ATTITUDE AND VIBRATION CONTROL OF A SATELLITE WITH A FLEXIBLE SOLAR PANEL USING LQR TRACKING WITH INFINITE TIME

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Abstract. *The objective of this work is to investigate the control of the attitude angles and the suppression of the vibration of the solar panel of a satellite in circular orbit around Earth .It is considered only the maneuver of the the satellite around its center of mass. The mathematical model for the complete system is derived using the lagrangean formalism. The control law considered is the LQR with infinite time (algebraic Ricatti equation) and the states are controlled in order to stay as close as possible of a specific set of reference states (tracking problem). The optimal gains are obtained considering only the linear part of the governing equations. The gains obtained in this way are then used to control the complete nonlinear model. Since LQR is a linear controller (and besides its robustness), one does not consider here great velocities and deflections which can transform the nonlinear mathematical model into a strongly nonlinear one.*

Keywords: *attitude control, vibration control, flexible structures, LQR, satellites*

1. INTRODUCTION

Communications satellites require high pointing accuracies for the antennas so that they may provide the desired coverage on the earth's surface. This requirement is achieved through an attitude control system which maintains the spacecraft, its orientation in space, within the allowable limits (Agraval, 1986). Numerical simulations of the attitude determination problem were developed by (Moro, 1983). The satellites, however, might have appendices (like a solar array) whose dynamics may affect the attitude behavior.

Solar panels are largely used in space applications and are important structures in systems like satellites and even in large space structures like the International Space Station (Wertz, 2002). The vibration control of these structures are a critical issue since it can excite other parts of the main structure. For example, the vibration of a solar panel can break down the attitude control of a spacecraft, conducing all the mission to a failure. Many and different methods can be used for this control problem. Neural nets, for instance, were investigated for satellite attitude control by (Carrara, 1997).

The objective of this work is to investigate the control of the attitude angles of a satellite and, at the same time, suppress the vibration of a beam-like flexible solar panel during some regular maneuver. The mathematical model of the system will be derived using the lagrangean formalism. Linear curvature is considered for the flexible beam. However, the interaction between the rigid and the flexible variables makes the complete system nonlinear. The control law adopted here is the Linear Quadratic Regulator (LQR) considering infinite time (Athans and Falb, 2007; Kirk, 2004; Stengel, 1994). No noise is considered here. No measurement and no real data are used in this work either. This paper is just a first approach to the problem. For the numerical simulations one uses the C language and the Matlab environment.

LQR is a linear controller but present some robustness to weak nonlinear perturbations in the linear model to be controlled (Fenili and Arantes, 2006). In this paper is not considered great angular velocities and neither large beam deflections which can transform the nonlinear mathematical model under investigation into a strongly nonlinear one. In this last case, the LQR control is prone to fail.

2. GOVERNING EQUATIONS OF MOTION

2.1. Geometric Model

The analysis developed here is planar. The satellite main body is geometrically modeled as a rigid body in the format of a box (only plane XY is considered here) whose sides measure one meter each, as illustrated in Fig. 1. At this box is clamped a flexible beam-like solar panel with 0.005 m x 0.15 m cross section and 1.2 m long. This beam is modeled as a Euler-Bernoulli beam and linear curvature is considered here.

The reference frame XY in Fig. 1 represents the inertial frame. The frames x_1y_1 and x_2y_2 are local reference frames. The inertial frame has its origin in the center of mass of the main body of the satellite.

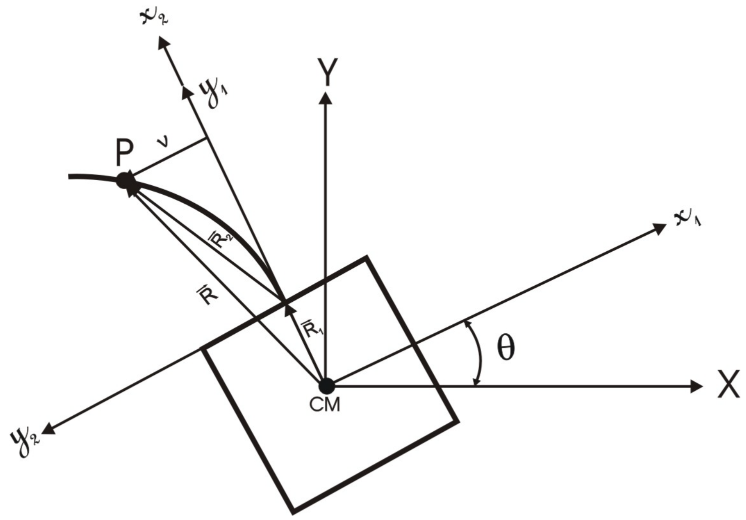

Figure 1. Geometric model

2.2. Mathematical model

The mathematical model for the main body and for the flexible solar panel clamped to it is obtained through the lagrangean formulation.

The beam deflection variable, $v(x,t)$, is discretized using the expansion:

$$v(x,t)=\sum_{i=1}^{n}\Phi_i(x)q_i(t) \tag{1}$$

The admissible functions $\Phi_i(x)$ are given by (Craig, 1981):

$$\Phi_i(x)=\cosh(a_ix)-\cos(a_ix)-\alpha_i(\mathrm{senh}(a_ix)-\mathrm{sen}(a_ix)) \tag{2}$$

where:

$$\alpha_i=\frac{\cosh(a_iL)+\cos(a_iL)}{\mathrm{senh}(a_iL)+\mathrm{sen}(a_iL)} \tag{3}$$

Considering the property of ortogonalization of the modes of vibration of the beam, it can be assumed that if $i=j$ one has:

$$\int_0^L\Phi_i(x)\Phi_j(x)dx=1 \tag{4}$$

and if $i\neq j$ one has:

$$\int_0^L\Phi_i(x)\Phi_j(x)dx=0 \tag{5}$$

This property will be used after the energies (kinetic and potential) of the system are defined. From here to the end of this work one will write $q_i(t)$ and $\Phi_i(x)$ simply q_i and Φ_i and $v(x,t)$ simply as v.

The Lagrange's equations associated to the problem investigated here are given by (Meirovitch, 1998):

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\frac{\partial L}{\partial \theta}=T_\theta \tag{6}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0 \tag{7}$$

where T_θ is the torque on the satellite responsible for attitude correction (coming, for example, from a momentum wheel) and L =T–V is the lagrangean. From here one considers i = 1 (one mode expansion in Eq. (1)).

For the complete system, the total kinetic energy T is given by $T_{satellite} + T_{panel}$ or

$$T=\frac{1}{2}I_{cube}\dot{\theta}^2+\frac{1}{2}\int_0^L \rho A\left|\dot{\vec{R}}\right|^2 dx \tag{8}$$

where

$$\vec{R}=\vec{R}_1+\vec{R}_2=-\left(R_1 \mathrm{sen}\theta+x\,\mathrm{sen}\theta+v\cos\theta\right)\vec{i}+\left(R_1\cos\theta+x\cos\theta-v\,\mathrm{sen}\theta\right)\vec{j} \tag{9}$$

Expanding *v* in Eq. (8) (coming from Eq. (9)) according to Eq. (1), using Eq. (2) and the ortogonalization property presented in Eqs. (4) and (5) and finally solving the integrals which do not depend on the admissible functions one obtains:

$$T=\frac{1}{2}I_{cube}\dot{\theta}^2+C_2\dot{\theta}^2L+C_5\dot{\theta}^2\frac{L^2}{2}+C_5\dot{\theta}\dot{q}_1\alpha_1+C_1\dot{\theta}^2\frac{L^3}{3}+C_4\dot{\theta}\dot{q}_1\beta_1+C_1\dot{\theta}^2q_1^2+C_1\dot{q}_1^2 \tag{10}$$

The potential energy stored in this system is of elastic type (no gravitational effects are considered) and is given by

$$V=\frac{1}{2}\int_0^L EI_{beam}v''^2dx \tag{11}$$

The same expansion as given in Eq. (1) is used in Eq. (11). Making to the potential energy the same operations executed before for the kinetic energy, the discretized lagrangean of the system is given by:

$$L=\frac{1}{2}I_{cube}\dot{\theta}^2+C_2\dot{\theta}^2L+C_5\dot{\theta}^2\frac{L^2}{2}+C_5\dot{\theta}\dot{q}_1\alpha_1+C_1\dot{\theta}^2\frac{L^3}{3}+C_4\dot{\theta}\dot{q}_1\beta_1+C_1\dot{\theta}^2q_1^2+C_1\dot{q}_1^2-C_1q_1^2\omega_1^2 \tag{12}$$

where:

$$C_1=\frac{1}{2}\rho A$$
$$C_2=\frac{1}{2}\rho A R_1^2$$
$$C_3=\frac{1}{2}\rho A R_1$$
$$C_4=\rho A$$
$$C_5=\rho A R_1$$
$$C_6=I_{cube}+2C_2L+C_5L^2+2C_1\frac{L^3}{3}$$

Substituting Eq. (12) in Eqs. (6) and (7) results the governing equations of motion given by:

$$C_6\ddot{\theta}+\left(C_5\alpha_1+C_4\beta_1\right)\ddot{q}_1+4C_1\dot{\theta}q_1\dot{q}_1=0 \quad (13)$$

$$C_5\ddot{\theta}\alpha_1+C_4\ddot{\theta}\beta_1+C_1 2\ddot{q}_1-2C_1\dot{\theta}^2 q_1+2C_1 q_1\omega_1^2=0 \quad (14)$$

In Eqs. (13) and (14), the variable θ represents the angular displacement of the main body of the satellite, the variable q_1 represents the time behavior or the deflection of the beam-like solar panel, I_{cube} represents the moment of inertia of the main body of the satellite about the inertia frame located at its center of mass (CM) and I_{beam} represents the moment of inertia of beam cross section about the neutral line.

2.3. State space form

The governing equations of motion given by Eqs. (13) and (14) are now written in state space form. The state vector is defined as:

$$x=\left\{x_1,x_2,x_3,x_4\right\}^T \quad (15)$$

where $x_1=\theta$, $x_2=\dot{\theta}$, $x_3=q_1$ and $x_4=\dot{q}_1$.

Using the vector defined in Eq. (15), the governing equations of motion in state space form are written as:

$$\dot{x}_1=x_2 \quad (16a)$$

$$\left(C_6+2C_1x_3^2\right)\dot{x}_2+\left(C_5\alpha+C_4\beta\right)\dot{x}_4=T_\theta-4C_1x_2x_3x_4 \quad (16b)$$

$$\dot{x}_3=x_4 \quad (17a)$$

$$\left(C_5\alpha+C_4\beta\right)\dot{x}_2+C_1 2\dot{x}_4=2C_1x_2^2x_3-2C_1x_3\omega^2 \quad (17b)$$

Equations (16) and (17) can be written in matrix form as:

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & C_6+2C_1x_3^2 & 0 & C_5\alpha+C_4\beta\\ 0 & 0 & 1 & 0\\ 0 & C_5\alpha+C_4\beta & 0 & C_1 2\end{bmatrix}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}+\begin{bmatrix}-x_2\\ 4C_1x_2x_3x_4\\ -x_4\\ -2C_1x_2^2x_3+2C_1x_3\omega^2\end{bmatrix}=\begin{bmatrix}0\\ T_\theta\\ 0\\ 0\end{bmatrix} \quad (18)$$

Multiplying Eq. (18) on the left by the inverse of the first matrix in this same equation, one will have:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}x_2\\ \dfrac{-8C_1^2x_2x_3x_4+\left(C_5\alpha+C_4\beta\right)\left(-2C_1x_2^2x_3+2C_1x_3\omega^2\right)}{2C_1C_6+4C_1^2x_3^2-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2}\\ x_4\\ \dfrac{4\left(C_5\alpha+C_4\beta\right)C_1x_2x_3x_4-\left(C_6+2C_1x_3^2\right)\left(-2C_1x_2^2x_3+2C_1x_3\omega^2\right)}{2C_1C_6+4C_1^2x_3^2-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2}\end{bmatrix}+\begin{bmatrix}0 & 0\\ \dfrac{2C_1}{2C_1C_6+4C_1^2x_3^2-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0\\ 0 & 0\\ \dfrac{-\left(C_5\alpha+C_4\beta\right)}{2C_1C_6+4C_1^2x_3^2-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0\end{bmatrix}\begin{bmatrix}T_\theta\\ 0\end{bmatrix} \quad (19)$$

Considering small deflections for the beam and small velocities, Eq. (19) assumes the form (Fenili and Arantes, 2006):

$$\left\{\dot{x}\right\}=\left[A\right]\left\{x\right\}+\left[B\right]\left\{u\right\} \quad (20)$$

where

$$[A]=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{(C_5\alpha+C_4\beta)2C_1\omega^2}{2C_1C_6-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{-C_6 2C_1\omega^2}{2C_1C_6-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0 \end{bmatrix} \tag{21}$$

$$[B]=\begin{bmatrix} 0 & 0 \\ \dfrac{2C_1}{2C_1C_6-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0 \\ 0 & 0 \\ \dfrac{-(C_5\alpha+C_4\beta)}{2C_1C_6-C_5^2\alpha^2-2C_5\alpha C_4\beta-C_4^2\beta^2} & 0 \end{bmatrix} \tag{22}$$

$$\{u\}=\begin{bmatrix} T \\ 0 \end{bmatrix} \tag{23}$$

3. THE LQR CONTROL LAW (STEADY STATE APPROACH)

The Linear Quadratic Regulator (LQR) is a controller widely used to control linear systems and some (weak) nonlinear system because of its reliability and robustness properties (Fenili and Arantes, 2006). Of course there is a limit for using this method to control nonlinear systems since the gains (for finite or infinite time) are obtained considering that the mathematical model for the dynamical system under investigation is linear. In the case presented here, and due to the nature of the system treated here, this limitation occurs in the angular velocity of the satellite. For high angular velocities, the LQR law can not control the system anymore. Of course the satellite does not develop such velocities in attitude correction, what is the same to say that the system analyzed here can be controlled in all possible real situations. The nonlinearities are present but act like small perturbations.

The LQR strategy is based on defining a cost function which must be minimized (Stengel R.F., 1994,). By minimizing this function one obtains a matrix of optimal gains to be used for feedback.

The LQR tracking control makes the system follow (or track) a desired trajectory over the entire time interval using a closed loop control law (Lewis, 1986). The cost function for tracking control is given by:

$$J(u)=\frac{1}{2}\int_{t_o}^{T}\left[\tilde{x}^T Q\tilde{x}+u^T P u\right]dt \tag{24}$$

where

$$\tilde{x}(t)=x(t)-x_d(t) \tag{25}$$

$x_d(t)$ is the vector of reference states, $x(t)$ is the vector of system states and **Q** and **P** are positive semi-definite weighting matrices. In this work, one considers steady state control (or control for infinite time) and the gain matrix is constant.

The optimum linear control law that minimizes the cost function J is given by (Stengel R.F., 1994,):

$$\{u\}=-[R]^{-1}[B]^T\left([K]\{x\}+\{s\}\right) \tag{26}$$

where K is a symmetric positive definite matrix that satisfies the matrix algebraic Ricatti equation given by

$$[A]^T[K]+[K][A]+[Q]-[K][B][R]^{-1}[B]^T[K]=0 \tag{27}$$

and s is the solution of

$$\left([A]^T-[K][B][R]^{-1}[B]^T\right)\{s\}-[Q]x_d=0 \quad (28)$$

were matrices A and B are the matrices (21) and (22).

4. NUMERICAL SIMULATIONS

In the numerical simulations presented here one considers that all the states are at disposal for feedback. In practical applications, it is very rare that you can measure all the states (the ones associated to the elastic modes of the solar panel, for instance) and one needs an observer to estimate the unmeasured states from the measurements.

A fourth order Runge-Kutta with time step of 0.001 s is used to the numerical integration of the governing equations of motion. The values considered for the physical parameters in the numerical simulations are presented in Tab. 1.

Table 1. Physical parameters

Parameter	Description	Value	Unit (S.I.)
ρ	Aluminum density	2700	kg/m^3
E	Young´s modulus	$0,7\cdot10^{11}$	N/m^2
a_1	Eigenvalue associated to the beam first mode of vibration	$1,878/L$	-
I_{beam}	Moment of inertia of the beam cross section about the neutral axis	$1,5625\cdot10^{-9}$	m^4
I_{cube}	Satellite’s main body moment of inertia about its center of mass	166	$kg\cdot m^2$
ω	Beam first mode of vibration	18,0001	rad/s

The weighting matrices considered here are:

$$[Q]=\begin{bmatrix}1000 & 0 & 0 & 0\\ 0 & 1000 & 0 & 0\\ 0 & 0 & 100 & 0\\ 0 & 0 & 0 & 100\end{bmatrix} \quad (29)$$

and

$$[R]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (30)$$

Two situations are considered in the numerical simulations. The first situation comprises case 1 and case 2. The second situation comprises case 3. In cases 1 and 2 are simulated typical situations of correction of attitude angles in a satellite. Usually the errors in pointing are not greater than three degrees in most satellites. The numerical simulation related to case 3 involves a great angular displacement and can be associated, for example, with some regular maneuver the satellite must realize in order to follow the sun. The basic idea here is to investigate the performance of the LQR controller when used to control weak nonlinear systems.

The kind of control represented by cases 1 and 2 can be related in real applications to the control of momentum wheels for attitude correction. As seen in Tab. 2, different errors are considered for θ (initial and reference states). The control situation represented by case 3 can be related to the control of propellants used to change the satellite position in space. The variation of θ is higher in this case.

Table 2. Conditions for numerical integration

State	Description	Initial State	Reference State	Unit (S.I.)
x_1: case 1	Variable θ	2.0	0.0	degrees
x_1: case 2	Variable θ	3.0	0.0	degrees
x_1: case 3	Variable θ	15.0	2.0	degrees
x_2, x_3, x_4: cases 1, 2 and 3	Variables $\dot{\theta}$, q and $\dot{q}$	0.0	0.0	rad/s, m and m/s

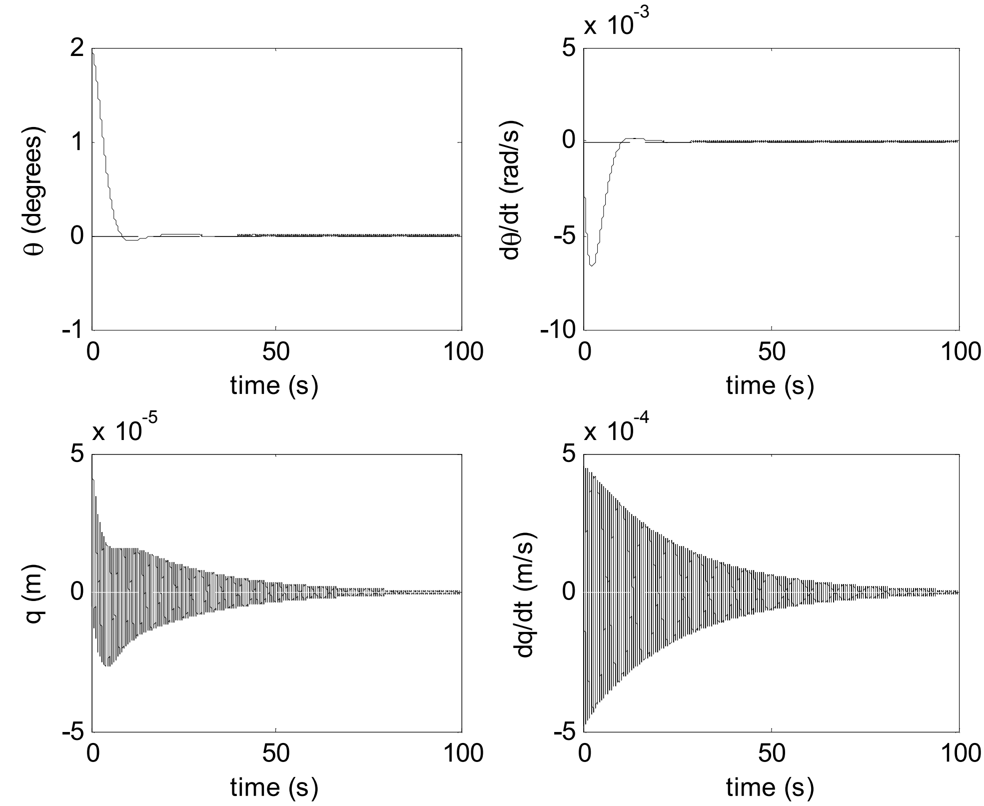

Figure 1. Results for case 1.

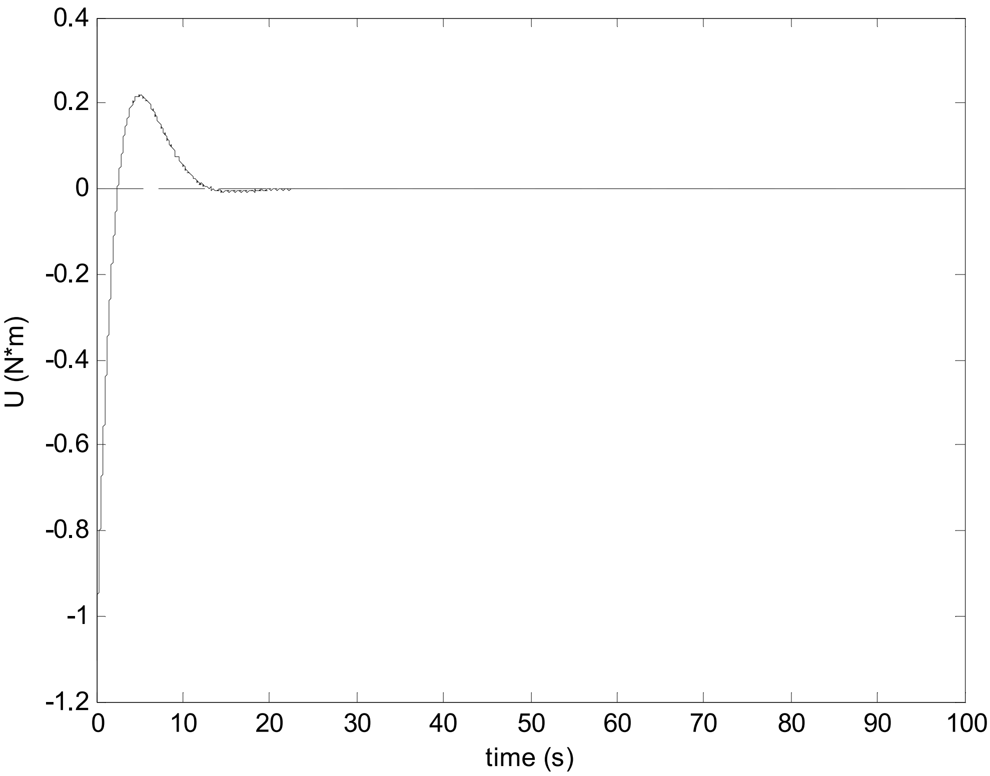

Figure 2. Control torque for case 1.

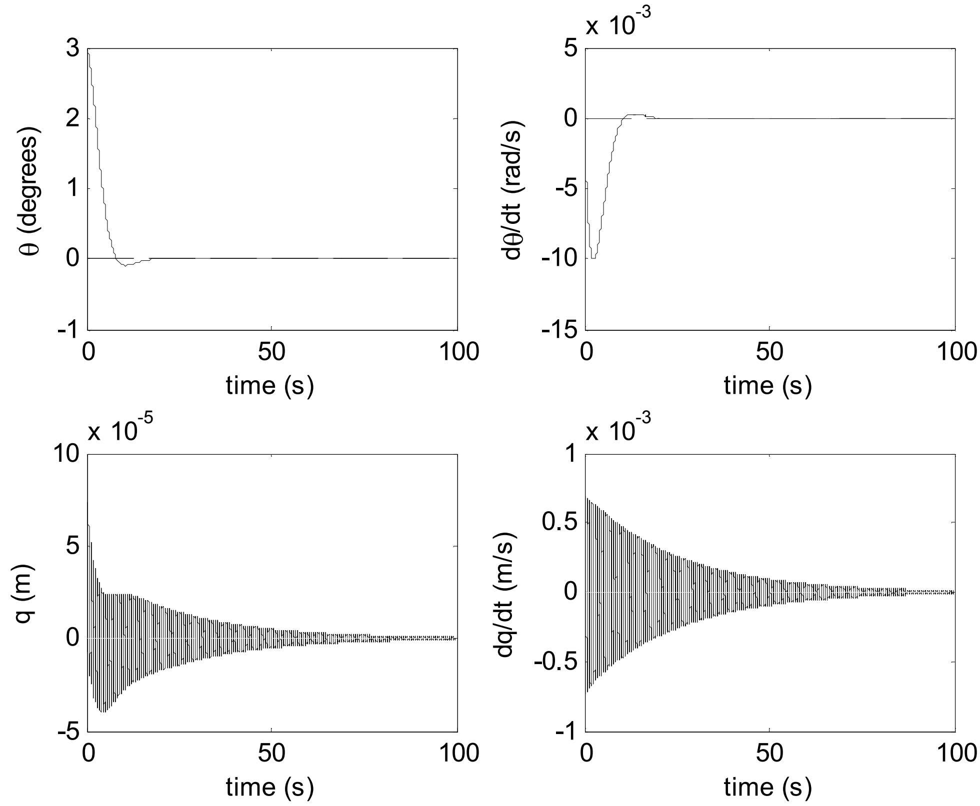

Figure 3. Results for case 2.

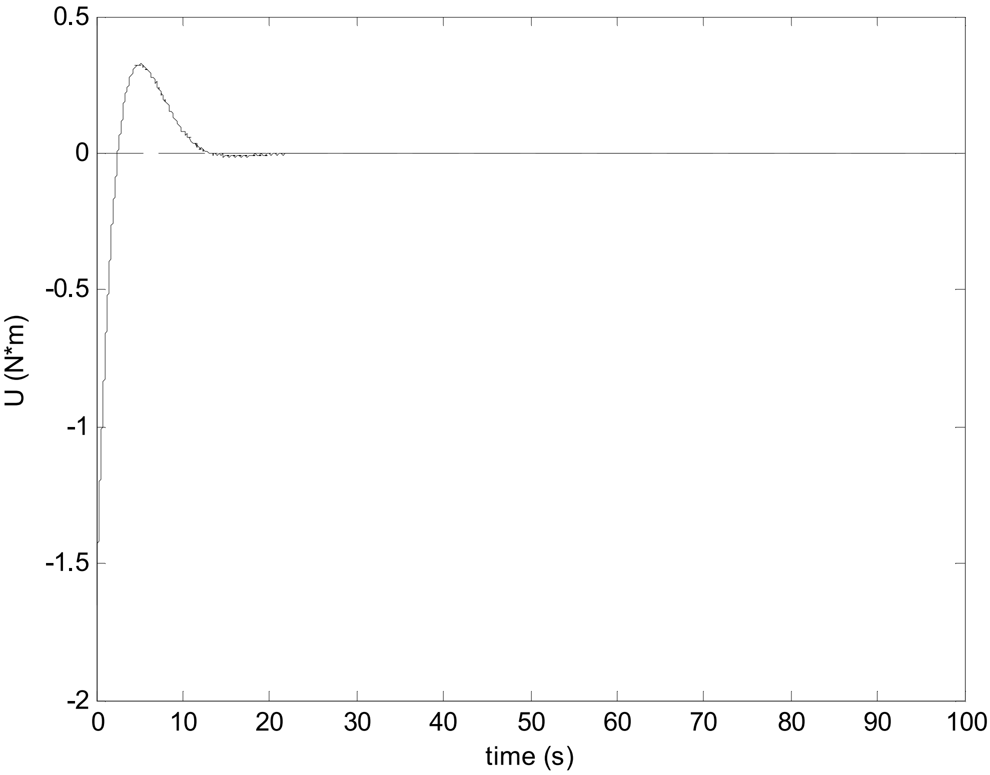

Figure 4. Control torque for case 2.

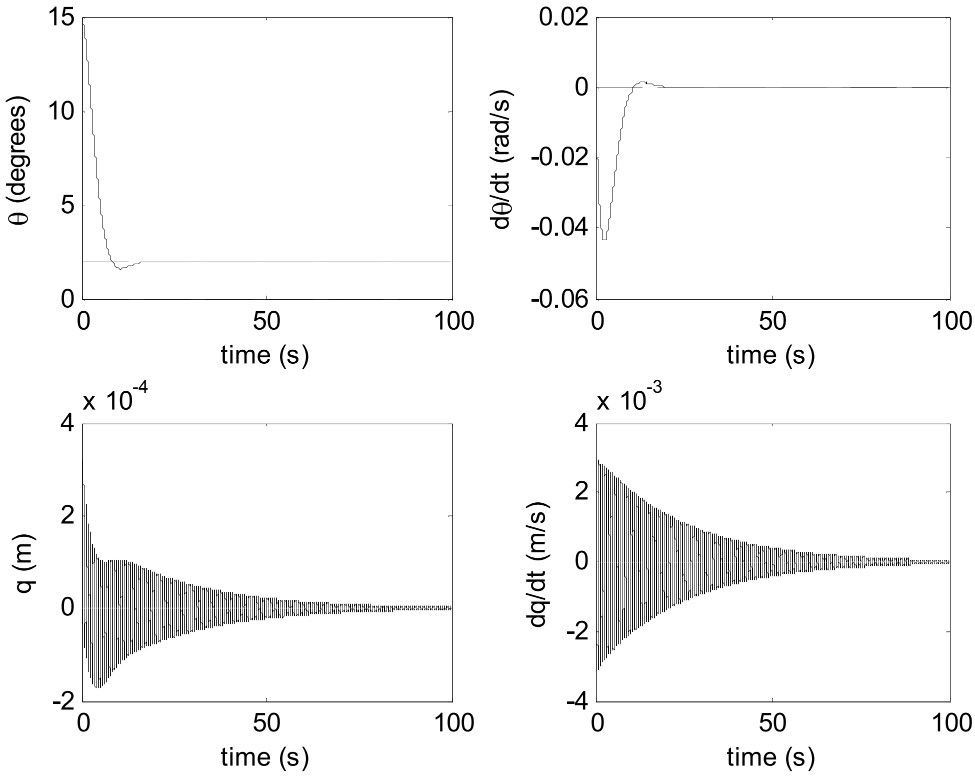

Figure 5. Results for case 3.

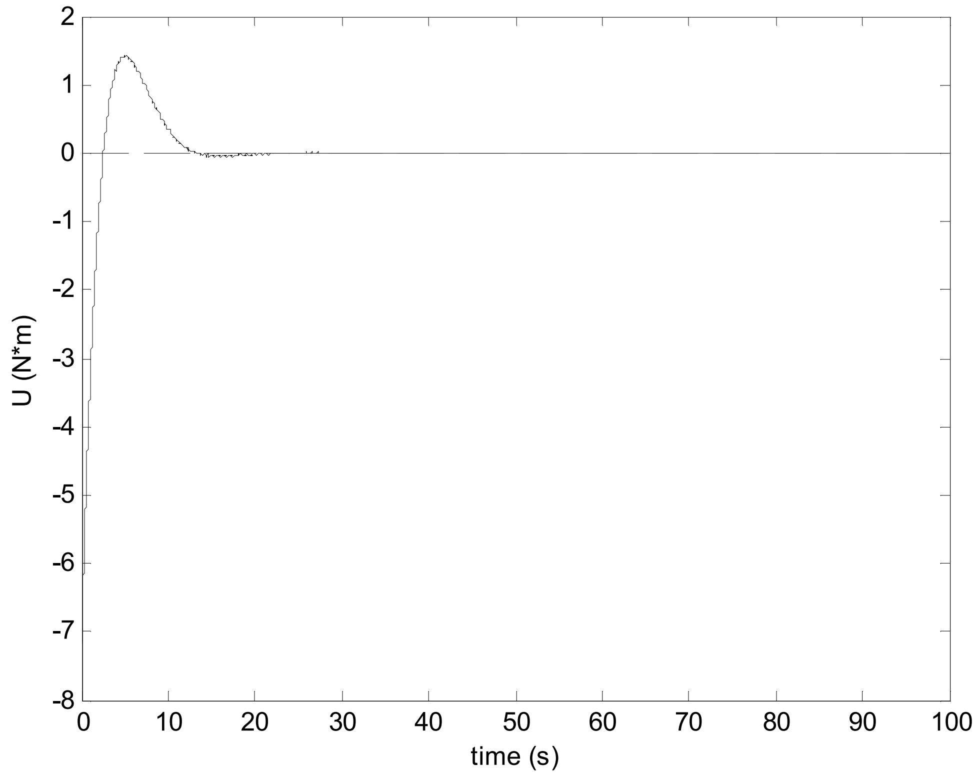

Figure 6. Control torque for case 3.

Figures 1 to 6 show satisfactory results. The actuator in not directly acting upon the beam (solar panel) and, for this reason, its vibration takes some time to fade. However, it can be verified in Figs. 1, 3 and 5 that its amplitude and velocity of deflection of the beam are converging to its desired states (zero deflection and zero velocity). In Figs. 2, 4 and 6 the control torques are converging to zero too.

The amplitude of the deflection of the beam, q_1, is small enough (in all the three cases) for the linear curvature assumption to be true.

5. CONCLUSIONS

The governing equations obtained in this work are nonlinear. The nonlinearities are weakly excited and the resulting weakly nonlinear governing equations are satisfactory controlled. To work with the LQR technique one needs a linear governing equation of motion. In this sense, small velocities and deflections are considered and the nonlinear governing equations are in this way linearized. The equilibrium states into which the control effort try to bring the system to are those related to the condition where all the velocities and panel deflections are equal to zero.

In the numerical simulations, two different situations were considered. In the first situation (cases 1 and 2) the deviation from the desired position is small and, therefore, for the time considered, sufficiently small angular velocities are developed. In this case, the final states are all zero. In the second situation (case 3), the angular deviation is greater (greater angular velocities involved) and the final state is not zero, but some desired angle different from zero. In both cases the desired final states are reached successfully.

Besides the fact that the control was designed not considering the nonlinearities, all the nonlinearities are considered in the numerical simulations and the LQR control is sufficiently robust to deal with them.

6. ACKNOWLEDGEMENTS

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