COMPARISON OF OPTIMIZATION METHODS APPLIED TO THE STEEL RISER PROBLEM

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Abstract. *The design of a riser system for offshore oil exploitation in deep water is both very challenging and time consuming, due to the several environmental conditions each riser is submitted to. To find a configuration that fullfills the design requirements under each environmental condition, many attempts are often required and in each one, many simulations are needed. The final result is also very dependent on the experience of the designer. To overcome such difficulties, helping the designer in obtaining a good solution in short time, one possible approach is the use of optimization algorithms, which try to find the configuration that best meets a certain objective, like minimum stress amplitude or minimum cost, subject to user defined restrictions such as top angle range, minimum tension and so on. This paper presents a comparison of the results obtained applying some of the most commom optimization algorithms to the problem of a catenary or lazy-wave riser subject to usual design criteria. The goal is to identify which algorithms are the most appropriate and in which situations each one should be used, by comparing both the configurations obtained and the time taken.*

Keywords*: optimization, risers, genetic algorithm, simulated annealing, sequential linear programming*

1. INTRODUCTION

The design of a riser system for offshore oil exploitation in deep waters is both very challenging and time consuming, due to the several environmental conditions each riser is submitted to. To find a configuration that fulfills the design requirements under each environmental condition, many attempts are often required and in each one, many simulations are needed and the final result is also very dependent on the experience of the designer. To overcome such difficulties, helping the designer in obtaining a good solution in short time, one possible approach is the use of optimization algorithms, which try to find the configuration that best meets a certain objective, like minimum stress amplitude or minimum cost, subject to user defined restrictions such as top angle range, minimum tension and so on.

The field of optimization has produced, over the years, a great collection of methods that are applicable to different classes of problems. Of these, some have become more popular, due to many reasons, like effectiveness, ease of implementation and speed. In this paper, three methods chosen due to their successful application to several problems will be compared on a specific problem, the optimization of a catenary or lazy-wave riser subject to usual design criteria. The goal is to identify which algorithms are the most appropriate and in which situations each one should be used, by comparing the configurations obtained, the objective function values and the time taken in some selected cases.

In the following sections, optimization basics as well as the studied methods will be briefly exposed. Then, the models used for static and dynamic analyses will be described, and, lastly, the case study will be presented, with results and conclusions.

2. OPTIMIZATION

Optimization is a study field concerned with finding algorithms that can solve automatically minimization or maximization problems. In the last decades, several different algorithms have been developed for such purposes, with varying degrees of success. As it is very difficult to solve a generic problem, these algorithms are usually tailored to solve specific types of problems.

The existing algorithms can be divided in two major categories, the mathematical and the probabilistic algorithms. Mathematical algorithms use information like the derivatives of the objective function to try to find its minimum. The probabilistic algorithms try to overcome this problem and find the global minimum of the problem, by using random generation of candidate configurations.

The mathematical algorithms tend to use a small number of objective function evaluations, but they are usually only capable of finding local minima, while the probabilistic algorithms are often (but not always) capable of finding global minima and don't need information about the specific problem, like gradients. However, they tend to take longer to run than mathematical methods, and their results are not always so much better as to justify their use.

Both categories of algorithms have been successfully used in several types of problems, and the aim of this paper is to compare the performance of some widely used algorithms in the specific case of a steel riser.

2.1. Objective Function

The objective function is the criterion by which the configurations will be compared. Most previous works applying optimization to the synthesis of riser configurations (Tanaka and Martins, 2006; Rodrigues, 2004; Larsen and Hanson, 1999) focused on the cost of the riser.

In this work a different approach is used, using the maximum stress amplitude of the dynamic problem solution as objective function to be minimized, as was the case in Tanaka and Martins, 2007. The idea is that minimizing such stress, fatigue life will be maximized. Even though only extreme dynamic cases are actually simulated, reducing stress amplitude in such cases will likely reduce these amplitudes in fatigue conditions also.

To perform an actual fatigue calculation during optimization would be computationally too costly, since many dynamic simulations per configuration would be needed, which motivates the use of this criterion for the optimization. The final configuration can later be tested for fatigue requirements in a dedicated software.

2.2. Constraints

The definition of the constraints is a problem specific decision and thus prior knowledge is of great importance. Based on standard design practice, the following constraints where enforced:

- Minimum curvature radius;
- Maximum stress;
- Minimum tension along the riser;
- Top angle range.

2.3. Penalty Functions

The optimization methods used in this paper requires that each set of design variables yields a solution which does not violate any of the constraints. It is, however, very likely that a configuration generated by the optimization process will not fulfill all the requirements. In this case, two major approaches are possible: The set of variables can be discarded or a penalty scheme created, so that the value of the objective function is increased. In the first case, the information contained in this solution is lost, while in the second case it is taken into account. Since it takes a relatively long time to perform each objective function and restrictions calculation (due to static and dynamic analyses), it is important to use this information to speed up the optimization process.

This also makes it possible to use initial configurations that violate some of the constraints, which is very important for this problem, since in many cases it is not trivial to find such configurations.

With this in mind, this work used a penalty scheme proportional to the rate of violation of each constraint, defined as follows:

For each constraint:

$$p(x) = w * \frac{c(x) - c}{c} * F_{obj} \; ,$$

if the constraint is violated and 0 otherwise, being $c(x)$ the value of the constraint for the actual configuration, c the threshold value for this constraint and w a weight, different for each constraint, defined so as to scale the penalty, making it high enough to avoid the problems indicated before.

The penalty term is the sum of the terms for each constraint;

$$P = \sum_i p_i(x)$$

No comparisons were made in this work to test the effect of the use of the penalties, but in a previous work (Tanaka and Martins, 2006) it was found that it was beneficial.

2.4. Optimization Algorithms

The use of three different algorithms will be addressed in this work: Genetic Algorithm (GA), Simulated Annealing (SA) and Sequential Linear Programming (SLP). They have been chosen due to their widespread use in many areas.

2.4.1 Genetic Algorithm

The Genetic Algorithm (GA) is an optimization method guided by random decisions, and thus capable of avoiding local minima in the search for the global one. It has already been used to address riser synthesis problems, as is the case of Tanaka and Martins (2006 and 2007), Rodrigues (2004), Cunliffe (2004) and Vieira et al. (2003).

Firstly, an initial population is created. A population is a collection of individuals, each one of these representing a different configuration. The number of individuals in the population is defined by the user and is a parameter of the GA.

Each individual has its own chromosomes, which are defined as a string of bits from the binary representations of the design variables. The design variables are the variables which represent a given solution, in this case, a configuration. Each individual (representing a configuration) is then evaluated and its objective function calculated.

This population evolves as generations passes, with new members formed through mutation and crossover replacing the older ones, generating (hopefully) better configurations.

The basic rule of evolution, the ´survival of the fittest´ is then applied, as the chromosomes are selected to mate and breed. The best individuals have greater probabilities of being selected.

New individuals are formed combining the old ones chromosomes' via crossover, when a randomly selected part of the chromosome of one parent is connected to a part of the chromosome of the other, generating two pairs of chromosomes. A mutation operator is also applied, inverting the corresponding bits, to keep the genetic diversity of the population.

This process creates a new population, with different chromosomes which represent different configurations. These new candidates are then evaluated and the process repeated until convergence is achieved.

The process requires no specific knowledge about the problem (like gradients or convexity). For further information on GA, the reader is referred to Goldberg (1989).

The GA is, by construction, a maximization method. To deal with minimization problems and some difficult maximization problems, a transform of the objective function, called fitness function, is defined, and the GA maximizes this function. In minimization problems, this function must increase as the objective function decreases.

One property of the fitness function is that it must maintain a reasonable relative difference among the better chromosomes in the population whilst not creating a very great difference among the best and the worst individuals. If this is not the case, the selection mechanism may fail and premature convergence occurs. To define such function, many different approaches have been proposed. In this work a simple form will be used:

$$Fit(x) = \frac{F_{\max} - F_{obj}}{F_{\max}}$$

Where F_{max} is a given large number, defined so that the fitness function is never negative and F_{obj} is the objective function value.

2.2.2 Simulated Annealing

The SA is another probabilistic method, guided by random decisions, which makes it more robust and capable of avoiding local minima, while looking for the global one. It has been used to optimize steel risers in a previous work by Tanaka and Martins (2006).

Simulated Annealing is usually described as a mathematical representation of the process of annealing, in which a metal that was very quickly cooled and developed less stable crystalline structures (characterized by a local minimum of potential energy) is heated to allow the movement of atoms in the solid solution in search for the global minimum of potential energy, the equilibrium configuration. In spite of this analogy with a natural phenomenon, the method can be derived in a purely mathematical basis using Markov chain theory. It can also be proved that it asymptotically converges to the global minimum if infinite iterations are allowed.

Each configuration's objective function represents the energy of the system in that particular state (set of design variables). The method uses a parameter called temperature, that controls the probability that an increase in energy will be accepted. This probability is defined as follows:

$$P(\Delta E) = e^{\left(\frac{-\Delta E}{kT}\right)} \qquad (1)$$

where k is the Boltzmann constant and ΔE the difference among energies from the last and actual states. As T decreases, so does the probability of acceptance of a configuration which increases the energy (and thus the objective function).

The way the temperature is varied during the method is called the cooling schedule and it is of great importance for the method's behavior. In this case a linear schedule will be used, so starting from a given initial temperature, each time equilibrium is found the temperature is decreased by a fixed amount, called cooling rate.

The method begins with a set of design variables, ideally randomly generated. This set of variables is then perturbed randomly, generating a new different set. The objective function (energy) is then calculated and if it is smaller than the previous one, the new set is accepted and the old one discarded, if the energy is bigger than that of the previous set, it has a probability calculated by Equation 1 of being accepted.

A new random perturbation is then performed on the set received from the previous iteration and the procedure repeated. This happens until equilibrium is found for the current temperature, which in our case is after a user defined number of accepted transitions. When equilibrium is found, the temperature is lowered according to the cooling schedule and the procedure continues until the final temperature is obtained. The final temperature should be near 0, so a small probability of acceptance of worse configurations exists near the end.

Since there is a probability that a configuration worse than the previous one is accepted, the method can avoid being trapped in local minima. This method uses only the objective function, not its derivatives, and it is affected by the choice of initial and final temperatures and by the number of configurations before equilibrium used for each temperature. It is also very sensitive to the cooling schedule and the perturbation rule, which is the rule used to generate the random perturbations.

When initial temperature is very high, it takes longer for the method to converge, but most of the solution space is searched. If it is too low, convergence is quicker, but the chance to find the minimum is much smaller. The bigger the number of configurations before equilibrium, the more likely that the minimum will be encountered, but also the longer it takes.

For the cooling schedule, many methods were proposed, as linear cooling (used here), logarithmic cooling and others based on statistical information derived from the method's evolution.

For the perturbation rule, it is quite difficult to find detailed information on its implementation, especially because efficient rules are problem specific. The rule used in this work consists in the random selection of the variables that are to be changed. The selected variables are then changed by a random amount proportional to the temperature, generating bigger changes at bigger temperatures. Even though good results were achieved, there is still room for improvement. For more on simulated annealing, the reader is referred to Laarhoven and Aarts(1987).

2.2.3 Sequential Linear Programming (SLP)

Sequential Linear Programming is a method from a more general class, the aproximation methods. The basic idea is to expand, in each iteration, both the objective function and the restrictions around the initial point of this iteration. In the linear programming case, only the first term of the expansion is used, so the functions are linearized. Thus, the non-linear problem becomes a sequence of linear problems, and each of them can be solved through a linear programming method. A similar method, which uses two expansion terms, called Sequential Quadratic Programming (SQP) was used by Larsen and Hanson (1999). In this work, SLP will be used instead due to the existing implementation readily available for the authors and also to the fact that Hessian matrix calculation for the problem is extremely expensive computationally.

Special care is needed to avoid that the solution, in each iteration, deviates too much from the linear area, so limits are imposed to the variable range in each iteration. The standard simplex method was used to solve each linear substep, due to its reliability and good speed.

One of SLP's disadvantages is that the initial configuration must respect the constraints, which may be difficult in some cases. The use of penalizations, however, makes it possible to use other configurations.

Other disadvantage is the use of the gradients of the objective function and of the constraints, which must be numerically calculated, by finite differences, taking a long time and many objective function evaluations.

3. MODELS

The aim of this paper is to study configurations ranging from the catenary to the lazy-wave, as in Figure 1. To allow for the optimizer to choose among these configurations, the riser was considered as having three different segments, each one with its own length. The second segment has a floater, whose diameter was also variable.

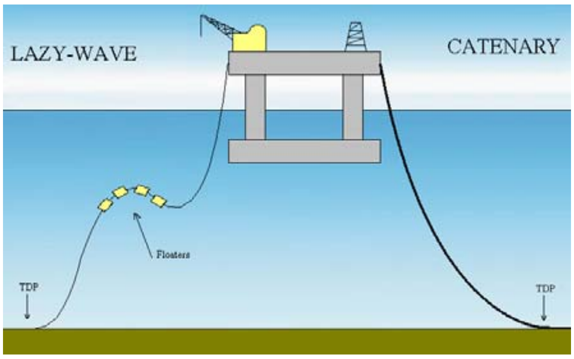

Figure 1. Riser Configurations

The materials for the pipe and the buoys are considered as given, as well as the inner and outer pipe diameter. The variables which the optimizer can change will be hereafter called design variables. For each segment, the length is a design variable, and for the second segment, the floater thickness is also a design variable.

To perform the analysis, environmental and field conditions also needs to be specified. The field conditions are the horizontal projection and the top connection height. The environment conditions are the current the riser is submitted to and the water depth and density. For the dynamic analysis, the movement imposed to the top of the riser by the floating unit also needs to be defined.

For each candidate solution, the resulting problem is solved for different specified environmental conditions.

The algorithm used for solving the static problem is based in the direct integration of the differential equations which govern the behavior of the riser without flexional rigidity, via a Runge-Kutta method. The flexional rigidity effect is introduced later, via a boundary-layer technique. Further information on the solution algorithm can be found in Martins (2000).

The dynamic behavior of the riser is modeled as a small perturbation of the static equilibrium position. The TDP is considered to be pinned and a linearization of the damping force is made, resulting in a linear finite element model, solved in the frequency domain. This linearization is based on the balance of dissipated energy per cycle. An iterative process is made so that the dissipated energy in the linear model is equal to that of the non-linear model. The effect of the movement of the TDP is introduced later, via a boundary-layer technique.

4. CASE STUDY

The data for the case study is the following:

Depth	1250	m
Horizontal projection	2000	m
Suspended height	1224.5	m
Riser inner diameter	0.3556	m
Riser outer diameter	0.4064	m
Riser material		
Density	7800	kg/m3
Young modulus	210	GPa
Floater material density	500	kg/m3

The length was left free to vary between 0 and 3000 m for each segment. The thickness of the floater on the second segment varied between 0 and 0.75 m. The precision used for the variables was 5 m for the lengths and 0.005 m for the floater thickness.

Each configuration was submitted to two environmental conditions. The first was the static only simulation of the riser without current and with the platform in its natural position (no offset), hereafter called neutral.

In the second condition, a dynamic analysis was performed on top of the previous static. The applied top movement was a 12 m vertical amplitude, which is taken from a real case of a FPSO turret system submitted to a centenary wave in the Campos Basin. Since this amplitude is very high, finding a viable solution for a steel riser in such conditions is a challenge for any designer, and a motivation for the use of optimization.

The movement's period is 14.2 s, following from the period of the centenary wave acting on the FPSO.

To ensure the viability of the riser obtained through the optimization process, some restrictions were imposed. The curvature radius must be bigger than 100 m and the maximum stress smaller than 530 MPa, with a safety factor of 1.2.

Also, the riser could not be under dynamic compression, and its top angle (measured from the vertical) in the neutral case should be 10^o, with a 1^o tolerance.

Three different initial configurations were tested, to illustrate the behaviour of each method in different conditions. The first configuration is a catenary with high dynamic compression and a 30^o top angle, much larger than specified. This case intends to test the methods' behaviour under unfavorable conditions. The second configuration is a lazy-wave and has the specified top angle combined with small dynamic compression. Finally, the third configuration, another lazy-wave, already meets all the restrictions. These cases were selected from a bigger pool as representative cases for situations that are likely to appear during the use of optimization to this specific problem.

4.1 Genetic Algorithm

The following parameters were used in the genetic algorithm:

Population	20
Crossover rate	0.5
Crossover type	2 point
Mutation rate	0.01
Selection type	Roulette Wheel + Elite (2 per gen.)

A Crossover rate of 0.5 means 1 out of 2 pairs of chromosomes suffers crossover. A 2 point crossover means the chromosome will be split in three parts. A mutation rate of 0.01 means the chance for mutations of each allele is 1/100. Roulette wheel selection gives each individual a reproducing chance proportional to its fitness, while elite selection automatically select the best of each generation to reproduce. Each configuration in the initial population is a random perturbation of the initial configuration for the corresponding case.

The algorithm stopped after 100 consecutive generations with a change in the objective function smaller than 0.1%.

4.2 Simulated Annealing

For the Simulated Annealing, the following parameters have been used:

Initial Temperature	10000
Final Temperature	0
Cooling Rate	333.33
Transitions per temperature	20

This set of parameters is both stable (leading to similar results over different runs) and capable of providing fast run times. The initial temperature is high enough so that most transitions are accepted even when reasonable increase in the objective function occurs, as is required by the method. The cooling rate is not fast enough to prevent convergence, but it also not too slow and, combined with the transitions per temperature, the total number of configurations simulated is not prohibitively large.

4.3 Sequential Linear Programming

For the SLP, the parameters are the maximum number of iterations, defined at 50 (but not reached in any of the simulations), and the definition of the limits for each iteration. The limits were defined as 10 times the variable's precision and were decremented when needed by a scale factor proportional to the gradient of each variable.

5. RESULTS

Figure 2 depicts the elastic lines for the first case, where the initial guess is a catenary. Table 1 gives information regarding the configurations and in Table 2 are presented the optimization results, including simulation time. The first segment is the one on the soil.

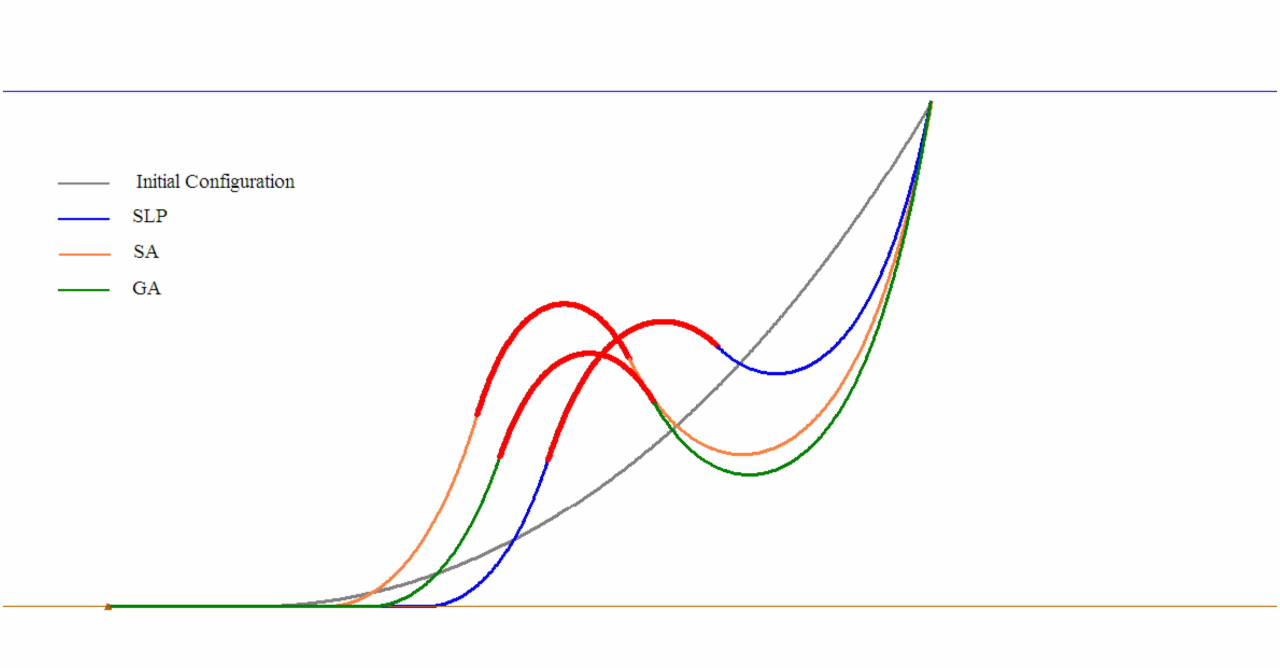

Figure 2. Optimum riser configurations for the first case

Table 1. Design variables for the configurations

	Initial Configuration	SLP	SA	GA
Segment 1 Length (m)	1150	1265	1160	1150
Segment 2 Length (m)	500	615	580	560
Segment 3 Length (m)	850	965	1410	1370
Floater Diameter (m)	---	0.83	0.93	0.89

Table 2. Optimization results for the first case

Method	Initial Configuration	SLP	SA	GA
Objection Function Value (MPa)	223.29 (+ Penalizations)	59.82	30.53	29.75
Computational Time (s)	---	960	3615	11220

From the presented results, it can be seen that the three methods were capable of finding viable solutions even starting from a configuration that was very far from fulfilling the design requirements. The results however varied among the methods, with GA and SA providing similar results, while SLP provided a configuration whose stress is almost the double of the other two methods. The reason for this can be readily explained by looking at Table 1. SLP found a configuration that is close to the initial one, except for the introduction of the floater, while both SA and GA changed significantly the third segment's length, besides introducing a floater. In fact, even though the configurations obtained through the GA and SA methods look reasonably different when their elastic lines are observed, the value of their design variables are very similar, leading to similar objective function values. Thus, as expected, SLP found a local minimum while SA and GA found some other minimum, which may be the global one.

As for the simulation time, SLP took much less time than SA, which in turn was faster than GA. In this problem, objective function evaluation takes much longer than any other computation, thus the number of objective function evaluations is the key factor to determine a method's performance. Since SLP is a local method, it is expected that it takes much fewer evaluations than the global methods. In fact, 80% of the computational time for the SLP was used to calculate the gradients (which also needs objective function evaluations), and so, if this calculation was faster, the method's time advantage would be even greater.

Figure 3 depicts the elastic lines for the second case, where the initial guess is a lazy-wave which violates the restrictions because of a slight dynamic compression. Table 3 gives information regarding the configurations and in Table 4 are presented the optimization results, including simulation time.

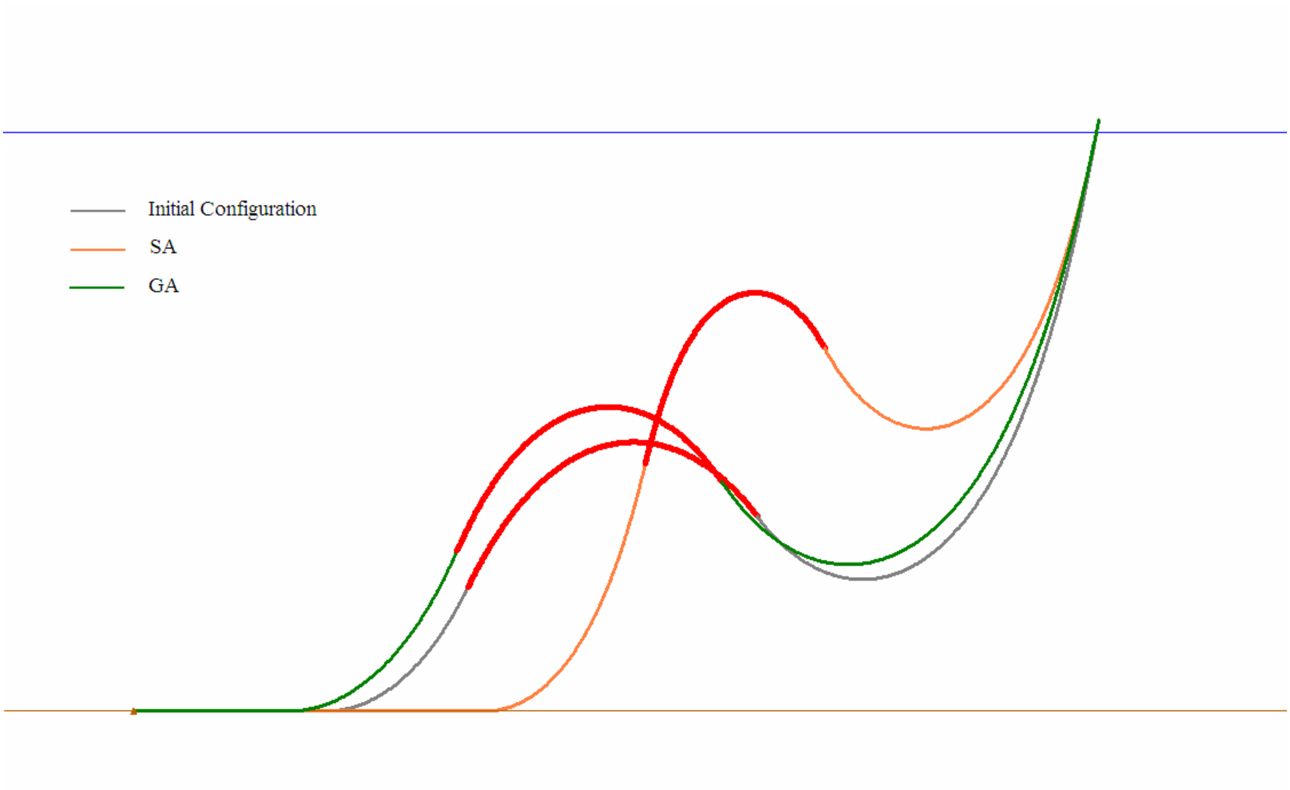

Figure 3. Optimum riser configurations for the first case

Table 3. Design variables for the configurations

	Initial Configuration	SLP	SA	GA
Segment 1 Length (m)	810	---	1380	830
Segment 2 Length (m)	790	---	640	750
Segment 3 Length (m)	1400	---	1060	1450
Floater Diameter (m)	0.80	---	0.89	0.84

Table 4. Optimization results for the first case

Method	Initial Configuration	SLP	SA	GA
Objection Function Value (MPa)	33.73 (+ Penalizations)	---	30.58	32.19
Computational Time (s)	---	---	3720	14040

The results for the SLP are not presented because this method was not capable of finding a viable solution in this case. This happened because the method got “trapped” in the initial configuration. From this configuration, all other trial configurations generated by SLP resulted either in greater dynamic compression or in an improvement in dynamic tension with violation of the top angle restriction. The probabilistic methods were capable of avoiding such problems and found viable configurations. The configuration resulting from SA is slightly better than the one from GA, but was obtained in a lot less computational time.

Figure 4 presents the elastic lines for the third case. Design variables values are in Table 5, while objective function and simulation times are presented in Table 6.

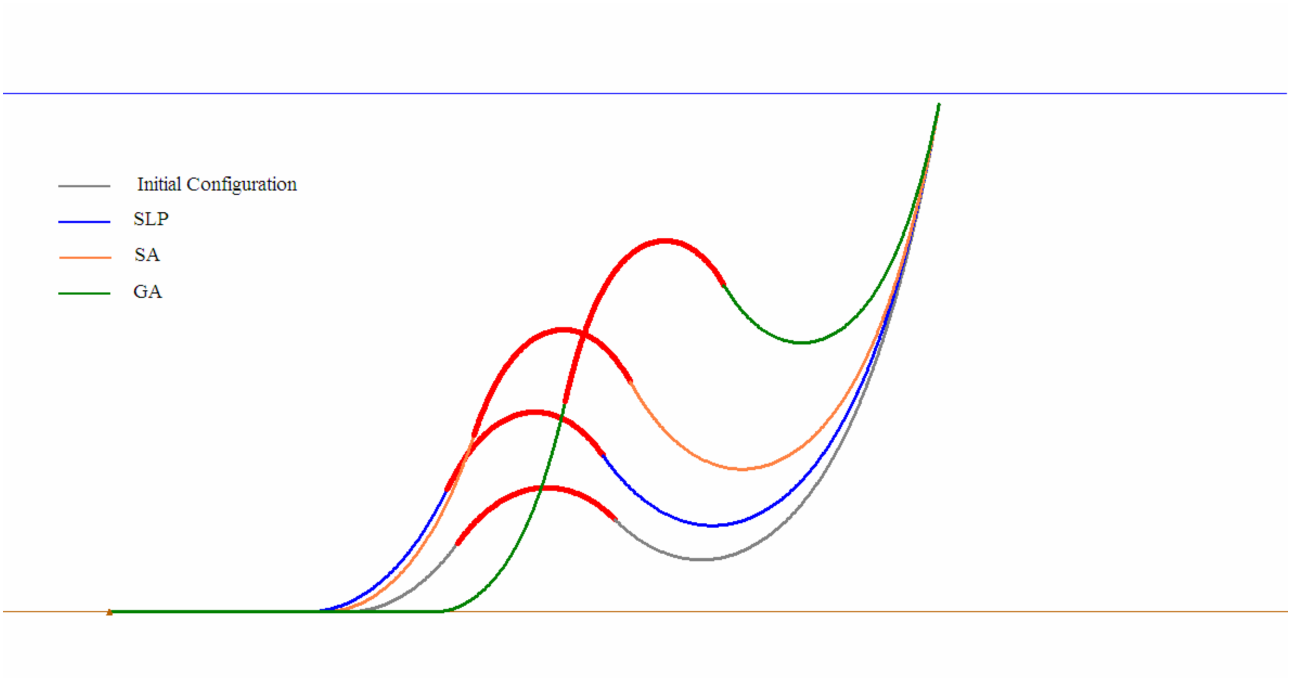

Figure 4. Optimum riser configurations for the third case

Table 5. Design variables' values

	Initial Configuration	SLP	SA	GA
Segment 1 Length (m)	900	950	1110	1420
Segment 2 Length (m)	450	500	570	670
Segment 3 Length (m)	1550	1550	1420	950
Floater Diameter (m)	0.86	0.91	0.92	0.87

Table 6. Optimization results for the first case

Method	Initial Configuration	SLP	SA	GA
Objection Function Value (MPa)	46.21	34.63	33.21	32.87
Computational Time (s)	---	720	3735	15480

In this case, the initial configuration was already viable, and had a relatively low objective function value. This is very favorable to the SLP method, since this configuration is initially closer to a minimum. Accordingly, in this case, the objective function value found by the SLP was very close to the one found by SA and GA, even tough the configurations are somewhat different. It is clear that in this case SLP is superior since it generates a configuration that is only slightly worse than the others, but in much less time.

The existence of many different configurations with similar stress amplitudes indicates that this is a hard problem, even for the global methods such as SA and GA. The fact that different configurations have similar objective function values is partly due to the use of a single dynamic condition. If more conditions, covering a wider movement frequency range, were used, the difference among the maximum stress amplitude in the worst case would grow and so would the difference among these configurations' objective function values.

6. CONCLUSIONS

The results presented show that the three methods are capable of improving existing designs in an efficient way and so they can be of great value in aiding the design phase of a steel riser. Even though the exploration is not comprehensive, some preliminary conclusions can be drawn, which are very similar to the ones obtained by Birk, Clauss and Lee (2004) studying the optimization of another offshore system, the shape of a hull, using GA, ASA (a variation of SA) and SQP.

SA and GA's performance perhaps could be improved by tuning their parameters. The authors, however, avoided such tuning, since in most real cases there wouldn't be such possibility, due to time constraints.

The advantages and disadvantages of each method are made clear through the study cases. SA and GA are both effective, but SA is much faster. This happens because it only needs one configuration at a time, unlike GA, that needs a population of them. Working with several configurations simultaneously has its advantages, and GA found smaller objective functions in some cases, but with much longer processing time. SA is, therefore, a better choice for such problems.

Generally speaking, SLP can only find minima in the neighborhood of the initial configuration, while GA and SA can search all the variable range to find a better solution, which in some cases may be significantly better. They do so at the cost of being much slower than SLP, so a tradeoff among computational time and final objective function value arises. In cases where the initial solution is already a good solution, the final objective function tends to be very close among the methods and SLP is the best choice. However, when the initial configuration does not fulfill the requirements, the difference between the results tend to grow, and so it is up to the designer to decide whether speed or lower stress is more important and choose the optimization method accordingly. There is also a chance that SLP may fail to find a viable solution, even if the starting configuration is not a very bad one, as shown in case two.

7. ACKNOWLEDGEMENTS

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