m_{a1}\Omega^2 + L_a \overline{G_a}(\Omega) \right] X_a(\Omega) \tag{16}$$

Substituting $Xa(\Omega)$ and $I(\Omega)$ in Eq. (16) the relation $X(\Omega)/F(\Omega)$ is obtained. In Eq. (17), A is a parameter that depends on the viscoelastic material and D a parameter that depends on the electrical circuit.

$$\frac{X(\Omega)}{F(\Omega)} = \frac{-\Omega^2 m_{a1} D + AD - \Theta^2\Omega^2}{\Omega^4 m_{a1} m_{a2} D - \Omega^2 AD(m_{a1} + m_{a2}) + \Theta^2\Omega^4 (m_{a1} + m_{a2})} \tag{17}$$

$$A = (L_a \overline{G_a}(\Omega))$$
$$D = (-\Omega^2 L + i\Omega R + 1/C)$$

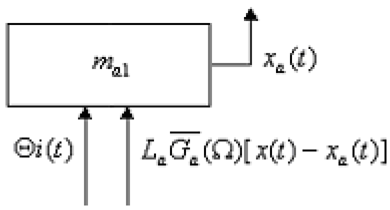

Figure 5. HEVDN free-body diagram for m_{a1}

From one degree of freedom the following functions, dynamic stiffness, mechanical impedance and dynamic mass are, respectively, obtained from Eq. (17):

$$K(\Omega) = \frac{F(\Omega)}{X(\Omega)} = \frac{\Omega^4 m_{a1} m_{a2} D - \Omega^2 AD(m_{a1} + m_{a2}) + \Theta^2\Omega^4 (m_{a1} + m_{a2})}{-\Omega^2 m_{a1} D + AD - \Theta^2\Omega^2} \tag{18}$$

$$Z(\Omega) = \frac{F(\Omega)}{i\Omega X(\Omega)} = \frac{\Omega^4 m_{a1} m_{a2} D - \Omega^2 AD(m_{a1} + m_{a2}) + \Theta^2\Omega^4 (m_{a1} + m_{a2})}{i\Omega(-\Omega^2 m_{a1} D + AD - \Theta^2\Omega^2)} \tag{19}$$

$$M(\Omega) = \frac{F(\Omega)}{-\Omega^2 X(\Omega)} = \frac{\Omega^4 m_{a1} m_{a2} D - \Omega^2 AD(m_{a1} + m_{a2}) + \Theta^2\Omega^4 (m_{a1} + m_{a2})}{-\Omega^2(-\Omega^2 m_{a1} D + AD - \Theta^2\Omega^2)} \tag{20}$$

3.3. HEVDN equivalent generalized quantities calculus

The equivalent generalized quantities are obtained from the system dynamic functions (Espíndola & Bavastri, 1999).

$$c_{eq}(\Omega) = \mathrm{Re}[Z(\Omega)] \tag{21}$$
$$m_{eq}(\Omega) = \mathrm{Re}[M(\Omega)] \tag{22}$$

Therefore, it is finally obtained an equivalent model to the HEVDN:

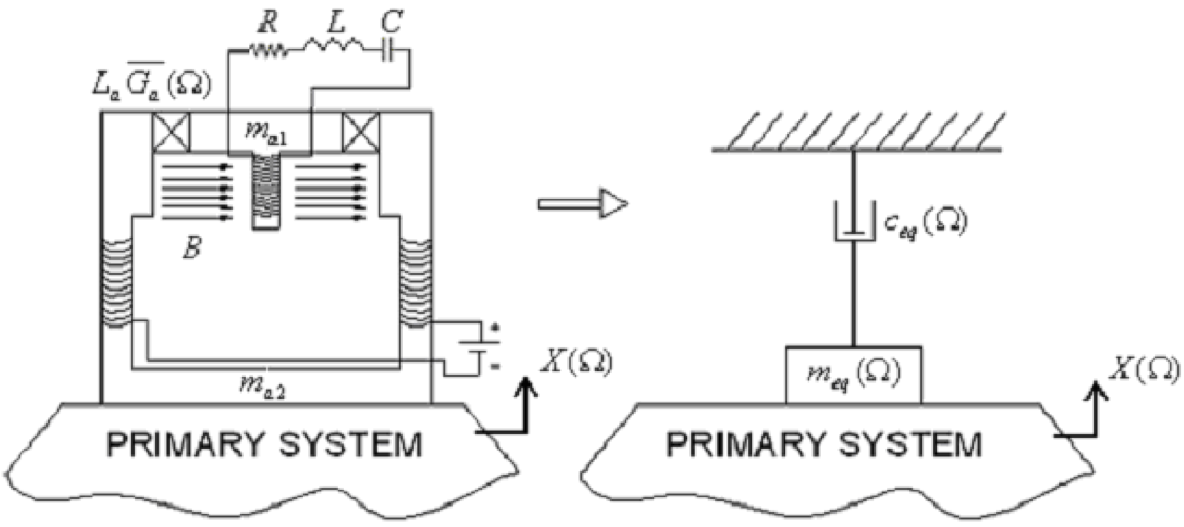

Figure 6. HEVDN equivalent generalized quantities and model equivalence

Now, it has been proved that both schemes shown in Fig. 4 are dynamically equivalent (Espíndola & Silva, 1992) in the sense that the dynamic stiffness "felt" by the primary system is the same in both cases.

The primary system "feels" the absorber as a $m_{eq}(\Omega)$ mass, frequency dependent, attached to it along a generalized coordinate $x(t)$ and a viscous dashpot (even if the damping is of viscoelastic type) of constant $c_{eq}(\Omega)$ (also frequency dependent) linked to a fixed reference. The dynamics of the resultant system (primary plus absorbers) can then be formulated in terms of the original physical generalized coordinates alone ($X(\Omega)$ in Fig. 6), although the new system has now additional degrees of freedom (one for each absorber). This is the main advantage of the concept of equivalent generalized quantities for the absorbers.

The motion equation for a compound system (primary system plus dynamic neutralizers) given by Fig. 6 is:

$$\left(m+m_{eq}\left(\Omega\right)\right)\ddot{x}\left(t\right)+\left(c+c_{eq}\left(\Omega\right)\right)\dot{x}\left(t\right)+k\,x\left(t\right)=f\left(t\right) \tag{23}$$

where $x(t)$ and $f(t)$ are the response and harmonic excitation with frequency Ω, respectively. The primary system is defined by the mass, m, damping, c, and stiffness, k. The dynamic neutralizer is defined by $m_{eq}(\Omega)$ and $c_{eq}(\Omega)$, generalized equivalent mass and damping, respectively. In the frequency domain, the Eq. (23) can be written

$$H\left(\Omega\right)=\frac{1}{-\Omega^2\left(m+m_{eq}\left(\Omega\right)\right)+i\Omega\left(c+c_{eq}\left(\Omega\right)\right)+k} \tag{24}$$

4. DYNAMIC NEUTRALIZER OPTIMUM DESIGN

The objective function, used to determine the optimum physical parameters of the neutralizer, is defined by:

$$f_{obj}\left(x\right)=R^n\rightarrow R \tag{25}$$

where

$$f_{obj}(x)=\max_{\Omega_1<\Omega<\Omega_2}\left|H(\Omega,x)\right| \tag{26}$$

and Ω_1 and Ω_2 are the lower and upper limits of the frequency range of concern. Therefore, the optimization problem is to minimize the objective function

$$min\quad f_{obj}\left(x\right) \tag{27}$$

subject to the following inequality constraints.

$$x_i^L < x_i < x_i^U \tag{28}$$

where $\boldsymbol{x}$ is the design vector, i is the i^{th} component, L is the lower constraint and U is the higher constraint.

For this methodology, the optimization procedure is divided in two parts. First, the optimization is made with the electric circuit turned off. Then, the optimum design of the viscoelastic dynamic neutralizer is found as Espíndola and Bavastri (1996 and 2001), Bavastri *et al.* (1998) and Espíndola *et al.* (2006). In this case, $m_{eq}(\Omega)$ and $c_{eq}(\Omega)$ are the same as Bavastri (1997).

After that, the viscoelastic dynamic neutralizer is exposed to temperature variation, which makes the neutralizer to detune and work in a non-optimum way. As result, the control system lacks in efficiency. Finally, to reset the optimum vibration control, the RLC electric circuit is turned on and, with the viscoelastic neutralizer physical parameters adapted to the new temperature, a non-linear optimization technique is used again to find the electric circuit optimum parameters to reduce the primary system vibration levels. In this way, the HEVDN part made with the viscoelastic material may be controlled by the electric circuit one.

5. NUMERICAL SIMULATION

For the HEVDN model, a numerical simulation over an one-degree-of-freedom cantilever beam primary system is presented. The frequency band of interest is a large one. The simulation goal is to reduce vibration amplitude in an optimal way. Table 1 show data for the cantilever beam and design specifications:

Table 1. Design specifications

Beam Length	0,5 m
Beam Width	0,1 m
Beam Height	0,005 m
Beam material Density	7850 kg/m^3
Beam material Elasticity Modulus	2E+11 N/m^2
Viscoelastic material	neoprene
Electric field intensity	4 T
Coil radius	0,025 m
Coil number of turns	1000
Magnet mass (m_{a2})	0,5 kg
Design temperature	25^{o}C
Detune temperatures	-5 and 50^{o}C

Figures 7 and 8 show simulation results. The primary system with HEVDN frequency response is shifted in relation to the primary system frequency response because of the magnet mass added to the system. It is shown the vibration amplitude reduction obtained by the designed HEVDN with the electrical circuit turned off. In this case, only the viscoelastic neutralizing effect is acting. Figures 7 and 8 also show detuning caused by a -30^{o}C and a +25^{o}C temperature variation, resulting in reduced performances, as well as the HEVDN performance with the electrical circuit turned on. The electrical circuit not only corrected detuning but improved vibration reduction by 4 dB (T=-5^oC detune) and 9 dB (T=50^oC detune), comparing to the initial designed specification. The optimum parameter values found for the electric circuit are $R=1k\Omega$; $L=130mH$; $C=210nF$ ($T=-5^oC$ detune) and $R=183k\Omega$; $L=459mH$; $C=2{,}66\mu F$ ($T=50^oC$ detune). For the first case, the resistor value reached lower restriction limit in optimization. In most simulation cases, inductance seems to play a non-important role in optimization. For both simulation cases, inductance values of 1mH and 1H do not cause significant change in frequency response. This means that, for $T=-5^oC$ detune temperature, the electric circuit is almost completely capacitive, adding only stiffness to the primary system.

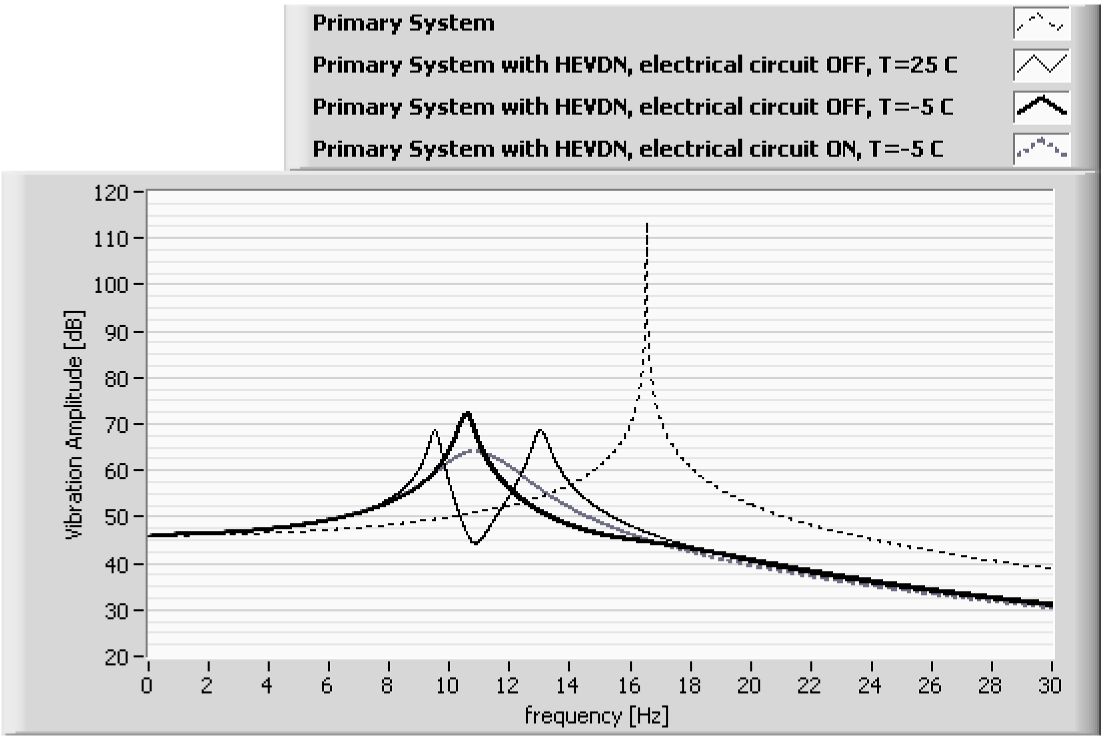

Figure 7. Frequency Response Functions

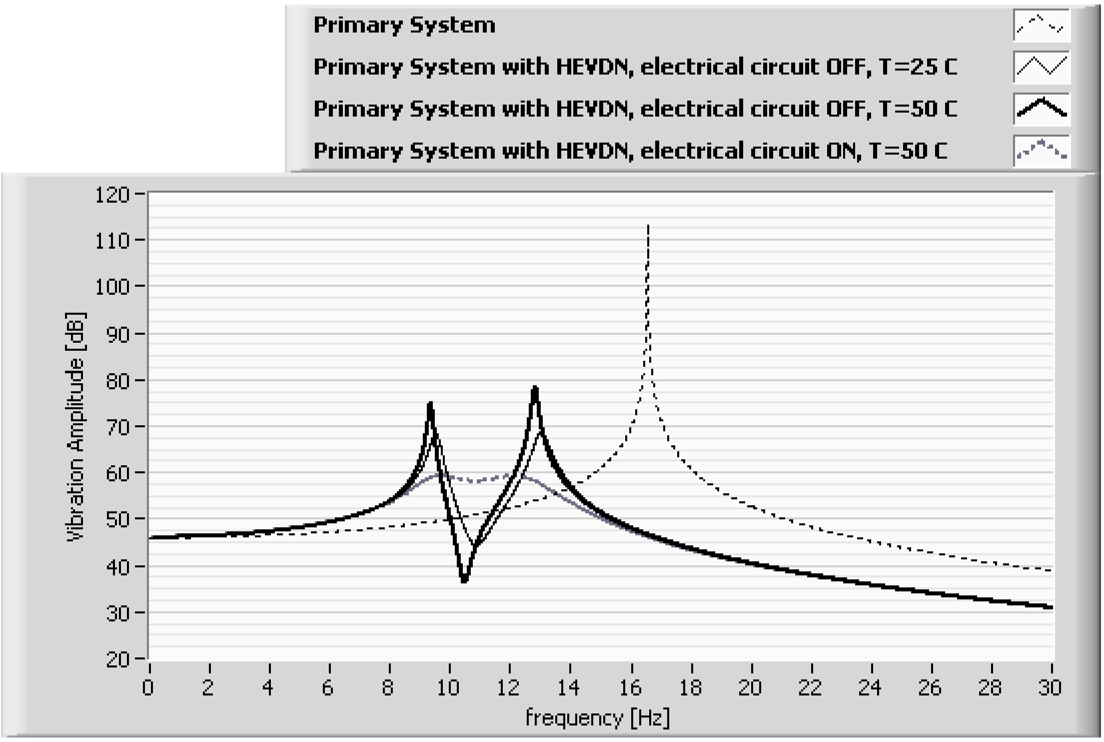

Figure 8. Frequency Response Functions

6. CONCLUSIONS

It was presented a new dynamic neutralizer model, made of a mechanical part (with viscoelastic material) and an electromechanical one (AC generator principle). This new model take benefits of both viscoelastic and electromechanical neutralizers, has minimum power consumption and do not need auxiliary support structures. In this new conception, the electromechanical neutralizer acts controlling the viscoelastic neutralizer, not the primary system.

Equivalent generalized quantities concept was exposed and used to model the HEVDN. It allows expressing whole system dynamics using only the primary system coordinates.

To demonstrate HEVDN performance in vibration control, an optimum device design methodology was introduced. It includes the HEVDN optimization with the electric circuit turned off. An intentional detune caused by temperature variation is simulated and a second optimization, now upon the electric circuit parameters is made to reduce vibration levels in a wide frequency range. The results show that the model can even improve the viscoelastic dynamic neutralizer performance for detuning temperatures, reducing amplitude vibration in at least 4 dB, comparing to the design temperature performance. It was also demonstrated with this model that vibration reduction optimization occurred for low and high detuning temperatures. The entire simulation proves the HEVDN model versatility. This study results show that it is possible to build a dynamic neutralizer able to work at different work regimes, in an active control configuration, not only to retune the viscoelastic neutralizer but also to improve its performance in many situations, including primary system structural characteristics time variation. Future studies include a wide range analysis of HEVDN behavior with variation of electric circuit parameters, detune temperatures, viscoelastic material characteristics and primary system natural frequency.

The simulation results shown that it may be possible to use only a *RC* electric circuit to control the viscoelastic dynamic neutralizer.

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