# OVERALL OPTIMIZATION OF FINNED ARRANGEMENTS IN TURBULENT FORCED CONVECTION 

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Abstract. This work presents the continuity of the studies of an experimental geometric optimization to maximize the total heat transfer rate between a bundle of finned tubes in a given volume and a given external flow both for circular and elliptic arrangements, for general staggered configurations. The optimization procedure started by establishing a fixed volume constraint to account for the design limited space availability. The experimental results were obtained for circular and elliptic configurations with a fixed number of twelve tubes, starting with an equilateral triangle configuration, which fitted uniformly into the fixed volume. A number of experimental configurations were built by reducing the tube-to-tube spacings, identifying the optimal spacing for maximum heat transfer. Similarly, it was possible to investigate the existence of optima with respect to other two geometric degrees of freedom, i.e., tube eccentricity and fin-to-fin spacing. The results are reported for air as the external fluid, in the range $2650 \leq R e_{2 b} \leq 10600$, where $2 b$ is the smaller ellipse axis. Circular and elliptic arrangements with the same air input velocity and flow obstruction cross-sectional area were compared on the basis of maximum total heat transfer. For low values of the free stream velocity, pressure drops are expected to be nearly equivalent with such criterion, but for higher velocities in the turbulent regime pressure drops are expected to vary with cross section shape, therefore the minimization of pumping power was not within the scope of the present study. Experimental optimization results for finned circular and elliptic tubes arrangements are presented. A relative heat transfer gain of up to $23 \%$ $\left(\operatorname{Re}_{2 b}=10600\right)$ is observed in the elliptic arrangement optimized with respect to tube-to-tube spacings, as compared to the optimal circular one. Such findings motivated the search for optima with respect to two additional degrees of freedom, i. e., eccentricity and fin-to-fin spacing. It is proposed a correlation for estimating the 3-way maximized total overall thermal conductance expected for any arrangement of the types studied in this paper.

Keyword: Heat Transfer, Pressure Drop, Optimization of Project Parameters.

## 1. INTRODUCTION

With the human societies in development, all the needs are increased. Food, water, shelter, energy are common necessities whatever the people or country throughout the world. Focusing in energy, its availability requires continuous investments by the governments to satisfy both industry and consumers needs. Besides generating more power and researching new sources of energy, many efforts have been directed to save energy through optimization of its use, mainly in industrial processes. Finned cross-flow heat exchangers are part of numerous engineering processes in industry and are unquestionably responsible for a large share of the total energy consumption wherever they are present.

The optimization of flow-system architecture is a widespread occurrence in engineering and nature. Many examples have been brought together under the title of constructal theory (Bejan, 2000), which is the thought that geometry (flow architecture) is generated by the pursuit of global performance subject to global constraints, in flow systems the geometry of which is free to change. According to constructal theory, the optimization of flow architecture starts at the smallest (elemental) scale, i.e., in this study, the shape of the heat exchanger flow channel. In principle, this procedure can be extended on a hierarchical ladder to larger and more complex systems, to explore multi-scale packings that use the available volume to the maximum.

Finned cross-flow heat exchangers are part of numerous engineering processes in industry and are unquestionably responsible for a large share of the total energy consumption wherever they are present in Bordalo and Saboya (1999), Saboya and Saboya (2001), Rosman et al. (1984), Khan et al. (2004), Elshazly et al. (2005), Elsayed et al. (2003), Min and Webb (2004), Kundu et al. (2006), O'Brien and Sohal (2005), O'Brien et al. (2004), Gao et al. (2003) and Kim et al. (1999).

In this work, the geometric optimization of design parameters for maximum heat transfer is pursued experimentally. The objective is to provide scientific information for the possible utilization of elliptical tubes instead of circular ones in the heat exchangers of practical applications and industrial processes (e.g., air conditioning and refrigeration, HVAC-R, systems, heaters, radiators) in the future. Therefore, it is necessary the investigation to be conducted for the turbulent
flow regime. The basic idea is to analyze the heat transfer gain using elliptic tubes heat exchangers as compared to the traditional circular ones when varying the following design parameters: $\delta=$ fin-to-fin spacing; $\mathrm{e}=$ ellipses eccentricity $(e=b / a)$, and $S=$ spacing between rows of tubes. Hence, the problem consists of identifying a configuration (internal architecture, shape) that provides maximum heat transfer for a given space (Bejan, 2000).

The main focus of the present study is on the experimental geometric optimization of staggered finned circular and elliptic tubes in a fixed volume. The paper describes a series of experiments conducted in the laboratory in the search for optimal geometric parameters in general staggered finned circular and elliptic configurations for maximum heat transfer in turbulent flow. Circular and elliptic arrangements, with the same flow obstruction cross-sectional area, are then compared on the basis of maximum total heat transfer and total mass of manufacturing material. Appropriate nondimensional groups are defined and the optimization results reported in dimensionless charts.

## 2. THEORY

A typical four-row tube and plate fin heat exchanger with a general staggered configuration is shown in Fig. 1. Fowler and Bejan (1994) showed that in the laminar regime, the flow through a large bank of cylinders could be simulated accurately by calculating the flow through a single channel, such as that illustrated by the unit cell seen in Fig. 1. Because of the geometric symmetries, there is no fluid exchange or heat transfer between adjacent channels, or at the top and side surfaces. At the bottom of each unit cell, no heat transfer is expected across the plate fin midplane. In Fig. 1, L, H and W are the length, height and width (tube length) of the array, respectively. The fins are identical, where $\mathrm{t}_{\mathrm{f}}$ is the thickness and $\delta$, is the fin-to-fin spacing.


Figure 1. General configuration of the arrangement of finned elliptic tubes

Dimensionless variables have been defined based on appropriate physical scales as follows:

$$
\begin{align*}
& (X, Y, Z)=\frac{(x, y, z)}{L} ; P=\frac{p}{\rho u_{\infty}^{2}}  \tag{1}\\
& (U, V, W)=\frac{(u, v, w)}{u_{\infty}} ; \theta=\frac{T-T_{\infty}}{T_{w}-T_{\infty}} ; \operatorname{Re}_{2 b}=\frac{u_{\infty}(2 b)}{v} \tag{2}
\end{align*}
$$

where ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) are the Cartesian coordinates, $\mathrm{m} ; \mathrm{p}$ the pressure, $\mathrm{Nm}^{-2} ; \rho$ the fluid density, $\mathrm{kg} \mathrm{m}^{-3} ; \mathrm{u}_{\infty}$ the free stream velocity, $\mathrm{m} \mathrm{s}^{-1} ;(\mathrm{u}, \mathrm{v}, \mathrm{w})$ the fluid velocities, $\mathrm{m} \mathrm{s}^{-1} ; \mathrm{T}$ the temperature, $\mathrm{K} ; \mathrm{T}_{\infty}$ the free stream temperature, $\mathrm{K} ; \mathrm{T}_{\mathrm{w}}$ the tubes surface temperature, K ; L the array length in the flow direction, $\mathrm{m}, \mathrm{H}$ the array height, $\mathrm{m}, \mathrm{W}$ the array width, m , and $v$. the fluid kinematic viscosity, $\mathrm{m}^{2} \mathrm{~s}^{-1}$. The objective is to find the optimal geometry, such that the volumetric heat transfer density is maximized, subject to a volume constraint. The engineering design problem starts by rec ognizing the finite availability of space, i.e., an available space $\mathrm{L} \times \mathrm{H} \times \mathrm{W}$ as a given volume that is to be filled with a heat exchanger. To maximize the volumetric heat transfer density means that the overall heat transfer rate between the fluid inside the tubes and the fluid outside the tubes will be maximized.

The dimensionless overall thermal conductance $\tilde{q}$, or volumetric heat transfer density is defined as follows (Matos et al., 2001, Matos et al., 2004a and Matos et al., 2004b):

$$
\begin{equation*}
\widetilde{\mathrm{q}}=\frac{\mathrm{Q} /\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\infty}\right)}{\mathrm{kLHW} /(2 \mathrm{~b})^{2}} \tag{3}
\end{equation*}
$$

where the overall heat transfer rate between the finned tubes and the free stream, Q , has been divided by the constrained volume, LHW; k is the fluid thermal conductivity, $\mathrm{W} \mathrm{m}^{-1} \mathrm{~K}^{-1}$, and $2 \mathrm{~b}=\mathrm{D}$ the ellipse smaller axis or tube diameter.

A balance of energy in one elemental channel states that:

$$
\begin{equation*}
\mathrm{Q}=\mathrm{N}_{\mathrm{ec}} \mathrm{Q}_{\mathrm{ec}}=\mathrm{N}_{\mathrm{ec}} \dot{\mathrm{~m}}_{\mathrm{ec}} \mathrm{c}_{\mathrm{p}}\left(\overline{\mathrm{~T}}_{\mathrm{out}}-\mathrm{T}_{\infty}\right) \tag{4}
\end{equation*}
$$

where $\mathrm{N}_{\mathrm{ec}}$ is the number of elemental channels. The elemental channel is defined as the sum of all unit cells in direction z . Therefore, the mass flow rate $\left(\mathrm{kg} \mathrm{s}^{-1}\right)$ entering one elemental channel is

$$
\begin{equation*}
\dot{\mathrm{m}}_{\mathrm{ec}}=\rho \mathrm{u}_{\infty}[(\mathrm{S}+2 \mathrm{~b}) / 2]\left(\mathrm{W}-\mathrm{n}_{\mathrm{f}} \mathrm{t}_{\mathrm{f}}\right) \tag{5}
\end{equation*}
$$

and $t_{f}$ is the fin thickness, $m, c_{p}$ is the fluid specific heat at constant pressure $\left[\mathrm{J} \mathrm{kg}^{-1} \mathrm{~K}^{-1}\right]$, and $\bar{T}_{\text {out }}$ is the average fluid temperature at the elemental channel outlet (K).

The number of fins in the arrangement is given by:

$$
\begin{equation*}
\mathrm{n}_{\mathrm{f}}=\frac{\mathrm{W}}{\mathrm{t}_{\mathrm{f}}+\delta} \tag{6}
\end{equation*}
$$

The dimensionless overall thermal conductance is rewritten utilizing Eqs. (3)-(6) as follows:

$$
\begin{equation*}
\widetilde{\mathrm{q}}=\frac{\mathrm{N}_{\mathrm{ec}}}{2} \operatorname{Pr} \operatorname{Re}_{2 \mathrm{~b}}\left[\frac{2 \mathrm{~b}}{\mathrm{~L}}\right]^{2} \frac{2 \mathrm{~b}}{\mathrm{H}}\left(\frac{\mathrm{~S}}{2 \mathrm{~b}}+1\right)\left(1-\phi_{\mathrm{f}}\right) \bar{\theta}_{\mathrm{out}} \tag{7}
\end{equation*}
$$

where $\phi_{\mathrm{f}}=\frac{\mathrm{n}_{\mathrm{f}} \mathrm{t}_{\mathrm{f}}}{\mathrm{W}}=\frac{\mathrm{t}_{\mathrm{f}}}{\mathrm{t}_{\mathrm{f}}+\delta}$, is the dimensionless fin density in direction $\mathrm{z}\left(0 \leq \mathrm{n}_{\mathrm{f}} \mathrm{t}_{\mathrm{f}} \leq \mathrm{W}\right)$, and $\operatorname{Pr}$ the fluid Prandtl number, $v / \alpha$.

For the sake of generalizing the results for all configurations of the type studied in this work, the dimensionless overall thermal conductance is alternatively defined as follows:

$$
\begin{equation*}
\widetilde{\mathrm{q}}_{*}=\frac{2}{\mathrm{~N}_{\mathrm{ec}}}\left[\frac{\mathrm{~L}}{2 \mathrm{~b}}\right]^{2} \frac{\mathrm{H}}{2 \mathrm{~b}} \widetilde{\mathrm{q}}=\operatorname{Pr}^{\operatorname{Re}} \mathrm{R}_{2 \mathrm{~b}}\left(\frac{\mathrm{~S}}{2 \mathrm{~b}}+1\right)\left(1-\phi_{\mathrm{f}}\right) \bar{\theta}_{\text {out }} \tag{8}
\end{equation*}
$$

The volume fraction occupied by solid material in the arrangement is given by

$$
\begin{equation*}
\tilde{V}=\frac{W}{L^{3}}\left[n_{t} \pi\left(a b-\left(a-t_{t}\right)\left(b-t_{t}\right)\right)+\phi_{f}\left(L H-n_{t} \pi a b\right)\right] \tag{9}
\end{equation*}
$$

where $t_{t}$ is the thickness of the tube wall, $m$, and $n_{t}$ is the total number tubes of the arrangement.

## 3. EXPERIMENTS

The same experimental rig that was utilized in previous studies for the laminar regime (Matos et al., 2001, Matos et al., 2004a and Matos et al., 2004b) was re-utilized in the laboratory to produce the necessary experimental data to perform the experimental optimization of finned arrangements. Figure 2 shows the experimental apparatus utilized in this study. The forced air flow was induced by suction with an axial electric fan, with a nominal power of 1 HP , and was capable of providing air free stream velocities, $u_{\infty}$, up to $20 \mathrm{~ms}^{-1}$.


Figure 2. Experimental apparatus
The objective of the experimental work was to evaluate the volumetric heat transfer density (or overall thermal conductance) of each tested arrangement by computing $\widetilde{\mathfrak{q}}_{*}$ with Eq. (8) through direct measurements of $u_{\infty}\left(\operatorname{Re}_{2 b}\right)$, and $\overline{\mathrm{T}}_{\text {out }}, \overline{\mathrm{T}}_{\mathrm{w}}$ and $\mathrm{T}_{\infty}\left(\bar{\theta}_{\text {out }}\right)$. The volume fraction occupied by solid material in the arrangement, $\widetilde{\mathrm{V}}$, was also evaluated according to Eq. (9), in order to compare the resulting total volume of solid material of the elliptic and circular arrangements.

Five runs were conducted for each experiment. Steady-state conditions were reached after 3 hours in all the experiments. The precision limit for each temperature point was computed as two times the standard deviation of the 5 runs (Editorial, 1992). It was verified that the precision limits of all variables involved in the calculation of $\tilde{\mathrm{q}}_{*}$ were negligible in comparison to the precision limit of $\bar{\theta}_{\text {out }}$, therefore $P_{\widetilde{q}_{*}} \cong P_{\bar{\theta}_{\text {out }}}$. The thermistors, anemometer, properties,
and lengths bias limits were found negligible in comparison with the precision limit of $\widetilde{\mathfrak{q}}_{*}$. As a result, the uncertainty of $\widetilde{\mathrm{q}}_{*}$ was calculated by:

$$
\begin{equation*}
\frac{\mathrm{U}_{\widetilde{\mathrm{q}}_{*}}}{\widetilde{\mathrm{q}}_{*}}=\left[\left(\frac{\mathrm{P}_{\widetilde{\mathrm{q}}_{*}}}{\widetilde{\mathrm{q}}_{*}}\right)^{2}+\left(\frac{\mathrm{B}_{\widetilde{\mathrm{q}}_{*}}}{\widetilde{\mathrm{q}}_{*}}\right)^{2}\right]^{1 / 2} \cong \frac{\mathrm{P}_{\bar{\theta}_{\text {out }}}}{\bar{\theta}_{\text {out }}} \tag{10}
\end{equation*}
$$

where $P_{\bar{\theta}_{\text {out }}}$ is the precision limit of $\bar{\theta}_{\text {out }}$.
The tested arrangements had a total of twelve tubes placed inside the fixed volume $\mathrm{L} \times \mathrm{H} \times \mathrm{W}$, with four tubes in each unit cell (four rows). For a particular tube and plate fin geometry, the tests started with an equilateral triangle configuration, which filled uniformly the fixed volume, with a resulting maximum dimensionless tube-to-tube spacing $\mathrm{S} / 2 \mathrm{~b}=1.5$. The spacing between tubes was then progressively reduced, i.e., $\mathrm{S} / 2 \mathrm{~b}=1.5,0.5,0.25$ and 0.1 , and in this interval an optimal spacing was found such that $\widetilde{\mathrm{q}}_{*}$ was maximum. All the tested arrangements had the aspect ratio $\mathrm{L} / 2 \mathrm{~b}=8.52$.

Several free stream velocities set points were tested, such that $u_{\infty}=2.5,5.0,7.5$ e $10.0 \mathrm{~ms}^{-1}$, corresponding to $\mathrm{Re}_{2 \mathrm{~b}}=2650,5300,7950$, and 10600 , respectively, which covered a significant portion of the air velocity range of interest for typical air conditioning applications, i.e., $1.8 \mathrm{~ms}^{-1} \leq \mathrm{u}_{\infty} \leq 18.2 \mathrm{~ms}^{-1}$, Bordalo and Saboya (1999). For those values of $\operatorname{Re}_{2 b}$, the turbulent flow regime is observed. The largest uncertainty calculated according to Eq. (10) in all tests was $\mathrm{U}_{\tilde{\mathrm{q}}_{*}} / \widetilde{\mathrm{q}}_{*}=0.075$.

## 3. RESULTS AND DISCUSSION

For each tested Reynolds number, $\operatorname{Re}_{2 b}$, the 3 -way optimization procedure was performed according to the following steps: i) for a given eccentricity, the dimensionless overall thermal conductance, $\widetilde{\mathrm{q}}_{*}$, was computed with Eq.
(8), for the range of tube-to-tube spacings $0.1 \leq \mathrm{S} / 2 \mathrm{~b} \leq 1.5$; ii) the same procedure was repeated for several eccentricities, i.e., $\mathrm{e}=0.4,0.5,0.6$ and 1 , and iii) steps i) and ii) were repeated for different fin-to-fin spacings configurations, i.e., $\phi_{\mathrm{f}}=0.006,0.094$, and 0.26 .

This study presents experimental optimization results for a higher range of Reynolds numbers than in previous optimization studies for finned elliptic tubes arrays (Matos et al., 2004a, Matos et al., 2004b and Matos et al., 2006), i.e., for $\mathrm{Re}_{2 \mathrm{~b}}=2650,5300,7950$ and 10600, therefore investigating the turbulent flow regime. The optima obtained in the experiments are sharp, stressing their importance in actual engineering design. The optimal tube-to-tube spacings found experimentally for $\mathrm{Re}_{2 \mathrm{~b}}=2650,5300,7950$ and 10600 , were in the range $0.5 \leq(\mathrm{S} / 2 \mathrm{~b})_{\text {opt }} \leq 0.6$, for $0.5 \leq \mathrm{e} \leq 1$.

The first step of the 3-way optimization procedure is documented by Matos et al. (2006), which show the experimental optimization of the tube-to-tube spacing and the tube eccentricity.

The continuity of the studies show the experimental optimization of the fin-to-fin spacings. Figure 3 illustrates the existence of a local optimal fin-to-fin spacing, $\phi_{\mathrm{f}}$, for $(\mathrm{S} / 2 \mathrm{~b})_{\text {opt }}=0.5$ and $\mathrm{e}=1$ (circular tubes). In this way, it is possible to investigate the effect of the variation of fin-to-fin spacing in isolation, on the heat transfer rate of the traditional circular arrangement. In all the experimental results shown so far, it was observed that as $\operatorname{Re}_{2 b}$ increases $\widetilde{\mathrm{q}}_{*, \mathrm{~m}}$ increases, with sharper maxima occurring at higher $\operatorname{Re}_{2 \mathrm{~b}}$.

Figure 4 reports the results of the 3-way global optimization with respect to the three degrees of freedom, $\mathrm{S} / 2 \mathrm{~b}$, e and $\phi_{f}$, obtained after performing the three steps of the optimization procedure. An optimal set of geometric parameters was determined experimentally such that $\widetilde{\mathrm{q}}_{*}$ was maximized three times, i.e.: $\left(\mathrm{S} / 2 \mathrm{~b}, \mathrm{e}, \phi_{\mathrm{f}}\right)_{\text {opt }} \cong(0.5,0.6$, 0.094), where the 3-way maximized dimensionless overall thermal conductance reads as $\tilde{\mathrm{q}}_{*, \mathrm{mmm}}$ at $\phi_{\mathrm{f}, \mathrm{opt}}$ for each tested $\mathrm{Re}_{2 \mathrm{~b}}$, in Fig. 4. A closer inspection of the results presented in Fig. 4 shows that the 3-way optimized internal configuration is "robust" with respect to the variation of the Reynolds number for the entire tested range. Therefore, it is proposed a correlation for estimating the 3-way maximized total overall thermal conductance expected for any arrangement of the types studied in this paper for $2650 \leq \operatorname{Re}_{2 \mathrm{~b}} \leq 10600$, as follows:

$$
\begin{equation*}
\widetilde{\mathrm{q}}_{*, \mathrm{mmm}}=1299.5+0.47003 \mathrm{Re}_{2 \mathrm{~b}}+0.000034064 \mathrm{Re}_{2 \mathrm{~b}}^{2} \quad, \quad \mathrm{R}=0.99053 \tag{11}
\end{equation*}
$$

where R is the statistics correlation coefficient.
Figure 5 shows with square symbols, the experimentally determined points for $\tilde{\mathrm{q}}_{*, \mathrm{mmm}}$ calculated with Eq. (8), and curve plotted with the correlation proposed by Eq. (11). The $\widetilde{\mathrm{q}}_{*, \mathrm{mmm}}$ trend with respect to the variation of $\operatorname{Re}_{2 \mathrm{~b}}$ is well approximated.

From all experimental results obtained in this study, it is important to stress that a heat transfer gain of up to $23 \%$ was observed in the 3 -way optimized elliptic arrangement of Fig. 4, as compared to the 2 -way optimized circular one (i.e., with respect to tube-to-tube and fin-to-fin spacings only, as shown in Fig. 3). This observation was made for the highest Reynolds number tested in the experiments, $\operatorname{Re}_{2 \mathrm{~b}}=10600$, which corresponded to an air free stream velocity $u_{\infty}=10 \mathrm{~ms}^{-1}$ in the experimental set-up.

Although the comparison of required pumping power between the elliptic and circular arrangements was not the objective of the present study, pressure drops were measured in all experimental runs. A pressure drop reduction of approximately $20 \%$ was observed in the 3-way optimized elliptic arrangement in comparison with the 2-way optimized circular one (i.e., with respect to tube-to-tube and fin-to-fin spacings only) for the highest Reynolds number tested in this study, i.e., $\operatorname{Re}_{2 b}=10600$. The measurements are consistent with previously reported pressure drop results for similar elliptic arrangements, Bordalo and Saboya (1999).

Finally, Figure 6 shows the volume fraction of solid material computed with Eq. (9) for the tested arrangements. The objective was to evaluate how the same flow obstruction cross-sectional area comparison criterion between elliptic and circular arrangements adopted in this study, affected total volume of solid material. It is observed that when the dimensionless fin density is small (small number of fins), the volume fraction of solid material, $\tilde{\mathrm{V}}$, increases as eccentricity decreases (from 0.033 at $\mathrm{e}=1$ to 0.053 at $\mathrm{e}=0.4$, for $\phi_{\mathrm{f}}=0.006$ ). However, such trend is inverted as the number of fins (or dimensionless fin density) increases. For example, the volume fraction $\widetilde{\mathrm{V}} \cong 0.104$ for $\mathrm{e}=0.5,0.6$ and 1 , for $\phi_{\mathrm{f}}=0.094$, and $\widetilde{\mathrm{V}}=0.215,0.222$ and 0.238 for $\mathrm{e}=0.5,0.6$ and 1 , respectively, for $\phi_{\mathrm{f}}=0.26$, as it is shown by Fig. 6. So, for the 3-way optimized elliptic configuration, with $\phi_{f, o p t}=0.094$, the volume fraction of solid material of the elliptic arrangement is the same as the circular one. Therefore, the same amount of material is required for manufacturing both the 3-way optimized elliptic arrangement and the circular one with the same dimensionless fin density.


Figure 3. Two-way optimization of finned circular arrangements with respect to tube-to-tube and fin-to-fin spacing.


Figure 4. three-way optimization of finned arrangements with respect to tube-to-tube spacing, eccentricity and fin-to-fin spacing.


Figure 5. The three-way maximized dimensionless heat transfer rate with respect to $\mathrm{Re}_{2 \mathrm{~b}}$.


Figure 6. The total solid volume fraction of the arrangements with respect to eccentricity and fin-to-fin spacing.

## 5. CONCLUSIONS

A theoretical and experimental study was presented in this paper for staggered finned circular and elliptic tubes heat exchangers to demonstrate that optimal design configurations can be found so that maximum heat transfer is observed, for a given fixed volume. Several experimental arrangements were built in the laboratory and many test runs were conducted in a wind tunnel in turbulent forced convection. The internal geometric structure of the arrangements was optimized for maximum heat transfer. Better global performance is achieved when flow and heat transfer resistances are minimized together, i.e., when the imperfection is distributed optimally in the available space. Optimal distribution of imperfection represents flow architecture, or constructal design (Bejan, 2000).

Appropriate dimensionless groups were identified to report the experimental results to allow for the general application to heat exchangers of the type treated in this study. A comparison criterion was adopted as in previous studies, Bordalo and Saboya (1999), Saboya and Saboya (2001), Rosman et al. (1984), Matos et al. (2004a), Matos et al. (2004b), i.e., establishing the same air input velocity and flow obstruction cross-sectional for the circular and elliptic arrangements, to compare the arrangements on the basis of maximum heat transfer in the most isolated way possible. Pressure drops for all arrangements were measured and the results agree with previously published results (Bordalo and Saboya, 1999). The arrangements were also compared in terms of total volume of solid material required for them to be manufactured.

The key conclusions of this study are listed as follows:

1. An optimal set of geometric parameters was determined experimentally such that $\tilde{\mathrm{q}}_{*}$ was maximized three times, i.e.: $\left(\mathrm{S} / 2 \mathrm{~b}, \mathrm{e}, \phi_{\mathrm{f}}\right)_{\text {opt }} \cong(0.5,0.6,0.094)$, where the 3-way maximized dimensionless overall thermal conductance is achieved;
2. The 3-way optimized elliptic arrangement exhibits a heat transfer gain of up to $23 \%$ relative to the optimal circular tube arrangement;
3. A compact analytical correlation was proposed to estimate the actual 3-way maximized overall thermal conductance in the design of elliptic tubes heat exchangers of the type studied in this paper;
4. For the 3-way optimized elliptic configuration, with $\phi_{\mathrm{f}, \mathrm{opt}}=0.094$, the volume fraction of solid material of the elliptic arrangement is the same as the circular one, and
5. The heat transfer gain, pressure drop reduction, and a similar amount of material to manufacture both arrangements show that the elliptic tubes optimized arrangement has the potential to deliver significantly higher global performance than the circular arrangement, with a similar investment cost.

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